# Effective Lagrangians for Higgs Physics

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Based on Arxiv: 1411.5026, 1311.1823, 1304.1151, 1211.4580, and 1207.1344

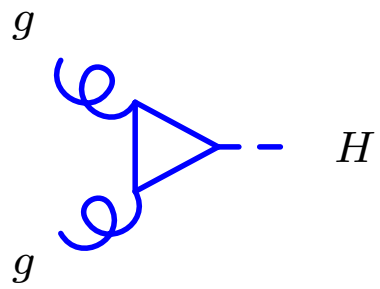
In collaboration with: MC Gonzalez-Garcia, J Gonzalez-Fraile, OJP Éboli, I Brivio, B Gavela, L Merlo, & S Rigolin

# Outline


1. Standard Model Higgs
2. Effective Lagrangians
3. Linear EFTs for Higgs Physics
   - Status after LHC runs
   - Unitarity Bounds
4. Chiral Expansion
5. Conclusions

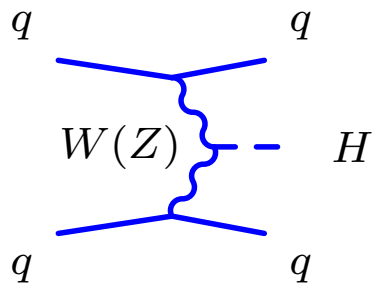
# The SM Higgs

- In the SM all Higgs couplings are fully determined, except $M_H$
- The Higgs is produced at the LHC via:




Gluon Gluon Fusion

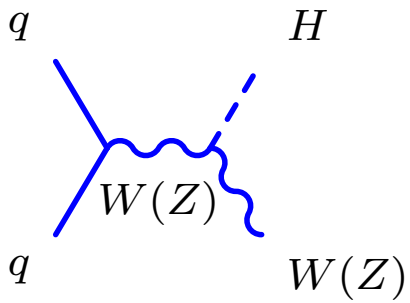
$\sim$ 15 pb




Vector Boson Fusion

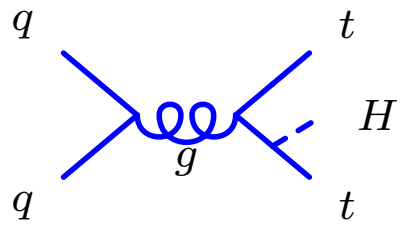
$\sim$ 1.2 pb

Associate Production :




With W(Z)

$\sim$ .6(.3) pb

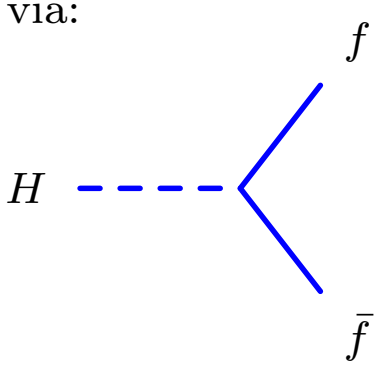


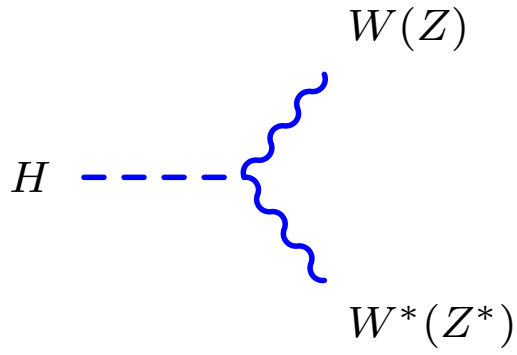

With $t\bar{t}$

$\sim$ .09 pb

# The SM Higgs

- In the SM all Higgs couplings are fully determined, except $M_H$
- The Higgs is produced at the LHC via: GGF, VBF, VH, $t\bar{t}$
- And decays via:



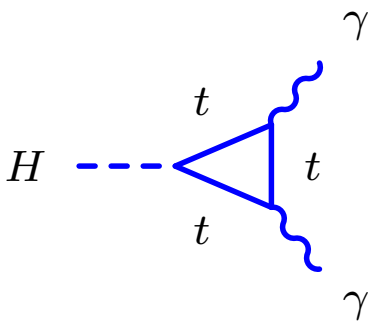



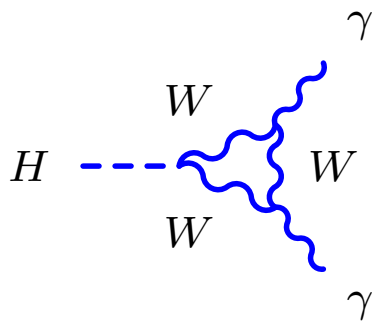

$\text{BR}_{\bar{b}b} \sim 58\%$

$\text{BR}_{WW^*} \sim 22\%$

$\text{BR}_{\bar{\tau}\tau} \sim 6\%$

$\text{BR}_{ZZ^*} \sim 3\%$

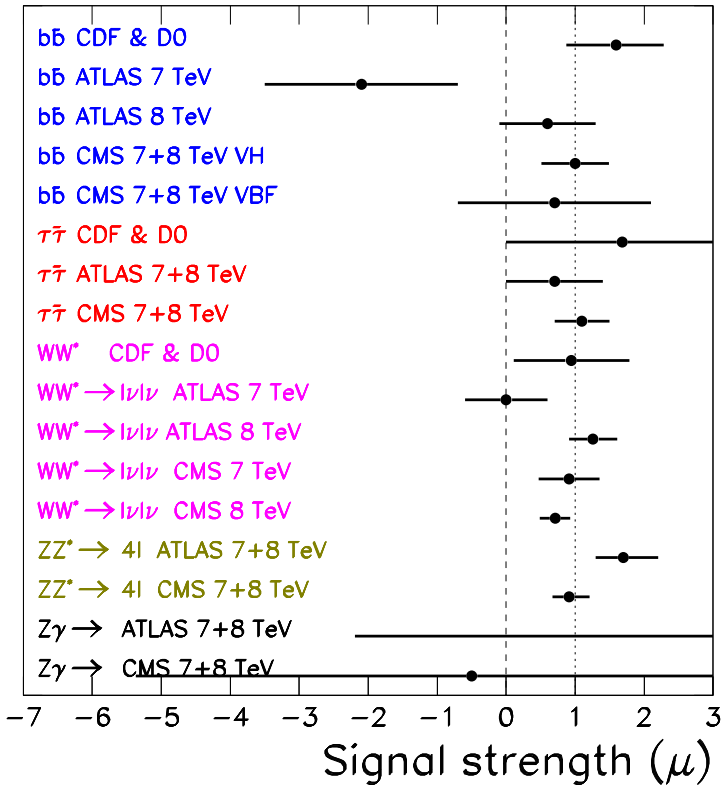


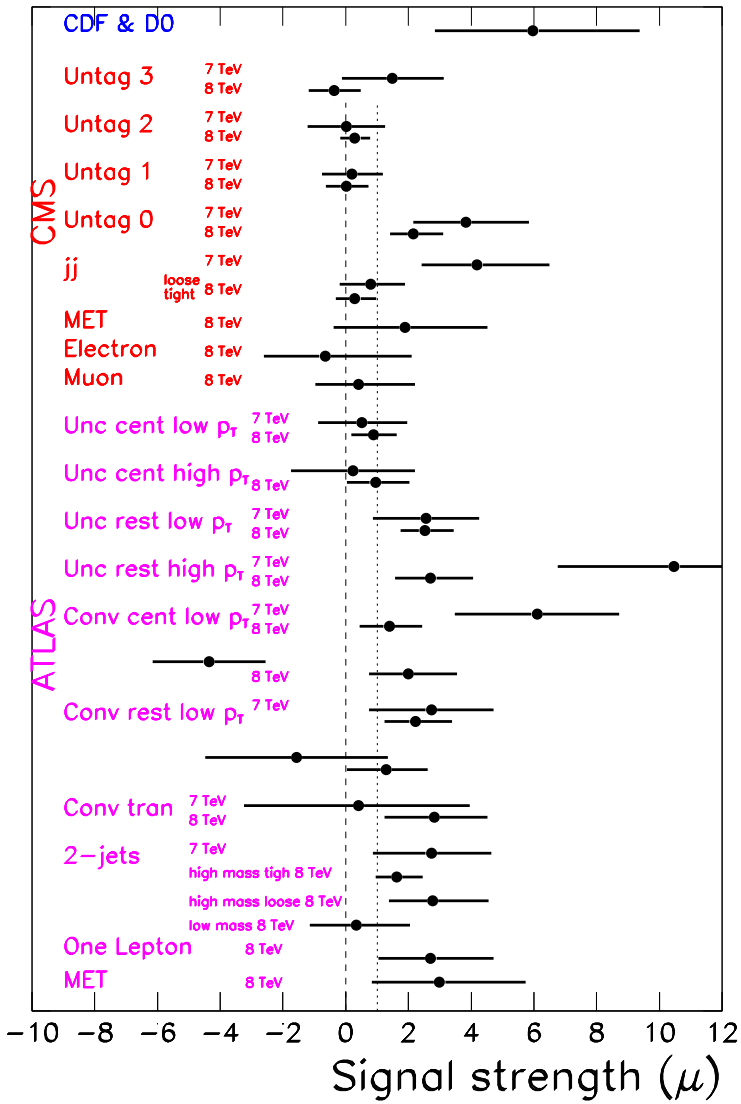

$\text{BR}_{\gamma\gamma} \sim .2\%$

# Higgs Discovery

In 2012 the first strong evidence of the discovery of the Higgs was released.




$\gamma\gamma$



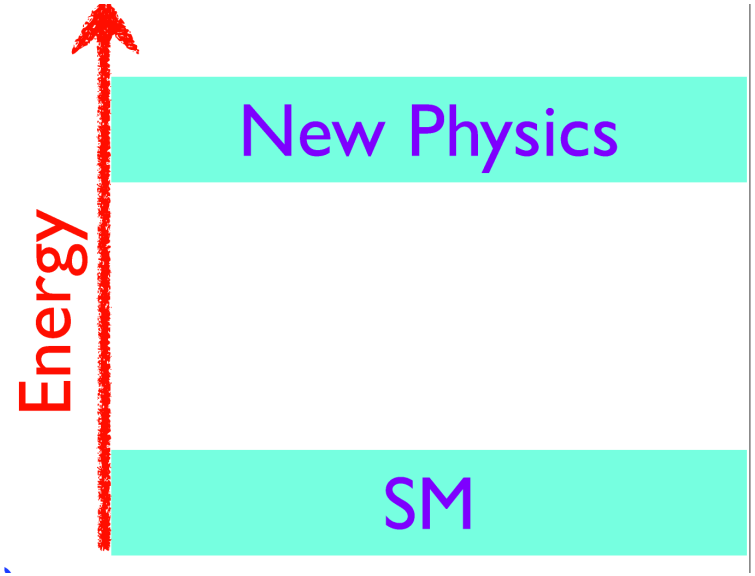

$$\mu \equiv \frac{(\sigma \times \mathrm{BR})_{\mathrm{Obs}}}{(\sigma \times \mathrm{BR})_{\mathrm{SM}}}$$

