from wires to cosmology

Mustafa Amin
with
Daniel Baumann
related work: condensed matter + cosmology

Anderson
Absence of diffusion in certain random matrices (1957)

Mello, Pereyra Kumar
Macroscopic approach to multichannel disordered wires (1987)

C. Beenakker,
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Green
Disorder in the early universe (2015)

+ many works on particle production during and after inflation.
motivation

• **observations**: early universe is simple
  - few numbers \( n_s \sim 1, r, T_{\text{reh}}, f_{NL} \lesssim 10 \)

• **theory**: not so much …
  - many interacting fields, many scales (\( E >> \text{TeV} \))

• **calculational tools**?

• **can the simplicity/universality be emergent?**
multifield inflation/reheating

- inflation/reheating: many interacting fields
- fluctuations: coupled, non-perturbative
complexity in time: cosmology

\[ \ddot{\chi}_k(\tau) + \left[ k^2 + m^2_{\text{eff}}(\tau) \right] \chi_k(\tau) = 0 \]

\[ m^2_{\text{eff}}(\tau) = -\frac{\ddot{a}(\tau)}{a(\tau)} + a^2(\tau)m^2_\phi + a^2(\tau)g^2(\phi(\tau) - \phi_*)^2 + \ldots \]

\[ m^2_{\text{eff}}(\tau) \]
complexity in space: wires

\[ \ddot{\chi}_k(\tau) + \left[ k^2 + m_{\text{eff}}^2(\tau) \right] \chi_k(\tau) = 0 \quad \text{(particle production)} \]

\[ \psi''(x) + \left[ k^2 - V(x) \right] \psi(x) = 0 \quad \text{(Schrodinger)} \]
Anderson localization!
complexity in space — emergent simplicity

\[ \psi''(x) + \left[ k^2 - V(x) \right] \psi(x) = 0 \]

\[ \psi(x) = e^{-x/2\xi} \]

caveat: wave function — transmission probability
Anderson Localization: chained scattering matrices

|ψ(L)⟩ = \(M_{N_s} \cdot M_{N_s-1} \cdots M_1|ψ(0)⟩\)

ψ(x) = e^{-x/2\xi}

phases are important!

caveat: wave function — transmission probability
complexity in space — Anderson localization
complexity in time — exponential particle production

\[ \ddot{\chi}_k(\tau) + \left[ k^2 + m_{\text{eff}}^2(\tau) \right] \chi_k(\tau) = 0 \]

\[ \psi''(x) + \left[ k^2 - V(x) \right] \psi(x) = 0 \]

\[ \chi_k(\tau) \sim e^{\mu_k \tau/2} \]

\[ \psi(x) = e^{-x/2\xi} \]

caveat: wave function — transmission probability, mode function — occupation number

MA & Baumann
occupation number performs
a drifted random walk

\[ \ddot{\chi}_k(\tau) + \left[k^2 + m_{\text{eff}}^2(\tau)\right] \chi_k(\tau) = 0 \]

\[ n(k, \tau) = \frac{1}{2\omega_k} (|\dot{\chi}_k|^2 + \omega_k^2 |\chi_k|^2) \]

\[ m_{\text{eff}}^2(t) \]
a Fokker Planck equation for the occupation number

\[
\frac{1}{\mu_k} \frac{\partial}{\partial \tau} P(n, \tau) = \frac{\partial}{\partial n} \left[ n(1 + n) \frac{\partial}{\partial n} P(n, \tau) \right]
\]

\(\mu_k\): local mean particle-production rate

analogue: mean free path

\[n(k, \tau)\]
the typical occupation number

\[ \frac{1}{\mu_k} \frac{\partial}{\partial \tau} P(n, \tau) = \frac{\partial}{\partial n} \left[ n(1 + n) \frac{\partial}{\partial n} P(n, \tau) \right] \]

\[ \mu_k : \] local mean particle-production rate

analogue: mean free path

\[ \langle n \rangle = \frac{1}{2} \left( e^{2\mu_k \tau} - 1 \right) \]

mean

\[ n_{\text{typ}} = e^{\langle \ln(1 + n) \rangle} \]

most probable

\[ = e^{\mu_k \tau} \]

not a fit!
many interacting fields (thick wires)

early universe: multiple interacting fields:

\[ \ddot{\chi}_a + \left[ k^2 \delta^b_a + \mathcal{M}^b_a(\tau) \right] \chi_b = 0 \]

\( a, b = 1, \ldots, N_f \)

real wires are not one-dimensional.
current conduction: multiple channels.
many interacting fields (thick wires)

\[ \ddot{\chi}_a + \left[ k^2 \delta^b_a + \mathcal{M}^b_a(\tau) \right] \chi_b = 0 \]

\[ \left( \frac{N_f + 1}{2} \right) \frac{1}{\mu_k} \frac{\partial}{\partial t} P(n_a, \tau) = \sum_{a=1}^{N_f} \frac{\partial}{\partial n_a} \left[ n_a (1 + n_a) J \frac{\partial}{\partial n_a} \left( J^{-1} P(n_a, \tau) \right) \right] \]

\[ n = \text{Tr}[\mathbf{n}] = \sum_{a=1}^{N_f} n_a \]

exact solutions!

