Anatomizing BSM effects with differential Higgsstrahlung observable

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Talk based on work w N. Craig, J. Gu and K. Wang, to appear
• Precision Higgs physics could directly probe new physics

• New physics can easily couples to Higgs, linking to hierarchy problem, electroweak baryogenesis, naturalness, dark matter, etc.

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Many proposals and studies, International Linear Collider (ILC) in Japan, Future Circular Collider (FCC-ee) by CERN, Circular Electron-Positron Collider (CEPC) in China, and many others.
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General purpose collider experiments require ~twenty years planning ahead.
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Comparing to HL-LHC
- Nearly one order of magnitude better precision, to (sub) percent level.
- Combining with the HL-LHC does improve, e.g. Higgs photon couplings (see more discussion for the case of the ILC, e.g., T. Han, ZL, J. Sayre, arXiv:1311.7155).

- Similar plots exist for FCC-ee, ILC, etc.
- Lepton collider Higgs factories also enable “model-independent” fits (not shown here.)
The Higgs physics potential is an essential physics piece for any proposal for future colliders, e.g. CEPC-circular electron-positron collider, preCDR (link) public. A future collider proposal in China, with a possible upgrade to hadron-collider (named SPPC) at 50-100 TeV. These Higgs projection results are one of my main contributions to CEPC at this stage. (open to discussion about results/improvements, etc.) A Higgs whitepaper out soon describing the details.

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- Similar plots exist for FCC-ee, ILC, etc.
- Lepton collider Higgs factories also enable “model-independent” fits (not shown here.)
Thus far, most works on Higgs physics focus on the simple \textit{kappa-scheme}, where BSM effects are captured by a scaling parameter,
\[
\kappa = \frac{g_{BSM}}{g_{SM}}
\]

In terms of higher-dimensional operators, these only capture the operators of type
\[
O_{\text{dim}6} = (H^+H)O_{SM}^h
\]

There are vast majority of other \textit{dim-6} operators, see, e.g. from Review of Particle Physics (Higgs section).
Operators involving bosons only

\[ \mathcal{O}_H = 1/(2v^2) \left( \partial^\mu \left( \Phi^\dagger \Phi \right) \right)^2 \]
\[ \mathcal{O}_T = 1/(2v^2) \left( \Phi^\dagger D^\mu \Phi \right)^2 \]
\[ \mathcal{O}_6 = -\lambda/(v^2) \left( \Phi^\dagger \Phi \right)^3 \]
\[ \mathcal{O}_B = (ig')/(2m_W^2) \left( \Phi^\dagger D^\mu \Phi \right) (\partial^\nu B_{\mu\nu}) \]
\[ \mathcal{O}_W = (ig)/(2m_W^2) \left( \Phi^\dagger \sigma^i \tilde{D}^\mu \Phi \right) (D^\nu W_{\mu\nu})^i \]
\[ \mathcal{O}_{HB} = (ig')/m_W^2 \left( D^\mu \Phi \right)^\dagger (D^\nu \Phi) B_{\mu\nu} \]
\[ \mathcal{O}_{HW} = (ig)/m_W^2 \left( D^\mu \Phi \right)^\dagger \sigma^i (D^\nu \Phi) W_{\mu\nu}^i \]
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\[ \mathcal{O}_{GG} = g_S^2/m_W^2 \Phi^\dagger \Phi G_{\mu\nu}^A G_A^{\mu\nu} \]
\[ \mathcal{O}_{H\tilde{B}} = (ig')/m_W^2 \left( D^\mu \Phi \right)^\dagger (D^\nu \Phi) \tilde{B}_{\mu\nu} \]
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\[ O_H = \frac{1}{2 v^2} \left( \partial^\mu (\Phi^\dagger \Phi) \right)^2 \]
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Ops. involving bosons and fermions

\[ O_t = y_t/v^2 (\Phi^\dagger \Phi) (\bar{q}_L \Phi^c t_R) \]
\[ O_b = y_b/v^2 (\Phi^\dagger \Phi) (\bar{q}_L \Phi b_R) \]
\[ O_{\tau} = y_\tau/v^2 (\Phi^\dagger \Phi) (\tilde{L}_L \Phi \tau_R) \]
\[ O_{Hq} = i/v^2 (\bar{q}_L \gamma^\mu q_L) (\Phi^\dagger \vec{D}_\mu \Phi) \]
\[ O_{Hq}^{(3)} = i/v^2 (\bar{q}_L \gamma^\mu \sigma^i q_L) (\Phi^\dagger \sigma^i \vec{D}_\mu \Phi) \]
\[ O_{Hu} = i/v^2 (\bar{u}_R \gamma^\mu u_R) (\Phi^\dagger \vec{D}_\mu \Phi) \]
\[ O_{Hd} = i/v^2 (\bar{d}_R \gamma^\mu d_R) (\Phi^\dagger \vec{D}_\mu \Phi) \]
\[ O_{Hud} = i y_u y_d/v^2 (\bar{u}_R \gamma^\mu d_R) (\Phi^c \vec{D}_\mu \Phi) \]
\[ O_{Hl} = i/v^2 (\bar{l}_R \gamma^\mu l_R) (\Phi^\dagger \vec{D}_\mu \Phi) \]
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Ops. involving bosons and fermions

