# Model Independent Analysis Of The Proton Magnetic Radius

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# Outline

- Motivation
- How do we extract the proton magnetic radius
- Our model-independent approach to extract the radius
- Parameterization and minimizing techniques
- Results
- Conclusion

# Motivation

- Proton is one of the fundamental constituent of matter.
- It is an extended object and should have a size which is not known to us precisely.
- The electric radius values extracted

  from e-p scattering e.g. $r_E^p$ = 0.871±0.009 fm, e.g. Hill and Paz(2010),

  from Muonic Hydrogen $r_E^p$ = 0.84184±0.0006 fm, Pohl et al.(2010).
- PDG 2014 reports the magnetic radius of proton as

  $r_M^p = 0.777 \pm 0.014$ fm according to Bernauer et al. (2010).

  Other values mentioned in PDG are :

  $r_M^p = 0.854 \pm 0.005$ fm Belushkin et al. (2007)

  $r_M^p = 0.876 \pm 0.020$ fm D. Borisyuk (2010)

# Form Factors

- In general there are two form factors known as Dirac and Pauli form factors , defined as

$$\langle N(p')|J_\mu^{em}|N(p)\rangle = \bar{u}(p')\left[\gamma_\mu F_1^N(q^2) + \frac{i\sigma_{\mu\nu}}{2m_N} F_2^N(q^2)q^\nu\right]u(p)$$

- The experimental data given by Sachs electric and magnetic form factors which are related to the Dirac and Pauli basis by

$$G_E^N(t) = F_1^N(t) + \frac{t}{4(m_N^2)}F_2^N(t)\,; \quad G_M^N(t) = F_1^N(t) + F_2^N(t)$$

where $t = q^2 = -Q^2$.

- $G_M^p(0) = \mu_p \approx 2.793 \;\; ; G_M^n(0) = \mu_n \approx -1.913$

- The isoscalar and isovector form factors are defined as

$$G_{M,E}^{I=0} = G_{M,E}^{p} + G_{M,E}^{n} \quad , \quad G_{M,E}^{I=1} = G_{M,E}^{p} - G_{M,E}^{n}$$

such that at t = 0, $G_M^{I=0}(0) = \mu_p + \mu_n \approx 0.88$ ;

$$G_M^{I=1}(0) = \mu_p - \mu_n \approx 4.7$$

# Measurement of Magnetic Radius

- In the Breit frame($q^2 = -\vec{q}^2$) the magnetic form factor is the Fourier transform of proton's magnetic moment density $\mu(\vec{r})$ :

$$G_M^p(q^2) = \int d^3x\, \mu(\vec{r})\, e^{i\vec{q}.\vec{r}} = \mu[1 - \frac{1}{6}\vec{q}^2\langle r^2\rangle_M + \cdots]$$

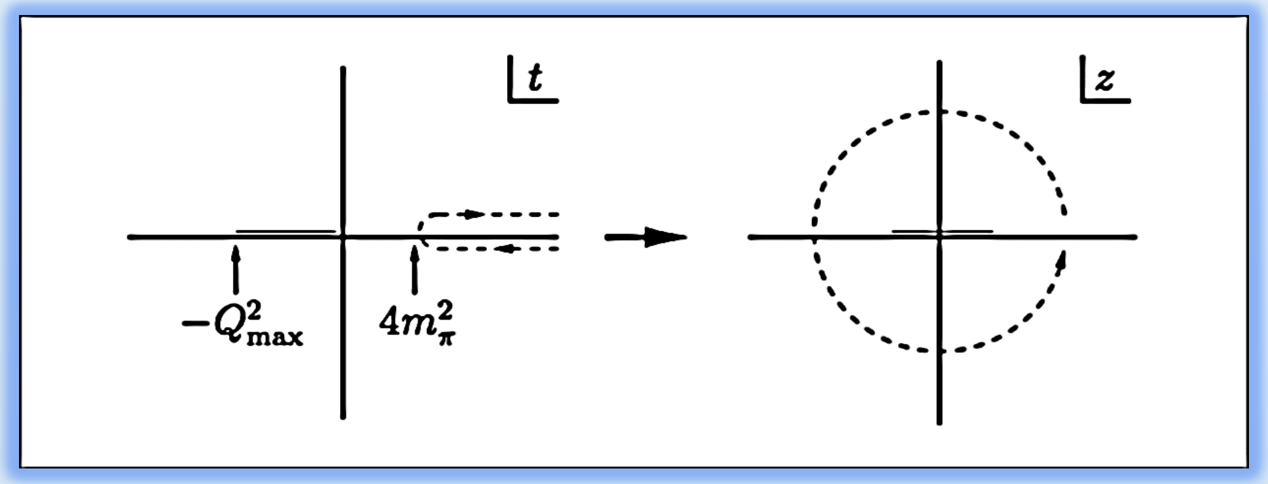
- The magnetic radius of proton is defined as

$$\langle r^2\rangle_M^p = \frac{6}{G_M^p(0)} \frac{dG_M^p\,(q^2)}{dq^2}\Big|_{q^2=0}$$

- Usually a form factor parameterization is chosen and then a fit is performed to experimentally measured form factors. The radius is then obtained from above equation.

