Position-dependent power spectrum: a new observable in the large-scale structure

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• main galaxy sample
• red luminous galaxy
• CMASS sample

credit: Michael Blanton and SDSS
What is the 3-point correlation function and bispectrum?

- **3-point correlation function:**
  \[ \langle \delta(r_1) \delta(r_2) \delta(r_3) \rangle = \zeta(r_1, r_2, r_3) \]

- **probability of finding triplets:**
  \[ dP = \bar{n}^3 [1 + \zeta(r_1, r_2, r_3)] d^3r_1 d^3r_2 d^3r_3 \]

- **bispectrum:**
  \[ \langle \delta(k_1) \delta(k_2) \delta(k_3) \rangle = (2\pi)^3 \delta_D(k_1 + k_2 + k_3) B(k_1, k_2, k_3) \]
Why study position-dependent power spectrum?

- Direct measurement of the 3-point correlation function (bispectrum) is difficult when the survey geometry and selection function are not ideal.
- There are only a few measurements of the bispectrum for the current large-scale structure surveys (e.g. Gil-Marín et al 2014 for SDSS DR11).
- This motivates us to find a new observable which is easier to model and gives similar information (especially in the squeezed limit).
What is the squeezed-limit bispectrum?

• $\langle \delta(k_1)\delta(k_2)\delta(k_3) \rangle = (2\pi)^3 \delta_D(k_1 + k_2 + k_3)B(k_1, k_2, k_3)$

• squeezed limit: $k_1 \approx k_2 \gg k_3$

• Squeezed-limit bispectrum measures the correlation of one long-wavelength (small-wavenumber) fluctuation and two short-wavelength (large-wavenumber) fluctuations.
How is the position-dependent $P(k,r)$ modulated by the bispectrum?

vanishing squeezed-limit bispectrum:
no correlation between power spectra and positions

non-vanishing squeezed-limit bispectrum:
correlation between power spectra and positions
How to measure $P(k, r_L)$?

Local mean

$$\bar{\delta}(r_L) = \frac{\bar{n}(r_L)}{\langle n \rangle} - 1$$

Global mean

$$P(k, r_L) = \frac{1}{V_L} |\delta(k, r_L)|^2$$

Local Fourier transform

$$\langle P(k, r_L)\bar{\delta}(r_L) \rangle$$
Small-scale power spectra depend on large-scale density fluctuations!

\[ P(k, r_L) \]

\[ V_{\text{tot}} = (2.4 \ h^{-1} \ \text{Gpc})^3 \]
\[ V_L = (300 \ h^{-1} \ \text{Mpc})^3 \]
Integrated bispectrum $iB(k)$

- $iB(k) = \langle P(k, \mathbf{r}_L) \bar{\delta}(\mathbf{r}_L) \rangle$
  \[ = \frac{1}{V_L^2} \int \frac{d^2 \hat{k}}{4\pi} \int \frac{d^3 q_1}{(2\pi)^3} \int \frac{d^3 q_2}{(2\pi)^3} B(k - q_1, -k + q_1 + q_2, -q_2) \]
  \[ \times W_L(q_1) W_L(-q_1 - q_2) W_L(q_2) \]

- $W_L(k) = V_L \ \text{sinc} \left( \frac{k_x L}{2} \right) \ \text{sinc} \left( \frac{k_y L}{2} \right) \ \text{sinc} \left( \frac{k_z L}{2} \right)$

- **squeezed limit:** $k \gg 1/L$ (e.g. $k=0.3 \ h \ \text{Mpc}^{-1}$ and $L=300 \ h^{-1}\text{Mpc}$)

- **normalized integrated bispectrum:**
  \[ \frac{iB(k)}{P(k) \sigma^2_L} \quad \text{where} \quad \sigma^2_L = \frac{1}{V_L^2} \int \frac{d^3 q_2}{(2\pi)^3} \ |W_L(q_2)|^2 P(q_2) \]
Measured normalized $iB(k)$ from $160 \ (2.4 \ h^{-1} \ Gpc)^3$ DM simulations

- BAO is damped more with smaller sub-volume size
- Nonlinear evolution also damps BAO
Separate universe approach

• The power spectrum in the universe with an infinite-wavelength density perturbation $\delta_0$ is

$$P(k|\delta_0) = P(k)|_{\delta_0=0} + P(k) \frac{d \ln P(k)}{d\delta_0} \bigg|_{\delta_0=0} \delta_0 + O(\delta_0^2)$$

$$\langle P(k|\delta_0)\delta_0 \rangle = iB(k) = P(k) \frac{d \ln P(k)}{d\delta_0} \bigg|_{\delta_0=0} \langle \delta_0^2 \rangle$$

