Effective Lagrangians for Higgs Physics

Tyler Corbett

YITP, SUNY Stony Brook
(Soon: ARC CoEPP, University of Melbourne)

Based on Arxiv: 1411.5026, 1311.1823, 1304.1151, 1211.4580, and 1207.1344

In collaboration with: MC Gonzalez-Garcia, J Gonzalez-Fraile, OJP Éboli, I Brivio,
B Gavela, L Merlo, & S Rigolin
Outline

1 Standard Model Higgs

2 Effective Lagrangians

3 Linear EFTs for Higgs Physics
   - Status after LHC runs
   - Unitarity Bounds

4 Chiral Expansion

5 Conclusions
The SM Higgs

- In the SM all Higgs couplings are fully determined, except $M_H$
- The Higgs is produced at the LHC via:

  - Gluon Gluon Fusion:
    - $g g \rightarrow H$
    - $g g \rightarrow W(Z) H$
    - $\sim 15\text{ pb}$
  - Vector Boson Fusion:
    - $W(Z) q q \rightarrow H$
    - $W(Z) g g \rightarrow H$
    - $\sim 1.2\text{ pb}$

  **Associate Production:**
  - $W(Z) q q \rightarrow H$
  - $W(Z) g g \rightarrow H$
  - $\sim 0.6(0.3)\text{ pb}$
  - $W(Z) g g \rightarrow H$
  - $\sim 0.09\text{ pb}$
The SM Higgs

- In the SM all Higgs couplings are fully determined, except $M_H$
- The Higgs is produced at the LHC via: GGF, VBF, VH, $t\bar{t}$
- And decays via:

  $H \rightarrow f\bar{f}$
  $BR_{\bar{b}b} \sim 58\%$
  $BR_{\tau\tau} \sim 6\%$

  $H \rightarrow W(Z)$
  $BR_{WW^*} \sim 22\%$
  $BR_{ZZ^*} \sim 3\%$

  $H \rightarrow t\bar{t}t$
  $BR_{\gamma\gamma} \sim 0.2\%$
Higgs Discovery

In 2012 the first strong evidence of the discovery of the Higgs was released.

\[ \mu \equiv \frac{(\sigma \times \text{BR})_{\text{Obs}}}{(\sigma \times \text{BR})_{\text{SM}}} \]

\[ \gamma \gamma \]
Effective Field Theories

We wish to quantify deviations from the predictions of the SM Higgs Mechanism, → consistent framework for doing so is an effective field theory (EFT)

We form an effective field theory via the assumptions:

- There is a mass gap between the low energy theory (SM) and the new physics
- The new physics occurs at some scale $\Lambda \sim M_{\text{new}}$
- For $\Lambda \sim M_{\text{new}} \to \infty$ the new physics decouples
  ⇒ induces corrections $\propto 1/\Lambda^n \to$ higher dimensional operators
Example: Muon Decay

Fermi theory of muon decay: the exchange of a $W$ boson → dimension–six operator,

$$\bar{\Psi}_L \Psi_L \left( \frac{1}{p^2 - M_W^2} \right) \bar{\Psi}_L \Psi_L \rightarrow -\bar{\Psi}_L \Psi_L \frac{1}{M_W^2} \left( 1 + \frac{p^2}{M_W^2} + \cdots \right) \bar{\Psi}_L \Psi_L$$

$$\rightarrow \frac{1}{M_W^2} \bar{\Psi}_L \Psi_L \bar{\Psi}_L \Psi_L$$

- To $O\left( \frac{1}{M_W^2} \right)$ this new dimension–six operator describes muon decay.
- For sufficiently low momenta, $p^2 \ll M_W^2$, this describes the process well.
- For better precision we can extend to the next order, dimension–eight, or $O\left( \frac{p^2}{M_W^2} \right)$ → well defined procedure to quantify deviations from low energy predicted behavior.
Example: Muon Decay

Fermi theory of muon decay: the exchange of a $W$ boson $\rightarrow$ dimension-six operator,

$$\bar{\Psi} L \Psi L \left( \frac{1}{p^2 - M_W^2} \right) \bar{\Psi} L \Psi L \quad \rightarrow \quad -\bar{\Psi} L \Psi L \frac{1}{M_W^2} \left( 1 + \frac{p^2}{M_W^2} + \cdots \right) \bar{\Psi} L \Psi L$$

$$\rightarrow \quad \frac{1}{M_W^2} \bar{\Psi} L \Psi L \bar{\Psi} L \Psi L$$

- To $O\left( \frac{1}{M_W^2} \right)$ this new dimension-six operator describes muon decay.
- For sufficiently low momenta, $p^2 \ll M_W^2$, this describes the process well.
- For better precision we can extend to the next order, dimension-eight, or $O\left( \frac{p^2}{M_W^4} \right)$ $\rightarrow$ well defined procedure to quantify deviations from low energy predicted behavior.
Effective Lagrangians

Example: Muon Decay

Fermi theory of muon decay: the exchange of a $W$ boson $\rightarrow$ dimension–six operator,

$$
\bar{\Psi}_L \Psi_L \left( \frac{1}{p^2 - M_W^2} \right) \bar{\Psi}_L \Psi_L \quad \rightarrow \quad -\bar{\Psi}_L \Psi_L \frac{1}{M_W^2} \left( 1 + \frac{p^2}{M_W^2} + \cdots \right) \bar{\Psi}_L \Psi_L
$$

$$
\rightarrow \quad \frac{1}{M_W^2} \bar{\Psi}_L \Psi_L \bar{\Psi}_L \Psi_L
$$

- To $\mathcal{O}(\frac{1}{M_W^2})$ this new dimension–six operator describes muon decay.
- For sufficiently low momenta, $p^2 \ll M_W^2$, this describes the process well.
- For better precision we can extend to the next order, dimension–eight, or $\mathcal{O}(\frac{p^2}{M_W^4})$ well defined procedure to quantify deviations from low energy predicted behavior.
Linear EFTs for Higgs Physics

We’re interested in the EFT for Higgs Physics.

Effective operators are formed from the field content,

\[ e_R, \quad u_R, \quad d_R, \quad L_L, \quad Q_L, \quad \Phi, \quad W_\mu, \quad Z_\mu, \quad A_\mu, \quad G_\mu \]

And their (covariant) derivatives.

The operators are then formed of all \( U(1)_{Y/2} \times SU(2)_L \times SU(3)_C \) invariant combinations.

Further

- restrict to dimension–six, i.e. assume all NP occurs at a sufficiently high \( \Lambda \)
- assume baryon and lepton numbers are conserved (\( \rightarrow \) no dimension–5 operators)
- assume operators are \( CP \)–even

Then we write the linear EFT to dimension–6 as:

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_n \frac{f_n}{\Lambda^2} \mathcal{O}_n \]
Linear EFTs for Higgs Physics

We’re interested in the EFT for Higgs Physics.

Effective operators are formed from the field content,

\[ e_R, \quad u_R, \quad d_R, \quad L_L, \quad Q_L, \quad \Phi, \quad W_\mu, \quad Z_\mu, \quad A_\mu, \quad G_\mu \]

And their (covariant) derivatives.

The operators are then formed of all \( U(1)_Y/2 \times SU(2)_L \times SU(3)_C \) invariant combinations.

Further

- restrict to dimension-six, i.e. assume all NP occurs at a sufficiently high \( \Lambda \)
- assume baryon and lepton numbers are conserved (\( \to \) no dimension-5 operators)
- assume operators are \( CP \)-even

Then we write the linear EFT to dimension-six as:

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_n \frac{f_n}{\Lambda^2} \mathcal{O}_n \]
We’re interested in the EFT for Higgs Physics.

Effective operators are formed from the field content,

\[ e_R, \quad u_R, \quad d_R, \quad L_L, \quad Q_L, \quad \Phi, \quad W_\mu, \quad Z_\mu, \quad A_\mu, \quad G_\mu \]

And their (covariant) derivatives.

The operators are then formed of all \( U(1)_Y/2 \times SU(2)_L \times SU(3)_C \) invariant combinations.

Further

- restrict to **dimension-six**, i.e. assume all NP occurs at a sufficiently high \( \Lambda \)
- assume **baryon and lepton numbers are conserved** (\( \rightarrow \) no dimension–5 operators)
- assume operators are **\( CP \)-even**

Then we write the linear EFT to dimension–6 as:

\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_n \frac{f_n}{\Lambda^2} \mathcal{O}_n
\]
The relevant Higgs–gauge boson operators are:

\[
\begin{align*}
\mathcal{O}_{BB} &= \Phi^\dagger \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} \Phi \\
\mathcal{O}_{GG} &= \Phi^\dagger \Phi G_a^{\mu\nu} G_a^{\mu\nu} \\
\mathcal{O}_{\Phi,1} &= (D_\mu \Phi)^\dagger \Phi \Phi^\dagger (D_\mu \Phi) \\
\mathcal{O}_{WWW} &= \text{Tr}[\hat{W}_\mu \hat{W}_\nu \hat{W}_\rho \hat{W}_\rho^{\mu}] \\
\mathcal{O}_{BW} &= \Phi^\dagger \hat{B}_{\mu\nu} \hat{W}^{\mu\nu} \Phi \\
\mathcal{O}_B &= (D_\mu \Phi)^\dagger \hat{B}^{\mu\nu} (D_\nu \Phi) \\
\mathcal{O}_{\Phi,2} &= \frac{1}{2} \partial_\mu (\Phi^\dagger \Phi) \partial^\mu (\Phi^\dagger \Phi) \\
\mathcal{O}_W &= (D_\mu \Phi)^\dagger \hat{W}^{\mu\nu} (D_\nu \Phi) \\
\mathcal{O}_{\Phi,4} &= (D_\mu \Phi)^\dagger (D_\mu \Phi) (\Phi^\dagger \Phi)
\end{align*}
\]
Higgs–Gauge Boson Operators

