Optimized evaluation approach for inflationary power spectra

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HM and Wayne Hu, arXiv:1503.04810

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Ade et al, 1502.02114

Planck TT, TE, EE + lowP

$$n_s = 0.9644 \pm 0.0049 \ (68\% CL)$$

$$\alpha = -0.009 \pm 0.008 \ (68\% CL)$$

$$r_{0.002} < 0.15 \ (95\% CL)$$

$$|\alpha| \lesssim |n_s - 1| = O(10^{-2})$$

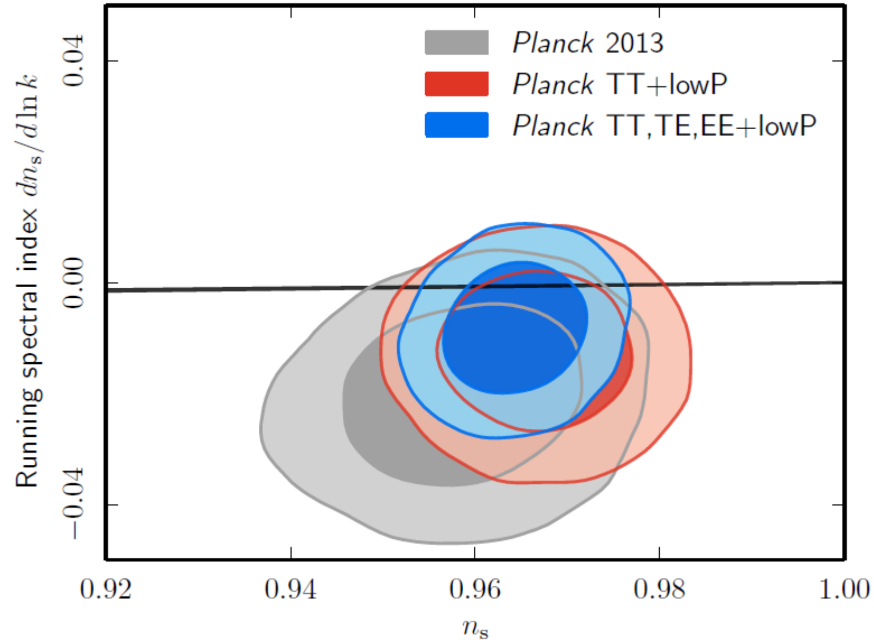


Fig. 5. Marginalized joint confidence contours for $(n_s, dn_s/d\ln k)$, at the 68 % and 95 % CL, in the presence of a non-zero tensor contribution, and using *Planck* TT+lowP or *Planck* TT,TE,EE+lowP. Constraints from the *Planck* 2013 data release are also shown for comparison. The thin black stripe shows the prediction of single-field monomial inflation models with $50 < N_* < 60$.

First order slow-roll approximation

Consistency relation

$$n_s - 1 = -6\epsilon_V + 2\eta_V$$

$$\alpha = 16\epsilon_V\eta_V - 24\epsilon_V^2 - 2\xi_V$$

$$r = 16\epsilon_V$$

Assumption

$$\epsilon_V, \eta_V = O(N^{-1})$$

$$\xi_V = O(N^{-2})$$

Suppose $|\alpha|, |n_s - 1| = O(10^{-2})$

→ $\epsilon_V, \eta_V, \xi_V = O(10^{-2})$

→ $n_s - 1$ needs ξ_V term ✖

→ Inappropriate order counting

Generalized slow-roll (GSR) approximation

Generalized slow-roll

Stewart, astro-ph/0110322
Choe, Gong, Stewart, hep-ph/0405155
Dvorkin, Hu, 0910.2237

Solve the Mukhanov-Sasaki equation

$$\frac{d^2y}{dx^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{f'' - 3f'}{f} \frac{y}{x^2}$$

de Sitter Deviation from dS

$$y = \sqrt{2k} a \delta\varphi, \quad x = k\eta, \quad f = 2\pi a \eta \sqrt{2\epsilon_H}, \quad ' = \frac{d}{d \ln \eta}$$

iteratively with Green function technique starting with the de Sitter solution

$$y_0 = \left(1 + \frac{i}{x}\right) e^{ix}$$

and using deviations from de Sitter as external source.

