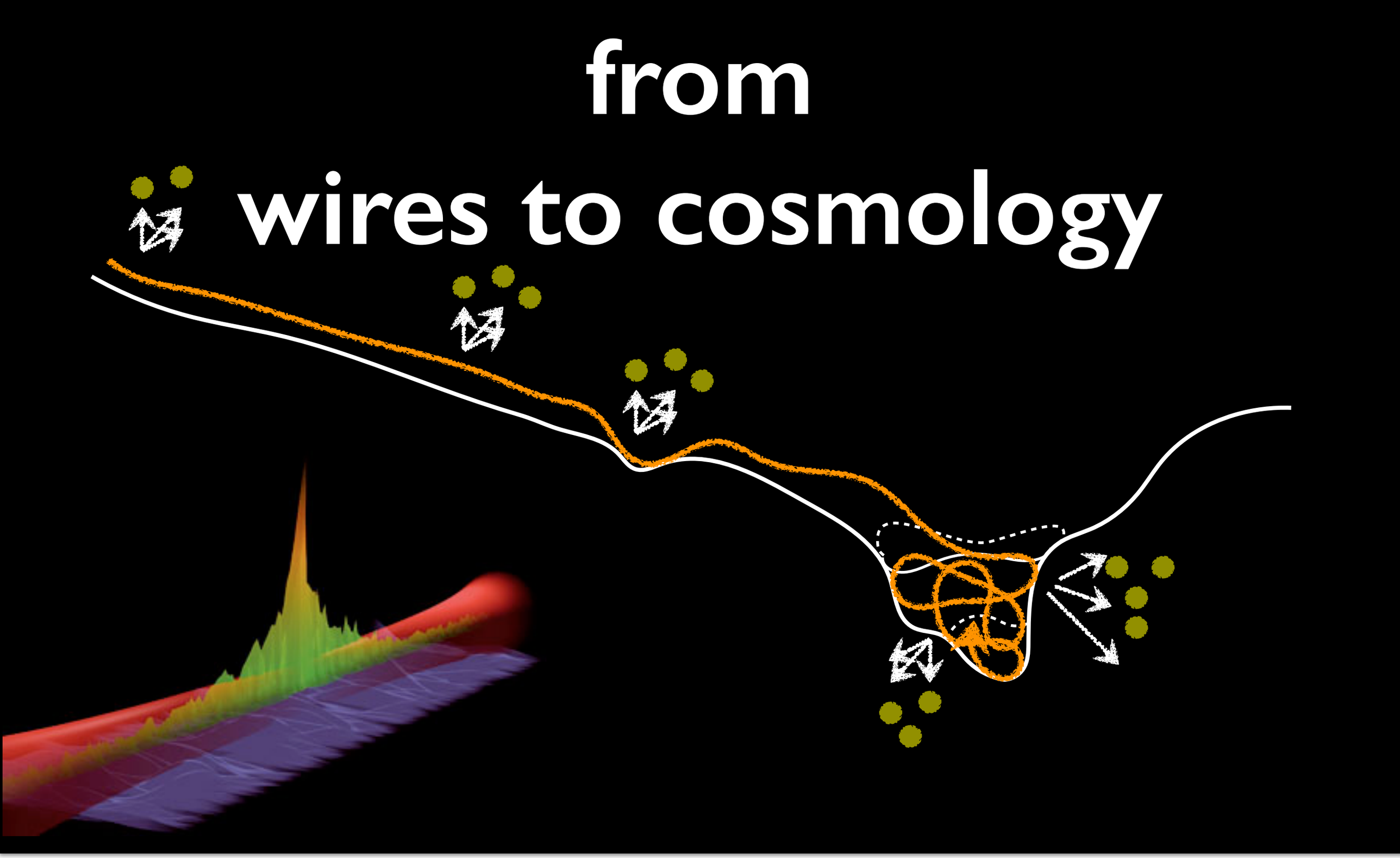
from wires to cosmology



RICE

Mustafa Amin
with
Daniel Baumann

UNIVERSITY OF CAMBRIDGE


WORK IN
PROGRESS

related work: condensed matter + cosmology

Anderson
Absence of diffusion in certain random matrices
(1957)

Mello, Pereyra Kumar
Macroscopic approach to multichannel disordered wires
(1987)

C. Beenakker,
Random matrix theory of quantum transport
(1997)

C. Muller and D. Delande,
Disorder and interference: localization phenomena
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Kofman, Linde & Starobinsky
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Traschen and Brandenberger
(1997)

Zanchin, Maia, Craig & Brandenberger
Reheating in the presence of noise
(1998)

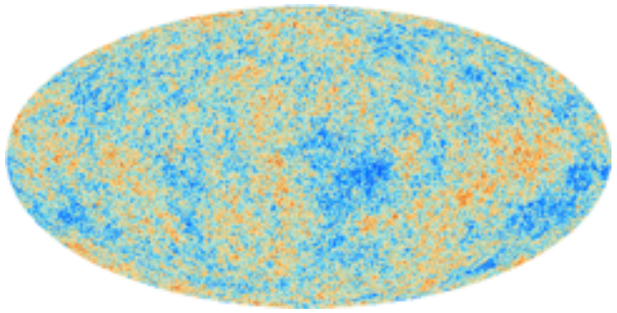
Nacir, Porto, Senatore and Zaldarriaga
Dissipative effects in the effective field theory of inflation
(2012)

Green
Disorder in the early universe
(2015)

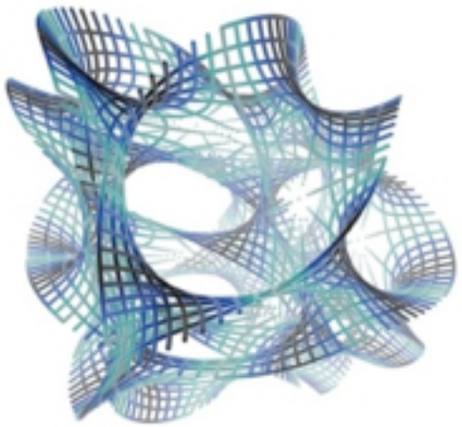
+ many works on particle production during and after inflation.

motivation

- **observations**: early universe is simple
  - few numbers $n_s \sim 1, r, T_{\rm reh}, f_{\rm NL} \lesssim 10$



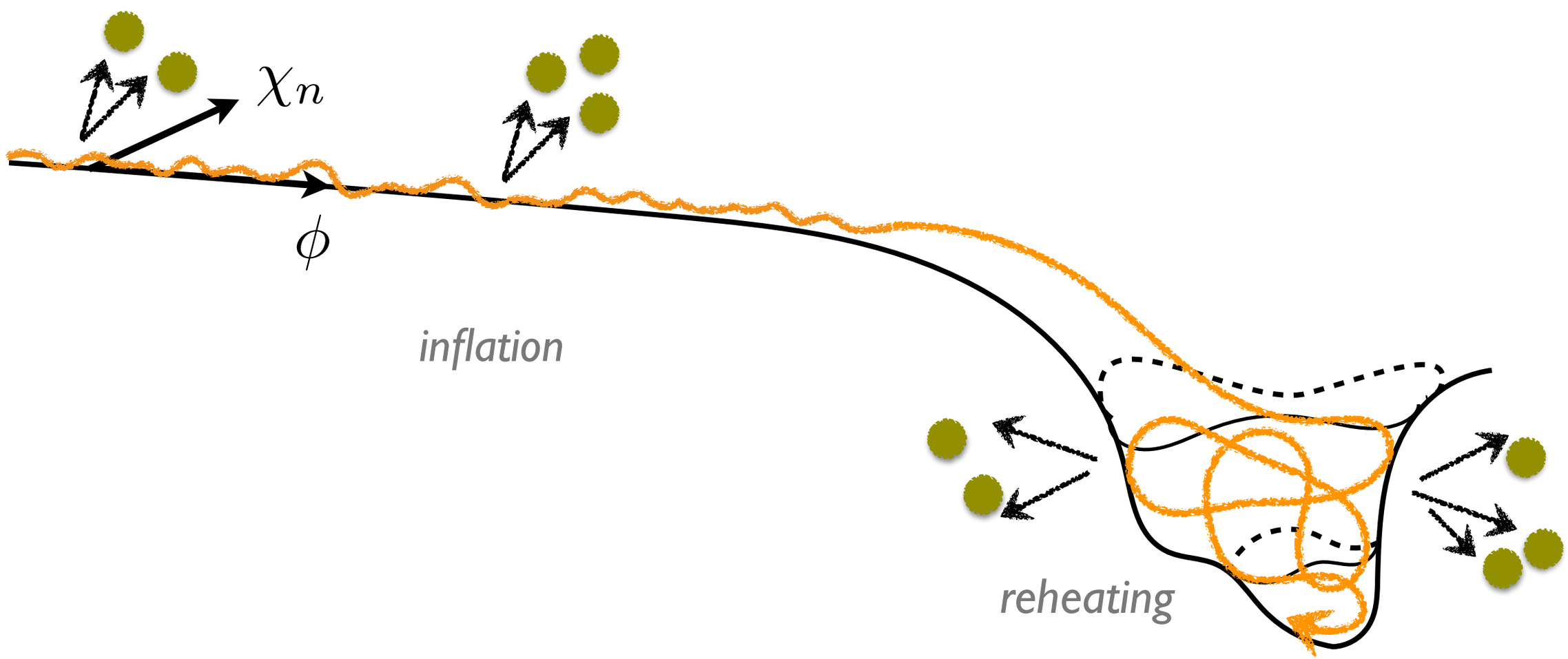
- **theory**: not so much …
  - many interacting fields, many scales (E >> TeV?)