The relevant Higgs–gauge boson operators are:

\[
\begin{align*}
O_{BB} &= \Phi^\dagger \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} \Phi \\
O_{GG} &= \Phi^\dagger \Phi G^a_{\mu\nu} G_a^{\mu\nu} \\
O_{\Phi,1} &= (D_\mu \Phi)^\dagger \Phi \Phi^\dagger (D^\mu \Phi) \\
O_{WW} &= \text{Tr}[\hat{W}_\mu^\nu \hat{W}_{\nu}^\rho \hat{W}_{\rho}^\mu] \\
O_{BW} &= \Phi^\dagger \hat{B}_{\mu\nu} \hat{W}^{\mu\nu} \Phi \\
O_B &= (D_\mu \Phi)^\dagger \hat{B}_{\mu\nu} (D^\nu \Phi) \\
O_{\Phi,2} &= \frac{1}{2} \partial_\mu (\Phi^\dagger \Phi) \partial^\mu (\Phi^\dagger \Phi) \\
O_{\Phi,4} &= (D_\mu \Phi)^\dagger (D^\mu \Phi)(\Phi^\dagger \Phi) \\
O_{WW} &= \Phi^\dagger \hat{W}_{\mu\nu} \hat{W}^{\mu\nu} \Phi
\end{align*}
\]

Unitary Gauge:

\[
\Phi \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}
\]

Operators in blue contain derivatives of \( H \rightarrow \) finite wavefunction renormalizations:

\[
H = h \left[ 1 + \frac{v^2}{2\Lambda^2} (f_{\phi,1} + 2f_{\phi,2} + f_{\phi,4}) \right]^{1/2}
\]

\( \rightarrow \) rescaling of all Higgs couplings.

\( \rightarrow \) correction to \( Z \) gauge–boson propagator (not \( W \)):

\[
Z \rightarrow \Delta T \propto f_{\Phi,1}
\]
Higgs–Gauge Boson Operators

The relevant Higgs–gauge boson operators are:

\[
\begin{align*}
\mathcal{O}_{BB} &= \Phi^\dagger \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} \Phi \\
\mathcal{O}_{GG} &= \Phi^\dagger \Phi G_{\mu\nu}^a G_a^{\mu\nu} \\
\mathcal{O}_{\Phi,1} &= (D_\mu \Phi)^\dagger \Phi \Phi^\dagger (D^\mu \Phi) \\
\mathcal{O}_{WW} &= \text{Tr} [\hat{W}_\mu \nu \hat{W}_\rho \rho \hat{W}^{\mu\nu}]
\end{align*}
\]

Green $\mathcal{O} \leftrightarrow$ loop effects for new particles charged under the SM gauge group:

Also $\mathcal{O}_{BW}$ results in tree level mixing between $Z$ and $\gamma$:

\[
Z \sim \sim \sim \gamma \rightarrow \Delta S \propto f_{BW}
\]
Higgs–Gauge Boson Operators

The relevant Higgs–gauge boson operators are:

\[ \mathcal{O}_{BB} = \Phi^\dagger \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} \Phi \]
\[ \mathcal{O}_{GW} = \Phi^\dagger \Phi G_{\mu\nu} G^{\mu\nu} \]
\[ \mathcal{O}_{\Phi,1} = (D_\mu \Phi)^\dagger \Phi \Phi^\dagger (D^\mu \Phi) \]
\[ \mathcal{O}_{WW} = \mathcal{O}_{WWW} = \text{Tr}[\hat{W}_{\mu} \hat{W}_{\nu} \hat{W}_{\rho} \hat{W}_{\sigma}] \]

Red \( \mathcal{O} \) → simultaneous shifts of \( HVV, VVV, \) and \( VVVV \) vertices:

\[ D_\mu \Phi = (\partial_\mu + \frac{1}{2} g' B_\mu + ig \frac{\tau^a}{2} W^a_\mu) \Phi \]
Higgs–Gauge Boson Operators

The relevant Higgs–gauge boson operators are:

\[ O_{BB} = \Phi^\dagger \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} \Phi \]
\[ O_{GG} = \Phi^\dagger \Phi G^a_{\mu\nu} G_a^{\mu\nu} \]
\[ O_{\Phi,1} = (D_\mu \Phi)^\dagger \Phi \Phi^\dagger (D^\mu \Phi) \]
\[ O_{WW} = \text{Tr}[\hat{W}_\mu^\nu \hat{W}_\nu^\rho \hat{W}_\rho^\mu] \]
\[ O_{BW} = \Phi^\dagger \hat{B}_{\mu\nu} \hat{W}^{\mu\nu} \Phi \]
\[ O_B = (D_\mu \Phi)^\dagger \hat{B}^{\mu\nu} (D^\nu \Phi) \]
\[ O_{\Phi,2} = \frac{1}{2} \partial_\mu (\Phi^\dagger \Phi) \partial^\mu (\Phi^\dagger \Phi) \]
\[ O_{\Phi,4} = (D_\mu \Phi)^\dagger (D^\mu \Phi) (\Phi^\dagger \Phi) \]

Purple $O \leftrightarrow$ Triple–Gauge and Quartic–Gauge couplings:
Higgs–Fermion Operators

The relevant Higgs–fermion operators are:

\[ O_{e\Phi,i,j} = (\Phi^\dagger \Phi)(\bar{L}_i \Phi e_{R_j}) \]
\[ O_{u\Phi,i,j} = (\Phi^\dagger \Phi)(\bar{Q}_i \Phi u_{R_j}) \]
\[ O_{d\Phi,i,j} = (\Phi^\dagger \Phi)(\bar{Q}_i \Phi d_{R_j}) \]

- Overall scaling of Yukawas \( \rightarrow \delta m_f \)
- Independent rescaling of \( Hff \) couplings
- Contributes to \( Vff \) couplings
- Strongly constrained by precision data

\[ O^{(1)}_{\Phi L,i,j} = \Phi^\dagger (i \not{D}_\mu \Phi)(\bar{L}_i \gamma^\mu L_j) \]
\[ O^{(1)}_{\Phi Q,i,j} = \Phi^\dagger (i \not{D}_\mu \Phi)(\bar{Q}_i \gamma^\mu Q_j) \]
\[ O^{(1)}_{\Phi e,i,j} = \Phi^\dagger (i \not{D}_\mu \Phi)(\bar{\epsilon}_R \gamma^\mu e_{R_j}) \]
\[ O^{(1)}_{\Phi u,i,j} = \Phi^\dagger (i \not{D}_\mu \Phi)(\bar{u}_R \gamma^\mu u_{R_j}) \]
\[ O^{(1)}_{\Phi d,i,j} = \Phi^\dagger (i \not{D}_\mu \Phi)(\bar{d}_R \gamma^\mu d_{R_j}) \]
\[ O^{(1)}_{\Phi u,d,i,j} = \Phi^\dagger (i \not{D}_\mu \Phi)(\bar{u}_R \gamma^\mu d_{R_j}) \]
\[ O^{(3)}_{\Phi L,i,j} = \Phi^\dagger (i \not{D}^a_\mu \Phi)(\bar{L}_i \gamma^\mu \tau_a L_j) \]
\[ O^{(3)}_{\Phi Q,i,j} = \Phi^\dagger (i \not{D}^a_\mu \Phi)(\bar{Q}_i \gamma^\mu \tau_a Q_j) \]
Equations of Motion

Not all operators are independent
→ they may be related by the EOM (Politzer ‘80; Georgi ‘91; Artz ‘98; Simma ‘94):

\[2\mathcal{O}_{\Phi,2} + 2\mathcal{O}_{\Phi,4} = \sum_{ij} \left( y^e_{ij} (\mathcal{O}_{e\Phi,ij})^\dagger + y^u_{ij} \mathcal{O}_{u\Phi,ij} + y^d_{ij} (\mathcal{O}_{d\Phi,ij})^\dagger + \text{h.c.} \right) - 2(\Phi^\dagger \Phi) \Phi^\dagger \frac{\partial V}{\partial \Phi^\dagger}\]

\[2\mathcal{O}_B + \mathcal{O}_{BW} + \mathcal{O}_{BB} + g'2 \left( \mathcal{O}_{\Phi,1} - \frac{1}{2} \mathcal{O}_{\Phi,2} \right) = -\frac{g'^2}{2} \sum_i \left( -\frac{1}{2} \mathcal{O}^{(1)}_{\Phi L,ii} + \frac{1}{6} \mathcal{O}^{(1)}_{\Phi Q,ii} \\ -\mathcal{O}^{(1)}_{\Phi e,ii} + \frac{2}{3} \mathcal{O}^{(1)}_{\Phi u,ii} - \frac{1}{3} \mathcal{O}^{(1)}_{\Phi d,ii} \right)\]

\[2\mathcal{O}_W + \mathcal{O}_{BW} + \mathcal{O}_{WW} + g^2 \left( \mathcal{O}_{\Phi,4} - \frac{1}{2} \mathcal{O}_{\Phi,2} \right) = -\frac{g^2}{4} \sum_i \left( \mathcal{O}^{(3)}_{\Phi L,ii} + \mathcal{O}^{(3)}_{\Phi Q,ii} \right)\]

With this we may remove 3 operators from our basis.