(cf. Born approximation)

Generalized slow-roll

First order of iteration

$$\ln \Delta_R^2(k) = -\int_0^\infty \frac{dx}{x} W'(x) G(\ln x)$$

Window function   Source function

$$W(x) = \frac{3\sin 2x}{2x^3} - \frac{3\ \cos 2x}{x^2} - \frac{3\sin 2x}{2x}$$

$$G(\ln x) = -2\ln f + \frac{2}{3}(\ln f)'$$

$$\approx \ln\left(\frac{H^2}{8\pi^2\epsilon_H}\right) - \frac{10}{3}\epsilon_H - \frac{2}{3}\delta_1$$

Optimized evaluation

HM, Hu, 1503.04810

First order of iteration

$$\ln \Delta_R^2(k) = -\int_0^\infty \frac{dx}{x} W'(x) G(\ln x)$$

Window function Source function

$$= G(\ln x_f) + \sum_{p=1}^{\infty} q_p(\ln x_f) G^{(p)}(\ln x_f)$$

Taylor expansion of G around evaluation point $\ln x_f$

$$q_p(\ln x_f) = -\frac{1}{p!}\int_0^\infty \frac{dx}{x} W'(x) \left(\ln \frac{x}{x_f}\right)^p$$

$\lim_{p\to\infty} \frac{q_p}{q_{p-1}} = -\frac{1}{2}$ → Convergence criteria $\lim_{p\to\infty} \left|\frac{G^{(p)}}{G^{(p-1)}}\right| = \frac{1}{\Delta N} < 2$

→ Low frequency oscillation

Optimized evaluation

HM, Hu, 1503.04810

$$\ln \Delta_R^2(k) = G(\ln x_f) + \sum_{p=1}^{\infty} q_p(\ln x_f) G^{(p)}(\ln x_f)$$

We can optimize the evaluation point $\ln x_f$ to minimize correction to a truncation.

$$x \equiv k\eta$$

Optimized evaluation

HM, Hu, 1503.04810

$$\ln \Delta_R^2(k) = G\left(\ln x_f\right) + \sum_{p=1}^{\infty} q_p\left(\ln x_f\right) G^{(p)}\left(\ln x_f\right)$$

We can optimize the evaluation point $\ln x_f$ to minimize correction to a truncation.

$$x \equiv k\eta$$

First order slow-roll approximation (no optimization)

$$\ln \Delta_R^2 = G(0) : \text{correction} = O(q_1(0)G'(0))$$

$\ln x_f = 0$: horizon exit

$q_1(0) \approx 1.06$

Optimized evaluation

HM, Hu, 1503.04810

$$\ln \Delta_R^2(k) = G(\ln x_f) + \sum_{p=1}^{\infty} q_p(\ln x_f) G^{(p)}(\ln x_f)$$

We can optimize the evaluation point $\ln x_f$ to minimize correction to a truncation.

$$x \equiv k\eta$$

First order slow-roll approximation (no optimization)

$\ln \Delta_R^2 = G(0)$: correction $= O(q_1(0)G'(0))$

$\ln x_f = 0$: horizon exit

$q_1(0) \approx 1.06$

Optimized leading order approximation

$\ln \Delta_R^2 = G(\ln x_1)$: correction $= O(q_2(\ln x_1)G''(\ln x_1))$

$\ln x_f = \ln x_1 \approx 1.06$ with $q_1(\ln x_1) = 0$

$q_2(\ln x_1) \approx -0.36$

~1 efold before horizon exit

Optimized evaluation

HM, Hu, 1503.04810

$$\ln \Delta_R^2 = G(\ln x_f) + \sum_{p=1}^{\infty} q_p(\ln x_f) G^{(p)}(\ln x_f)$$

$$n_s - 1 = -G'(\ln x_f) - \sum_{p=1}^{\infty} q_p(\ln x_f) G^{(p+1)}(\ln x_f)$$

$$\alpha = G''(\ln x_f) + \sum_{p=1}^{\infty} q_p(\ln x_f) G^{(p+2)}(\ln x_f)$$

We can optimize the evaluation point $\ln x_f = \ln x_p$ where $q_p(\ln x_p) = 0$ and make a correction to a truncation at q_{p-1} as small as $O\left(q_{p+1}(\ln x_p) G^{(p+1)}(\ln x_p)\right)$.