# Effective Field Theories

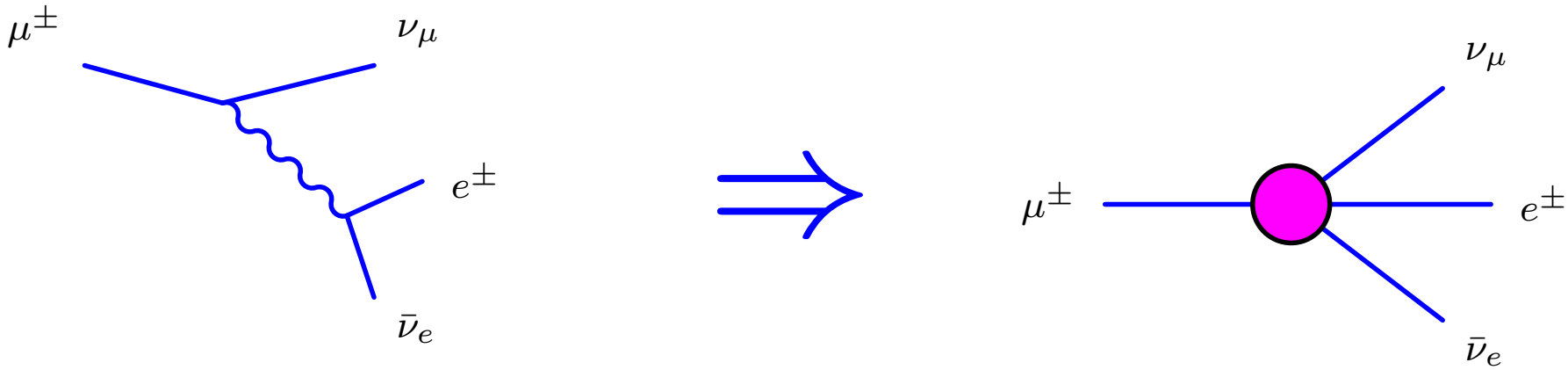
We wish to quantify deviations from the predictions of the SM Higgs Mechanism,
$\rightarrow$ consistent framework for doing so is an effective field theory (EFT)

We form an effective field theory via the assumptions:

- There is a mass gap between the low energy theory (SM) and the new physics
- The new physics occurs at some scale $\Lambda \sim M_{\text{new}}$
- For $\Lambda \sim M_{\text{new}} \rightarrow \infty$ the new physics decouples
  $\Rightarrow$ induces corrections $\propto 1/\Lambda^n \rightarrow$ higher dimensional operators



# Example: Muon Decay




Fermi theory of muon decay: the exchange of a $W$ boson $\rightarrow$ dimension–six operator,

$$\bar{\Psi}_L\Psi_L\left(\frac{1}{p^2-M_W^2}\right)\bar{\Psi}_L\Psi_L \quad \rightarrow \quad -\bar{\Psi}_L\Psi_L\frac{1}{M_W^2}\left(1+\frac{p^2}{M_W^2}+\cdots\right)\bar{\Psi}_L\Psi_L$$

$$\rightarrow \quad \frac{1}{M_W^2}\bar{\Psi}_L\Psi_L\bar{\Psi}_L\Psi_L$$

- To $\mathcal{O}(\frac{1}{M_W^2})$ this new dimension–six operator describes muon decay.
- For sufficiently low momenta, $p^2 \ll M_W^2$, this describes the process well.
- For better precision we can extend to the next order, dimension–eight, or $\mathcal{O}(\frac{p^2}{M_W^4})$ $\rightarrow$ well defined procedure to quantify deviations from low energy predicted behavior.

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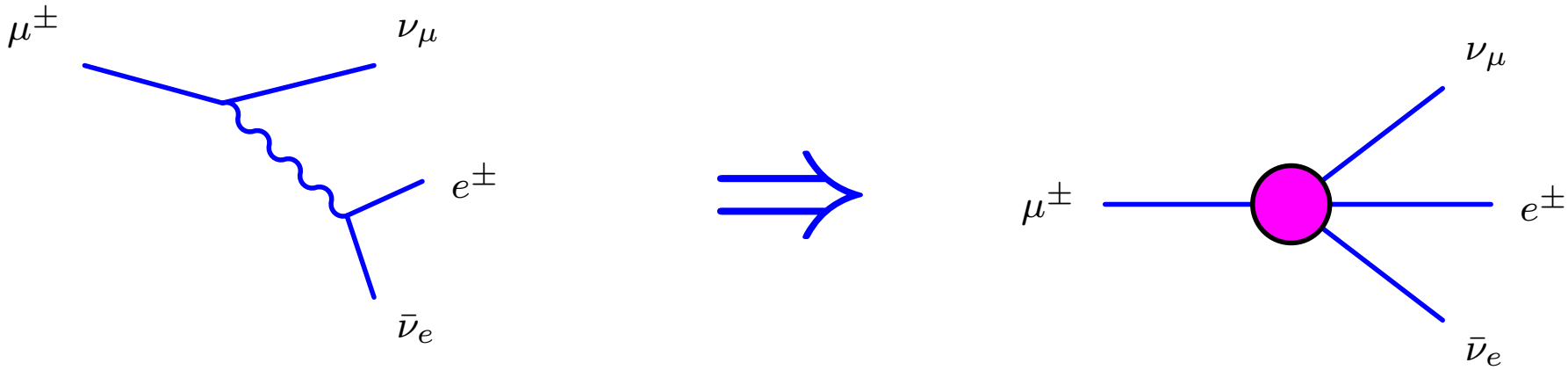


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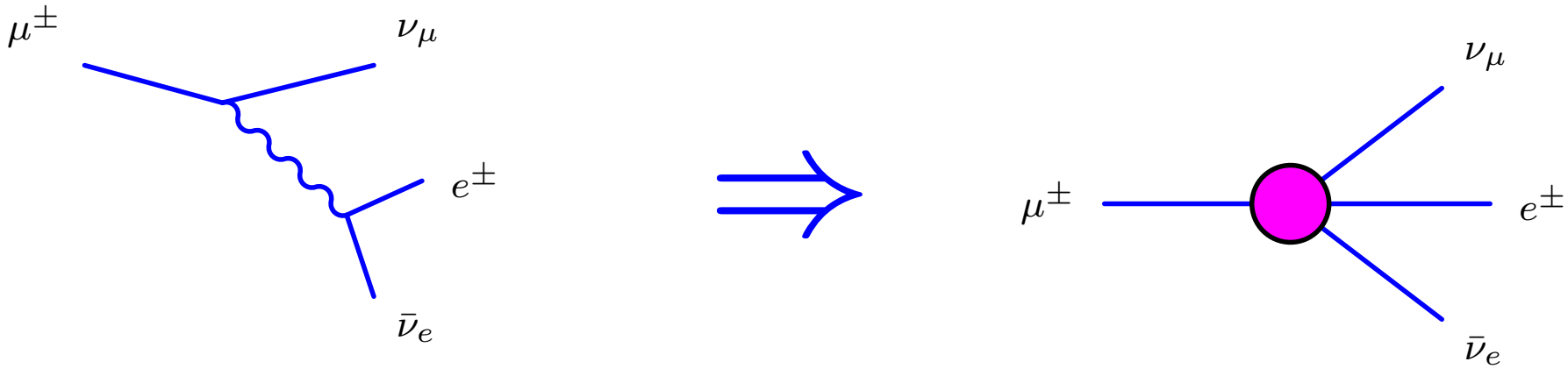


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# Linear EFTs for Higgs Physics

We're interested in the EFT for Higgs Physics.

Effective operators are formed from the field content,

$$e_R, \quad u_R, \quad d_R, \quad L_L, \quad Q_L, \quad \Phi, \quad W_\mu, \quad Z_\mu, \quad A_\mu, \quad G_\mu$$

And their (covariant) derivatives.

The operators are then formed of all $U(1)_{Y/2} \times SU(2)_L \times SU(3)_C$ invariant combinations.

Further

- restrict to dimension–six, i.e. assume all NP occurs at a sufficiently high $\Lambda$
- assume baryon and lepton numbers are conserved ($\rightarrow$ no dimension–5 operators)
- assume operators are $CP-$even

Then we write the linear EFT to dimension–6 as:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_n \frac{f_n}{\Lambda^2} \mathcal{O}_n$$

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# Higgs–Gauge Boson Operators

The relevant Higgs–gauge boson operators are:

$$\mathcal{O}_{BB} = \Phi^\dagger \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} \Phi \qquad \mathcal{O}_{BW} = \Phi^\dagger \hat{B}_{\mu\nu} \hat{W}^{\mu\nu} \Phi \qquad \mathcal{O}_{WW} = \Phi^\dagger \hat{W}_{\mu\nu} \hat{W}^{\mu\nu} \Phi$$

$$\mathcal{O}_{GG} = \Phi^\dagger \Phi G^a_{\mu\nu} G_a^{\mu\nu} \qquad \mathcal{O}_{B} = (D_\mu \Phi)^\dagger \hat{B}^{\mu\nu} (D_\nu \Phi) \qquad \mathcal{O}_{W} = (D_\mu \Phi)^\dagger \hat{W}^{\mu\nu} (D_\nu \Phi)$$

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Unitary Gauge:

$$\Phi \to \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+H \end{pmatrix}$$

Operators in blue contain derivatives of H $\to$ finite wavefunction renormalizations:

$$H = h\left[1 + \frac{v^2}{2\Lambda^2}(f_{\phi,1} + 2f_{\phi,2} + f_{\phi,4})\right]^{1/2}.$$

$\to$ rescaling of all Higgs couplings.

$\to$ correction to $Z$ gauge–boson propagator (not $W$):

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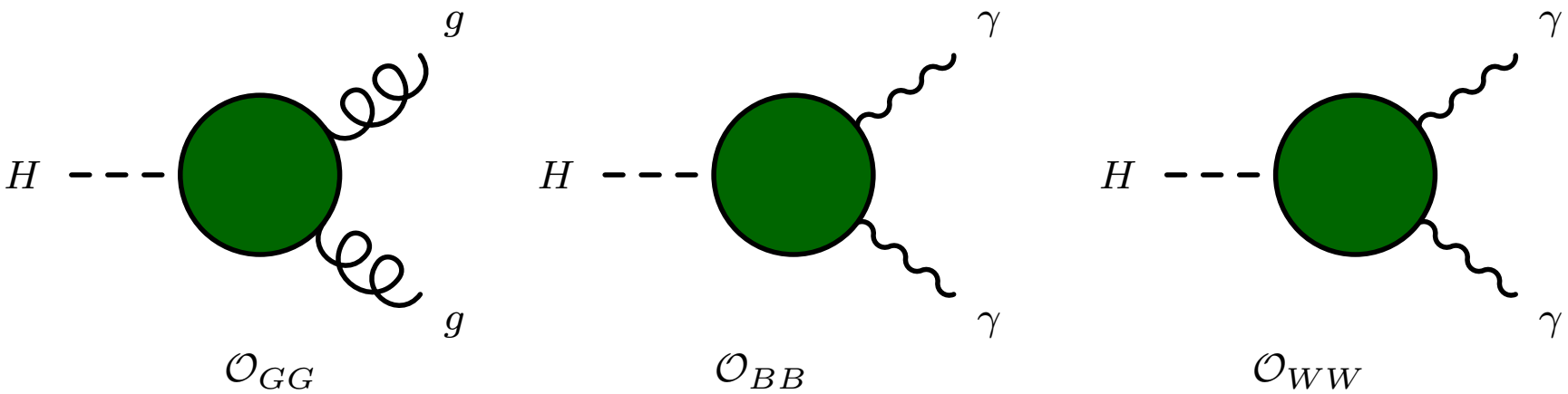
Green $\mathcal{O}$ $\leftrightarrow$ loop effects for new particles charged under the SM gauge group:




Also $\mathcal{O}_{BW}$ results in tree level mixing between $Z$ and $\gamma$:

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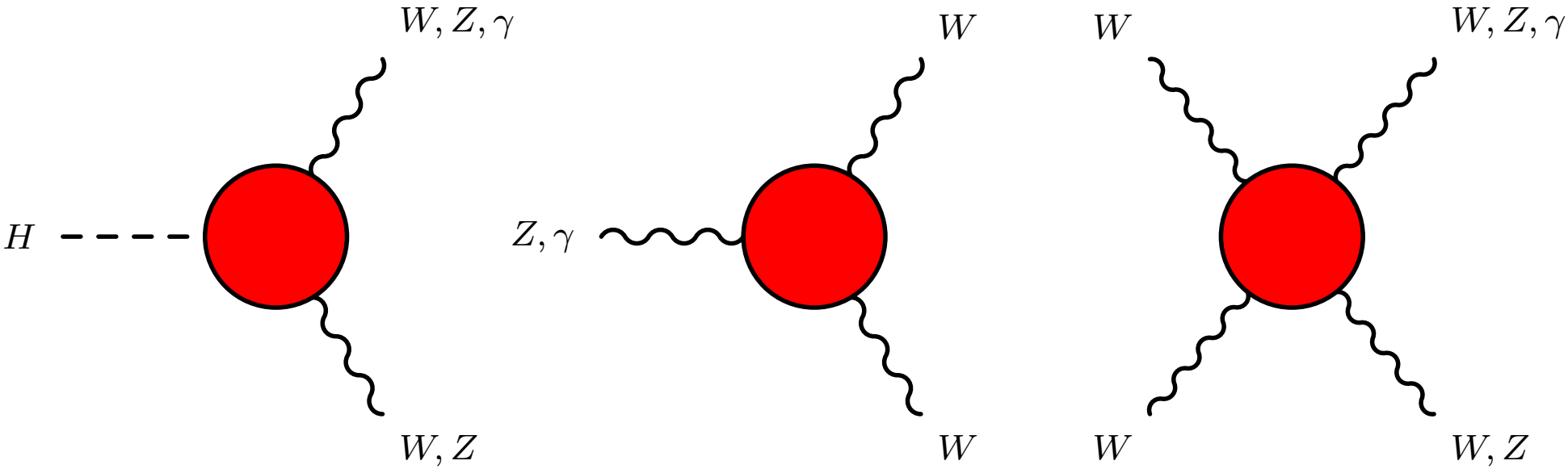
Red $\mathcal{O}$ $\rightarrow$ simultaneous shifts of $HVV$, $VVV$, and $VVVV$ vertices:

$$D_\mu\Phi = (\partial_\mu + i\frac{1}{2}g'B_\mu + ig\frac{\tau_a}{2}W^a_\mu)\Phi$$



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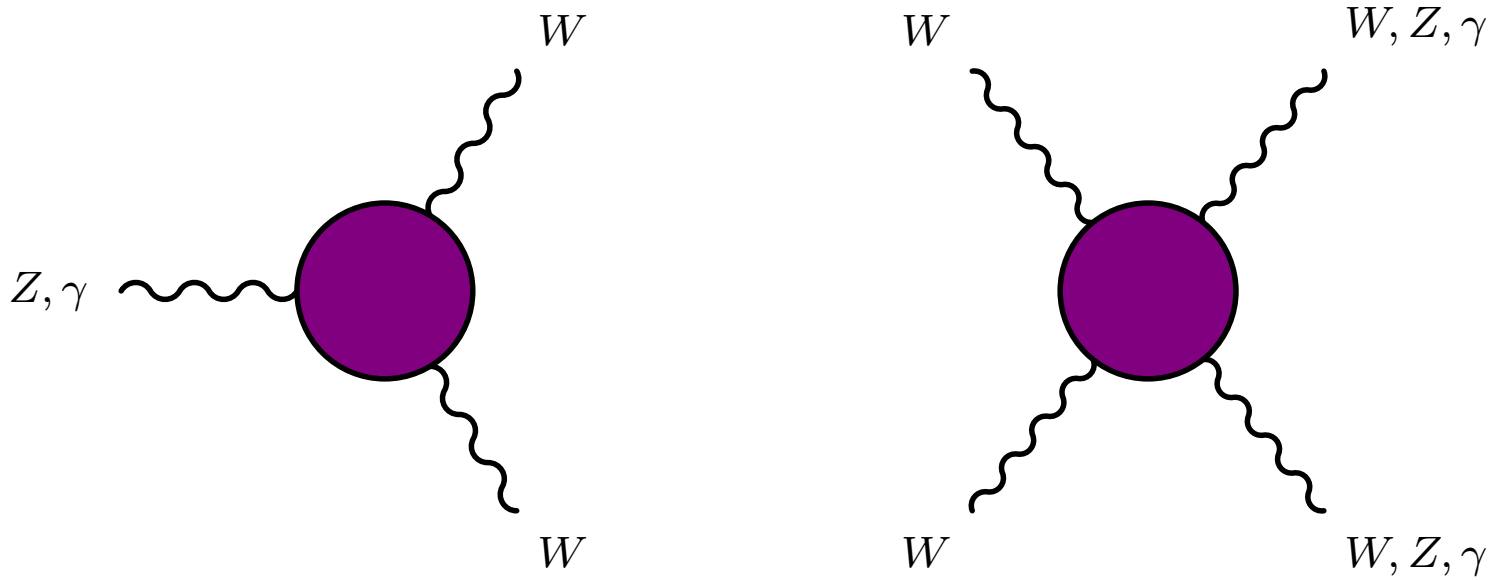
Purple $\mathcal{O}$ $\leftrightarrow$ Triple–Gauge and Quartic–Gauge couplings:



# Higgs–Fermion Operators

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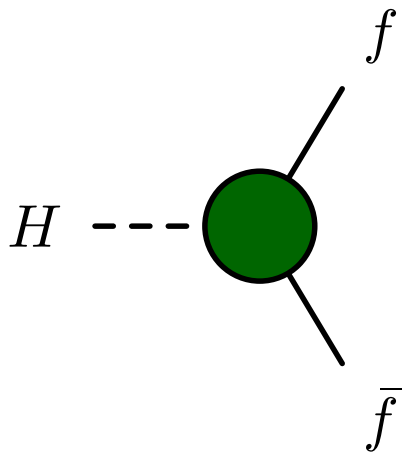
$$\mathcal{O}_{e\Phi,ij} = (\Phi^\dagger\Phi)(\bar{L}_{Li}\Phi e_{R_j})$$
$$\mathcal{O}_{u\Phi,ij} = (\Phi^\dagger\Phi)(\bar{Q}_{Li}\tilde{\Phi}u_{R_j})$$
$$\mathcal{O}_{d\Phi,ij} = (\Phi^\dagger\Phi)(\bar{Q}_{Li}\Phi d_{Rj})$$




$$\mathcal{O}^{(1)}_{\Phi L,ij} = \Phi^\dagger(i\overleftrightarrow{D}_\mu\Phi)(\bar{L}_{Li}\gamma^\mu L_{Lj})$$
$$\mathcal{O}^{(1)}_{\Phi Q,ij} = \Phi^\dagger(i\overleftrightarrow{D}_\mu\Phi)(\bar{Q}_{Li}\gamma^\mu Q_{Lj})$$
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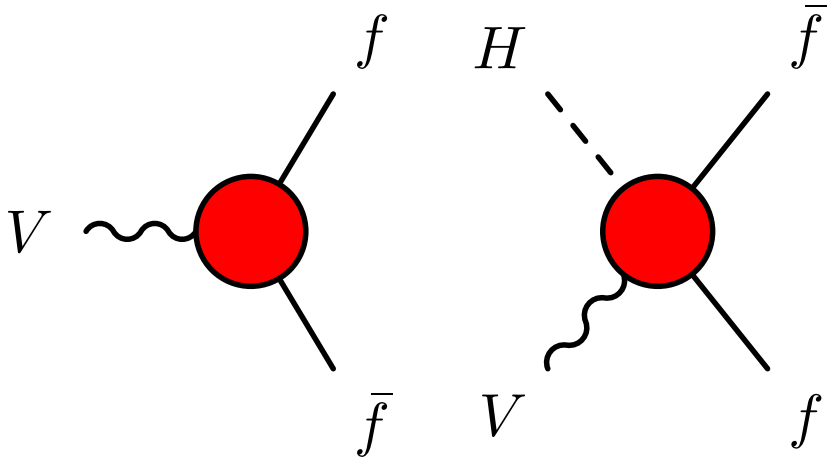
$$\mathcal{O}^{(1)}_{\Phi ud,ij} = \tilde{\Phi}^\dagger(i\overleftrightarrow{D}_\mu\Phi)(\bar{u}_{R_i}\gamma^\mu d_{R_j})$$
$$\mathcal{O}^{(3)}_{\Phi L,ij} = \Phi^\dagger(i\overleftrightarrow{D}^a_\mu\Phi)(\bar{L}_{Li}\gamma^\mu\tau_a L_{Lj})$$
$$\mathcal{O}^{(3)}_{\Phi Q,ij} = \Phi^\dagger(i\overleftrightarrow{D}^a_\mu\Phi)(\bar{Q}_{Li}\gamma^\mu\tau_a Q_{Lj})$$

- Overall scaling of Yukawas $\rightarrow \delta m_f$
- Independent rescaling of $Hff$ couplings

- Contributes to $Vff$ couplings
- Strongly constrained by precision data



# Equations of Motion

Not all operators are independent
→ they may be related by the EOM (Politzer '80;Georgi '91;Artz '98;Simma '94):

$$2\mathcal{O}_{\Phi,2} + 2\mathcal{O}_{\Phi,4} = \sum_{ij}\left(y^e_{ij}(\mathcal{O}_{e\Phi,ij})^\dagger + y^u_{ij}\mathcal{O}_{u\Phi,ij} + y^d_{ij}(\mathcal{O}_{d\Phi,ij})^\dagger + \text{h.c.}\right) - 2(\Phi^\dagger\Phi)\Phi^\dagger\frac{\partial V}{\partial\Phi^\dagger}$$

$$2\mathcal{O}_B + \mathcal{O}_{BW} + \mathcal{O}_{BB} + g'^2\left(\mathcal{O}_{\Phi,1} - \tfrac{1}{2}\mathcal{O}_{\Phi,2}\right) = -\frac{g'^2}{2}\sum_i\Big(-\tfrac{1}{2}\mathcal{O}^{(1)}_{\Phi L,ii} + \tfrac{1}{6}\mathcal{O}^{(1)}_{\Phi Q,ii} - \mathcal{O}^{(1)}_{\Phi e,ii} + \tfrac{2}{3}\mathcal{O}^{(1)}_{\Phi u,ii} - \tfrac{1}{3}\mathcal{O}^{(1)}_{\Phi d,ii}\Big)$$

$$2\mathcal{O}_W + \mathcal{O}_{BW} + \mathcal{O}_{WW} + g^2\left(\mathcal{O}_{\Phi,4} - \tfrac{1}{2}\mathcal{O}_{\Phi,2}\right) = -\frac{g^2}{4}\sum_i\left(\mathcal{O}^{(3)}_{\Phi L,ii} + \mathcal{O}^{(3)}_{\Phi Q,ii}\right)$$

With this we may remove 3 operators from our basis.