# Analyticity of Form Factors

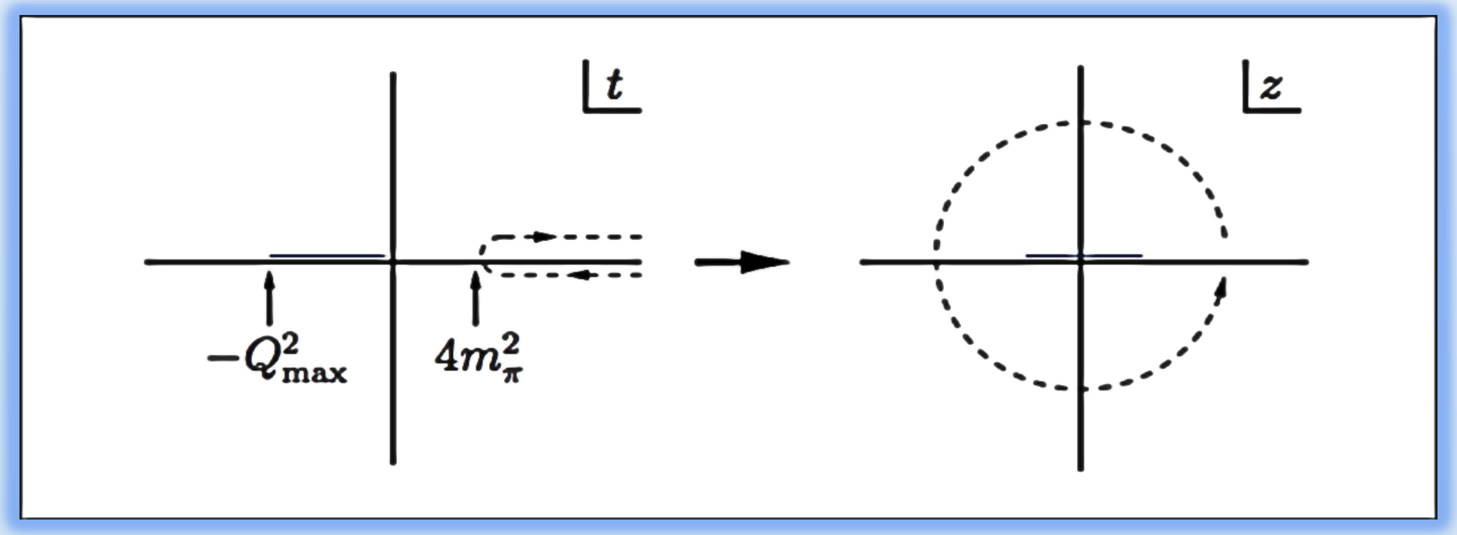
- Form Factors are analytic in the cut plane.




Conformal mapping of the cut plane to the unit circle

- If any complex function $f(z)$ is analytic, we can express it as a series expansion $f(z) = \sum_{k=0}^{\infty} a_k z^k$ .

- Choosing $z(t, t_{cut}, t_0) = \frac{\sqrt{t_{cut}-t} - \sqrt{t_{cut}-t_0}}{\sqrt{t_{cut}-t} + \sqrt{t_{cut}-t_0}}$, where $t_0$ is a free parameter (determines the location of $z = 0$) and $t_{cut} = 4m_\pi^2$ , maps the cut plane to the unit circle.




- For $t_0$ = 0, $t_{cut}$ = $4m_\pi^2$=0.0784 GeV$^2$ and $t$= $-0.5$ GeV$^2$ , $z_{max}$ = 0.70.

- We can expand the magnetic form factor as

$$G_M(q^2) = \sum_{k=0}^{\infty} a_k z(q^2)^k$$

which always converges.

- Advantage of z expansion is that its coefficients can be bound using the knowledge of imaginary part of $G_M$.