Optimized evaluation

HM, Hu, 1503.04810

Second order of iteration

$$\ln \Delta_R^2 = I_0 + \ln\left[\left(1 + \frac{I_1^2}{4} + \frac{I_2}{2}\right)^2 + \frac{I_1^2}{2}\right]$$

$$I_0 = -\int_0^\infty \frac{dx}{x} W'(x) G(\ln x)$$

$$I_1 = \frac{1}{\sqrt{2}} \int_0^\infty \frac{dx}{x} X(x) G'(\ln x)$$

$$I_2 = -4 \int_0^\infty \frac{dx}{x}\left(X + \frac{X'}{3}\right)\frac{f'}{f}\int_0^x \frac{du}{u^2}\frac{f'}{f}$$

Differs from standard second-order slow-roll approximation as order counting is different.

Second order in $1/\Delta N$ expansion

$$\ln \Delta_R^2 \approx G(\ln x_f) + q_1 G'(\ln x_f) + q_2 G''(\ln x_f) + \frac{\pi^2}{8}\left[G'(\ln x_f)\right]^2 - 4\left[\frac{f'}{f}(\ln x_f)\right]^2$$

Monodromy case study

We consider monodromy inflation

Silverstein et al, 0803.3085

$$V(\phi) = \bar{V}(\phi) + \delta V(\phi)$$

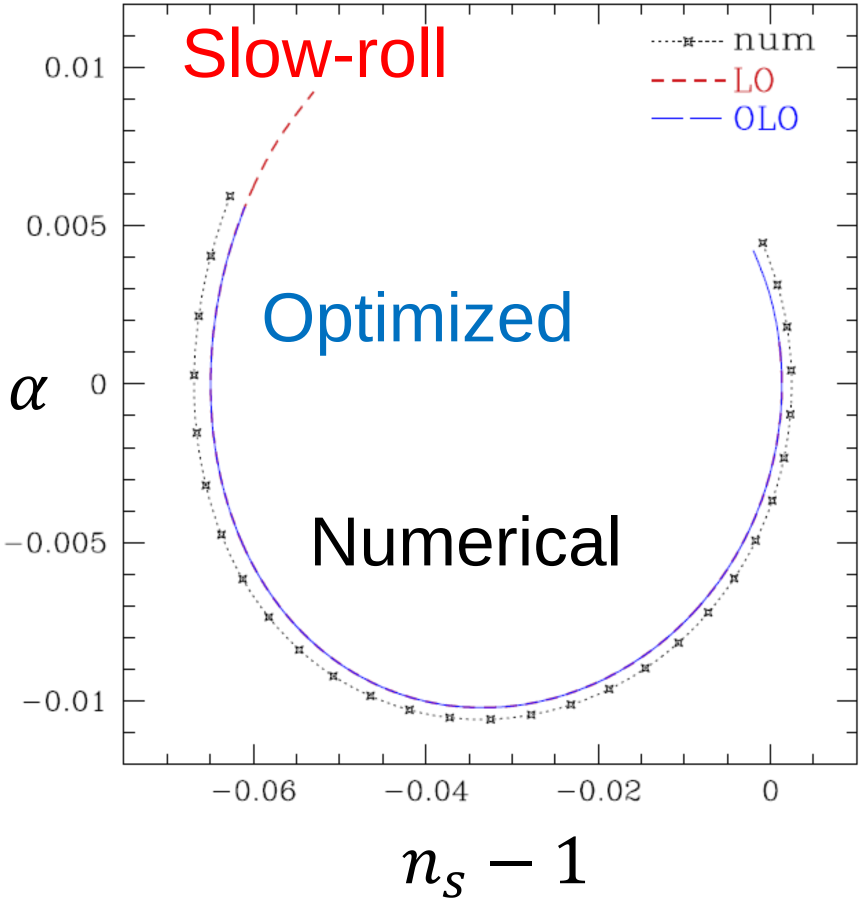
$$\bar{V}(\phi) = \lambda\phi$$

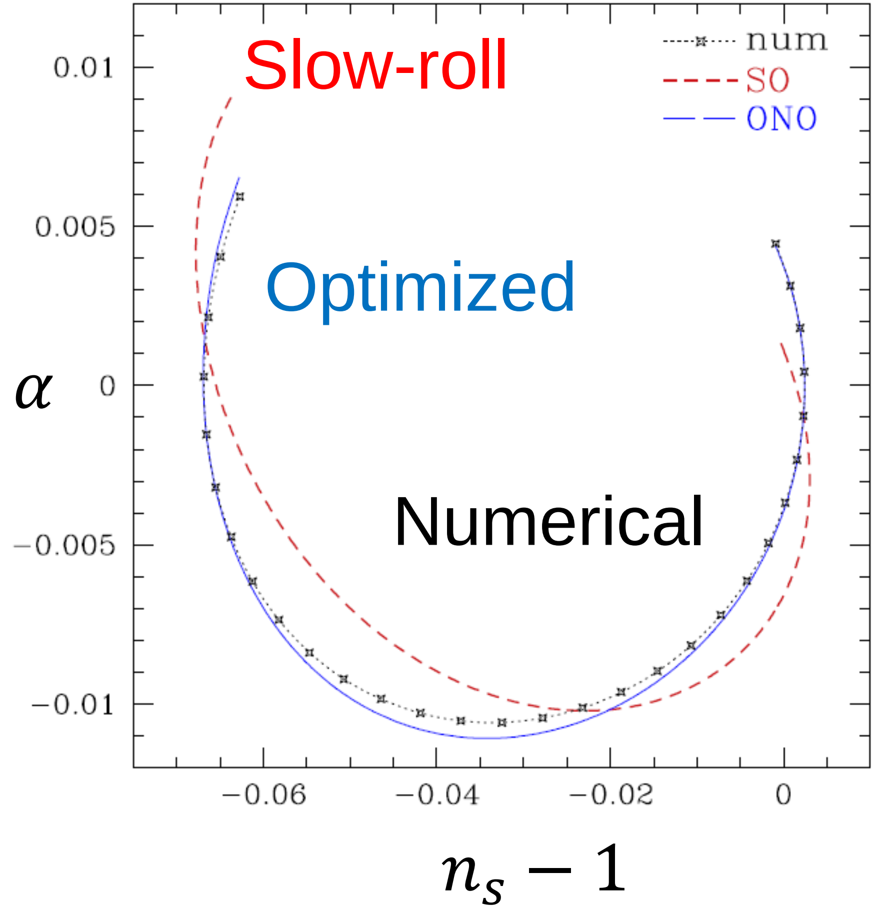