In doing so we use a data driven approach:
- Avoid theoretical prejudice – assumptions on tree vs. loop level of individual operators
- Choose a basis which is more easily related to available data
Equations of Motion

Not all operators are independent
→ they may be related by the EOM (Politzer ‘80; Georgi ‘91; Artz ‘98; Simma ‘94):

\[ 2\mathcal{O}_{\Phi,2} + 2\mathcal{O}_{\Phi,4} = \sum_{ij} \left( y_{ij}^e (\mathcal{O}_{e\Phi,ij}^\dagger) + y_{ij}^u \mathcal{O}_{u\Phi,ij} + y_{ij}^d (\mathcal{O}_{d\Phi,ij}^\dagger) + \text{h.c.} \right) - 2(\Phi^\dagger \Phi) \frac{\partial V}{\partial \Phi^\dagger} \]

\[ 2\mathcal{O}_B + \mathcal{O}_{BW} + \mathcal{O}_{BB} + g'^2 (\mathcal{O}_{\Phi,1} - \frac{1}{2} \mathcal{O}_{\Phi,2}) = -\frac{g'^2}{2} \sum_i \left( -\frac{1}{2} \mathcal{O}^{(1)}_{\Phi L,ii} + \frac{1}{6} \mathcal{O}^{(1)}_{\Phi Q,ii} \right) \]

\[ -\mathcal{O}^{(1)}_{\Phi e,ii} + \frac{2}{3} \mathcal{O}^{(1)}_{\Phi u,ii} - \frac{1}{3} \mathcal{O}^{(1)}_{\Phi d,ii} \]

\[ 2\mathcal{O}_W + \mathcal{O}_{BW} + \mathcal{O}_{WW} + g^2 (\mathcal{O}_{\Phi,4} - \frac{1}{2} \mathcal{O}_{\Phi,2}) = -\frac{g^2}{4} \sum_i \left( \mathcal{O}^{(3)}_{\Phi L,ii} + \mathcal{O}^{(3)}_{\Phi Q,ii} \right) \]

With this we may remove 3 operators from our basis.

In doing so we use a data driven approach:
- Avoid theoretical prejudice – assumptions on tree vs. loop level of individual operators
- Choose a basis which is more easily related to available data
**Most Constrained Operators:**

We recall (constrained by electroweak precision data):

\[
Z \rightarrow \Delta T \propto f_{\Phi,1} \quad Z \rightarrow \Delta S \propto f_{BW}
\]

Additionally some of our fermionic operators are strongly constrained \((V_{ff})\):

(by \(Z\)–pole physics, atomic parity violation, etc)

\[
\begin{align*}
O^{(1)}_{\Phi L,ij} &= \Phi^\dagger (i \slashed{\partial}_\mu \Phi)(\bar{L}_i \gamma^\mu L_j) \\
O^{(1)}_{\Phi Q,ij} &= \Phi^\dagger (i \slashed{\partial}_\mu \Phi)(\bar{Q}_i \gamma^\mu Q_j) \\
O^{(1)}_{\Phi e,ij} &= \Phi^\dagger (i \slashed{\partial}_\mu \Phi)(\bar{e}_{R_i} \gamma^\mu e_{R_j}) \\
O^{(1)}_{\Phi u,ij} &= \Phi^\dagger (i \slashed{\partial}_\mu \Phi)(\bar{u}_{R_i} \gamma^\mu u_{R_j}) \\
O^{(1)}_{\Phi d,ij} &= \Phi^\dagger (i \slashed{\partial}_\mu \Phi)(\bar{d}_{R_i} \gamma^\mu d_{R_j}) \\
O^{(1)}_{\Phi ud,ij} &= 3\Phi^\dagger (i \slashed{\partial}_\mu \Phi)(\bar{u}_{R_i} \gamma^\mu d_{R_j})
\end{align*}
\]

But EWPD does not constrain \(V_{ff}\) and gauge–boson self–energies independently!

→ “Blind Directions” (de Rujula, Gavela, Masso, Hernandez, ‘94; Elias-Miro et al ‘13)
→ We are able to constrain all but two fermionic operators simultaneously using EWPD

(see arXiv: 1311.1823)
Most Constrained Operators:

We recall (constrained by electroweak precision data):

\[
Z \rightarrow \Delta T \propto f_{\Phi,1} \quad Z \rightarrow \Delta S \propto f_{BW}
\]

Additionally some of our fermionic operators are strongly constrained (\(Vff\)):
(by \(Z\)-pole physics, atomic parity violation, etc)

\[
O_{\Phi L,ij}^{(1)} = \Phi^\dagger (i \bar{D}_\mu \Phi) (\bar{L}_i \gamma^\mu L_j)
\]
\[
O_{\Phi Q,ij}^{(1)} = \Phi^\dagger (i \bar{D}_\mu \Phi) (\bar{Q}_i \gamma^\mu Q_j)
\]
\[
O_{\Phi e,ij}^{(1)} = \Phi^\dagger (i \bar{D}_\mu \Phi) (\bar{\epsilon}_R_i \gamma^\mu \epsilon_R_j)
\]
\[
O_{\Phi u,ij}^{(1)} = \Phi^\dagger (i \bar{D}_\mu \Phi) (\bar{u}_R_i \gamma^\mu u_R_j)
\]
\[
O_{\Phi d,ij}^{(1)} = \Phi^\dagger (i \bar{D}_\mu \Phi) (\bar{d}_R_i \gamma^\mu d_R_j)
\]
\[
O_{\Phi ud,ij}^{(1)} = \tilde{\Phi}^\dagger (i \bar{D}_\mu \Phi) (\bar{u}_R_i \gamma^\mu d_R_j)
\]

But EWPD does not constrain \(Vff\) and gauge–boson self–energies independently!

→ “Blind Directions” (de Rujula, Gavela, Masso, Hernandez, ‘94; Elias-Miro et al ‘13)

→ We are able to constrain all but two fermionic operators simultaneously using EWPD
(see arXiv: 1311.1823)
Most Constrained Operators:

We recall (constrained by electroweak precision data):

\[ Z \overset{\text{Electroweak}}{\longrightarrow} \Delta T \propto f_{\Phi,1} \quad Z \overset{\text{Electroweak}}{\longrightarrow} \Delta S \propto f_{BW} \]

Additionally some of our fermionic operators are strongly constrained ($V_{ff}$):
(by $Z$–pole physics, atomic parity violation, etc)

\[
\begin{align*}
O_{\Phi L,ij}^{(1)} &= \Phi^\dagger(i \not{D}_\mu \Phi)(\bar{L}_L i \gamma^\mu L_{Lj}) \\
O_{\Phi Q,ij}^{(1)} &= \Phi^\dagger(i \not{D}_\mu \Phi)(\bar{Q}_L i \gamma^\mu Q_{Lj}) \\
O_{\Phi e,ij}^{(1)} &= \Phi^\dagger(i \not{D}_\mu \Phi)(\bar{e}_R i \gamma^\mu e_{Rj}) \\
O_{\Phi u,ij}^{(1)} &= \Phi^\dagger(i \not{D}_\mu \Phi)(\bar{u}_R i \gamma^\mu u_{Rj}) \\
O_{\Phi d,ij}^{(1)} &= \Phi^\dagger(i \not{D}_\mu \Phi)(\bar{d}_R i \gamma^\mu d_{Rj}) \\
O_{\Phi ud,ij}^{(1)} &= \tilde{\Phi}^\dagger(i \not{D}_\mu \Phi)(\bar{u}_R i \gamma^\mu d_{Rj})
\end{align*}
\]

But EWPD does not constrain $V_{ff}$ and gauge–boson self–energies independently!

→ “Blind Directions” (de Rujula, Gavela, Masso, Hernandez, ‘94; Elias-Miro et al ‘13)
→ We are able to constrain all but two fermionic operators simultaneously using EWPD
(see arXiv: 1311.1823)
Recalling $O_B$ and $O_W$ affect $HVV$ and $VVV$ vertices,

$$O_B = (D^\mu \Phi)^\dagger \hat{B}_{\mu\nu}(D^\nu \Phi) \quad O_W = (D^\mu \Phi)^\dagger \hat{W}_{\mu\nu}(D^\nu \Phi)$$
Triple Gauge Couplings

Recalling $O_W$ and $O_B$ affect $HVV$ and $VVV$ vertices,

$$O_B = (D^\mu \Phi)^\dagger \hat{B}_{\mu\nu} (D^\nu \Phi) \quad O_W = (D^\mu \Phi)^\dagger \hat{W}_{\mu\nu} (D^\nu \Phi)$$

→ keep in the basis as can be constrained from $VVV$(TGC) measurements.

$$\mathcal{L}_{WWV} = -ig_{WWV} \left[ g_1^V (W_{\mu\nu}^+ W^{-\mu} V^\nu - W_{\mu}^+ V_{\nu} W^{-\mu\nu}) + \kappa V W_{\mu}^+ W_{\nu}^- V^{\mu\nu} + \frac{\lambda V}{m_W^2} W_{\mu\nu}^+ W^{-\nu\rho} V_{\mu}^\rho \right]$$

\[
\begin{align*}
\Delta g_1^Z &= g_1^Z - 1 = \frac{g_2^2 v^2}{8 c_W^2 \Lambda^2} \left( f_W + 2 \frac{s_W^2}{c_W^2 - s_W^2} f_{BW} \right) - \frac{1}{4(c_W^2 - s_W^2)} f_{\Phi,1} \frac{v^2}{\Lambda^2} \\
\Delta \kappa_\gamma &= \kappa_\gamma - 1 = \frac{g_2^2 v^2}{8 \Lambda^2} (f_W + f_B - 2 f_{BW}) \\
\Delta \kappa_Z &= \kappa_Z - 1 = \frac{g_2^2 v^2}{8 c_W^2 \Lambda^2} \left( c_W^2 f_W - s_W^2 f_B + \frac{4 s_W^2 c_W^2}{c_W^2 - s_W^2} f_{BW} \right) - \frac{1}{4(c_W^2 - s_W^2)} f_{\Phi,1} \frac{v^2}{\Lambda^2}
\end{align*}
\]

(Hagiwara, Peccei, Zeppenfeld, Hikasa ‘86)
Reduction of Basis

Using the EOM we eliminate $O_{\Phi,4}$ and two fermionic operators, → remaining operator basis is then:

$$O_{GG}, O_{BW}, O_{WW}, O_{BB}, O_{W}, O_{B}, O_{\Phi,1}, O_{\Phi,2}, O_{f\Phi}, O_{\Phi f}^{(1)}, O_{\Phi f}^{(3)}$$
Reduction of Basis