- calculational tools?
- can the simplicity/universality be emergent?

multifield inflation/reheating

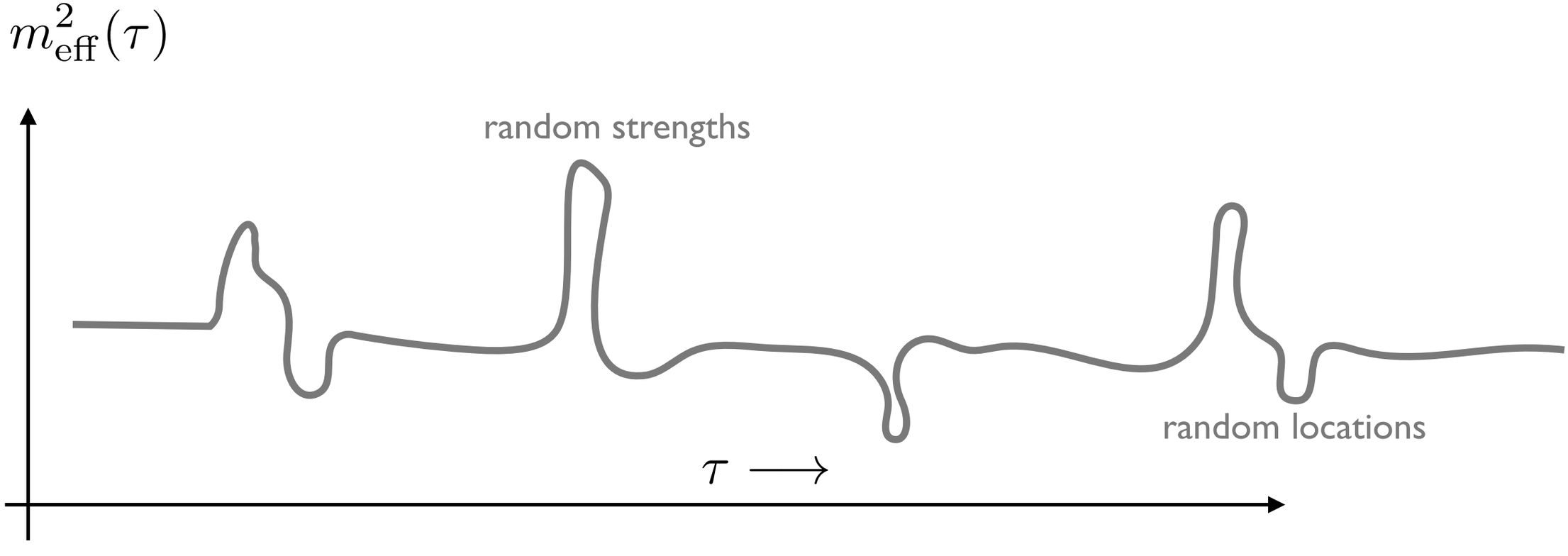
- inflation/reheating: many interacting fields
- fluctuations: coupled, non-perturbative



complexity in time: cosmology

$$\ddot{\chi}_k(\tau) + \left[k^2 + m_{\rm eff}^2(\tau)\right]\chi_k(\tau) = 0$$

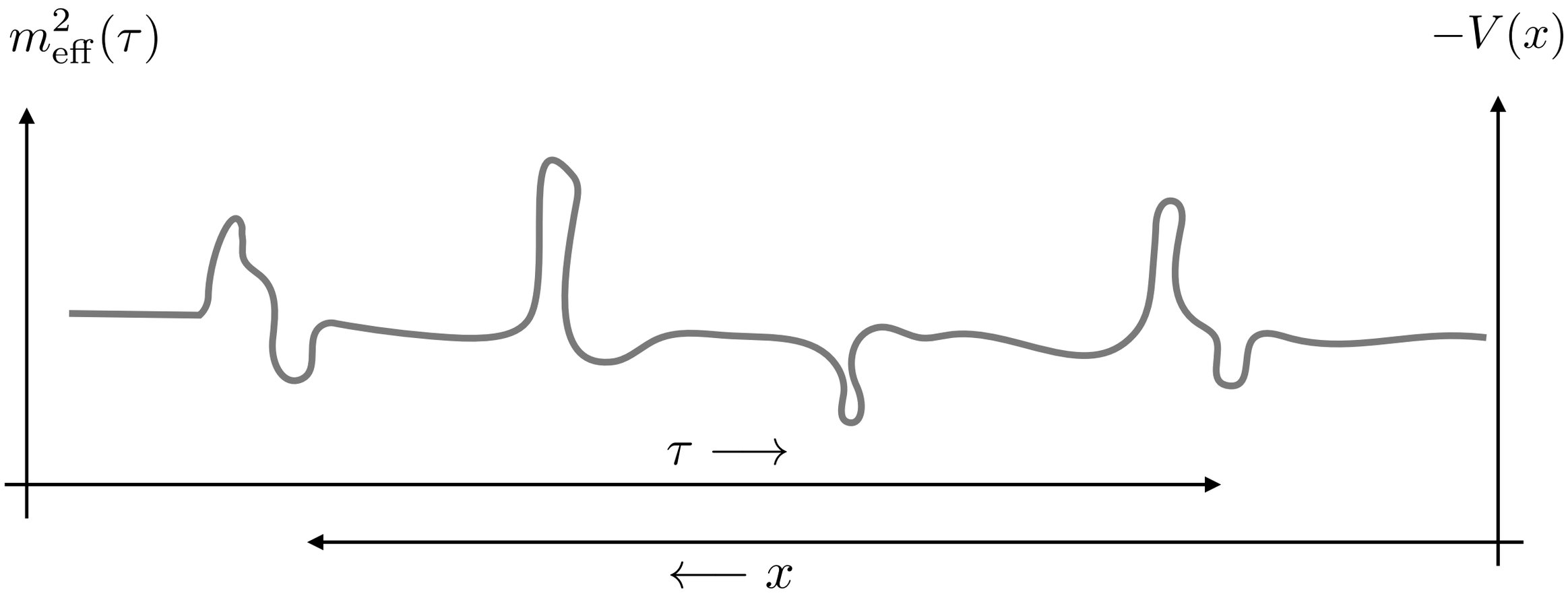
$$m_{\rm eff}^2(\tau) = -\frac{\ddot{a}(\tau)}{a(\tau)} + a^2(\tau)m_\varphi^2 + a^2(\tau)g^2(\phi(\tau)-\phi_*)^2 + \ldots$$