$$\delta V(\phi) = \Lambda^4 \cos\left(\frac{\phi}{f} + \theta\right)$$

HM, Hu, 1503.04810

Monodromy case study – low frequency

Leading order $\omega \sim \Delta N^{-1} \sim 0.3$ Next-to-leading order





Same formula with different evaluation point

Different formulae with different evaluation point

HM, Hu, 1503.04810

Monodromy case study – high frequency

Evaluate integrals with stationary phase approximation

$$\ln \Delta_R^2 = I_0 + \ln\left[\left(1 + \frac{I_1^2}{4} + \frac{I_2}{2}\right)^2 + \frac{I_1^2}{2}\right]$$

$$\delta \ln \Delta_R^2 \approx \delta I_0 + 2\bar{I}_1 \delta I_1 + \delta I_1^2$$

$$\bar{I}_1 = \frac{\pi}{2\sqrt{2}}(1 - \bar{n}_s)$$

$$\delta I_0 = -A\left[\cos(\psi - \beta) - \frac{3f}{2\phi_*}\sin(\psi - \beta)\right]$$

$$\delta I_1 = \frac{A}{\sqrt{2}}\tanh\left(\frac{\pi\omega_*}{2}\right)\left[\sin(\psi - \beta) + \frac{3f}{2\phi_*}\cos(\psi - \beta)\right]$$

- $O(A^2)$

cf. Flauger et al, 0907.2916
Flauger et al, 1412.1814 →
- $O(A)$
- Phase is free parameter