In doing so we use a data driven approach:

- Avoid theoretical prejudice – assumptions on tree vs. loop level of individual operators
- Choose a basis which is more easily related to available data

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Not all operators are independent
→ they may be related by the EOM (Politzer '80;Georgi '91;Artz '98;Simma '94):

$$2\mathcal{O}_{\Phi,2} + 2\mathcal{O}_{\Phi,4} = \sum_{ij}\left(y^e_{ij}(\mathcal{O}_{e\Phi,ij})^\dagger + y^u_{ij}\mathcal{O}_{u\Phi,ij} + y^d_{ij}(\mathcal{O}_{d\Phi,ij})^\dagger + \text{h.c.}\right) - 2(\Phi^\dagger\Phi)\Phi^\dagger\frac{\partial V}{\partial\Phi^\dagger}$$

$$2\mathcal{O}_B + \mathcal{O}_{BW} + \mathcal{O}_{BB} + g'^2\left(\mathcal{O}_{\Phi,1} - \tfrac{1}{2}\mathcal{O}_{\Phi,2}\right) = -\frac{g'^2}{2}\sum_i\Big(-\tfrac{1}{2}\mathcal{O}^{(1)}_{\Phi L,ii} + \tfrac{1}{6}\mathcal{O}^{(1)}_{\Phi Q,ii} - \mathcal{O}^{(1)}_{\Phi e,ii} + \tfrac{2}{3}\mathcal{O}^{(1)}_{\Phi u,ii} - \tfrac{1}{3}\mathcal{O}^{(1)}_{\Phi d,ii}\Big)$$

$$2\mathcal{O}_W + \mathcal{O}_{BW} + \mathcal{O}_{WW} + g^2\left(\mathcal{O}_{\Phi,4} - \tfrac{1}{2}\mathcal{O}_{\Phi,2}\right) = -\frac{g^2}{4}\sum_i\left(\mathcal{O}^{(3)}_{\Phi L,ii} + \mathcal{O}^{(3)}_{\Phi Q,ii}\right)$$

With this we may remove 3 operators from our basis.

In doing so we use a data driven approach:

- Avoid theoretical prejudice – assumptions on tree vs. loop level of individual operators
- Choose a basis which is more easily related to available data

# Most Constrained Operators:

We recall (constrained by electroweak precision data):

Additionally some of our fermionic operators are strongly constrained ($Vff$):
(by $Z$–pole physics, atomic parity violation, etc)

$$\mathcal{O}^{(1)}_{\Phi L,ij} = \Phi^\dagger (i \overleftrightarrow{D}_\mu \Phi)(\bar{L}_{Li} \gamma^\mu L_{Lj}) \qquad \mathcal{O}^{(3)}_{\Phi L,ij} = \Phi^\dagger (i \overleftrightarrow{D}^a_\mu \Phi)(\bar{L}_{Li} \gamma^\mu \tau_a L_{Lj})$$
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But EWPD does not constrain $Vff$ and gauge–boson self–energies independently!
→ "Blind Directions" (de Rujula, Gavela, Masso, Hernandez, '94; Elias-Miro et al '13)
→ We are able to constrain all but two fermionic operators simultaneously using EWPD
(see arXiv: 1311.1823)

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$\rightarrow \Delta T \propto f_{\Phi,1}$

$\rightarrow \Delta S \propto f_{BW}$

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# Triple Gauge Couplings

Recalling $\mathcal{O}_W$ and $\mathcal{O}_B$ affect $HVV$ and $VVV$ vertices,

$$\mathcal{O}_B = (D^\mu \Phi)^\dagger \hat{B}_{\mu\nu} (D^\nu \Phi) \qquad \mathcal{O}_W = (D^\mu \Phi)^\dagger \hat{W}_{\mu\nu} (D^\nu \Phi)$$

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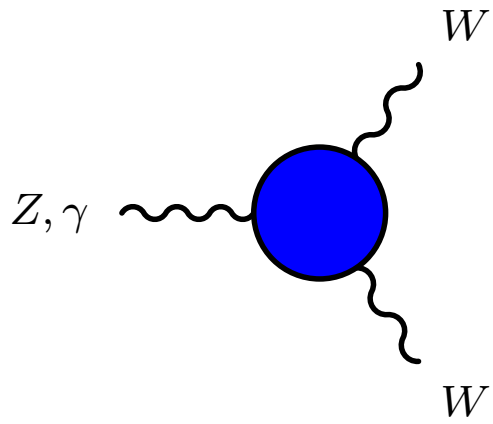
$\rightarrow$ keep in the basis as can be constrained from $VVV$(TGC) measurements.

$$\mathcal{L}_{WWV} = -ig_{WWV}\left[g_1^V (W^+_{\mu\nu} W^{-\mu} V^\nu - W^+_\mu V_\nu W^{-\mu\nu}) + \kappa_V W^+_\mu W^-_\nu V^{\mu\nu} + \frac{\lambda_V}{m_W^2} W^+_{\mu\nu} W^{-\nu\rho} V^\mu_\rho\right]$$




$$\begin{aligned}
\Delta g_1^Z &= g_1^Z - 1 = \frac{g^2 v^2}{8 c_W^2 \Lambda^2}\left(f_W + 2\frac{s_W^2}{c_W^2 - s_W^2} f_{BW}\right) - \frac{1}{4(c_W^2 - s_W^2)} f_{\Phi,1} \frac{v^2}{\Lambda^2} \\
\Delta\kappa_\gamma &= \kappa_\gamma - 1 = \frac{g^2 v^2}{8\Lambda^2}(f_W + f_B - 2 f_{BW}) \\
\Delta\kappa_Z &= \kappa_Z - 1 = \frac{g^2 v^2}{8 c_W^2 \Lambda^2}\left(c_W^2 f_W - s_W^2 f_B + \frac{4 s_W^2 c_W^2}{c_W^2 - s_W^2} f_{BW}\right) - \frac{1}{4(c_W^2 - s_W^2)} f_{\Phi,1} \frac{v^2}{\Lambda^2}
\end{aligned}$$

(Hagiwara, Peccei, Zeppenfeld, Hikasa '86)

# Reduction of Basis

Using the EOM we eliminate $\mathcal{O}_{\Phi,4}$ and two fermionic operators,
$\rightarrow$ remaining operator basis is then:

$$\mathcal{O}_{GG},\ \mathcal{O}_{BW},\ \mathcal{O}_{WW},\ \mathcal{O}_{BB},\ \mathcal{O}_{W},\ \mathcal{O}_{B},\ \mathcal{O}_{\Phi,1},\ \mathcal{O}_{\Phi,2},\ \mathcal{O}_{f\Phi},\ \mathcal{O}^{(1)}_{\Phi f},\ \mathcal{O}^{(3)}_{\Phi f}$$

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Pre–Higgs EWPD strongly constrained: $\times$

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FCNC constrain off diagonal $\mathcal{O}_{f\Phi}$:

Then we have 15 operators:

- 6 involving gauge bosons: $\mathcal{O}_{GG},\ \mathcal{O}_{WW},\ \mathcal{O}_{BB},\ \mathcal{O}_{W},\ \mathcal{O}_{B},\ \mathcal{O}_{\Phi,2}$
- 9 involving fermions: $\mathcal{O}_{e\Phi,ii},\ \mathcal{O}_{u\Phi,ii},\ \mathcal{O}_{d\Phi,ii}$
- Neglect fermion couplings to $1^{\text{st}}$ and $2^{\text{nd}}$ generations: Not resolvable at current precision
- Insufficient data on $ttH$ production to resolve $\mathcal{O}_{u\Phi,33}$ from $\mathcal{O}_{WW},\ \mathcal{O}_{BB},\ \mathcal{O}_{\Phi,2},\ \mathcal{O}_{GG}$
  $\Rightarrow$Only two fermionic operators accessible with current precision: $\mathcal{O}_{d\Phi,33}$ & $\mathcal{O}_{e\Phi,33}$

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* w/ most recent data ttH coupling may be resolved independently,
  arXiv: 1505.05516, TC, J Gonzalez-Fraile, OJP Éboli, D Gonçalves, T Plehn, M Rauch

# Final Basis

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} - \frac{\alpha_s v}{8\pi}\frac{f_g}{\Lambda^2}\mathcal{O}_{GG} + \frac{f_{WW}}{\Lambda^2}\mathcal{O}_{WW} + \frac{f_{BB}}{\Lambda^2}\mathcal{O}_{BB} + \frac{f_W}{\Lambda^2}\mathcal{O}_W + \frac{f_B}{\Lambda^2}\mathcal{O}_B + \frac{f_{\Phi,2}}{\Lambda^2}\mathcal{O}_{\Phi,2}$$
$$+ \frac{f_{\text{bot}}}{\Lambda^2}\mathcal{O}_{d\Phi,33} + \frac{f_\tau}{\Lambda^2}\mathcal{O}_{e\Phi,33}$$

Operator effects on observables (correlations result from gauge invariance):

| | $\mathcal{O}_{GG}$ | $\mathcal{O}_{\text{BB}}$ | $\mathcal{O}_{\text{WW}}$ | $\mathcal{O}_{\text{B}}$ | $\mathcal{O}_{\text{W}}$ | $\mathcal{O}_{\Phi,2}$ |
|---|---|---|---|---|---|---|
| $hgg$ | ✘ | | | | | |
| $h\gamma\gamma$ | | ✘ | ✘ | | | |
| $hZ\gamma$ | | ✘ | ✘ | ✘ | ✘ | |
| $hZZ$ | | ✘ | ✘ | ✘ | ✘ | ✘ |
| $HW^+W^-$ | | | ✘ | | ✘ | ✘ |
| $\gamma W^+W^-$ | | | | ✘ | ✘ | |
| $ZW^+W^-$ | | | | ✘ | ✘ | |

+ shifts for bottom and $\tau$ Yukawa couplings

→ Different Lorentz structures from SM tree level
→ includes correlations required by gauge invariance

# Analysis Framework

We will constrain the effective operator basis using a $\chi^2$–statistic:

$$\chi^2 = \min_{\xi_{\text{pull}}} \sum_j \frac{(\mu_j - \mu_j^{\text{exp}})^2}{\sigma_j^2} + \sum_{\text{pull}} \left( \frac{\xi_{\text{pull}}}{\sigma_{\text{pull}}} \right)^2$$

where:

$$\mu_F = \frac{\epsilon_{gg}^F \sigma_{gg}^{\text{ano}}(1+\xi_g) + \epsilon_{\text{VBF}}^F \sigma_{\text{VBF}}^{\text{ano}}(1+\xi_{\text{VBF}}) + \epsilon_{WH}^F \sigma_{WH}^{\text{ano}}(1+\xi_{VH}) + \epsilon_{ZH}^F \sigma_{ZH}^{\text{ano}}(1+\xi_{VH}) + \epsilon_{t\bar{t}H}^F \sigma_{t\bar{t}H}^{\text{ano}}}{\epsilon_{gg}^F \sigma_{gg}^{\text{SM}} + \epsilon_{\text{VBF}}^F \sigma_{\text{VBF}}^{\text{SM}} + \epsilon_{WH}^F \sigma_{WH}^{\text{SM}} + \epsilon_{ZH}^F \sigma_{ZH}^{\text{SM}} + \epsilon_{t\bar{t}H}^F \sigma_{t\bar{t}H}^{\text{SM}}} \cdot \frac{\text{BR}_F^{\text{ano}}}{\text{BR}_F^{\text{SM}}}$$

- $\epsilon_{\text{Prod}}^F$ given by experiments for SM kinematics (∼same for anomalous, Banerjee et al.)
- $\sigma_i^{\text{SM}}$ and $\Gamma_i^{\text{SM}}$ are known to 1 or 2 loops

To obtain higher orders in the anomalous contributions, scale by SM higher order effects:

$$\sigma_Y^{\text{ano}} = \left. \frac{\sigma_Y^{\text{ano}}}{\sigma_Y^{\text{SM}}} \right|_{\text{tree}} \sigma_Y^{\text{SM}}|_{\text{soa}}$$

$$\Gamma^{\text{ano}}(h \to F) = \left. \frac{\Gamma^{\text{ano}}(h \to F)}{\Gamma^{\text{SM}}(h \to F)} \right|_{\text{tree}} \Gamma^{\text{SM}}(h \to F)|_{\text{soa}}$$