complexity in space: wires

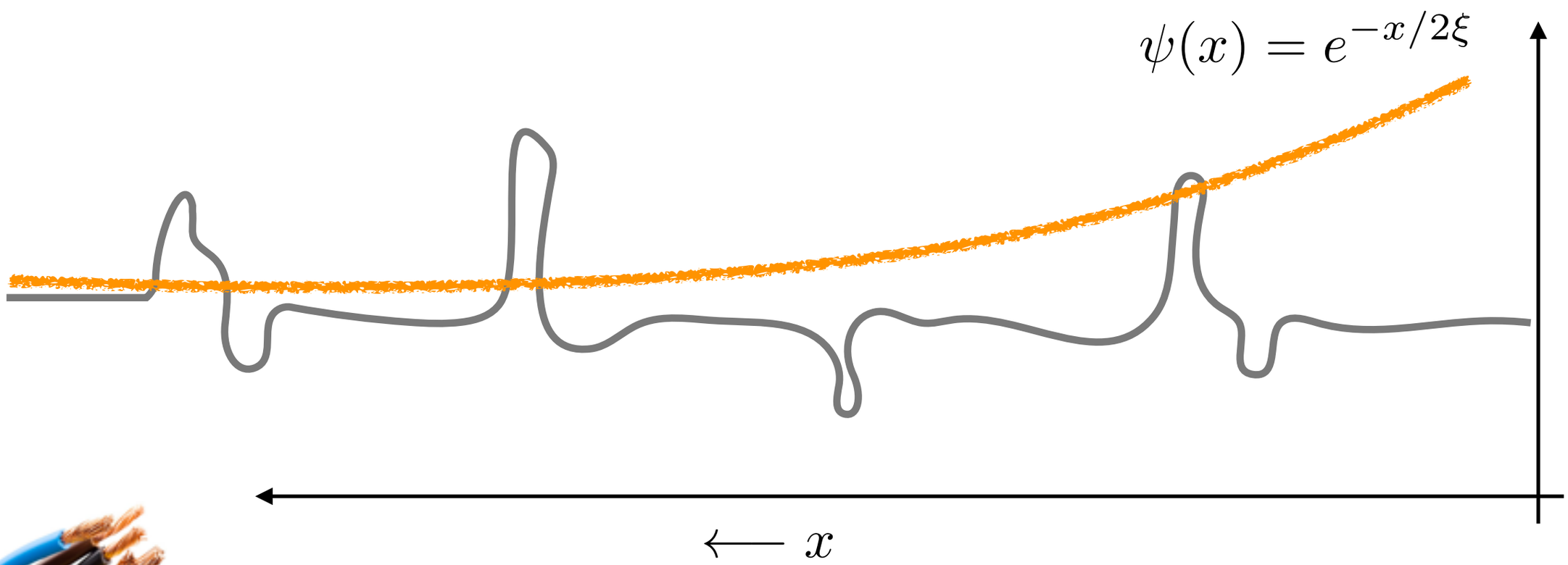
$$\ddot{\chi}_k(\tau) + \left[k^2 + m^2_{\text{eff}}(\tau)\right]\chi_k(\tau) = 0$$ particle production

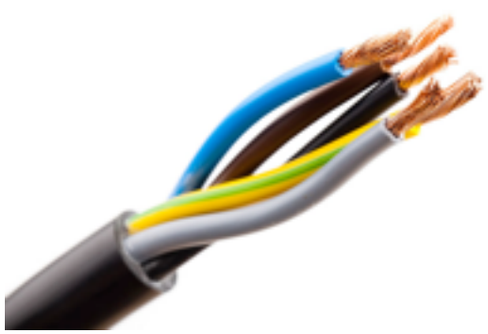
$$\psi''(x) + \left[k^2 - V(x)\right]\psi(x) = 0$$ Schrodinger



Anderson localization!
complexity in space — emergent simplicity

$$\psi''(x) + \left[k^2 - V(x)\right]\psi(x) = 0$$





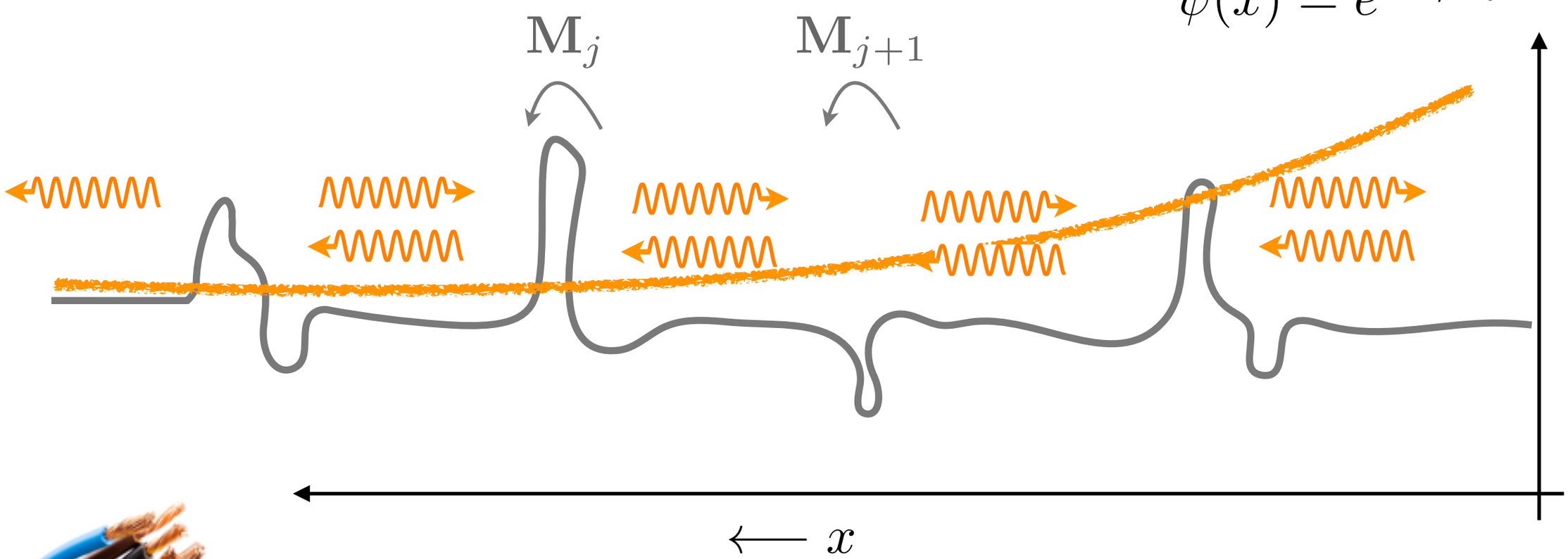
caveat: wave function — transmission probability

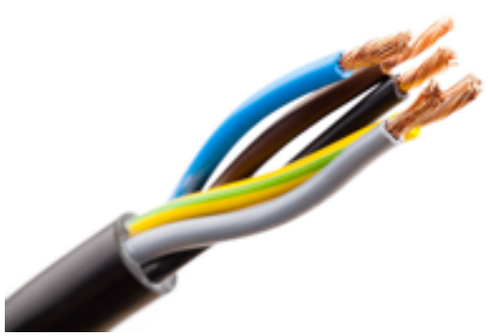
Anderson Localization: chained scattering matrices

phases are important!

$$|\psi(L)\rangle = \mathbf{M}_{N_s} \cdot \mathbf{M}_{N_s-1} \ldots \mathbf{M}_1 |\psi(0)\rangle$$

$$\psi(x) = e^{-x/2\xi}$$



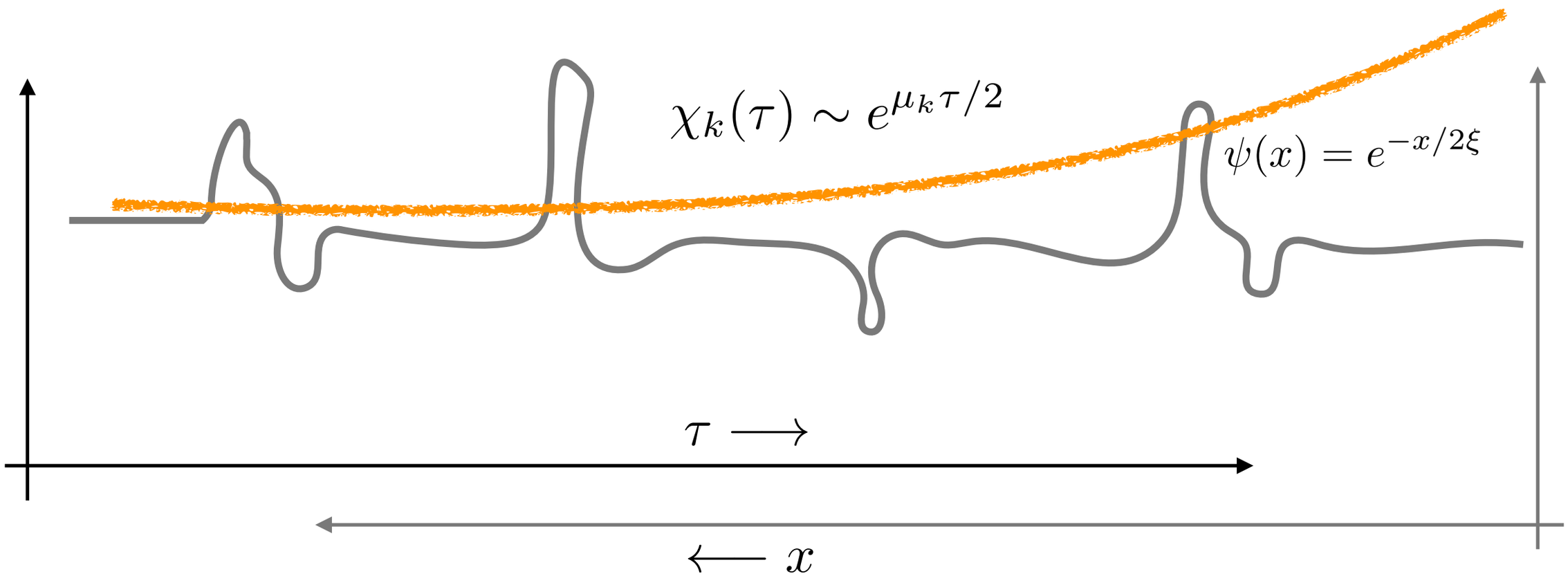


caveat: wave function — transmission probability

complexity in space — Anderson localization
complexity in time — exponential particle production