Using the EOM we eliminate $O_{Φ,4}$ and two fermionic operators, → remaining operator basis is then:

$$O_{GG}, O_{WW}, O_{WW}, O_{BB}, O_{W}, O_{B}, O_{Φ,1}, O_{Φ,2}, O_{fΦ}, O_{1f}, O_{2f}$$

Pre–Higgs EWPD strongly constrained: X
Reduction of Basis

Using the EOM we eliminate $O_{\Phi,4}$ and two fermionic operators, → remaining operator basis is then:

$$O_{GG}, \times O_{AW}, \times O_{WW}, \times O_{BB}, \times O_{W}, \times O_{B}, \times O_{\Phi,1}, \times O_{\Phi,2}, \times O_{f\Phi}, \times O_{(1)^{\Phi}}, \times O_{(2)^{\Phi}}$$

Pre-Higgs EWPD strongly constrained: $\times$

FCNC constrain off diagonal $O_{f\Phi}$: $\bigcirc$
Reduction of Basis

Using the EOM we eliminate $O_{\Phi,4}$ and two fermionic operators, \[ \rightarrow \] remaining operator basis is then:

\[ O_{GG}, O_{WW}, O_{BB}, O_W, O_B, O_{\Phi,1}, O_{\Phi,2}, O_{f\Phi}, O_{i_{1}^{(1)}}, O_{i_{1}^{(2)}} \]

Pre–Higgs EWPD strongly constrained: \[ \times \]

FCNC constrain off diagonal $O_{f\Phi}$: \[ \bigcirc \]

Then we have 15 operators:

- 6 involving gauge bosons: $O_{GG}$, $O_{WW}$, $O_{BB}$, $O_W$, $O_B$, $O_{\Phi,2}$
- 9 involving fermions: $O_{e\Phi,ii}$, $O_{u\Phi,ii}$, $O_{d\Phi,ii}$
- Neglect fermion couplings to 1\textsuperscript{st} and 2\textsuperscript{nd} generations: \textbf{Not resolvable at current precision}
- Insufficient data on $ttH$ production to resolve $O_{u\Phi,33}$ from $O_{WW}$, $O_{BB}$, $O_{\Phi,2}$, $O_{GG}$ ⇒ \textbf{Only two fermionic operators} accessible with current precision: $O_{d\Phi,33}$ & $O_{e\Phi,33}$
Reduction of Basis

Using the EOM we eliminate $O_{\Phi,4}$ and two fermionic operators, remaining operator basis is then:

$O_{GG}, O_{WW}, O_{BB}, O_W, O_B, O_{\Phi,1}, O_{\Phi,2}, O_{f\Phi}, O_{f1}^{(1)}, O_{f2}^{(2)}$

Pre-Higgs EWPD strongly constrained:

FCNC constrain off diagonal $O_{f\Phi}$:

Then we have 15 operators:

- 6 involving gauge bosons: $O_{GG}, O_{WW}, O_{BB}, O_W, O_B, O_{\Phi,2}$
- 9 involving fermions: $O_{e\Phi,ii}, O_{u\Phi,ii}, O_{d\Phi,ii}$
- Neglect fermion couplings to 1st and 2nd generations: Not resolvable at current precision

- Insufficient data on $ttH$ production to resolve $O_{u\Phi,33}$ from $O_{WW}, O_{BB}, O_{\Phi,2}, O_{GG}$
  $\Rightarrow$ Only two fermionic operators accessible with current precision: $O_{d\Phi,33} \& O_{e\Phi,33}$

* w/ most recent data $ttH$ coupling may be resolved independently,
arXiv: 1505.05516, TC, J Gonzalez-Fraile, OJP Éboli, D Gonçalves, T Plehn, M Rauch
Final Basis

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} - \frac{\alpha_s v}{8\pi} \frac{f_g}{\Lambda^2} O_{GG} + \frac{f_{WW}}{\Lambda^2} O_{WW} + \frac{f_{BB}}{\Lambda^2} O_{BB} + \frac{f_{W}}{\Lambda^2} O_{W} + \frac{f_{B}}{\Lambda^2} O_{B} + \frac{f_{\Phi,2}}{\Lambda^2} O_{\Phi,2} \]

\[ + \frac{f_{\text{bot}}}{\Lambda^2} O_{d\Phi,33} + \frac{f_{\tau}}{\Lambda^2} O_{e\Phi,33} \]

Operator effects on observables (correlations result from gauge invariance):

<table>
<thead>
<tr>
<th>$O_{GG}$</th>
<th>$O_{BB}$</th>
<th>$O_{WW}$</th>
<th>$O_{B}$</th>
<th>$O_{W}$</th>
<th>$O_{\Phi,2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_{gg}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h\gamma\gamma$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h\gamma Z$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h ZZ$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H W^+ W^-$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma W^+ W^-$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Z W^+ W^-$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

+ shifts for bottom and $\tau$ Yukawa couplings

→ Different Lorentz structures from SM tree level

→ includes correlations required by gauge invariance
Analysis Framework

We will constrain the effective operator basis using a $\chi^2$–statistic:

$$\chi^2 = \min_{\xi_{\text{pull}}} \sum_j \left( \frac{\mu_j - \mu_{\text{exp}}^j}{\sigma_j^2} \right)^2 + \sum_{\text{pull}} \left( \frac{\xi_{\text{pull}}}{\sigma_{\text{pull}}} \right)^2$$

where:

$$\mu_F = \frac{\epsilon_{\text{Prod}}^F \sigma_{\text{SM}}^F + \epsilon_{\text{VBF}}^F \sigma_{\text{VBF}}^F}{\sigma_{\text{SM}}^F + \sigma_{\text{VBF}}^F} \cdot \frac{\sigma_{\text{ano}}^F}{\sigma_{\text{SM}}^F}$$

- $\epsilon_{\text{Prod}}^F$ given by experiments for SM kinematics ($\sim$same for anomalous, Banerjee et al.)
- $\sigma_{i}^{\text{SM}}$ and $\Gamma_{i}^{\text{SM}}$ are known to 1 or 2 loops

To obtain higher orders in the anomalous contributions, scale by SM higher order effects:

$$\sigma_{Y}^{\text{ano}} = \left. \frac{\sigma_{Y}^{\text{ano}}}{\sigma_{Y}^{\text{SM}}} \right|_{\text{tree}} \left. \sigma_{Y}^{\text{SM}} \right|_{\text{soa}}$$

$$\Gamma_{\text{ano}}(h \to F) = \left. \frac{\Gamma_{\text{ano}}(h \to F)}{\Gamma_{\text{SM}}(h \to F)} \right|_{\text{tree}} \left. \Gamma_{\text{SM}}(h \to F) \right|_{\text{soa}}$$
Inclusion of TGCs

Additionally we will use TGC measurements to further constrain the operators:

\[ \mathcal{L}_{WWV} = -ig_{WWV} \left[ g_1^V (W_{\mu} W^{-\mu} V^{\nu} - W_{\mu}^+ V_{\nu} W^{-\mu \nu}) + \kappa V W_{\mu}^+ W_{\nu}^- V^{\mu \nu} \right] \]

\[ \Delta g_1^Z = g_1^Z - 1 = \frac{g^2 v^2}{8 c_W^2 \Lambda^2} f_W \]

\[ \Delta \kappa_\gamma = \kappa_\gamma - 1 = \frac{g^2 v^2}{8 \Lambda^2} (f_W + f_B) \]

\[ \Delta \kappa_Z = \kappa_Z - 1 = \frac{g^2 v^2}{8 c_W^2 \Lambda^2} \left( c_W^2 f_W - s_W^2 f_B \right) \]

From LEP data, the best fit values and correlation \( \rho \) given by:

\[ \kappa_\gamma = 0.984^{+0.049}_{-0.049}, \quad g_1^Z = 1.004^{+0.024}_{-0.025}, \quad \rho = 0.11 \]

* Also include EWPD effects at one loop in arXiv: 1211.4580
Results: Higgs + TGC + EWPD ($\mu = \Lambda = 10$ TeV)

First consider no fermionic operators (i.e. $f_W$, $f_B$, $f_{WW}$, $f_{BB}$, $f_\Phi, 2$, $f_{\text{bot}} = f_{\tau} = 0$):

Best fit near $f_i = 0$ (SM)

Higgs data only

Higgs + TGV

Higgs + TGV + EWPD
Results: Higgs + TGC + EWPD ($\mu = \Lambda = 10$ TeV)

First consider no fermionic operators (i.e. $f_W$, $f_B$, $f_{WW}$, $f_{BB}$, $f_\Phi$, $f_{bot} = f_\tau = 0$):

Interference with SM $\rightarrow$ degeneracy
Results: Higgs + TGC + EWPD ($\mu = \Lambda = 10$ TeV)

First consider no fermionic operators (i.e. $f_W, f_B, f_{WW}, f_{BB}, f_{\Phi^2}, f_{\text{bot}} = f_{\tau} = 0$):

Interference with SM $\rightarrow$ degeneracy

Anticorrelation due to $H \rightarrow \gamma\gamma$
Results: Higgs $+$ TGC $+$ EWPD (II)

Scenario 1: $f_W, f_B, f_{WW}, f_{BB}, f_{\Phi,2}, f_{\text{bot}} = f_\tau = 0$

Scenario 2: $f_W, f_B, f_{WW} = -f_{BB}, f_{\Phi,2}, f_{\text{bot}}, f_\tau$
Results: Higgs + TGC + EWPD (II)

Scenario 1: $f_W, f_B, f_{WW}, f_{BB}, f_{\Phi,2}, f_{\text{bot}} = f_\tau = 0$

Scenario 2: $f_W, f_B, f_{WW} = -f_{BB}, f_{\Phi,2}, f_{\text{bot}}, f_\tau$
Best Fit and 90% CL regions

For Tevatron+LHC+TGC:

<table>
<thead>
<tr>
<th></th>
<th>Fit with $f_{\text{bot}} = f_{\tau} = 0$</th>
<th>Fit with $f_{\text{bot}}$ and $f_{\tau}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Best fit</td>
<td>90% CL allowed range</td>
</tr>
<tr>
<td>$f_g/\Lambda^2\ (\text{TeV}^{-2})$</td>
<td>1.1, 22</td>
<td>$[-3.3, 5.1] \cup [19, 26]$</td>
</tr>
<tr>
<td>$f_{WW}/\Lambda^2\ (\text{TeV}^{-2})$</td>
<td>1.5</td>
<td>$[-3.2, 8.2]$</td>
</tr>
<tr>
<td>$f_{BB}/\Lambda^2\ (\text{TeV}^{-2})$</td>
<td>-1.6</td>
<td>$[-7.5, 5.3]$</td>
</tr>
<tr>
<td>$f_W/\Lambda^2\ (\text{TeV}^{-2})$</td>
<td>2.1</td>
<td>$[-5.6, 9.6]$</td>
</tr>
<tr>
<td>$f_B/\Lambda^2\ (\text{TeV}^{-2})$</td>
<td>-10</td>
<td>$[-29, 8.9]$</td>
</tr>
<tr>
<td>$f_{\phi,2}/\Lambda^2\ (\text{TeV}^{-2})$</td>
<td>-1.0</td>
<td>$[-10, 8.5]$</td>
</tr>
<tr>
<td>$f_{\text{bot}}/\Lambda^2\ (\text{TeV}^{-2})$</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$f_{\tau}/\Lambda^2\ (\text{TeV}^{-2})$</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

SM lays well within $1\sigma$ CL regions
Observables: Cross Sections and Branching Ratios

From $\chi^2(f_i)$ we are able to construct the allowed ranges for $\sigma_i^{ano}$ and $BR_i^{ano}$:
TGC Projections from Higgs Measurements

TGC ↔ HVV from gauge invariance:

\[
\begin{align*}
\Delta g_1^Z &= g_1^Z - 1 = \frac{g^2 v^2}{8 c_W^2 \Lambda^2} f_W \\
\Delta \kappa_\gamma &= \kappa_\gamma - 1 = \frac{g^2 v^2}{8 \Lambda^2} (f_W + f_B) \\
\Delta \kappa_Z &= \kappa_Z - 1 = \frac{g^2 v^2}{8 c_W^2} (c_W^2 f_W - s_W^2 f_B)
\end{align*}
\]

→ project Higgs data onto TGC bounds:
(before we did the reverse!)

Complementary to direct searches at:
- LEP
- Tevatron
- & ATLAS

Marginalized over unshown parameters,
\((f_g, f_{WW}, f_{BB}, f_{\Phi,2})\)

⇒ Complementary bounds
⇒ becoming competitive w/ direct measurement!
TGC Projections from Higgs Measurements

TGC ↔ HVV from gauge invariance:

\[
\begin{align*}
\Delta g_1^Z &= g_1^Z - 1 = \frac{g^2 v^2}{8 c_W^2 \Lambda^2} f_W \\
\Delta \kappa_\gamma &= \kappa_\gamma - 1 = \frac{g^2 v^2}{8 \Lambda^2} (f_W + f_B) \\
\Delta \kappa_Z &= \kappa_Z - 1 = \frac{g^2 v^2}{8 c_W^2 \Lambda^2} (c_W^2 f_W - s_W^2 f_B)
\end{align*}
\]

→ project Higgs data onto TGC bounds: (before we did the reverse!)

Complementary to direct searches at:

- LEP
- Tevatron
- & ATLAS

Marginalized over unshown parameters,
\[(f_g, f_{WW}, f_{BB}, f_{\Phi,2})\]

⇒ Complementary bounds
⇒ becoming competitive w/ direct measurement!
Partial Wave Unitarity

Dimension–six operators $\rightarrow$ growth of amplitude $w/ E_{\text{COM}}$
$\rightarrow$ violation of S–matrix unitarity

We impose the unitarity of the $S$–Matrix to constrain the operators:

- bounds derived from the optical theorem

- imply the necessity of new physics
  $\rightarrow$ new fundamental particles (SM Higgs fixes unitarity issues of GB scattering)
  $\rightarrow$ new composite degree of freedom ($\rho$ meson fixes unitarity issues of $\pi$ scattering)

- We will obtain bounds on the COM energy ($\sqrt{s}$) as a function of $f_i/\Lambda$
  $\rightarrow$ we can combine these with our Higgs fit results!
Partial Wave Unitarity

Decomposing the amplitudes for $VV \rightarrow VV$ & $f\bar{f} \rightarrow VV$ into partial waves:

\[
\mathcal{M}(V_1 \lambda_1 V_2 \lambda_2 \rightarrow V_3 \lambda_3 V_4 \lambda_4) = 16\pi \sum_J \left( J + \frac{1}{2} \right) \sqrt{1 + \delta_{V_1 \lambda_1} \lambda_{\lambda_2}} \sqrt{1 + \delta_{V_3 \lambda_3} \lambda_{\lambda_4}} d_{\lambda \mu}(\theta) e^{iM\phi} T^J (V_1 V_2 \rightarrow V_3 V_4)
\]

\[
\mathcal{M}(f_1 \sigma_1 \bar{f}_2 \sigma_2 \rightarrow V_3 \lambda_3 V_4 \lambda_4) = 16\pi \sum_J \left( J + \frac{1}{2} \right) \delta_{\sigma_1, \bar{\sigma}_2} d_{\sigma_1 \lambda_1} \lambda_{\lambda_2} \lambda_{\lambda_3} \lambda_{\lambda_4} (\theta) T^J (f_1 \bar{f}_2 \rightarrow V_3 V_4)
\]

Using the optical theorem one may derive the unitarity limit for the $T^J$’s:

\[
|T^J (V_1 \lambda_1 V_2 \lambda_2 \rightarrow V_1 \lambda_1 V_2 \lambda_2)| \leq 2,
\]

\[
\sum_{V_3 \lambda_3, V_4 \lambda_4} |T^J (f_1 \sigma_1 \bar{f}_2 \sigma_2 \rightarrow V_3 \lambda_3 V_4 \lambda_4)|^2 \leq 1
\]
Partial Wave Unitarity

Decomposing the amplitudes for $VV \to VV$ & $f\bar{f} \to VV$ into partial waves:

$$\mathcal{M}(V_{1\lambda_1} V_{2\lambda_2} \to V_{3\lambda_3} V_{4\lambda_4}) = 16\pi \sum_J \left( J + \frac{1}{2} \right) \sqrt{1 + \delta_{V_{1\lambda_1} V_{2\lambda_2}}} \sqrt{1 + \delta_{V_{3\lambda_3} V_{4\lambda_4}}} d_J^{\lambda \mu}(\theta) e^{iM\phi} T^J (V_1 V_2 \to V_3 V_4)$$

$$\mathcal{M}(f_{\sigma_1} \bar{f}_{\sigma_2} \to V_{3\lambda_3} V_{4\lambda_4}) = 16\pi \sum_J (J + \frac{1}{2}) \delta_{\sigma_1, -\sigma_2} d_J^{\sigma_1 - \sigma_2, \lambda_3 - \lambda_4}(\theta) T^J (f_1 \bar{f}_2 \to V_3 V_4)$$

Using the optical theorem one may derive the unitarity limit for the $T^J$’s:

$$|T^J (V_{1\lambda_1} V_{2\lambda_2} \to V_{1\lambda_1} V_{2\lambda_2})| \leq 2,$$

$$\sum_{V_{3\lambda_3}, V_{4\lambda_4}} |T^J (f_{\sigma_1} \bar{f}_{\sigma_2} \to V_{3\lambda_3} V_{4\lambda_4})|^2 \leq 1$$
**VV → VV Unitarity Violating Amplitudes: \( O_{\Phi,2} \& O_{\Phi,4} \)**

The high energy \( (\sqrt{s} \gg M_W, M_Z, M_H) \) behavior for \( O_{\Phi,2} \& O_{\Phi,4} \) → same up to a sign.

We define: \( f_{\Phi,2,4} \equiv f_{\Phi,2} - f_{\Phi,4} \)

\[
\begin{array}{|c|c|}
\hline
\text{Process} & (\times \frac{f_{\Phi,2,4}}{\Lambda^2} \times s) \\
\hline
W^+W^+ \rightarrow W^+W^+ & -1 \\
W^+Z \rightarrow W^+Z & -\frac{1}{2}X \\
W^+H \rightarrow W^+H & -\frac{1}{2}X \\
W^+W^- \rightarrow W^+W^- & \frac{1}{2}Y \\
W^+W^- \rightarrow ZZ & 1 \\
W^+W^- \rightarrow HH & -1 \\
ZZ \rightarrow HH & -1 \\
ZH \rightarrow ZH & -\frac{1}{2}X \\
\hline
\end{array}
\]

\( \mathcal{M}(s, M_W, M_Z, M_H) \rightarrow \mathcal{M}(s \gg M_W^2, M_Z^2, M_H^2) \)

\( X \equiv 1 - \cos \theta \)

\( Y \equiv 1 + \cos \theta \)

Only grow as \( s \), result of gauge symmetry!

Same Lorentz structures as SM

→ violation only in \( V_L V_L \rightarrow V_L V_L \)
**VV → VV Unitarity Violating Amplitudes: $\mathcal{O}_{WWW}$**

Even more Lorentz structures → some more helicity combinations violating unitarity...