# Inclusion of TGCs

Additionally we will use TGC measurements to further constrain the operators:

$$\mathcal{L}_{WWV} = -ig_{WWV}\left[g_1^V(W^+_{\mu\nu}W^{-\mu}V^{\nu} - W^+_{\mu}V_{\nu}W^{-\mu\nu}) + \kappa_V W^+_{\mu}W^-_{\nu}V^{\mu\nu}\right]$$

$$\begin{aligned}
\Delta g_1^Z &= g_1^Z - 1 &&= \frac{g^2v^2}{8c_{\rm W}^2\Lambda^2} f_W \\
\Delta\kappa_\gamma &= \kappa_\gamma - 1 &&= \frac{g^2v^2}{8\Lambda^2}(f_W + f_B) \\
\Delta\kappa_Z &= \kappa_Z - 1 &&= \frac{g^2v^2}{8c_{\rm W}^2\Lambda^2}\left(c_{\rm W}^2 f_W - s_{\rm W}^2 f_B\right)
\end{aligned}$$

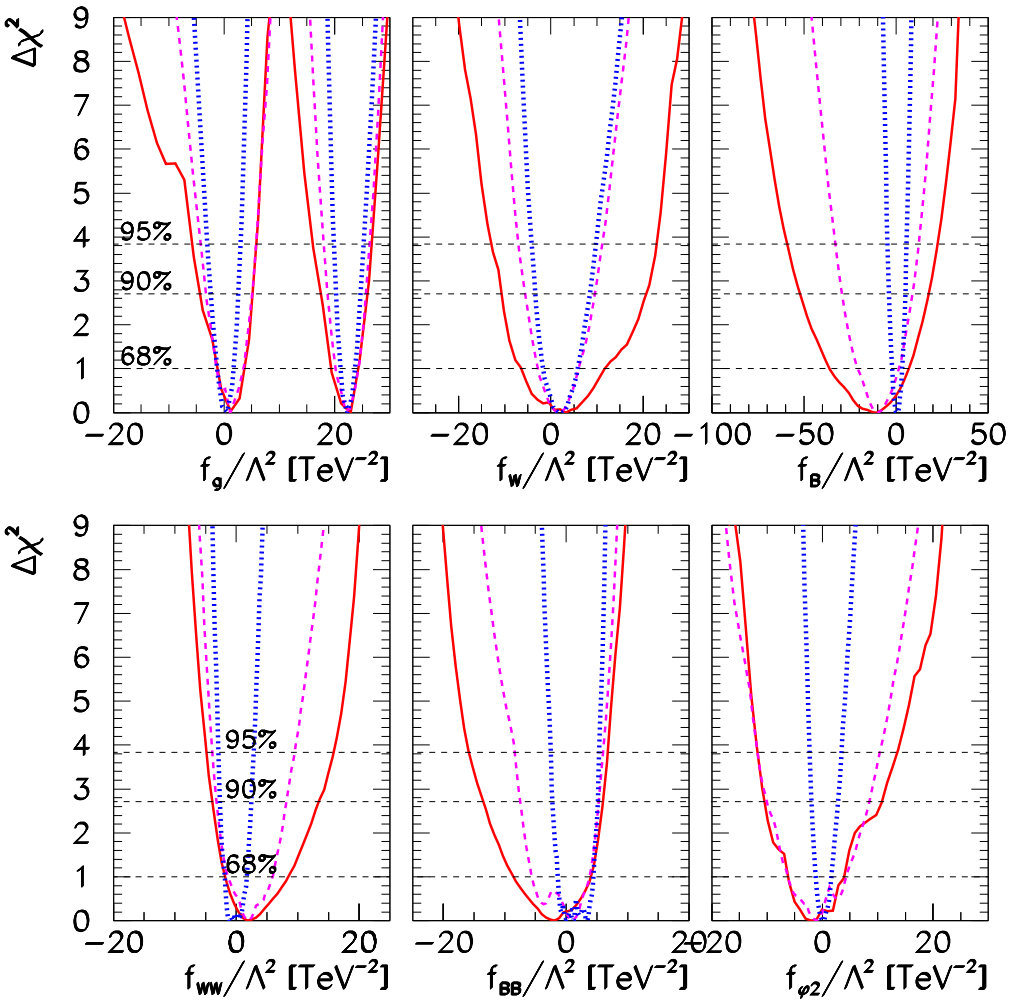
From LEP data, the best fit values and correlation $\rho$ given by:

$$\kappa_\gamma = 0.984^{+0.049}_{-0.049}, \qquad g_1^Z = 1.004^{+0.024}_{-0.025}, \qquad \rho = 0.11$$

* Also include EWPD effects at one loop in arXiv: 1211.4580

# Results: Higgs + TGC + EWPD ($\mu = \Lambda = 10$ TeV)

First consider no fermionic operators (i.e. $f_W$, $f_B$, $f_{WW}$, $f_{BB}$, $f_{\Phi,2}$, $f_{\rm bot} = f_\tau = 0$):




Best fit near $f_i = 0$ (SM)

Higgs data only

Higgs + TGV

Higgs + TGV + EWPD

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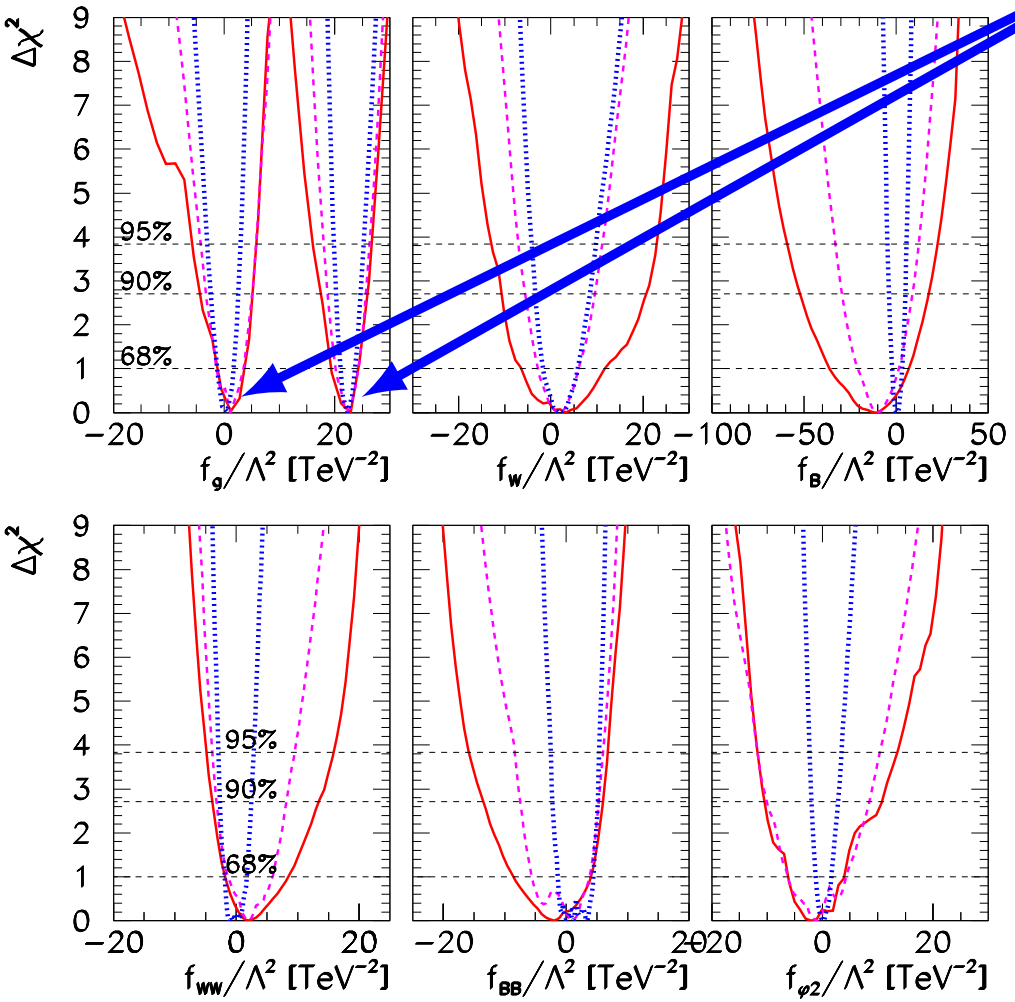
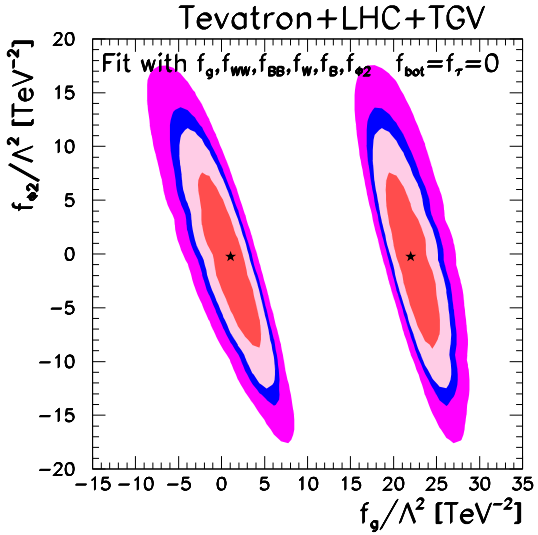
Interference with SM $\rightarrow$ degeneracy



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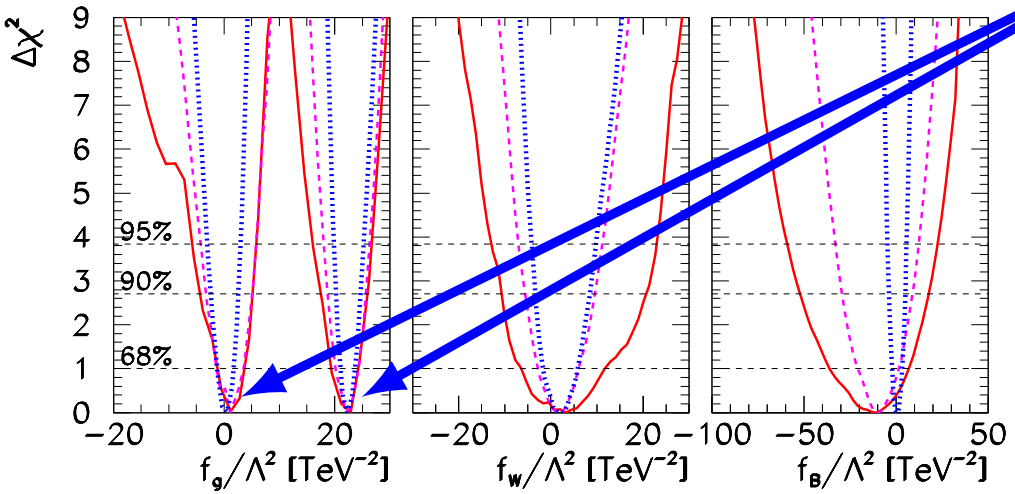
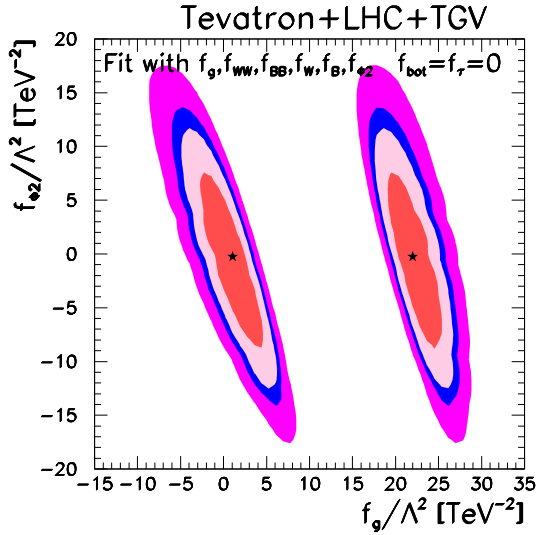
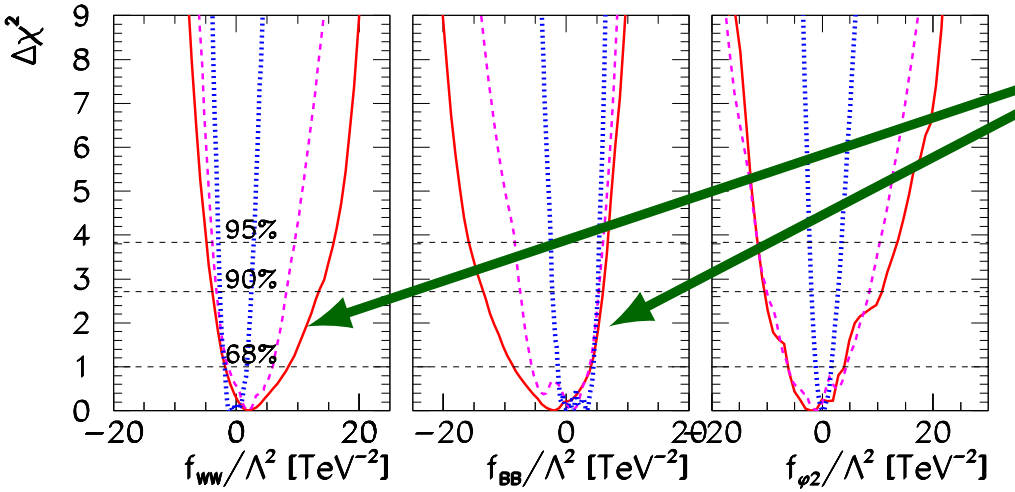
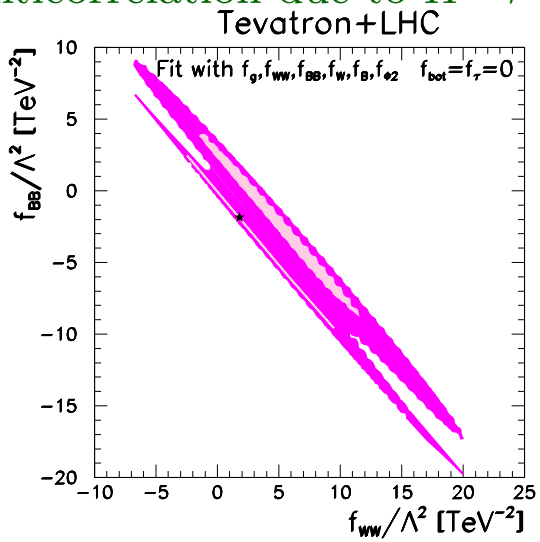