$$\ddot{\chi}_k(\tau) + \left[k^2 + m_{\rm eff}^2(\tau)\right]\chi_k(\tau) = 0$$

$$\psi''(x) + \left[k^2 - V(x)\right]\psi(x) = 0$$

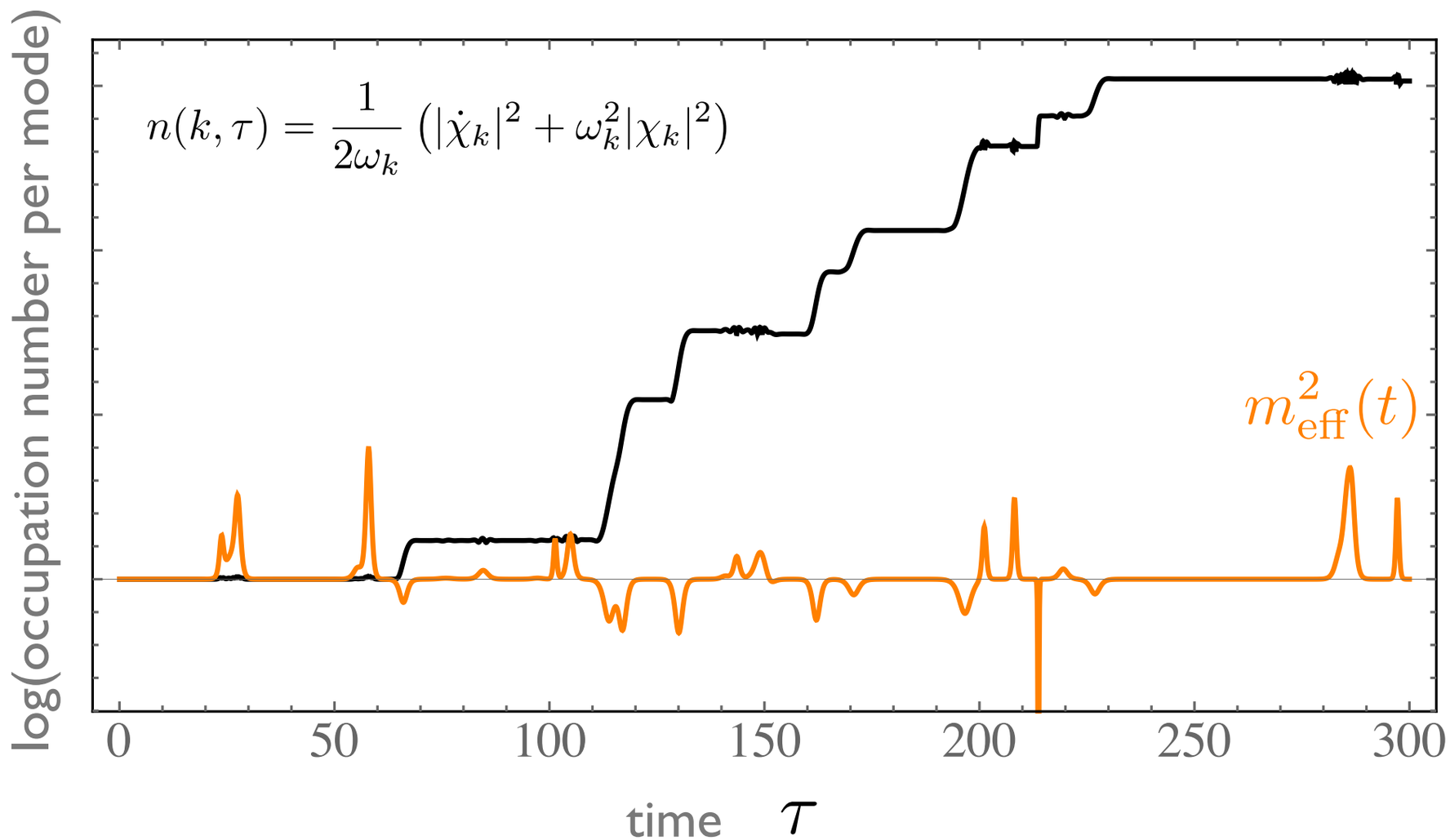


caveat: wave function — transmission probability, mode function — occupation number

MA & Baumann

occupation number performs a drifted random walk

$$\ddot{\chi}_k(\tau) + \left[k^2 + m_{\rm eff}^2(\tau)\right]\chi_k(\tau) = 0$$



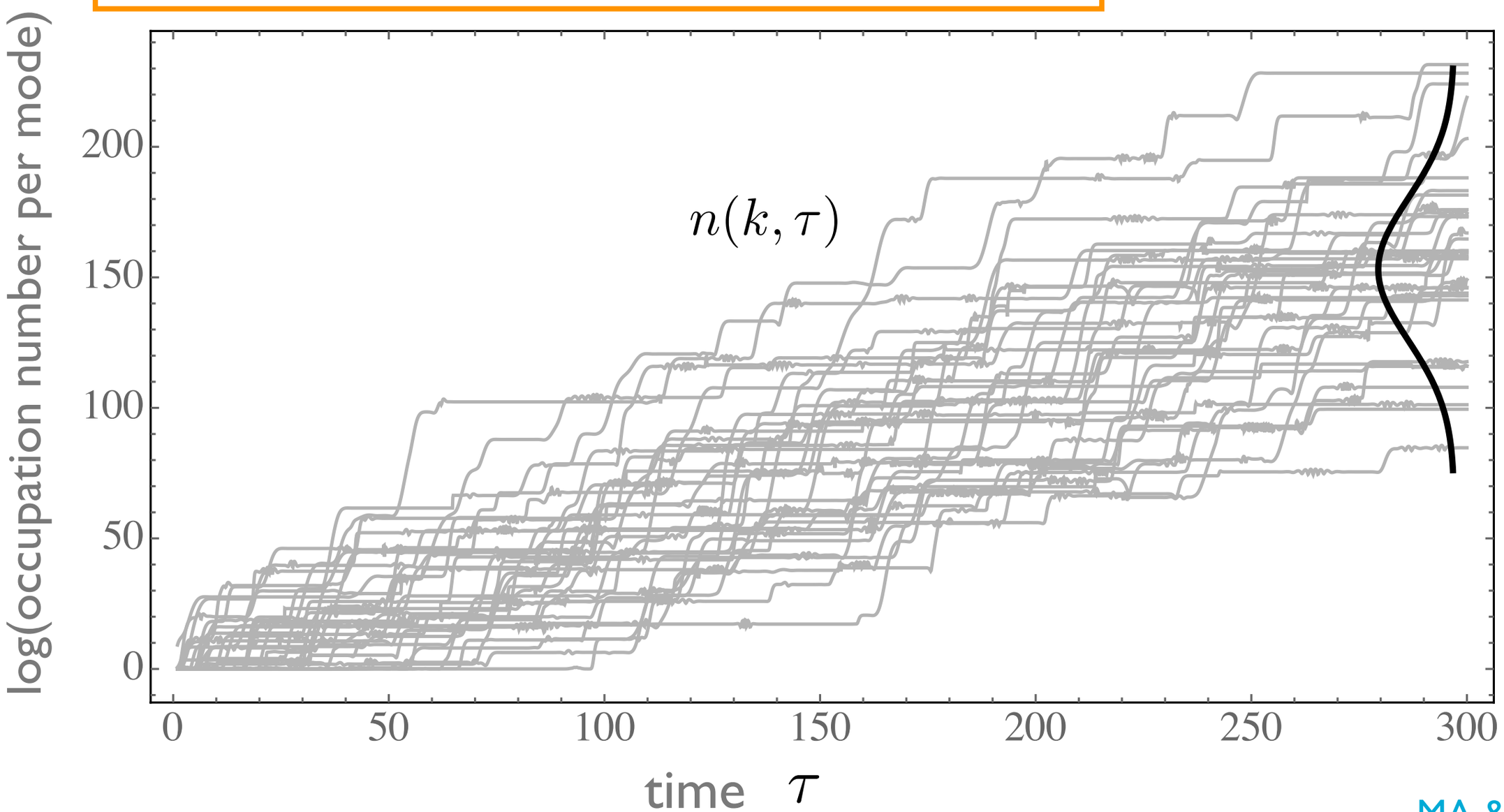
MA & Baumann

a Fokker Planck equation for the occupation number

$$\frac{1}{\mu_k}\frac{\partial}{\partial\tau}P(n,\tau) = \frac{\partial}{\partial n}\left[n(1+n)\frac{\partial}{\partial n}P(n,\tau)\right]$$