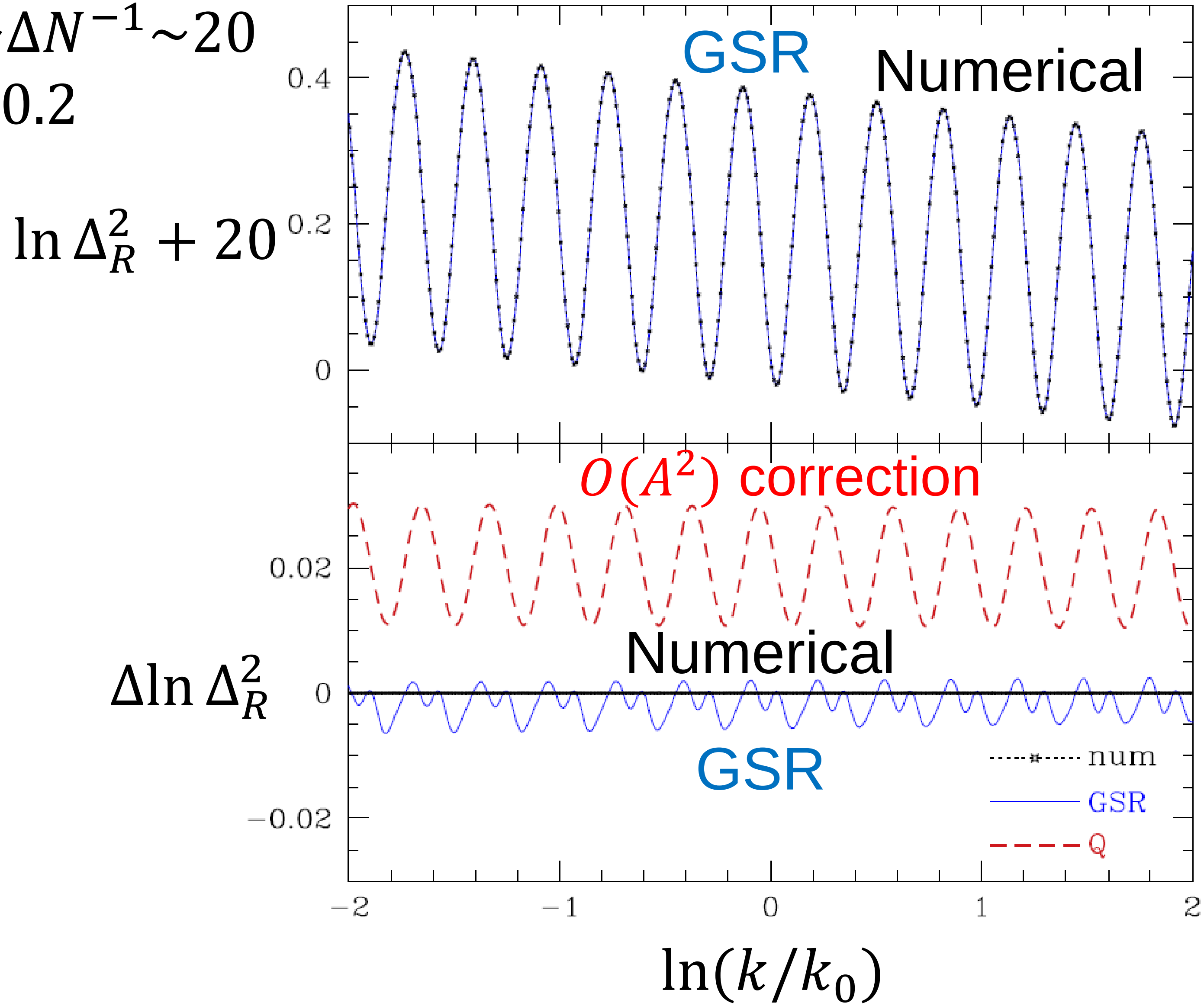
- All terms are written in terms of model parameters

HM, Hu, 1503.04810

Monodromy case study – high frequency

$\omega \sim \Delta N^{-1} \sim 20$

$A \sim 0.2$



Summary

- Using GSR approximation, we developed optimized evaluation of inflationary power spectra that are written by Taylor expansion of features.
- Optimized leading order approximation reabsorb next-to-leading order corrections in Taylor expansion by evaluating the same formulae as the standard slow-roll at ~ 1 efold before horizon exit.
- We revealed that standard second-order slow-roll approximation fails while optimized next-to-leading order leads to 10^{-3} accuracy.
- For high frequency monodromy, we obtained $\ln \Delta_R^2$ up to $O(A^2)$ and clarified model parameter dependencies.