\[ \langle n \rangle = \frac{N_f}{2} \left( e^{2\mu_k \tau} - 1 \right) \]

\[ n_{\text{typ}} \rightarrow e^{\frac{2N_f}{N_f+1} \mu_k \tau} \]

most probable

MA & Baumann
simplicity/universality

\( \mu_k \)  local mean particle production rate
\( N_f \)  number of fields

\( l_{mf} \)  mean ballistic mean free path
\( N_f \)  number of channels
simplicity/universality

\( \mu_k \)  local mean particle production rate
\( N_f \)  number of fields

\( l_{mf} \)  mean ballistic mean free path
\( N_c \)  number of channels

\( \mu_k \) - calculate from ‘local” microphysics or parametrize
\( N_f \) - regimes exist where dependence vanishes

**universality**: regimes exist where dependence on both vanishes!
universality from Random Matrix Theory

\[ n = \text{Tr}[n] = \sum_{a=1}^{N_f} n_a \]

large number of fields: \( N_f \)

large number of interactions: \( N_s \)

\( n \) is obtained from a product of \( N_s \) matrices of dimension \( N_f \) with random entries

\[ M = M_{N_s-1} \cdot M_{N_s-1} \ldots M_1 \]

Random Matrix Theory

products of large number of random matrices \( MM^\dagger \)

self averaging, non-random eigenvalues

eigenvalue repulsion — largest eigenvalue dominates in \( n \)
prediction for exponential behavior!
applications
applications: reheating

reheating:
many interacting fields, non-perturbative

- **new** statistical tools (Fokker Planck + RMT)
- generalization of classic results:
applications: inflation

Nacir, Porto, Senatore, and Zaldarriaga
Green, Horn, Senatore, and Silverstein

\[ \mathcal{L} = \mathcal{L}_{\text{sr}} - m^2(t + \pi) \chi^2 \]

**Goldstone boson**
\[ \zeta = -H \pi \]

\[ \left( \partial_t^2 + 3H \partial_t + \frac{k^2}{a^2} \right) \pi = \frac{dm^2}{dt} \chi^2 \]

**source**

\[ (\chi^2)_{S} \equiv \langle \chi^2 \rangle_{\pi=0} \]

**stochastic noise**

\[ (\chi^2)_{R} \equiv \int_{t}^{t'} dt' \; G_{\text{ret}}^{(\chi^2)}(t, t') \pi(t') \]

**linear response**

**background dynamics**

**particle production**

\[ \langle n_{k_1} n_{k_2} \cdots \rangle \]

**curvature fluctuations**

\[ \langle \zeta_{k_1} \zeta_{k_2} \cdots \rangle \]
summary

complex theoretical models

inflation/reheating:
many interacting fields, non-perturbative

map to disordered wire
statistical tools: Fokker Planck + Random Matrices

\[
\frac{1}{\mu_k} \frac{\partial}{\partial \tau} P(n, \tau) = \frac{\partial}{\partial n} \left[ n(1+n) \frac{\partial}{\partial n} P(n, \tau) \right] \quad \mathbf{M} = \mathbf{M}_{N_s-1} \cdot \mathbf{M}_{N_s-1} \cdots \mathbf{M}_1
\]

emergent universality/simplicity

\[
n_{\text{typ}} \rightarrow e^{\frac{2N_f}{N_f+1} \mu_k \tau}
\]

could simplicity of observations hinting at emergent universality?
paper coming soon … stay tuned!

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Extra Slides
**Time-dependent Klein-Gordon**

\[
\ddot{\chi}_k(\tau) + \left[ k^2 + m_{\text{eff}}^2(\tau) \right] \chi_k(\tau) = 0
\]

**Time-independent Schrödinger**

\[
\frac{d^2\psi}{dx^2} + (E - V(x))\psi = 0
\]

**Particle production**

\[
\langle n \rangle = \frac{1}{2} \left( e^{2\mu_k \tau} - 1 \right)
\]

**Anderson localization**

\[
\langle \rho \rangle = e^{L/l}
\]

**Fokker-Planck equation**

\[
P(n, T)
\]

**Multiple fields (Random Matrix Theory)**

**Fokker-Planck equation**

\[
P(\rho, L)
\]

**Multiple channels (Random Matrix Theory)**
inflation
\[
\frac{1}{\mu_k} \frac{\partial \langle n \rangle}{\partial \tau} = N_f + 2\langle n \rangle
\]

\[
\frac{(N_f + 1)}{2} \frac{1}{\mu_k} \frac{\partial \langle n^2 \rangle}{\partial \tau} = (N_f^2 + N_f + 2)\langle n \rangle + 2(N_f + 1)\langle n^2 \rangle + 2\langle n_2 \rangle
\]

\[
\frac{(N_f + 1)}{2} \frac{1}{\mu_k} \frac{\partial \langle n_2 \rangle}{\partial \tau} = (2N_f + 2)\langle n \rangle + \langle n^2 \rangle + (2N_f + 3)\langle n_2 \rangle
\]

where \( n = \sum_{a=1}^{N_f} n_a \) and \( n_2 = \sum_{a=1}^{N_f} n_a^2 \).

exact solutions!

\[
\langle n \rangle = \frac{N_f}{2} \left( e^{+2\mu_k \tau} - 1 \right) \quad \frac{\langle \text{Var}(n) \rangle}{\langle n \rangle^2} \xrightarrow{\mu_k \tau \to \infty} \frac{N_f + 1}{3N_f} e^{+4(N_f+1)^{-1} \mu_k \tau}
\]