μ_k : local mean particle-production rate

analogue: mean free path



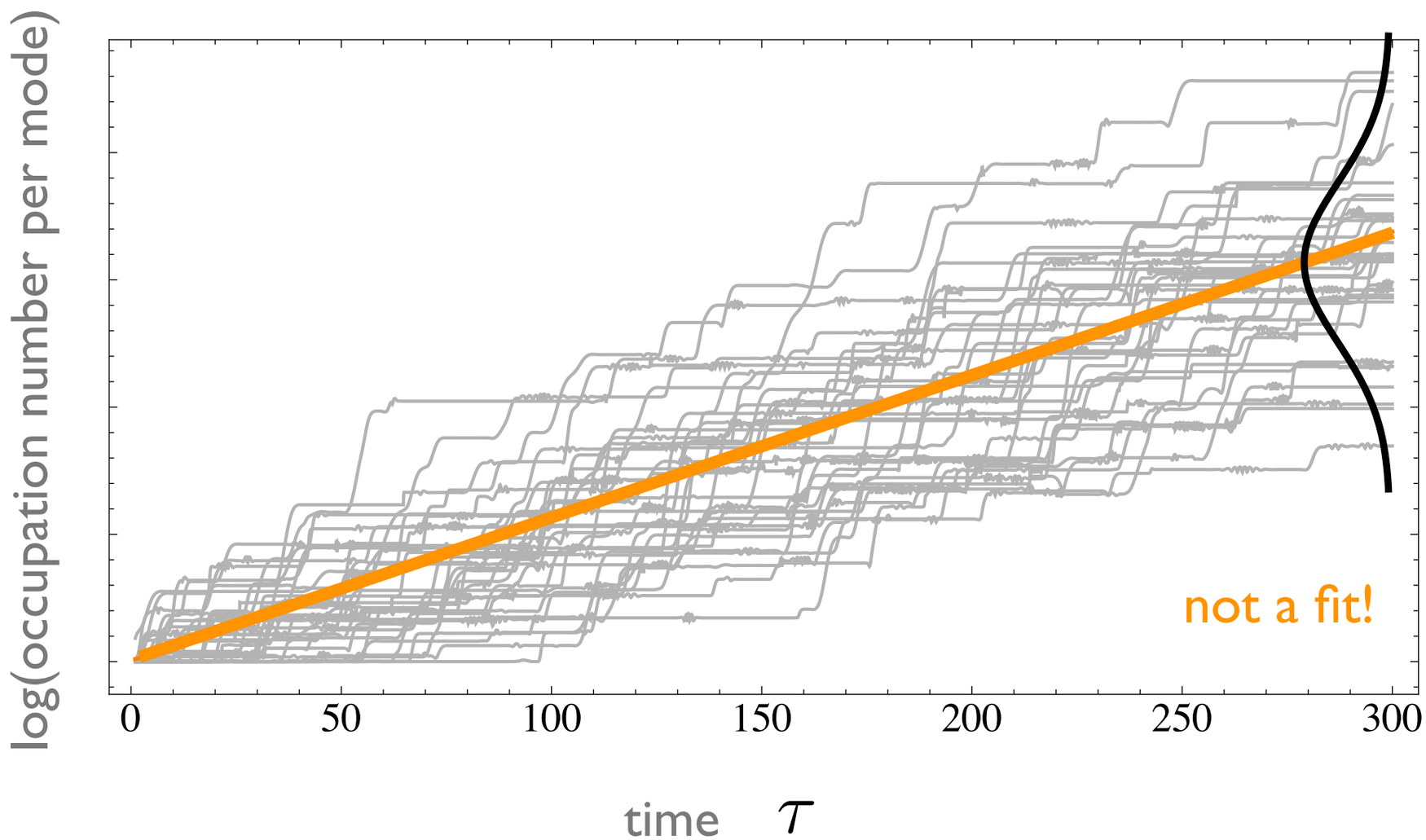
MA & Baumann

the typical occupation number

$$\frac{1}{\mu_k}\frac{\partial}{\partial\tau}P(n,\tau) = \frac{\partial}{\partial n}\left[n(1+n)\frac{\partial}{\partial n}P(n,\tau)\right]$$

μ_k : local mean particle-production rate

analogue: mean free path



$$\langle n\rangle = \frac{1}{2}\left(e^{2\mu_k\tau}-1\right)$$

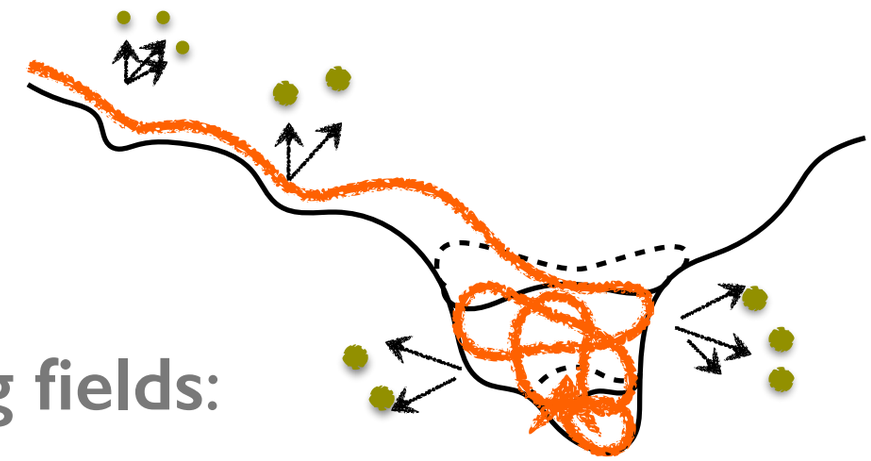
mean

$$n_{\rm typ} = e^{\langle \ln(1+n)\rangle}$$

most probable $= e^{\mu_k\tau}$

MA & Baumann

many interacting fields (thick wires)



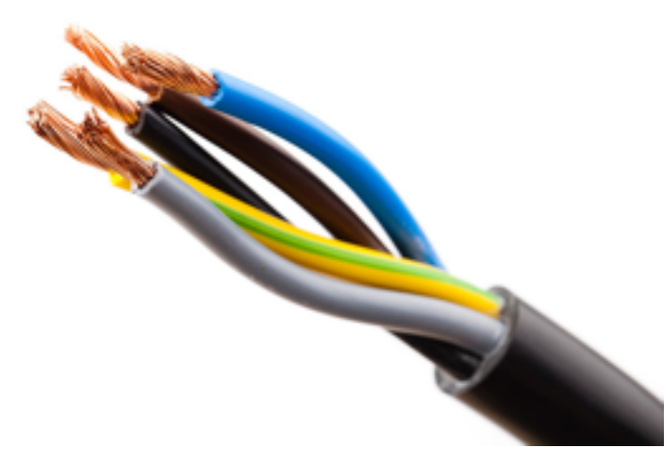
early universe: **multiple interacting fields**:

$$\ddot{\chi}_a + \left[k^2 \delta_a^b + \mathcal{M}^b{}_a(\tau)\right] \chi_b = 0$$

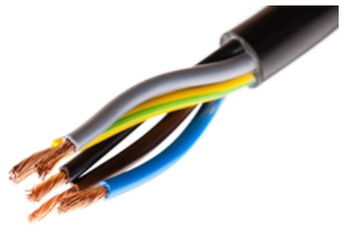
$$a, b = 1, \ldots, N_{\mathrm{f}}$$

real wires are not one-dimensional.