# Coefficients

- The analytic structure in the $t$-plane implies a dispersion relation

$$G^p(t) = \frac{1}{\pi}\int_{t_{cut}}^{\infty} dt' \frac{\mathrm{Im}G^p(t'+i0)}{t'-t}$$

- Parameterizing by $z(t) = e^{i\theta}$ and solving $\mathrm{z} = \frac{\sqrt{t_{cut}-t} - \sqrt{t_{cut}-t_0}}{\sqrt{t_{cut}-t} + \sqrt{t_{cut}-t_0}}$ for $t$, we get

$$t = t_0 + \frac{2(t_{cut} - t_0)}{1-\cos\theta} \equiv t(\theta)$$

- Using the orthogonality over the unit circle, we get

$$a_k = -\frac{2}{\pi}\int_0^{\pi} \mathrm{Im}G[t(\theta) + i0]\ \sin(k\theta)\, d\theta$$

- Using $\theta = 2\ \sin^{-1}\sqrt{\frac{t_{cut} - t_0}{t - t_{cut}}}$ we derive the final expression for the coefficients

$$\boxed{a_k = -\frac{2}{\pi}\int_{t_{cut}}^{\infty} \frac{dt}{(t - t_0)}\sqrt{\frac{t_{cut} - t_0}{t - t_{cut}}}\,\mathrm{Im}G[t(\theta) + i0]\sin((k\theta(t))}$$

# Bounds on coefficients

- To extract $r_E^p$ Hill and Paz (2010) used bounds of $|a_k| \leq 5$ ,10.

- Since $a_0 = G_M^{I=1}(0) = \mu_p - \mu_n \approx 4.7$ , bound of 5 is too stringent.

- Using vector dominance ansatz we find:

$$I = 1 \ (\rho \text{ exchange}) \ |a_k| \leq 5.1$$

- Using $\pi\pi$ continuum we find:

  $a_0 \approx 7.9, a_1 \approx -5.5, a_2 \approx -6.1, a_4 \approx 1.1.$

  Using $|\sin(k\theta)| \leq 1$ gives $|a_k| \lesssim 7.2$ for $k \geq 1$.

- Above $t = 4m_N^2$ use $e^+e^- \rightarrow N\bar{N}$: negligible contribution to $a_k$

- We conclude:

$|a_k| \leq 5$ is not conservative enough, use $|a_k| \leq 10$ and $|a_k| \leq 15$

- We used higher bounds like 20 , but did not observe significant changes in the extracted values.

# Minimizing technique

- We fit our data by ‘z expansion’ method as described

$$G_M^p = a_0 + a_1 z(q^2) + a_2 z^2(q^2) + \cdots$$

with $z(q^2) = z(q^2, t_{cut}, t_0 = 0)$.

- We minimize a $\chi^2$ function

$$\chi^2 = \sum (\text{data} - \text{theory})^2 / (\delta G_M)^2$$

for ‘data’ we used proton, neutron and $\pi\pi$ datasets, $\delta G_M$ is the corresponding error bar and z-expansion provided our ‘theory’.

# Datasets used for $r_M^p$ Extraction

- For the Proton Data we use

$$t_{cut} = 4m_\pi^2 \Longrightarrow z(q^2) \equiv z(q^2, 4m_\pi^2, 0)$$

- Including the Neutron data allows us to separate the $I = 1$ and $I = 0$ isospin component.

$$I = 0\,,\ t_{cut} = 9m_\pi^2 \Longrightarrow \text{decrease } z_{max}$$

- Inclusion of $\pi\pi$ data raise the effective threshold for the isovector. In this case

$$I = 1,\ t_{cut} = 16m_\pi^2 \Longrightarrow \text{decrease } z_{max}$$

# Results for Proton Data

*Arrington et.al(2007)*

$Q^2 \leq 0.5\ GeV^2$

| Bound on $a_k$ | $r_M^p$ | +σ | -σ |
|---|---|---|---|
| 5 | 0.89 | 0.03 | 0.05 |
| 10 | 0.91 | 0.03 | 0.06 |
| 15 | 0.92 | 0.04 | 0.07 |
| 20 | 0.93 | 0.04 | 0.07 |

$Q^2 \leq 1.0\ GeV^2$

| Bound on $a_k$ | $r_M^p$ | +σ | -σ |
|---|---|---|---|
| 5 | 0.89 | 0.02 | 0.05 |
| 10 | 0.90 | 0.03 | 0.07 |
| 15 | 0.91 | 0.04 | 0.07 |
| 20 | 0.91 | 0.05 | 0.08 |