<table>
<thead>
<tr>
<th></th>
<th>$00++$</th>
<th>$0+0-$</th>
<th>$0+0-$</th>
<th>$++00$</th>
<th>$++00$</th>
<th>$+++-$</th>
<th>$+--+  $</th>
<th>$++-+$</th>
<th>$++--$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W^{+}W^{+} \rightarrow W^{+}W^{+}$</td>
<td>0</td>
<td>$-\frac{3(2+Y)}{32s_{W}^4}$</td>
<td>$\frac{3(2+X)}{32s_{W}^4}$</td>
<td>$\frac{3(2+X)}{32s_{W}^4}$</td>
<td>$-\frac{3(2+Y)}{32s_{W}^4}$</td>
<td>0</td>
<td>$-\frac{3}{8s_{W}^4}$</td>
<td>$\frac{3}{4s_{W}^3}$</td>
<td>$\frac{3}{8s_{W}^3}$</td>
</tr>
<tr>
<td>$W^{+}Z \rightarrow W^{+}Z$</td>
<td>$\frac{3(Y-X)c_{W}}{32s_{W}^2}$</td>
<td>0</td>
<td>$\frac{3(X+2)c_{W}}{32s_{W}^4}$</td>
<td>$\frac{3(X+2)c_{W}}{32s_{W}^4}$</td>
<td>0</td>
<td>$\frac{3(Y-X)c_{W}}{32s_{W}^4}$</td>
<td>$-\frac{3c_{W}}{8s_{W}^3}$</td>
<td>$\frac{3c_{W}}{4s_{W}^2}$</td>
<td>$\frac{3}{8s_{W}^2}$</td>
</tr>
<tr>
<td>$W^{+}\gamma \rightarrow W^{+}\gamma$</td>
<td>0</td>
<td>0</td>
<td>$\frac{3(X+2)c_{W}}{32s_{W}^4}$</td>
<td>$\frac{3(Y-X)c_{W}}{32s_{W}^4}$</td>
<td>0</td>
<td>0</td>
<td>$\frac{3c_{W}}{8s_{W}^3}$</td>
<td>$\frac{3c_{W}}{4s_{W}^2}$</td>
<td>$\frac{3}{8s_{W}^2}$</td>
</tr>
<tr>
<td>$W^{+}Z \rightarrow W^{+}\gamma$</td>
<td>$-\frac{3(Y-X)}{32s_{W}^2}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$-\frac{3}{32s_{W}^2}$</td>
<td>$\frac{3}{32s_{W}^2}$</td>
<td>$\frac{3}{32s_{W}^2}$</td>
<td>$\frac{3}{32s_{W}^2}$</td>
<td>$\frac{3}{32s_{W}^2}$</td>
</tr>
<tr>
<td>$W^{+}Z \rightarrow W^{+}H$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$-\frac{3}{32s_{W}^2}$</td>
<td>$\frac{3}{32s_{W}^2}$</td>
<td>$\frac{3}{32s_{W}^2}$</td>
<td>$\frac{3}{32s_{W}^2}$</td>
</tr>
<tr>
<td>$W^{+}\gamma \rightarrow W^{+}H$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$-\frac{3}{32s_{W}^2}$</td>
<td>$\frac{3}{32s_{W}^2}$</td>
<td>$\frac{3}{32s_{W}^2}$</td>
<td>$\frac{3}{32s_{W}^2}$</td>
</tr>
<tr>
<td>$W^{+}W^{-} \rightarrow W^{+}W^{-}$</td>
<td>$\frac{3(Y-X)}{32s_{W}^2}$</td>
<td>$\frac{3(2+Y)}{32s_{W}^4}$</td>
<td>0</td>
<td>0</td>
<td>$-\frac{3(2+Y)c_{W}}{32s_{W}^4}$</td>
<td>$\frac{3(Y-X)c_{W}}{32s_{W}^4}$</td>
<td>$\frac{3c_{W}}{8s_{W}^3}$</td>
<td>$\frac{3c_{W}}{4s_{W}^2}$</td>
<td>$\frac{3}{8s_{W}^2}$</td>
</tr>
<tr>
<td>$W^{+}W^{-} \rightarrow ZZ$</td>
<td>0</td>
<td>$\frac{3(2+Y)c_{W}}{32s_{W}^4}$</td>
<td>$-\frac{3(2+Y)c_{W}}{32s_{W}^4}$</td>
<td>$\frac{3(2+Y)c_{W}}{32s_{W}^4}$</td>
<td>0</td>
<td>0</td>
<td>$\frac{3c_{W}}{8s_{W}^3}$</td>
<td>$\frac{3c_{W}}{4s_{W}^2}$</td>
<td>$\frac{3}{8s_{W}^2}$</td>
</tr>
<tr>
<td>$W^{+}W^{-} \rightarrow \gamma\gamma$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$-\frac{3}{32s_{W}^2}$</td>
<td>$\frac{3}{32s_{W}^2}$</td>
<td>$\frac{3}{32s_{W}^2}$</td>
<td>$\frac{3}{32s_{W}^2}$</td>
</tr>
<tr>
<td>$W^{+}W^{-} \rightarrow ZZ$</td>
<td>0</td>
<td>$\frac{3(2+Y)c_{W}}{32s_{W}^4}$</td>
<td>$-\frac{3(2+Y)c_{W}}{32s_{W}^4}$</td>
<td>$\frac{3(2+Y)c_{W}}{32s_{W}^4}$</td>
<td>0</td>
<td>0</td>
<td>$\frac{3c_{W}}{8s_{W}^3}$</td>
<td>$\frac{3c_{W}}{4s_{W}^2}$</td>
<td>$\frac{3}{8s_{W}^2}$</td>
</tr>
<tr>
<td>$W^{+}W^{-} \rightarrow HH$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$-\frac{3}{32s_{W}^2}$</td>
<td>$\frac{3}{32s_{W}^2}$</td>
<td>$\frac{3}{32s_{W}^2}$</td>
<td>$\frac{3}{32s_{W}^2}$</td>
</tr>
<tr>
<td>$W^{+}W^{-} \rightarrow \gamma H$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$-\frac{3}{32s_{W}^2}$</td>
<td>$\frac{3}{32s_{W}^2}$</td>
<td>$\frac{3}{32s_{W}^2}$</td>
<td>$\frac{3}{32s_{W}^2}$</td>
</tr>
</tbody>
</table>
All Couplings Simultaneously

In arXiv:1411.5026 we give bounds for one operator different from zero at a time.

With no model of NP to guide us, we must consider more than one coupling non–zero.

- search for largest allowed value for each coefficient while varying others

→ Most conservative constraints on a parameter allowing for cancellations in others

| $\left| \frac{f_{\Phi 2,4}}{\Lambda^2} s \right|$ | $\leq$ | 105 |
| $\left| \frac{f_{W}}{\Lambda^2} s \right|$ | $\leq$ | 205 |
| $\left| \frac{f_{B}}{\Lambda^2} s \right|$ | $\leq$ | 640 |
| $\left| \frac{f_{WW}}{\Lambda^2} s \right|$ | $\leq$ | 200 |
| $\left| \frac{f_{BB}}{\Lambda^2} s \right|$ | $\leq$ | 880 |
| $\left| \frac{f_{WWW}}{\Lambda^2} s \right|$ | $\leq$ | 82 |

Note: $f \sim 1 \rightarrow s \sim 100\Lambda^2$, current bounds may be unreliable for some $f$. 

Tyler Corbett (YITP)
Combined Results:

We did not constrain $f_{W W W}$ from Higgs data, but TGC bounds give:

$$-0.041 \leq \lambda_{\gamma} \leq -0.003 \quad \lambda_{\gamma} = \frac{3g^2 M_W^2}{2\Lambda^2} f_{W W W}$$

Combining the Higgs data fits with the unitarity bounds obtained we find:

$$-10 \leq \frac{f_{\phi,2}}{\Lambda^2} (\text{TeV}^{-2}) \leq 8.5 \quad \Rightarrow \quad \sqrt{s} \leq 3.2 \, \text{TeV}$$

$$-5.6 \leq \frac{f_{W}}{\Lambda^2} (\text{TeV}^{-2}) \leq 9.6 \quad \Rightarrow \quad \sqrt{s} \leq 4.6 \, \text{TeV}$$

$$-29 \leq \frac{f_{B}}{\Lambda^2} (\text{TeV}^{-2}) \leq 8.9 \quad \Rightarrow \quad \sqrt{s} \leq 4.7 \, \text{TeV}$$

$$-3.2 \leq \frac{f_{W W}}{\Lambda^2} (\text{TeV}^{-2}) \leq 8.2 \quad \Rightarrow \quad \sqrt{s} \leq 4.9 \, \text{TeV}$$

$$-7.5 \leq \frac{f_{B B}}{\Lambda^2} (\text{TeV}^{-2}) \leq 5.3 \quad \Rightarrow \quad \sqrt{s} \leq 11 \, \text{TeV}$$

$$-15 \leq \frac{f_{W W W}}{\Lambda^2} (\text{TeV}^{-2}) \leq 3.9 \quad \Rightarrow \quad \sqrt{s} \leq 2.4 \, \text{TeV}$$
Linear EFTs for Higgs Physics

We’re interested in the EFT for Higgs Physics.

Effective operators are formed from the field content,

\[ e_R, \quad u_R, \quad d_R, \quad L_L, \quad Q_L, \quad \Phi, \quad W_\mu, \quad Z_\mu, \quad A_\mu, \quad G_\mu \]

And their (covariant) derivatives.

The operators are then formed of all \( U(1)_{Y/2} \times SU(2)_L \times SU(3)_C \) invariant combinations.

Further

- restrict to \textit{dimension–six}, i.e. assume all NP occurs at a sufficiently high \( \Lambda \)
- assume \textit{baryon and lepton numbers are conserved} (\( \rightarrow \) no dimension–5 operators)
- assume operators are \textit{CP–even}

Then we write the linear EFT to dimension–6 as:

\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_n \frac{f_n}{\Lambda^2} \mathcal{O}_n
\]
We’re interested in the EFT for Higgs Physics.

Effective operators are formed from the field content,

\[ e_R, \ u_R, \ d_R, \ L_L, \ Q_L, \ \Phi, \ W_\mu, \ Z_\mu, \ A_\mu, \ G_\mu \]

And their (covariant) derivatives.

The operators are then formed of all \( U(1)_{Y/2} \times SU(2)_L \times SU(3)_C \) invariant combinations.

Further

- restrict to dimension–six, i.e. assume all NP occurs at a sufficiently high \( \Lambda \)
- assume baryon and lepton numbers are conserved (→ no dimension–5 operators)
- assume operators are \( CP–even \)

Then we write the linear EFT to dimension–6 as:

\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_n \frac{f_n}{\Lambda^2} \mathcal{O}_n
\]
We’re interested in the EFT for Higgs Physics.