current conduction: **multiple channels**.



many interacting fields (thick wires)



$$\ddot{\chi}_a + \left[k^2\delta_a^b + \mathcal{M}^b{}_a(\tau)\right]\chi_b = 0$$

$$\left(\frac{N_\mathrm{f}+1}{2}\right)\frac{1}{\mu_k}\frac{\partial}{\partial t}P(n_a,\tau) = \sum_{a=1}^{N_\mathrm{f}}\frac{\partial}{\partial n_a}\left[n_a(1+n_a)J\frac{\partial}{\partial n_a}\left(J^{-1}P(n_a,\tau)\right)\right]$$

equivalent to DMPK equation

$$n = \mathrm{Tr}[\mathbf{n}] = \sum_{a=1}^{N_\mathrm{f}} n_a$$

exact solutions!

$$\langle n\rangle = \frac{N_\mathrm{f}}{2}\left(e^{2\mu_k\tau}-1\right)$$

$$n_\mathrm{typ} \to e^{\frac{2N_\mathrm{f}}{N_\mathrm{f}+1}\mu_k\tau}$$

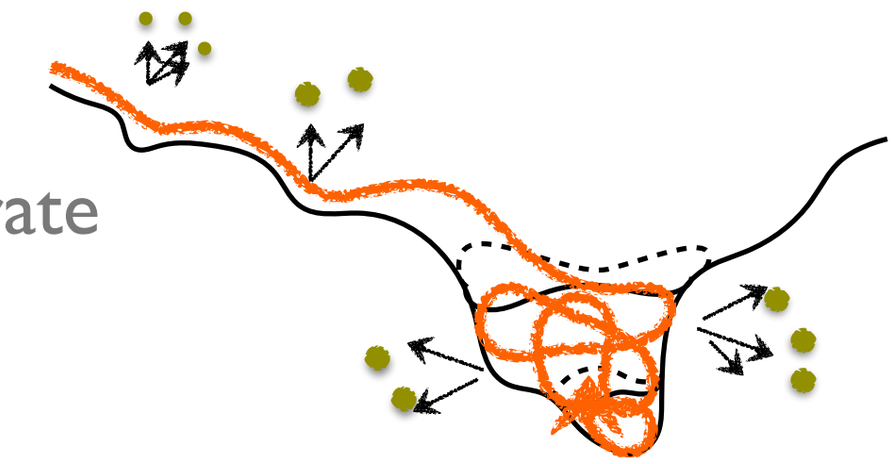
most probable

MA & Baumann

simplicity/universality

μ_k local mean particle production rate

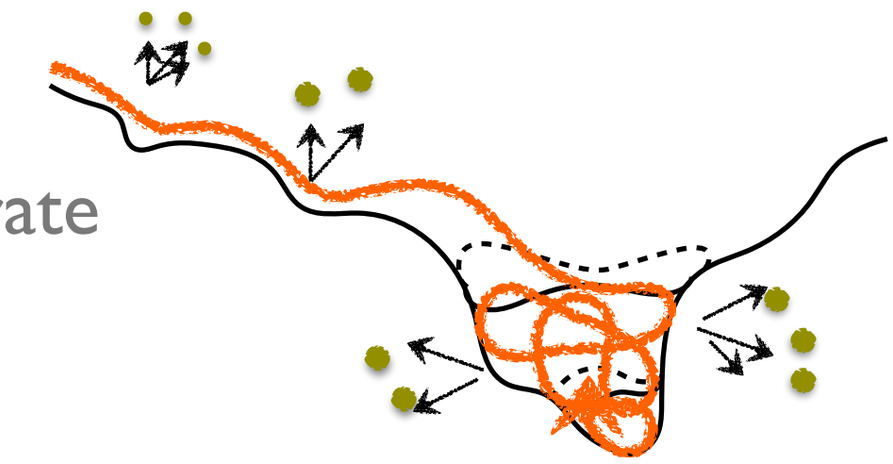
N_f number of fields



l_mf mean ballistic mean free path

N_f number of channels

simplicity/universality



μ_k local mean particle production rate

N_f number of fields

l_mf mean ballistic mean free path

N_c number of channels

μ_k - calculate from 'local" microphysics or parametrize

N_f - regimes exist where dependence vanishes

universality: regimes exist where dependence on both vanishes!

universality from Random Matrix Theory

$$n = \mathrm{Tr}[\mathbf{n}] = \sum_{a=1}^{N_\mathrm{f}} n_a$$

large number of fields: N_f

large number of interactions: N_s

n is obtained from a product of N_s matrices of dimension N_f with random entries

$$\mathbf{M} = \mathbf{M}_{N_s-1} \cdot \mathbf{M}_{N_s-1} \ldots \mathbf{M}_1$$

Random Matrix Theory

products of large number of random matrices $\mathbf{M}\mathbf{M}^\dagger$

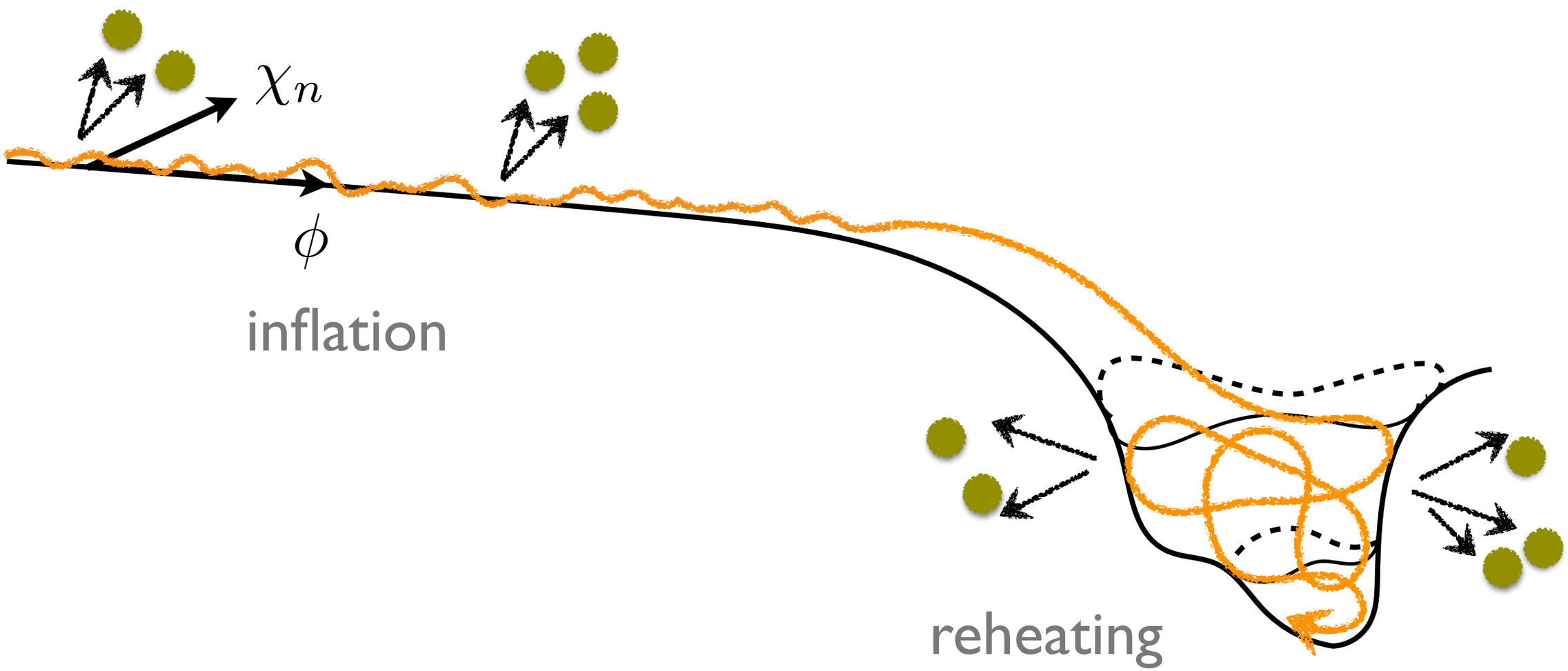
self averaging, non-random eigenvalues

eigenvalue repulsion — largest eigenvalue dominates in n

prediction for exponential behavior!