Interference with SM → degeneracy



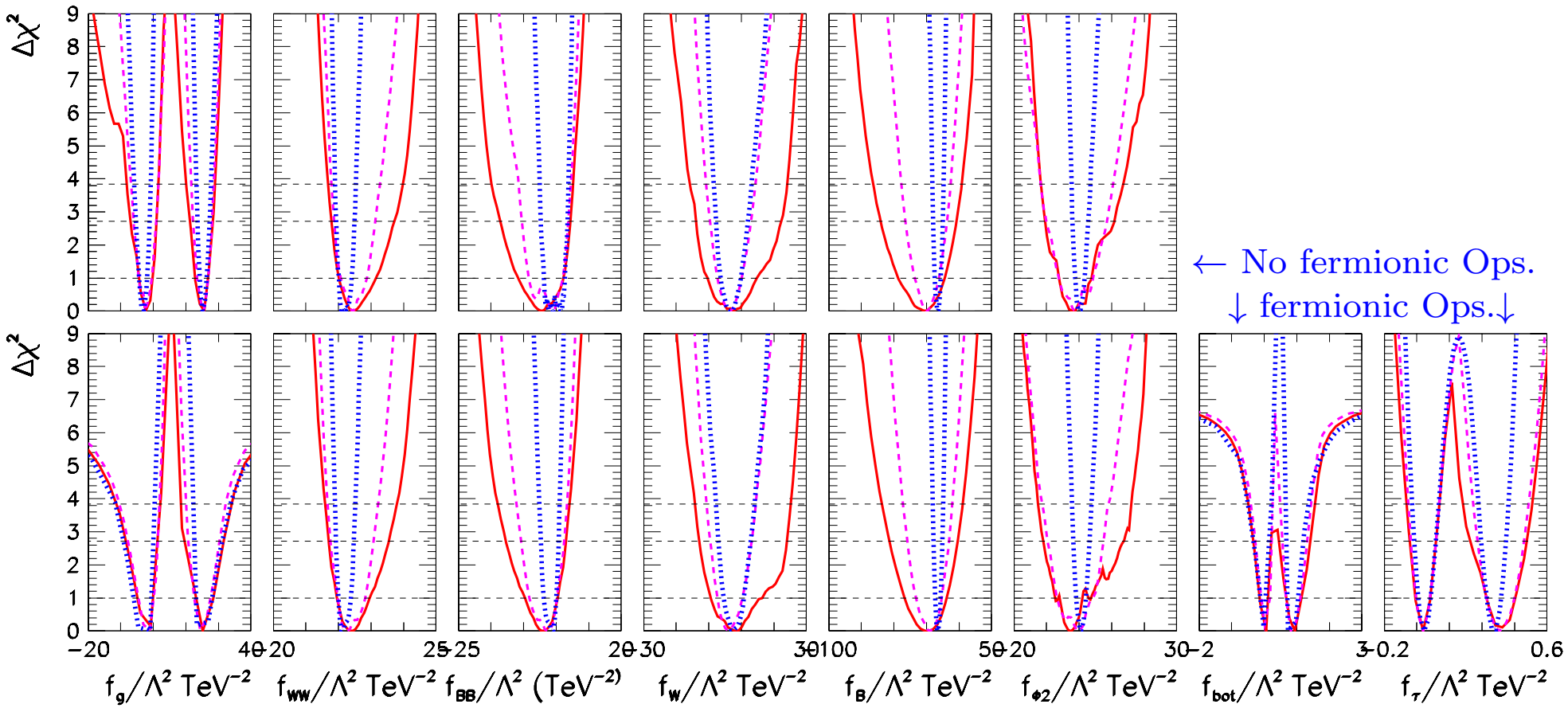

Anticorrelation due to $H \to \gamma\gamma$



# Results: Higgs + TGC + EWPD (II)

Scenario 1: $f_W,\ f_B,\ f_{WW},\ f_{BB},\ f_{\Phi,2},\ f_{\rm bot} = f_\tau = 0$

Scenario 2: $f_W,\ f_B,\ f_{WW} = -f_{BB},\ f_{\Phi,2},\ f_{\rm bot},\ f_\tau$



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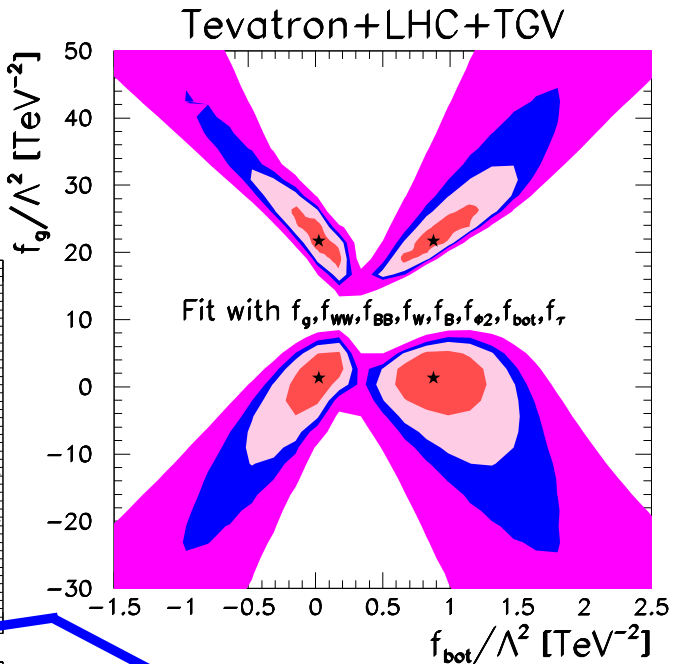
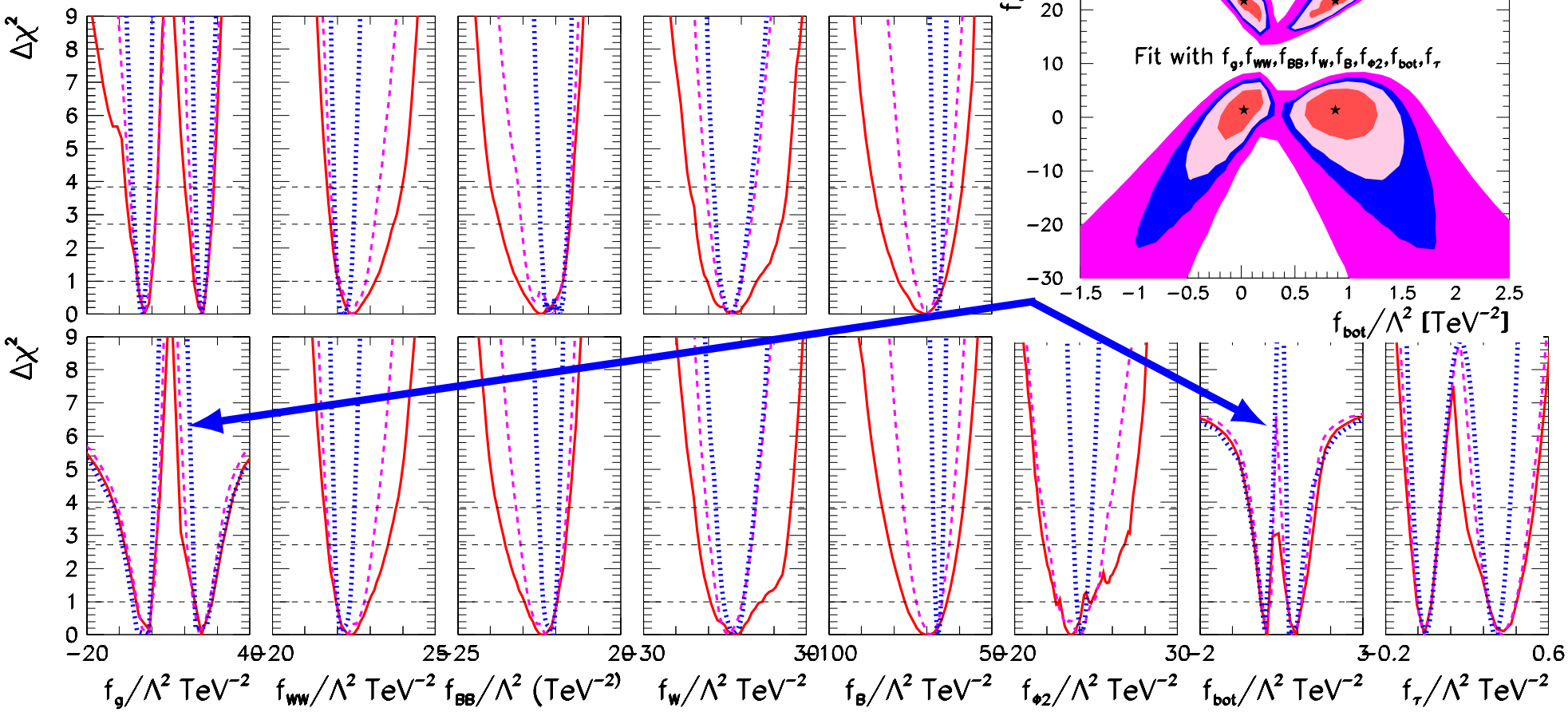
Strong correlation $f_g \otimes f_{\rm bot}$