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>[ O_{uB} = (g' y_u)/m_W^2 (\bar{q}<em>L \Phi^c \sigma^\mu \nu u_R) B</em>{\mu\nu} ]</td>
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<tr>
<td>[ O_{uW} = (g y_u)/m_W^2 (\bar{q}<em>L \sigma^i \Phi^c \sigma^\mu \nu u_R) W</em>{\mu\nu}^i ]</td>
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<td>[ O_{uG} = (g S y_u)/m_W^2 (\bar{q}<em>L \Phi^c \sigma^\mu \nu t^A u_R) G</em>{\mu\nu}^A R ]</td>
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<td>[ O_{dB} = (g' y_d)/m_W^2 (\bar{q}<em>L \Phi \sigma^\mu \nu d_R) B</em>{\mu\nu} ]</td>
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<td>[ O_{dW} = (g y_d)/m_W^2 (\bar{q}<em>L \sigma^i \Phi \sigma^\mu \nu d_R) W</em>{\mu\nu}^i ]</td>
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<td>[ O_{LB} = (g' y_l)/m_W^2 (\bar{L}<em>L \Phi \sigma^\mu \nu l_R) B</em>{\mu\nu} ]</td>
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<td>[ O_{lW} = (g y_l)/m_W^2 (\bar{L}<em>L \sigma^i \Phi \sigma^\mu \nu l_R) W</em>{\mu\nu}^i ]</td>
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Some efforts trying to capture the Higgs physics with different Lorentz structure (comparing to SM), includes,

**Off-shell, Higgs plus X (jet, W, Z) as function of scale parameter such as pT, etc., measurements.**


(Top, Tau, VV) CP (beyond scaling)


Or some other inclusive measurements, such as total rates.
INCLUSIVE MEASUREMENT

N. Craig, M. Farina, M. McCullough and M. Perelstein \textit{arXiv:1411.0676}
\[ \mathcal{O}_{WW} = g^2 |H|^2 W_{\mu\nu}^a W^{a,\mu\nu} \]
\[ \mathcal{O}_{BB} = g'^2 |H|^2 B_{\mu\nu} B^{\mu\nu} \]
\[ \mathcal{O}_{WB} = gg' H^\dagger \sigma^a H W_{\mu\nu}^a B^{\mu\nu} \]
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\[ \mathcal{O}_{L}^{(3)e} = \left( i H^\dagger \sigma^a \overset{\leftrightarrow}{D}_\mu H \right) (\bar{L}_L \gamma^\mu \sigma^a L_L) \]
\[ \mathcal{O}_{LL}^{(3)e} = (\bar{L}_L \gamma^\mu \sigma^a L_L) (\bar{L}_L \gamma^\mu \sigma^a L_L) \]
\[ \mathcal{O}_L^e = \left( i H^\dagger \overset{\leftrightarrow}{D}_\mu H \right) (\bar{L}_L \gamma^\mu L_L) \]
\[ \mathcal{O}_R^e = \left( i H^\dagger \overset{\leftrightarrow}{D}_\mu H \right) (\bar{e}_R \gamma^\mu e_R) \]
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\begin{align*}
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\end{align*}
IN THE MODEL SPACE...

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<thead>
<tr>
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<th>$\Phi(1, 3, 0)$</th>
<th>$\Phi(1, 2, -\frac{1}{2})$</th>
<th>$\Phi(1, 1, 0)$</th>
<th>$A^\mu_{U(1)}$</th>
<th>$A^\mu_{SU(2)}$</th>
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- Corrections in terms of higher dimensional operators usually **coexist**.
- The relative sizes depend on the dynamics, overall sizes depend on new physics scale.
- Thus, difficult to extract new physics scale nor distinguish (needs extra information on) different models.

\[ M_{HZ\ell\ell}^\mu = \frac{1}{m_H} \bar{u}(p_3, s_3) \left[ \gamma^\mu (H_{1, V} + H_{1, A} \gamma_5) + \frac{q^{\mu}q}{m_H^2} (H_{2, V} + H_{2, A} \gamma_5) \right. \\

+ \left. \frac{\epsilon^{\mu\nu\sigma\rho} p_\nu q_\sigma}{m_H^2} \gamma_\rho (H_{3, V} + H_{3, A} \gamma_5) \right] v(p_4, s_4), \]
Many operators (twelve) contribute. Amplitude decomposed into six different currents. Expanding over dim-6 operators in LO (interfering with SM), only six matrix elements are independent. Construct six asymmetry observable.