MA & Baumann

applications

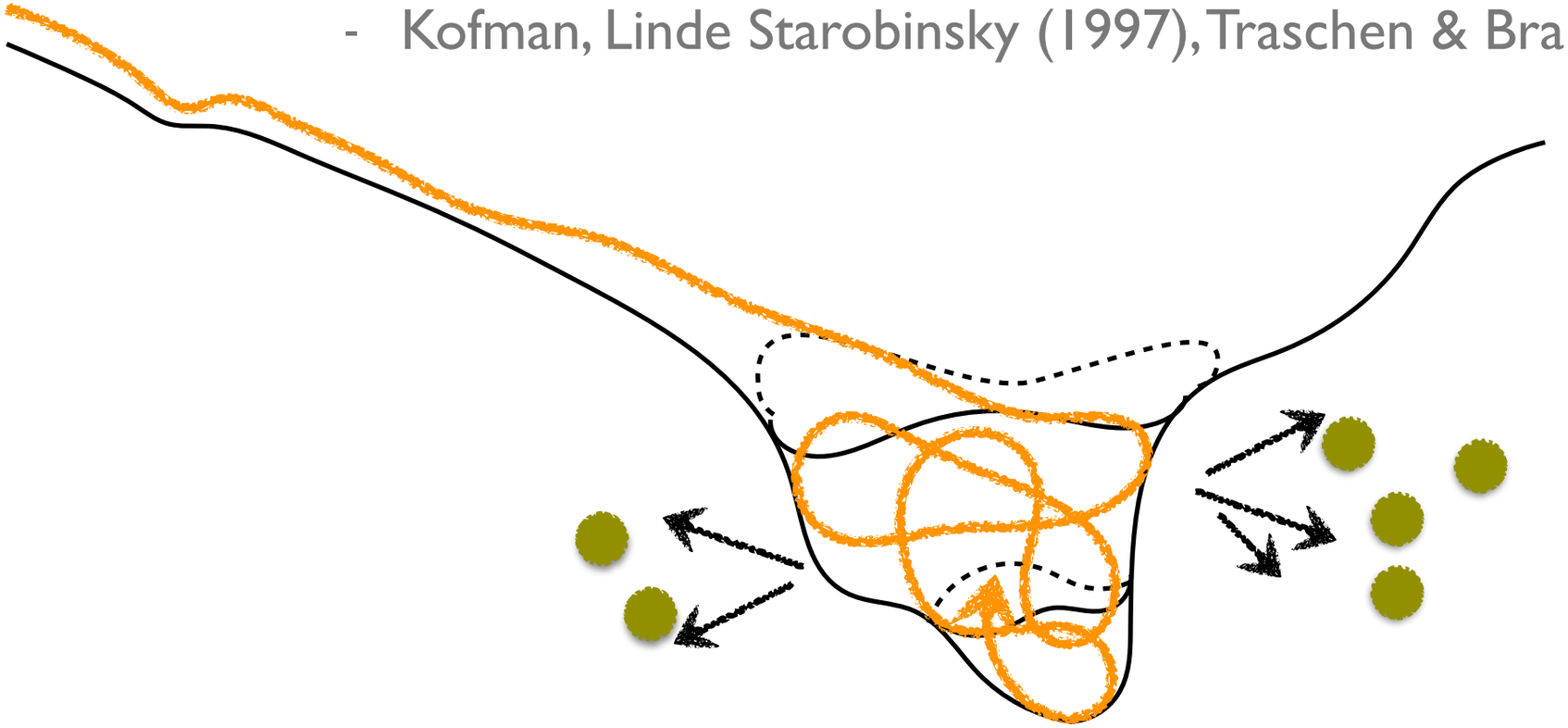


WORK IN PROGRESS

applications : reheating

reheating:
many interacting fields, non-perturbative

- **new** statistical tools (Fokker Planck + RMT)
- generalization of classic results:
  - Kofman, Linde Starobinsky (1997), Traschen & Brandenberger (1997)



MA & Baumann

WORK IN PROGRESS

applications: inflation

Nacir, Porto, Senatore, and Zaldarriaga
Green, Horn, Senatore, and Silverstein

$$\mathcal{L} = \mathcal{L}_{\rm sr} - m^2(t+\pi)\,\chi^2$$

Goldstone boson $\zeta = -H\pi$

$$\left(\partial_t^2 + 3H\partial_t + \frac{k^2}{a^2}\right)\pi = \frac{dm^2}{dt}\,\chi^2$$

source

stochastic noise

$$(\chi^2)_S \equiv \langle\chi^2\rangle_{\pi=0}$$

linear response

$$(\chi^2)_R \equiv \int^t \mathrm{d}t'\, G_{\rm ret}^{\langle\chi^2\rangle}(t,t')\,\pi(t')$$

background dynamics

particle production

$$\langle n_{k_1} n_{k_2} \ldots\rangle$$

curvature fluctuations

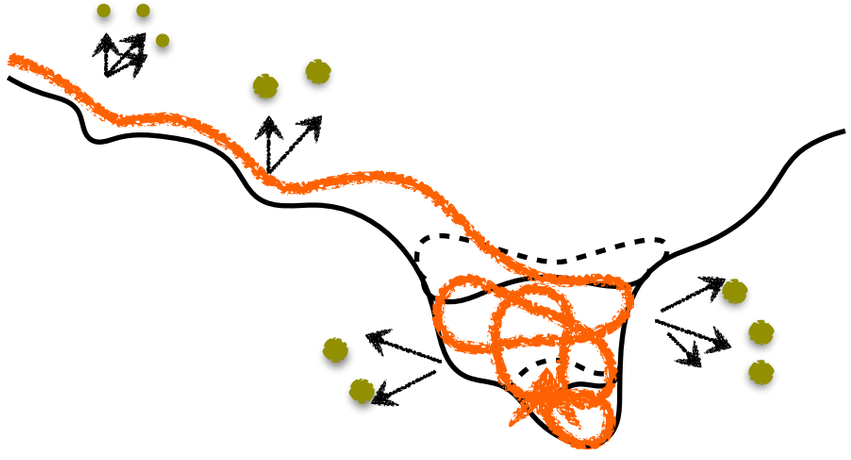
$$\langle\zeta_{k_1}\zeta_{k_2}\ldots\rangle$$

MA, Baumann & Green

summary

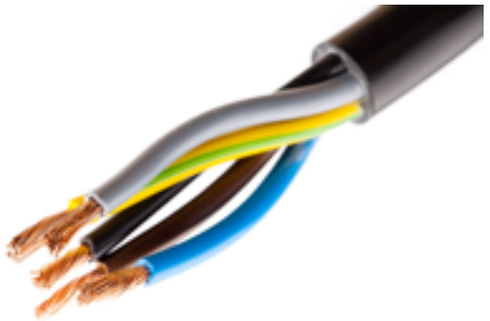
complex theoretical models

inflation/reheating:
many interacting fields, non-perturbative



map to disordered wire
statistical tools: Fokker Planck + Random Matrices

$$\frac{1}{\mu_k}\frac{\partial}{\partial\tau}P(n,\tau) = \frac{\partial}{\partial n}\left[n(1+n)\frac{\partial}{\partial n}P(n,\tau)\right] \qquad \mathbf{M} = \mathbf{M}_{N_s-1}\cdot\mathbf{M}_{N_s-1}\ldots\mathbf{M}_1$$



emergent universality/simplicity

$$n_{\rm typ} \to e^{\frac{2N_{\rm f}}{N_{\rm f}+1}\mu_k\tau}$$

could simplicity of observations hinting at emergent universality?
paper coming soon … stay tuned!

MA & Baumann

related work: condensed matter + cosmology

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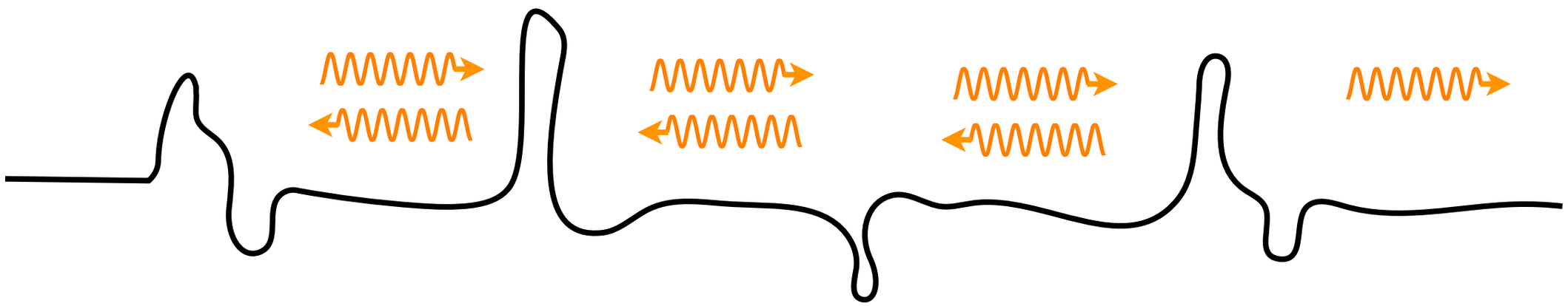
+ many works on particle production during and after inflation.