Effective operators are formed from the field content,

\[ e_R, \ u_R, \ d_R, \ L_L, \ Q_L, \ \Phi, \ W_\mu, \ Z_\mu, \ A_\mu, \ G_\mu \]

And their (covariant) derivatives.

The operators are then formed of all \( U(1)_{Y/2} \times SU(2)_L \times SU(3)_C \) invariant combinations.

Further

- restrict to dimension-six, i.e. assume all NP occurs at a sufficiently high \( \Lambda \)
- assume baryon and lepton numbers are conserved \( (\rightarrow \) no dimension-five operators)
- assume operators are \( CP\)–even

Then we write the linear EFT to dimension-six as:

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_n \frac{f_n}{\Lambda^2} \mathcal{O}_n \]
Chiral EFTs for Higgs Physics

We’re interested in the EFT for Higgs Physics.

Effective operators are formed from the field content, 

\[ e_R, \ u_R, \ d_R, \ L_L, \ Q_L, \ \Phi, \ W_\mu, \ Z_\mu, \ A_\mu, \ G_\mu \]

And their (covariant) derivatives.

The operators are then formed of all \( U(1)_{Y/2} \times SU(2)_L \times SU(3)_C \) invariant combinations.

Further

- restrict to 4 derivatives, i.e. assume all NP occurs for \( p^2 \ll \Lambda_{NP} \)
- assume baryon and lepton numbers are conserved
- assume operators are \( CP \)-even

Then we write the chiral EFT to 4 derivatives as:

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_n \frac{f_n}{\Lambda^2} \mathcal{O}_n \]

\[ h, U = \exp(-i\pi^a \sigma^a/v) \]
We’re interested in the EFT for Higgs Physics.

Effective operators are formed from the field content,

\[ e_R, \quad u_R, \quad d_R, \quad L_L, \quad Q_L, \quad \Phi, \quad W_\mu, \quad Z_\mu, \quad A_\mu, \quad G_\mu \]

And their (covariant) derivatives.

The operators are then formed of all \( U(1)_{Y/2} \times SU(2)_L \times SU(3)_C \) invariant combinations.

Further

- restrict to 4 derivatives, i.e. assume all NP occurs for \( p^2 \ll \Lambda_{NP} \)
- assume baryon and lepton numbers are conserved
- assume operators are \( CP \)-even

Then we write the chiral EFT to 4 derivatives as:

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_n c_n \mathcal{P}_n \]
Chiral Expansion to 4–Derivatives

Less symmetry → more operators,
Defining: $U(x) = e^{i\sigma_a \pi^a(x)/v}$, $V_\mu \equiv (D_\mu U)U^\dagger$, $T \equiv U\sigma_3 U^\dagger$.

$\mathcal{P}_2(h) = i g' B_{\mu\nu} \text{Tr}(T[V^\mu, V^\nu]) F_2$

$\mathcal{P}_4 = i g' B_{\mu\nu} \text{Tr}(TV^\mu) \partial^\nu F_4$

Where, for $\mathcal{O}_B = \frac{g'}{2} (D_\mu \Phi)^\dagger B^{\mu\nu} (D_\nu \Phi)$, these were correlated.

$\Rightarrow$ Potential for discrimination between expansions!

$\Rightarrow$ A lot more in arXiv:1311.1823.
Conclusions

- SM Higgs mechanism completely determines the Higgs properties given $M_H$

- To test deviations model independently $\rightarrow$ EFT

- We have explored the linear EFT expansion (at dim=6), arXiv:1211.4580 & 1411.5026
  $\rightarrow$ Relevant for an elementary Higgs
  $\rightarrow$ Data driven approach, choose basis such that it is best constrained by data
  $\rightarrow$ Compatible within $1\sigma$ with SM predictions, still room for NP
  $\rightarrow$ HVV and TGC correlation
  $\rightarrow$ Indirect TGC bounds becoming competitive with direct TGC measurements
  $\rightarrow$ Lowest energies at which perturbative unitarity is violated: $\sim 2 - 3$ TeV

- Briefly mentioned the chiral expansion (at $p^4$), more in arXiv:1311.1823
  $\rightarrow$ Relevant for a light composite Higgs
  $\rightarrow$ HVV and TGC uncorrelated: possible discrimination between expansions

- Most up to date fit is in 1505.05516, includes more than presented:
  $\rightarrow$ Anomalous $Ht\bar{t}$
  $\rightarrow$ Kinematic distributions
  $\rightarrow$ Future impact of off shell measurements

Although consistent with the SM there is still large room for deviations which will be/are being probed at LHC run II
Backup: Gluon Fusion – $\text{BR}_{\gamma\gamma}$ Correlation

Inclusion of $f_{\text{bot}} \rightarrow$ loosen bounds on GGF $\rightarrow$ more parameter space for $\text{BR}_{\gamma\gamma} < \text{BR}_{\gamma\gamma}^{\text{SM}}$
Backup: Optical Theorem

The optical theorem states:

$$\text{Im}T^J(12 \rightarrow 34) = \sum_{12 \rightarrow 1'2'} \frac{1}{\sqrt{s}} |\vec{p}_{1'2'}| T^J(12 \rightarrow 1'2')T^J(1'2' \rightarrow 34),$$

with:

$$|\vec{p}_{ij}| = \sqrt{[s - (m_i + m_j)^2][s - (m_i - m_j)^2]} \cdot \frac{2}{2\sqrt{s}}.$$ 

Which we may rewrite as:

$$\text{Im}T^J(12 \rightarrow 12) = \frac{|\vec{p}_{12}|}{\sqrt{s}} |T^J(12 \rightarrow 12)|^2 + \sum_{1'2' \neq 12} \frac{|\vec{p}_{1'2'}|}{\sqrt{s}} |T^J(12 \rightarrow 1'2')|^2.$$ 

For only one intermediate channel we obtain:

$$\text{Im}T^J(12 \rightarrow 12) = \frac{|\vec{p}_{12}|}{\sqrt{s}} |T^J(12 \rightarrow 12)|^2.$$ 

Rewriting $T^J$ as,

$$T^J(12 \rightarrow 12) = \frac{\sqrt{s}}{|\vec{p}_{12}|} e^{i\delta} \sin \delta,$$

Gives the result for elastic scattering:

$$|T^J(12 \rightarrow 12)| \leq \frac{\sqrt{s}}{|\vec{p}_{12}|} \rightarrow 2.$$
Backup: Optical Theorem (II)

Alternatively considering fermion scattering we obtain from the optical theorem:

\[
2 \text{Im}[T^J(f_1 \sigma_1 \bar{f}_2 \sigma_2 \rightarrow f_1 \sigma_1 \bar{f}_2 \sigma_2)] = \left| T^J(f_1 \sigma_1 \bar{f}_2 \sigma_2 \rightarrow f_1 \sigma_1 \bar{f}_2 \sigma_2) \right|^2 \\
+ \sum_{V_3 \lambda_3, V_4 \lambda_4} \left| T^J(f_1 \sigma_1 \bar{f}_2 \sigma_2 \rightarrow V_3 \lambda_3 V_4 \lambda_4) \right|^2 \\
+ \sum_N \left| T^J(f_1 \sigma_1 \bar{f}_2 \sigma_2 \rightarrow N) \right|^2 ,
\]

Defining,

\[
T^J(12 \rightarrow 12) \equiv y + ix, \quad d \equiv \sum_{1'2' \neq 12} \frac{|\vec{p}_{1'2'}|}{\sqrt{s}} \left| T^J(12 \rightarrow 1'2') \right|^2 ,
\]

Allows us to rewrite the above as:

\[
x = \frac{|\vec{p}_{12}|}{\sqrt{s}} (x^2 + y^2) + d.
\]

Finally in order for the assumption that x is real to hold, the quadratic equation requires:

\[
2 \sum_{1'2' \neq 12} \frac{|\vec{p}_{1'2'}|}{\sqrt{s}} \left| T^J(12 \rightarrow 1'2') \right|^2 \leq 1 \rightarrow \sum_{V_3 \lambda_3, V_4 \lambda_4} \left| T^J(f_1 \sigma_1 \bar{f}_2 \sigma_2 \rightarrow V_3 \lambda_3 V_4 \lambda_4) \right|^2 \leq 1
\]
Inclusion of EWPD \((S, T, \text{ and } U)\)

Additionally we include the effects of \(f_{WW}, f_{BB}, f_W, f_B, \text{ and } f_{\Phi,2}\) at 1–loop: (Hagiwara et al.; Alam, Dawson, Szalapski)

\[
\alpha_{EM} \Delta S = \frac{1}{6} \frac{e^2}{16\pi^2} \left\{ 3(f_W + f_B) \frac{m_H^2}{\Lambda^2} \log \left( \frac{\mu^2}{m_H^2} \right) \\
+ 2 \left[ (5c_W^2 - 2)f_W - (5c_W^2 - 3)f_B \right] \frac{m_Z^2}{\Lambda^2} \log \left( \frac{\mu^2}{m_Z^2} \right) \\
- \left[ (22c_W^2 - 1)f_W - (30c_W^2 + 1)f_B \right] \frac{m_Z^2}{\Lambda^2} \log \left( \frac{\mu^2}{m_Z^2} \right) \\
- 24 \left( c_W^2 f_{WW} + s_W^2 f_{BB} \right) \frac{m_Z^2}{\Lambda^2} \log \left( \frac{\mu^2}{m_Z^2} \right) \right\}
\]

\[
\alpha_{EM} \Delta T = \frac{3}{4c_W^2} \frac{e^2}{16\pi^2} \left\{ f_B \frac{m_H^2}{\Lambda^2} \log \left( \frac{\mu^2}{m_H^2} \right) + \left( c_W^2 f_W + f_B \right) \frac{m_Z^2}{\Lambda^2} \log \left( \frac{\mu^2}{m_Z^2} \right) \\
+ \left[ 2c_W^2 f_W + (3c_W^2 - 1)f_B \right] \frac{m_Z^2}{\Lambda^2} \log \left( \frac{\mu^2}{m_Z^2} \right) \\
- f_{\Phi,2} \frac{v^2}{\Lambda^2} \log \left( \frac{\mu^2}{m_Z^2} \right) \right\}
\]

\[
\alpha_{EM} \Delta U = -\frac{1}{3} \frac{e^2 s_W^2}{16\pi^2} \left\{ (-4f_W + 5f_B) \frac{m_Z^2}{\Lambda^2} \log \left( \frac{\mu^2}{m_Z^2} \right) + (2f_W - 5f_B) \frac{m_Z^2}{\Lambda^2} \log \left( \frac{\mu^2}{m_Z^2} \right) \right\}
\]