• One can then consider the normalized integrated bispectrum as how the power spectrum responds to the density perturbation $\delta_0$, i.e.

$$\frac{iB(k)}{P(k)\sigma_L^2} = \frac{d \ln P(k)}{d\delta_0}$$
Mapping the universe

\[ \Omega_m, \Omega_\Lambda, \Omega_k, H_0, \rho_m \]

\[ \Omega_m(1 + \delta_0) \]

(\textit{e.g.} Sirko 2005, Baldauf et al 2011, Sherwin & Zaldarriaga 2012, Li et al, 2014)
Change of $P(k)$ with $\delta_0$

- dilation effect (change of scale):
  \[
  \tilde{P}(\tilde{k}, t) \rightarrow \left[ 1 - \frac{1}{3} \frac{d \ln k^3 P(k, t)}{d \ln k} \frac{D(t)}{D(t_0)} \delta_0 \right] P(k, t)
  \]

- reference density effect ($\rho \rightarrow \rho(1 + \delta_0)$):
  \[
  \tilde{P}(\tilde{k}, t) \rightarrow \left[ 1 + 2 \frac{D(t)}{D(t_0)} \delta_0 \right] P(\tilde{k}, t)
  \]

- growth effect:
  \[
  \tilde{P}(\tilde{k}, t) \rightarrow \left[ 1 + \frac{26}{21} \frac{D(t)}{D(t_0)} \delta_0 \right] P(\tilde{k}, t) \quad \text{for linear } P(k)
  \]
  separate universe simulations for nonlinear $P(k)$
Separate universe modeling

\[ \frac{iB(k)}{P(k)\sigma^2_L} \]

\[ \frac{iB(k)}{P(k)\sigma^2_L} \]

\[ k \quad h^{-1} \text{Mpc} \]

\[ \text{z=0} \]

\[ \text{z=1} \]

\[ \text{z=2} \]

\[ \text{z=3} \]

\[ 300 \quad h^{-1} \text{Mpc} \]
Application to the SDSS-III BOSS DR10 CMASS sample

- Correlation function is easier to measure than power spectrum for galaxy surveys.
- Galaxy 3-point function $\zeta_g = b_1^3 \zeta_{\text{SPT}} + b_1^2 b_2 \zeta b_2$ is sensitive to the nonlinear bias at the leading order (unlike the 2-point function).
How to measure $\xi(r, r_L)$?

**local mean**

$$\bar{\delta}(r_L) = \frac{\bar{n}(r_L)}{\langle n \rangle} - 1$$

**global mean**

$$\xi(r, r_L)$$

$$\langle \xi(r, r_L) \bar{\delta}(r_L) \rangle$$

$$\equiv i\zeta(r)$$
Divide the sample in NGC

- ~400,000 galaxies at 0.43<z<0.7 (~ 2 h^{-3} Gpc^{3})
- weighting: $w = (w_{cp} + w_{zf} - 1)w_{star}w_{see}w_{FKP}$
- 600 PTHalos mock catalogs (Manera et al 2014)
Mocks in redshift space

$r^2 \xi(r)$

$d.o.f. = 36$

$b_1 = 1.931 \pm 0.077$

$b_2 = 0.54 \pm 0.35$

$i \zeta(r) / \sigma_L^2$

$r \ [h^{-1} \text{ Mpc}]$

$220 \ h^{-1} \text{ Mpc}$

$120 \ h^{-1} \text{ Mpc}$
BOSS DR10 CMASS sample

$d.o.f. = 38$

$\chi^2(\text{data}) = 46.43$
Combining with other probes

- $i\zeta_g$ is sensitive to $b_2$ at the leading order
- Anisotropic clustering (Samushia et al 2013): 
  $$[(b_1\sigma_8)+(f\sigma_8)\mu^2]^2 \rightarrow (b_1\sigma_8), (f\sigma_8)$$
- Weak lensing (More et al 2015): $\sigma_8$
- Running MCMC on $i\zeta_g$ with Gaussian priors of $(b_1\sigma_8)$, $(f\sigma_8)$, and $\sigma_8$, we obtain $b_2=0.41\pm0.41$ for BOSS CMASS sample.
Conclusions

• Three-point function contains extra information (on top of two-point function) and so can be used to constrain the cosmological parameters.

• Position-dependent power spectrum and correlation function are useful methods to extract the squeezed-limit bispectrum.

• Combining with anisotropic clustering and weak lensing, we can constrain the nonlinear bias of BOSS CMASS sample to be $b_2 = 0.41 \pm 0.41$. 