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Response function—how much the observable would change as a function of the coefficients of relevant operators.

Total rate sees most operators, CP-odd ones at quadratic level, while different asymmetries pick up corresponding “modes”.
When studying all these asymmetries simultaneously, we are to see the relative sizes of the operators, and get much better sense of possible underlying new physics.
For simplicity and without much lose of generality, we study the “background-free” channel of $ee \rightarrow ZH$, $Z \rightarrow ll$, $H \rightarrow bb$ at lepton colliders. ~few million of events will be selected, we impose selection cuts on these channels following the detailed studies done for CEPC preCDR. Study includes various collider effects including beamstrahlung, collider cuts, input parameter dependence, and their relative contribution to uncertainty.
PHENOMENOLOGY

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E.g, how the parton level simulation on the asymmetries (which agrees well with our analytic calculation) change under collider environment. Blue/red before/after cuts. Green/yellow band, projected 1-/2- sigma sensitivity on individual observable.
Asymmetry breaks the flat direction of inclusive rate measurement, and provides novel information about the underlying operator structures.

Many plots of similar types are obtained by our studies. We are to demonstrate these diagnostic power in model space as well.
To gain more from going to higher energies, we propose to study the ZZ-fusion channel for inclusive measurement.

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SUMMARY AND OUTLOOK

We explore lepton collider Higgs factory physics potential from asymmetry observable.

They provide novel information *not* captured in usual precision study.

Very important in improving sensitivities of precision measurement, distinguishing contributions from different new physics, and probing the underlying dynamics of new physics.
SUMMARY AND OUTLOOK

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Thank you!
\[ J(q^2, \theta_1, \theta_2, \phi) = J_1(1 + \cos^2 \theta_1 \cos^2 \theta_2 + \cos^2 \theta_1 + \cos^2 \theta_2) \]
\[ + J_2 \sin^2 \theta_1 \sin^2 \theta_2 + J_3 \cos \theta_1 \cos \theta_2 \]
\[ + (J_4 \sin \theta_1 \sin \theta_2 + J_5 \sin 2\theta_1 \sin 2\theta_2) \sin \phi \]
\[ + (J_6 \sin \theta_1 \sin \theta_2 + J_7 \sin 2\theta_1 \sin 2\theta_2) \cos \phi \]
\[ + J_8 \sin^2 \theta_1 \sin^2 \theta_2 \sin 2\phi + J_9 \sin^2 \theta_1 \sin^2 \theta_2 \cos 2\phi. \]
\[ J_1 = 2r s \left( g_A^2 + g_V^2 \right) \left( |H_{1,V}|^2 + |H_{1,A}|^2 \right), \]

\[ J_2 = \kappa \left( g_A^2 + g_V^2 \right) \left[ \kappa \left( |H_{1,V}|^2 + |H_{1,A}|^2 \right) + \lambda \text{Re} \left( H_{1,V} H_{2,V}^* + H_{1,A} H_{2,A}^* \right) \right], \]

\[ J_3 = 32r s g_A g_V \text{Re} \left( H_{1,V} H_{1,A}^* \right), \]

\[ J_4 = 4\kappa \sqrt{r s} \lambda g_A g_V \text{Re} \left( H_{1,V} H_{3,A}^* + H_{1,A} H_{3,V}^* \right), \]

\[ J_5 = \frac{1}{2} \kappa \sqrt{r s} \lambda \left( g_A^2 + g_V^2 \right) \text{Re} \left( H_{1,V} H_{3,V}^* + H_{1,A} H_{3,A}^* \right), \]

\[ J_6 = 4\sqrt{r s} g_A g_V \left[ 4\kappa \text{Re} \left( H_{1,V} H_{1,A}^* \right) + \lambda \text{Re} \left( H_{1,V} H_{2,A}^* + H_{1,A} H_{2,V}^* \right) \right], \]

\[ J_7 = \frac{1}{2} \sqrt{r s} \left( g_A^2 + g_V^2 \right) \left[ 2\kappa \left( |H_{1,V}|^2 + |H_{1,A}|^2 \right) + \lambda \text{Re} \left( H_{1,V} H_{2,V}^* + H_{1,A} H_{2,A}^* \right) \right], \]

\[ J_8 = 2r s \sqrt{\lambda} \left( g_A^2 + g_V^2 \right) \text{Re} \left( H_{1,V} H_{3,V}^* + H_{1,A} H_{3,A}^* \right), \]

\[ J_9 = 2r s \left( g_A^2 + g_V^2 \right) \left( |H_{1,V}|^2 + |H_{1,A}|^2 \right). \]