# Best Fit and 90% CL regions

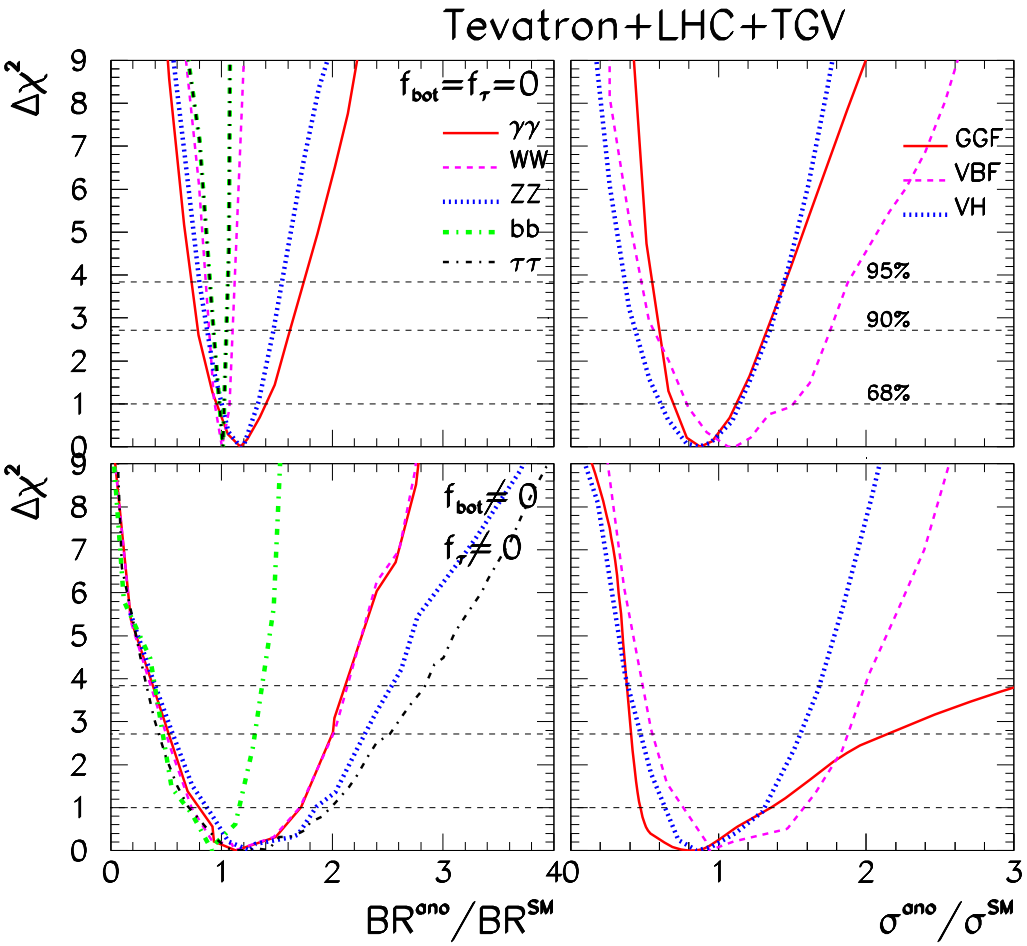
For Tevatron+LHC+TGC:

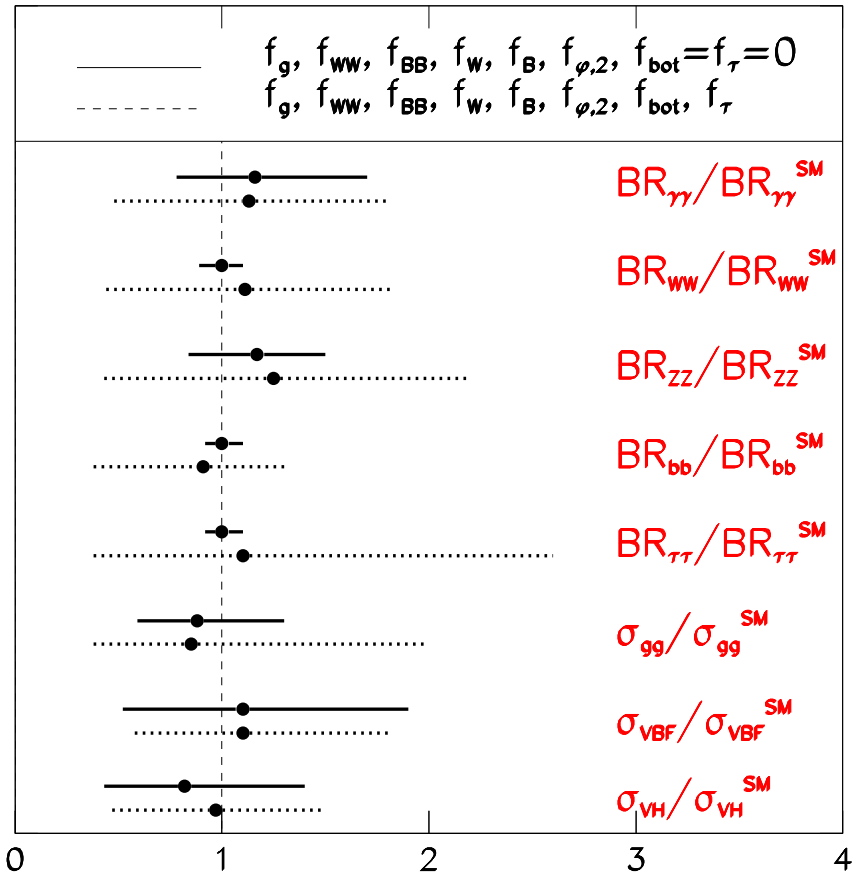
| | Fit with $f_{\rm bot} = f_\tau = 0$ | | Fit with $f_{\rm bot}$ and $f_\tau$ | |
|---|---|---|---|---|
| | Best fit | 90% CL allowed range | Best fit | 90% CL allowed range |
| $f_g/\Lambda^2$ $(\text{TeV}^{-2})$ | 1.1, 22 | $[-3.3, 5.1] \cup [19, 26]$ | 2.1, 21 | $[-5.3, 5.8] \cup [17, 22]$ |
| $f_{WW}/\Lambda^2$ $(\text{TeV}^{-2})$ | 1.5 | $[-3.2, 8.2]$ | 0.65 | $[-4.2, 7.7]$ |
| $f_{BB}/\Lambda^2$ $(\text{TeV}^{-2})$ | -1.6 | $[-7.5, 5.3]$ | -0.65 | $[-7.7, 4.2]$ |
| $f_W/\Lambda^2$ $(\text{TeV}^{-2})$ | 2.1 | $[-5.6, 9.6]$ | 1.7 | $[-5.4, 9.8]$ |
| $f_B/\Lambda^2$ $(\text{TeV}^{-2})$ | -10 | $[-29, 8.9]$ | -7.9 | $[-28, 11]$ |
| $f_{\phi,2}/\Lambda^2$ $(\text{TeV}^{-2})$ | -1.0 | $[-10, 8.5]$ | -1.3 | $[-9.8, 7.5]$ |
| $f_{\rm bot}/\Lambda^2$ $(\text{TeV}^{-2})$ | — | — | 0.01, 0.84 | $[-0.28, 0.24] \cup [0.55, 1.3]$ |
| $f_\tau/\Lambda^2$ $(\text{TeV}^{-2})$ | — | — | -0.01, 0.37 | $[-0.07, 0.05] \cup [0.26, 0.49]$ |

SM lays well within $1\sigma$ CL regions

# Observables: Cross Sections and Branching Ratios

From $\chi^2(f_i)$ we are able to construct the allowed ranges for $\sigma_i^{\text{ano}}$ and $\text{BR}_i^{\text{ano}}$:



# TGC Projections from Higgs Measurements

TGC $\leftrightarrow$ $HVV$ from gauge invariance:

$$\begin{aligned}
\Delta g_1^Z &= g_1^Z - 1 &&= \frac{g^2 v^2}{8 c_{\rm W}^2 \Lambda^2} f_W \\
\Delta \kappa_\gamma &= \kappa_\gamma - 1 &&= \frac{g^2 v^2}{8 \Lambda^2} (f_W + f_B) \\
\Delta \kappa_Z &= \kappa_Z - 1 &&= \frac{g^2 v^2}{8 c_{\rm W}^2 \Lambda^2} (c_{\rm W}^2 f_W - s_{\rm W}^2 f_B)
\end{aligned}$$

$\rightarrow$ project Higgs data onto TGC bounds:
(before we did the reverse!)

Complementary to direct searches at:

- LEP
- Tevatron
- & ATLAS

Marginalized over unshown parameters,
$(f_g,\ f_{WW},\ f_{BB},\ f_{\Phi,2})$

$\Rightarrow$Complementary bounds
$\Rightarrow$becoming competitive w/ direct measurement!

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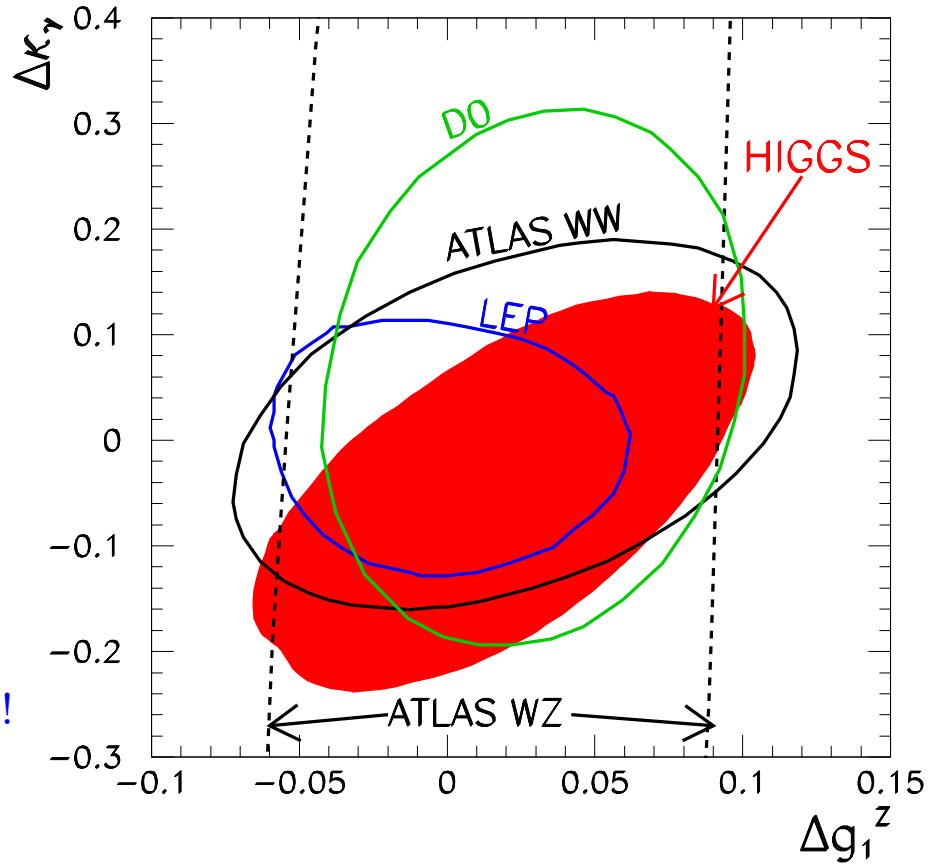

# Partial Wave Unitarity

Dimension–six operators $\rightarrow$ growth of amplitude w/ $E_{\rm COM}$
$\rightarrow$ violation of S–matrix unitarity

We impose the unitarity of the $S$–Matrix to constrain the operators:

- bounds derived from the optical theorem
- imply the necessity of new physics
  $\rightarrow$ new fundamental particles (SM Higgs fixes unitarity issues of GB scattering)
  $\rightarrow$ new composite degree of freedom ($\rho$ meson fixes unitarity issues of $\pi$ scattering)
- We will obtain bounds on the COM energy ($\sqrt{s}$) as a function of $f_i/\Lambda$
  $\rightarrow$ we can combine these with our Higgs fit results!