# Results for (Proton + Neutron) Data

$Q^2 \leq 0.5\ GeV^2$

| Bound on $a_k$ | $r_M^p$ | +σ | -σ |
|---|---|---|---|
| 5 | 0.86 | 0.02 | 0.01 |
| 10 | 0.87 | 0.04 | 0.05 |
| 15 | 0.87 | 0.05 | 0.05 |
| 20 | 0.88 | 0.04 | 0.06 |

$Q^2 \leq 1.0\ GeV^2$

| Bound on $a_k$ | $r_M^p$ | +σ | -σ |
|---|---|---|---|
| 5 | 0.87 | 0.02 | 0.02 |
| 10 | 0.88 | 0.02 | 0.05 |
| 15 | 0.88 | 0.04 | 0.05 |
| 20 | 0.88 | 0.05 | 0.06 |

# Results for (Proton+ Neutron+ $\pi\pi$) Data

$Q^2 \leq 0.5\ GeV^2$

| Bound on $a_k$ | $r_M^p$ | +σ | -σ |
|---|---|---|---|
| 5 | 0.867 | 0.010 | 0.013 |
| 10 | 0.871 | 0.011 | 0.015 |
| 15 | 0.873 | 0.012 | 0.016 |
| 20 | 0.876 | 0.012 | 0.018 |

$Q^2 \leq 1.0\ GeV^2$

| Bound on $a_k$ | $r_M^p$ | +σ | -σ |
|---|---|---|---|
| 5 | 0.867 | 0.006 | 0.008 |
| 10 | 0.874 | 0.008 | 0.015 |
| 15 | 0.874 | 0.012 | 0.014 |
| 20 | 0.875 | 0.013 | 0.016 |

# Extraction of the Neutron Magnetic Radius

- We can extract the radius of the neutron similarly as proton, using

$$\langle r^2 \rangle_M^n = \frac{6}{G_M^n(0)} \frac{d}{dq^2} G_M^n\,(q^2)\Big|_{q^2=0}$$

Neutron Data

| $Q^2$ $(GeV^2)$ | Bound | $r_M^n$ | +σ | -σ |
|---|---|---|---|---|
| 0.5 | 10 | 0.74 | 0.13 | 0.06 |
| | 15 | 0.65 | 0.21 | 0.07 |
| 1.0 | 10 | 0.77 | 0.17 | 0.09 |
| | 15 | 0.74 | 0.20 | 0.11 |

$r_M^n$ = 0.7±0.2

Neutron + Proton Data

| $Q^2$ $(GeV^2)$ | Bound | $r_M^n$ | +σ | -σ |
|---|---|---|---|---|
| 0.5 | 10 | 0.89 | 0.06 | 0.09 |
| | 15 | 0.88 | 0.08 | 0.09 |
| 1.0 | 10 | 0.88 | 0.06 | 0.08 |
| | 15 | 0.89 | 0.07 | 0.10 |

$r_M^n$ = 0.9±0.1

Neutron + Proton + $\pi\pi$ Data

| $Q^2$ $(GeV^2)$ | Bound | $r_M^n$ | +σ | -σ |
|---|---|---|---|---|
| 0.5 | 10 | 0.89 | 0.03 | 0.03 |
| | 15 | 0.89 | 0.03 | 0.03 |
| 1.0 | 10 | 0.88 | 0.03 | 0.01 |
| | 15 | 0.88 | 0.03 | 0.02 |

$r_M^n$ = 0.89±0.03

# Conclusion

- Calculations using the proton data gives

$$r_M^p = 0.91^{+0.03}_{-0.06} \pm 0.02 \text{ fm.}$$

- Calculations including the neutron data finds the radius as

$$r_M^p = 0.87^{+0.04}_{-0.05} \pm 0.01 \text{ fm.}$$

- Calculations including $\pi\pi$ data along with proton and neutron data sets gives

$$r_M^p = 0.87^{+0.02}_{-0.02} \text{ fm.}$$

- Our results are more consistent with the values of 0.876 ± 0.02 fm or 0.854 ± 0.005 fm, in contrast to the PDG 2014 value 0.777 ± 0.014 fm.

- Even for higher bounds like $|a_k| \leq 20$, the values don't change significantly.

- The extracted magnetic radius is independent of the numbers of parameters we fit or the cut on $Q^2$.

- It is interesting to note that within errors, magnetic radius of the neutron ($r_M^n$= 0.89 ± 0.03 fm) is consistent with the magnetic radius of the proton ($r_M^p$= 0.87 ± 0.02 fm).

# Thank you !