Some issues - finite part? renormalization scale \(\mu = ?\)
\( \mathcal{L}_{\text{eff}}, \) to Four–Derivatives

\[
\mathcal{L}_{\text{eff}} = \xi \left[ c_B \mathcal{P}_B(h) + c_W \mathcal{P}_W(h) + c_G \mathcal{P}_G(h) + c_C \mathcal{P}_C(h) + c_T \mathcal{P}_T(h) + c_H \mathcal{P}_H(h) \right] + \xi \sum_{i=1}^{10} c_i \mathcal{P}_i(h) + \xi^2 \sum_{i=1}^{25} c_i \mathcal{P}_i(h) + \xi^4 c_{26} \mathcal{P}_{26}(h) + \sum_i \xi^{n_i} c_{HH} \mathcal{P}_{iHH}(h)
\]

Define: \( U(x) = e^{i\sigma_a \pi^a(x)/v} \), \( V_\mu \equiv (D_\mu U)U^\dagger \), \( T \equiv U \sigma_3 U^\dagger \), \( \xi \equiv \frac{v^2}{f^2} \).

\( \xi \) helps us to compare between the linear and chiral expansions.

**Weighted by \( \xi \):**

\[
\begin{align*}
\mathcal{P}_C(h) &= -\frac{v^2}{4} \text{Tr}(V^\mu V_\mu) \mathcal{F}_C \\
\mathcal{P}_T(h) &= \frac{v^2}{4} \text{Tr}(TV_\mu) \text{Tr}(TV^\mu) \mathcal{F}_T \\
\mathcal{P}_B(h) &= -\frac{g^2}{4} B_{\mu\nu} B^{\mu\nu} \mathcal{F}_B \\
\mathcal{P}_W(h) &= -\frac{g^2}{4} W_\mu^a W^a_{\mu\nu} \mathcal{F}_W \\
\mathcal{P}_G(h) &= -\frac{g^2}{4} G_\mu^a G^{a\mu\nu} \mathcal{F}_G \\
\mathcal{P}_1(h) &= gg' B_{\mu\nu} \text{Tr}(TW_{\mu\nu}) \mathcal{F}_1 \\
\mathcal{P}_2(h) &= ig' B_{\mu\nu} \text{Tr}(\mathcal{T}[V^\mu, V^\nu]) \mathcal{F}_2 \\
\mathcal{P}_3(h) &= ig \text{Tr}(W_{\mu\nu}[V^\mu, V^\nu]) \mathcal{F}_3 \\
\mathcal{P}_4 &= ig' B_{\mu\nu} \text{Tr}(TV^\mu) \partial^\nu \mathcal{F}_4 \\
\mathcal{P}_5 &= ig \text{Tr}(W_{\mu\nu} V^\mu) \partial^\nu \mathcal{F}_5 \\
\mathcal{P}_6 &= (\text{Tr}(V^\mu V_\mu))^2 \mathcal{F}_6 \\
\mathcal{P}_7 &= \text{Tr}(V^\mu V_\mu)^2 \partial^\nu \partial^\nu \mathcal{F}_7 \\
\mathcal{P}_8 &= \text{Tr}(V^\mu V_\mu) \partial^\nu \mathcal{F}_8(h) \partial^\nu \mathcal{F}_8' \\
\mathcal{P}_9 &= \text{Tr}((D_\mu V^\mu)^2) \partial^\nu \mathcal{F}_9 \\
\mathcal{P}_{10} &= \text{Tr}(V^\nu D_\mu V^\mu) \mathcal{F}_{10}
\end{align*}
\]
(Very) Brief Comparison of Bases

Order in $\xi$ of an operator $\rightarrow$ minimum canonical dimension to which one must expand in the linear expansion necessary to generate the corresponding effects of the operator in the chiral expansion.

For small $\xi$ we can relate the Linear basis to the Chiral (more operators):

$$O_{WW} = \frac{g^2}{4} \Phi^\dagger W^{\mu\nu} W_{\mu\nu} \Phi \quad P_W = -\frac{g^2}{2} \text{Tr}(W_{\mu\nu} W^{\mu\nu}) F_W(h)$$

$$O_{BB} = \frac{g'}{4} \Phi^\dagger B^{\mu\nu} B_{\mu\nu} \Phi \quad P_B = -\frac{g'^2}{4} \text{Tr}(B_{\mu\nu} B^{\mu\nu}) F_B(h)$$

$$O_{BW} = \frac{gg'}{4} \Phi^\dagger B^{\mu\nu} W_{\mu\nu} \Phi \quad P_1 = gg' B_{\mu\nu} \text{Tr}(T W^{\mu\nu}) F_1(h)$$

$$O_{B} = \frac{g'}{2} (D_\mu \Phi)^\dagger B^{\mu\nu} (D_\nu \Phi) \quad P_2 = ig' B_{\mu\nu} \text{Tr}(T[V^\mu, V^{\nu}]) F_2(h)$$

$$O_{W} = \frac{g}{2} (D_\mu \Phi)^\dagger W^{\mu\nu} (D_\nu \Phi) \quad P_3 = ig \text{Tr}(W_{\mu\nu} [V^\mu, V^{\nu}]) F_3(h)$$

$$O_{\phi,1} = (D_\mu \Phi)^\dagger \Phi \Phi^\dagger (D_\nu \Phi) \quad P_T = \frac{v^2}{4} \text{Tr}(TV_\mu) \text{Tr}(TV^\mu) F_T(h)$$

$$O_{\phi,2} = \frac{1}{2} \partial_\mu (\Phi^\dagger \Phi) \partial^\mu (\Phi^\dagger \Phi) \quad P_H = \frac{1}{2} (\partial_\mu h)(\partial^\mu h) F_H(h)$$

$$O_{\phi,4} = (D_\mu \Phi)^\dagger (D_\nu \Phi) (\Phi^\dagger \Phi) \quad P_C = -\frac{v^2}{4} \text{Tr}(V_\mu V^\mu) F_C(h)$$

Higgs–Gauge, Triple Gauge, and Quartic Gauge couplings are no longer correlated!
(Very) Brief Comparison of Bases II

Each operator contributes to particular HVV and TGCs. In particular\(^1\):

<table>
<thead>
<tr>
<th></th>
<th>(O_{BB})</th>
<th>(O_{WW})</th>
<th>(O_B)</th>
<th>(O_W)</th>
<th>(P_B)</th>
<th>(P_W)</th>
<th>(P_2)</th>
<th>(P_4)</th>
<th>(P_3)</th>
<th>(P_5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(h\gamma\gamma)</td>
<td>(\times)</td>
<td>(\times)</td>
<td></td>
<td></td>
<td>(\times)</td>
<td>(\times)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(h\gamma Z)</td>
<td>(\times)</td>
<td>(\times)</td>
<td>(\times)</td>
<td>(\times)</td>
<td>(\times)</td>
<td>(\times)</td>
<td>(\times)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(hZZ)</td>
<td>(\times)</td>
<td>(\times)</td>
<td>(\times)</td>
<td>(\times)</td>
<td>(\times)</td>
<td>(\times)</td>
<td>(\times)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(HW^+W^-)</td>
<td>(\times)</td>
<td></td>
<td></td>
<td></td>
<td>(\times)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\gamma W^+W^-)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(\times)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ZW^+W^-)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(\times)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notice that green illustrates the decorrelation of TGC from Higgs data for

\[
O_B = \frac{v^2}{16} P_2 + \frac{v^2}{8} P_4
\]

And blue illustrates a similar decorrelation for

\[
O_W = \frac{v^2}{8} P_3 - \frac{v^2}{4} P_5
\]

\(^1\) \(O_{\phi_4,\phi_2} (P_H,T)\) give shifts to all SM HVV & Hff vertices
Higgs Phenomenology in Chiral Expansion

For comparison w/ linear expansion relevant ops. are: $\mathcal{P}_G \; \mathcal{P}_4 \; \mathcal{P}_5 \; \mathcal{P}_B \; \mathcal{P}_W \; \mathcal{P}_H \; \mathcal{P}_C$

Taking: $c_i \mathcal{F}(h) = c_i + 2a_i \frac{h}{v} + b_i \frac{h^2}{v^2} + \cdots$

Bounds from Higgs Analysis (Set A → SM–like $HVV$ and $Hff$ shifted simultaneously):

Higgs Only, *No TGC bounds*
Best fit near $c_i = 0$ (SM)
$Hgg$ degeneracy from $a_G \sim -2$SM-loop
$H\gamma\gamma \rightarrow$ anticorrelation between $a_W$ & $a_B$
Discriminating Linear from Chiral Realizations

Recalling the relation between $O_B$ and $P_2, P_4$ and $O_W$ and $P_3, P_5$:

$$O_B = \frac{v^2}{16} P_2 + \frac{v^2}{8} P_4 \quad O_W = \frac{v^2}{8} P_3 - \frac{v^2}{4} P_5$$

We define the discriminating quantities (using a similar relation for $O_W$ and $P_3, P_5$):

$$\Sigma_B \equiv 4(2c_2 + a_4) \quad \Sigma_W \equiv 2(c_3 - a_5), \quad \Sigma_{B(W)} \rightarrow \frac{f_{B(W)} v^2}{\Lambda^2}$$

$$\Delta_B \equiv 4(2c_2 - a_4) \quad \Delta_W \equiv 2(c_3 + a_5), \quad \Delta_B = \Delta_W \rightarrow 0$$