Extra Slides

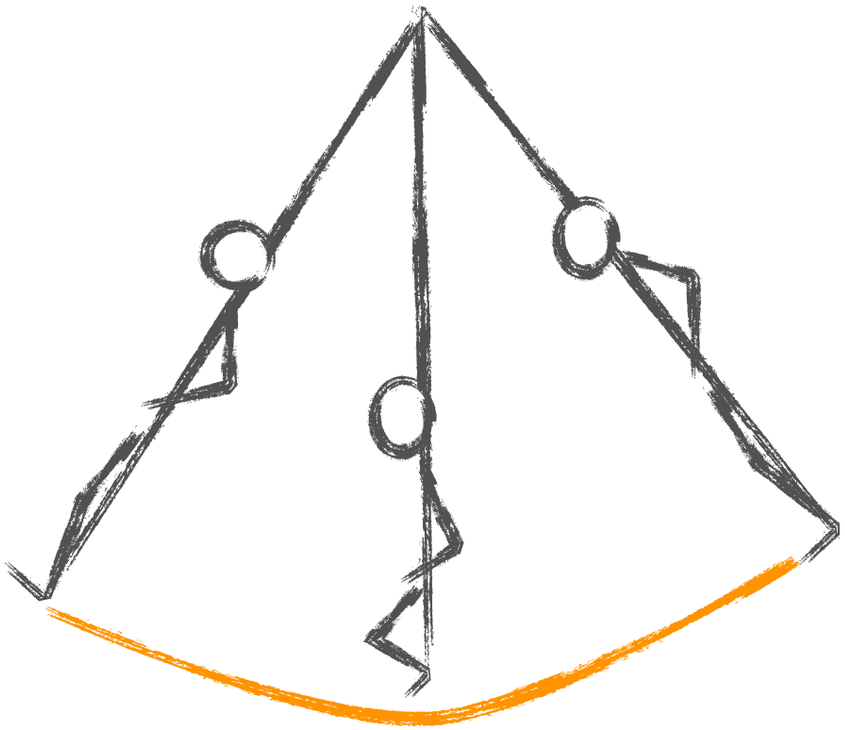
dictionary

Time-dependent Klein-Gordon	Time-independent Schrödinger
$\ddot{\chi}_k(\tau) + \left[k^2 + m_{\rm eff}^2(\tau)\right]\chi_k(\tau) = 0$	$\frac{d^2\psi}{dx^2} + \big(E - V(x)\big)\psi = 0$

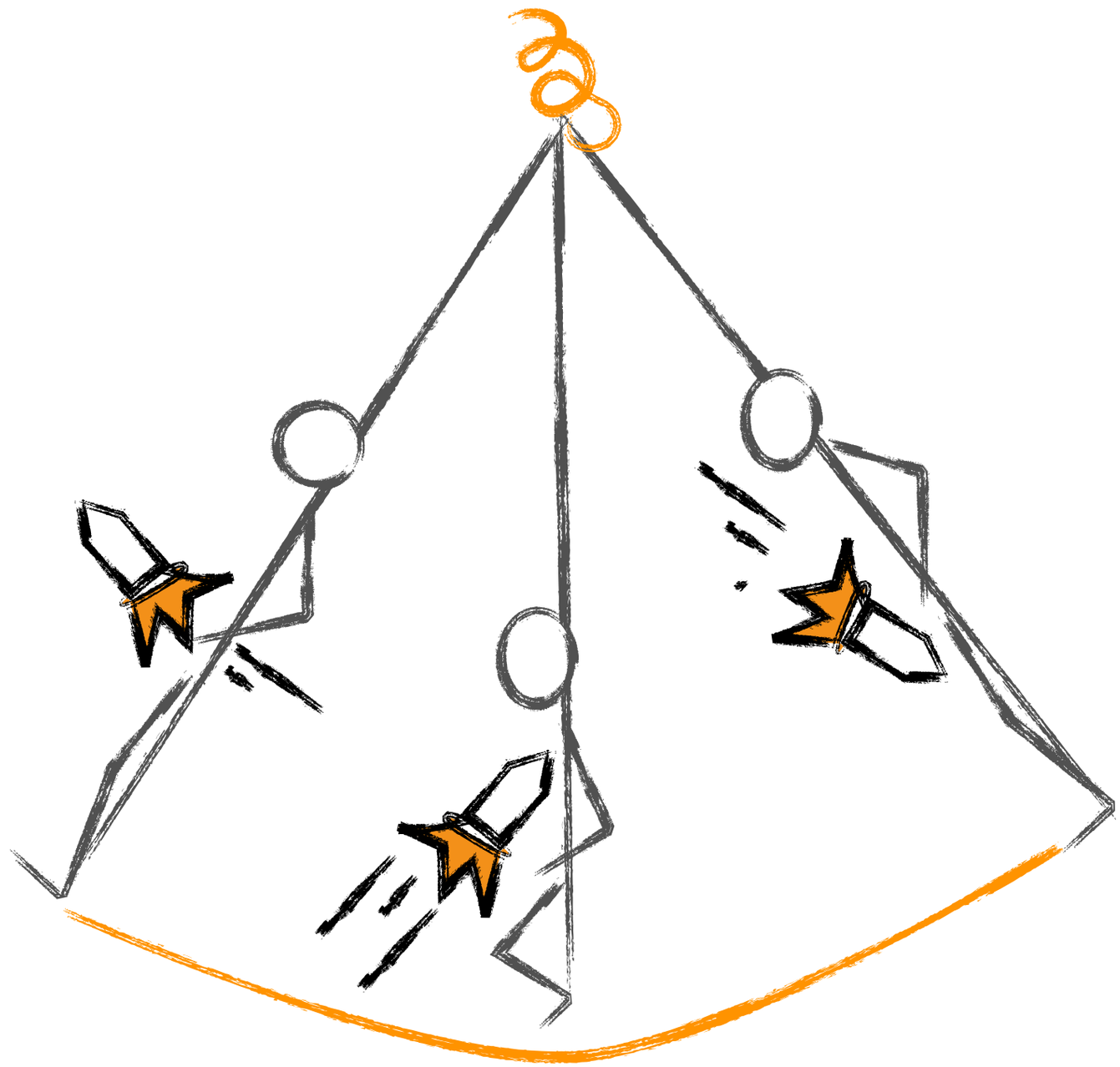


particle production $\langle n\rangle = \frac{1}{2}\left(e^{2\mu_k\tau} - 1\right)$	Anderson localization $\langle\rho\rangle = e^{L/l}$
Fokker-Planck equation $P(n, T)$	Fokker-Planck equation $P(\rho, L)$
multiple fields (Random Matrix Theory)	multiple channels (Random Matrix Theory)


inflation
TeV
MeV
eV



Old



Particle Production 2.0

Mustafa Amin

moments

$$\frac{1}{\mu_k}\frac{\partial\langle n\rangle}{\partial\tau} = N_{\rm f} + 2\langle n\rangle$$

$$\frac{(N_{\rm f}+1)}{2}\frac{1}{\mu_k}\frac{\partial\langle n^2\rangle}{\partial\tau} = (N_{\rm f}^2 + N_{\rm f} + 2)\langle n\rangle + 2(N_{\rm f}+1)\langle n^2\rangle + 2\langle n_2\rangle$$

$$\frac{(N_{\rm f}+1)}{2}\frac{1}{\mu_k}\frac{\partial\langle n_2\rangle}{\partial\tau} = (2N_{\rm f}+2)\langle n\rangle + \langle n^2\rangle + (2N_{\rm f}+3)\langle n_2\rangle$$

where $n = \sum_{a=1}^{N_{\rm f}} n_a$ and $n_2 = \sum_{a=1}^{N_{\rm f}} n_a^2$.

exact solutions!

$$\langle n\rangle = \frac{N_{\rm f}}{2}\left(e^{+2\mu_k\tau} - 1\right) \qquad \frac{\langle {\rm Var}(n)\rangle}{\langle n\rangle^2} \xrightarrow{\mu_k\tau\to\infty} \frac{N_{\rm f}+1}{3N_{\rm f}}\, e^{+4(N_{\rm f}+1)^{-1}\mu_k\tau}$$