# Partial Wave Unitarity

Decomposing the amplitudes for $VV \to VV$ & $f\bar{f} \to VV$ into partial waves:

$$\mathcal{M}(V_{1\lambda_1} V_{2\lambda_2} \to V_{3\lambda_3} V_{4\lambda_4}) = 16\pi \sum_J \left(J + \frac{1}{2}\right) \sqrt{1 + \delta^{V_{2\lambda_2}}_{V_{1\lambda_1}}} \sqrt{1 + \delta^{V_{4\lambda_4}}_{V_{3\lambda_3}}} \, d^J_{\lambda\mu}(\theta) e^{iM\phi} T^J(V_1 V_2 \to V_3 V_4)$$

$$\mathcal{M}(f_{1\sigma_1} \bar{f}_{2\sigma_2} \to V_{3\lambda_3} V_{4\lambda_4}) = 16\pi \sum_J (J + \frac{1}{2}) \delta_{\sigma_1, -\sigma_2} \, d^J_{\sigma_1 - \sigma_2, \lambda_3 - \lambda_4}(\theta) T^J(f_1 \bar{f}_2 \to V_3 V_4)$$

Using the optical theorem one may derive the unitarity limit for the $T^J$'s:

$$|T^J(V_{1\lambda_1} V_{2\lambda_2} \to V_{1\lambda_1} V_{2\lambda_2})| \leq 2,$$

$$\sum_{V_{3\lambda_3}, V_{4\lambda_4}} \left| T^J(f_{1\sigma_1} \bar{f}_{2\sigma_2} \to V_{3\lambda_3} V_{4\lambda_4}) \right|^2 \leq 1$$

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# $VV \to VV$ Unitarity Violating Amplitudes: $\mathcal{O}_{\Phi,2}$ & $\mathcal{O}_{\Phi,4}$

The high energy ($\sqrt{s} \gg M_W, M_Z, M_H$) behavior for $\mathcal{O}_{\Phi,2}$ & $\mathcal{O}_{\Phi,4}$ $\to$ same up to a sign.

We define: $f_{\Phi 2,4} \equiv f_{\Phi,2} - f_{\Phi,4}$

| | $(\times \frac{f_{\Phi,2,4}}{\Lambda^2} \times s)$ |
|---|---|
| $W^+W^+ \to W^+W^+$ | $-1$ |
| $W^+Z \to W^+Z$ | $-\frac{1}{2}X$ |
| $W^+H \to W^+H$ | $-\frac{1}{2}X$ |
| $W^+W^- \to W^+W^-$ | $\frac{1}{2}Y$ |
| $W^+W^- \to ZZ$ | $1$ |
| $W^+W^- \to HH$ | $-1$ |
| $ZZ \to HH$ | $-1$ |
| $ZH \to ZH$ | $-\frac{1}{2}X$ |

$\mathcal{M}(s, M_W, M_Z, M_H) \to \mathcal{M}(s \gg M_W^2, M_Z^2, M_H^2)$

$X \equiv 1 - \cos\theta$

$Y \equiv 1 + \cos\theta$

Only grow as $s$, result of gauge symmetry!

Same Lorentz structures as SM

$\to$violation only in $V_L V_L \to V_L V_L$

# $VV \to VV$ Unitarity Violating Amplitudes: $\mathcal{O}_{WWW}$

Even more Lorentz structures → some more helicity combinations violating unitarity...

| | $(\times 2e^4 \frac{f_{WWW}}{\Lambda^2} \times s)$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $00++$ | $0+0-$ | $0+-0$ | $+00-$ | $+0-0$ | $++00$ | $+++-$<br>$+--+$<br>$+-++$<br>$++-+$ | $++--$ |
| $W^+W^+ \to W^+W^+$ | $0$ | $-\frac{3(2+Y)}{32s_W^4}$ | $\frac{3(2+X)}{32s_W^4}$ | $\frac{3(2+X)}{32s_W^4}$ | $-\frac{3(2+Y)}{32s_W^4}$ | $0$ | $-\frac{3}{4s_W^4}$ | $\frac{3}{2s_W^4}$ |
| $W^+Z \to W^+Z$ | $\frac{3(Y-X)c_W}{32s_W^4}$ | $0$ | $\frac{3(X+2)c_W}{32s_W^4}$ | $\frac{3(X+2)c_W}{32s_W^4}$ | $0$ | $\frac{3(Y-X)c_W}{32s_W^4}$ | $-\frac{3c_W^2}{8s_W^4}X$ | $\frac{3c_W^2}{4s_W^4}X$ |
| $W^+\gamma \to W^+\gamma$ | $-$ | $0$ | $-$ | $-$ | $-$ | $-$ | $-\frac{3}{8s_W^2}X$ | $\frac{3}{4s_W^2}X$ |
| $W^+Z \to W^+\gamma$ | $-\frac{3(Y-X)}{32s_W^3}$ | $0$ | $-$ | $\frac{3(X+2)}{32s_W^3}$ | $-$ | $-$ | $-\frac{3c_W}{8s_W^3}X$ | $\frac{3c_W}{4s_W^3}X$ |
| $W^+Z \to W^+H$ | $-$ | $-$ | $\frac{3(X+2)c_W}{32s_W^4}$ | $-$ | $\frac{3(2+Y)}{32s_W^4}$ | $-\frac{3(Y-X)c_W}{32s_W^4}$ | $-$ | $-$ |
| $W^+\gamma \to W^+H$ | $-$ | $-$ | $\frac{3(X+2)}{32s_W^3}$ | $-$ | $-$ | $-\frac{3(Y-X)}{32s_W^3}$ | $-$ | $-$ |
| $W^+W^- \to W^+W^-$ | $\frac{3(Y-X)}{32s_W^4}$ | $\frac{3(2+Y)}{32s_W^4}$ | $0$ | $0$ | $\frac{3(2+Y)}{32s_W^4}$ | $\frac{3(Y-X)}{32s_W^4}$ | $\frac{3}{8s_W^4}Y$ | $-\frac{3}{4s_W^4}Y$ |
| $W^+W^- \to ZZ$ | $0$ | $\frac{3(2+Y)c_W}{32s_W^4}$ | $-\frac{3(X+2)c_W}{32s_W^4}$ | $-\frac{3(X+2)c_W}{32s_W^4}$ | $\frac{3(2+Y)c_W}{32s_W^4}$ | $0$ | $\frac{3c_W^2}{4s_W^4}$ | $-\frac{3c_W^2}{2s_W^4}$ |
| $W^+W^- \to \gamma\gamma$ | $0$ | $-$ | $-$ | $-$ | $-$ | $-$ | $\frac{3}{4s_W^2}$ | $-\frac{3}{2s_W^2}$ |
| $W^+W^- \to Z\gamma$ | $0$ | $\frac{3(2+Y)}{32s_W^3}$ | $-$ | $-\frac{3(2+X)}{32s_W^3}$ | $-$ | $-$ | $\frac{3c_W}{4s_W^3}$ | $-\frac{3c_W}{2s_W^3}$ |
| $W^+W^- \to ZH$ | $-$ | $-$ | $-\frac{3(2+X)c_W}{32s_W^4}$ | $-$ | $-\frac{3(2+Y)c_W}{32s_W^4}$ | $\frac{3(Y-X)}{32s_W^4}$ | $-$ | $-$ |
| $W^+W^- \to \gamma H$ | $-$ | $-$ | $-\frac{3(X+2)}{32s_W^3}$ | $-$ | $-\frac{3(2+Y)}{32s_W^3}$ | $-$ | $-$ | $-$ |

# All Couplings Simultaneously

In arXiv:1411.5026 we give bounds for one operator different from zero at a time.

With no model of NP to guide us, we must consider more than one coupling non–zero.

- search for largest allowed value for each coefficient while varying others
  → Most conservative constraints on a parameter allowing for cancellations in others

$$\begin{aligned}
\left|\frac{f_{\Phi 2,4}}{\Lambda^2}s\right| &\leq 105 \\
\left|\frac{f_W}{\Lambda^2}s\right| &\leq 205 \\
\left|\frac{f_B}{\Lambda^2}s\right| &\leq 640 \\
\left|\frac{f_{WW}}{\Lambda^2}s\right| &\leq 200 \\
\left|\frac{f_{BB}}{\Lambda^2}s\right| &\leq 880 \\
\left|\frac{f_{WWW}}{\Lambda^2}s\right| &\leq 82
\end{aligned}$$

Note: $f \sim 1 \to s \sim 100\Lambda^2$, current bounds may be unreliable for some $f$.

# Combined Results:

We did not constrain $f_{WWW}$ from Higgs data, but TGC bounds give:

$$-0.041 \leq \lambda_\gamma \leq -0.003 \quad \lambda_\gamma = \frac{3g^2 M_W^2}{2\Lambda^2} f_{WWW}$$

Combining the Higgs data fits with the unitarity bounds obtained we find:

$$\begin{aligned}
-10 \leq \tfrac{f_{\Phi,2}}{\Lambda^2}(\text{TeV}^{-2}) \leq 8.5 \quad &\Rightarrow \quad \sqrt{\mathbf{s}} \leq \mathbf{3.2}\ \text{TeV} \\
-5.6 \leq \tfrac{f_W}{\Lambda^2}(\text{TeV}^{-2}) \leq 9.6 \quad &\Rightarrow \quad \sqrt{s} \leq 4.6\ \text{TeV} \\
-29 \leq \tfrac{f_B}{\Lambda^2}(\text{TeV}^{-2}) \leq 8.9 \quad &\Rightarrow \quad \sqrt{s} \leq 4.7\ \text{TeV} \\
-3.2 \leq \tfrac{f_{WW}}{\Lambda^2}(\text{TeV}^{-2}) \leq 8.2 \quad &\Rightarrow \quad \sqrt{s} \leq 4.9\ \text{TeV} \\
-7.5 \leq \tfrac{f_{BB}}{\Lambda^2}(\text{TeV}^{-2}) \leq 5.3 \quad &\Rightarrow \quad \sqrt{s} \leq 11\ \text{TeV} \\
-15 \leq \tfrac{f_{WWW}}{\Lambda^2}(\text{TeV}^{-2}) \leq 3.9 \quad &\Rightarrow \quad \sqrt{\mathbf{s}} \leq \mathbf{2.4}\ \text{TeV}
\end{aligned}$$

# Linear EFTs for Higgs Physics

We're interested in the EFT for Higgs Physics.

Effective operators are formed from the field content,

$$e_R, \quad u_R, \quad d_R, \quad L_L, \quad Q_L, \quad \Phi, \quad W_\mu, \quad Z_\mu, \quad A_\mu, \quad G_\mu$$

And their (covariant) derivatives.

The operators are then formed of all $U(1)_{Y/2} \times SU(2)_L \times SU(3)_C$ invariant combinations.

Further

- restrict to dimension–six, i.e. assume all NP occurs at a sufficiently high $\Lambda$
- assume baryon and lepton numbers are conserved ($\rightarrow$ no dimension–5 operators)
- assume operators are $CP-$even

Then we write the linear EFT to dimension–6 as:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_n \frac{f_n}{\Lambda^2} \mathcal{O}_n$$

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And their (covariant) derivatives.

$$h, U = \exp(-i\pi^a \sigma^a / v)$$

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Further

- restrict to 4 derivatives, i.e. assume all NP occurs for $p^2 \ll \Lambda_{\rm NP}$
- assume baryon and lepton numbers are conserved
- assume operators are $CP-$even

Then we write the chiral EFT to 4 derivatives as:

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \sum_n \frac{f_n}{\Lambda^2}\mathcal{O}_n$$

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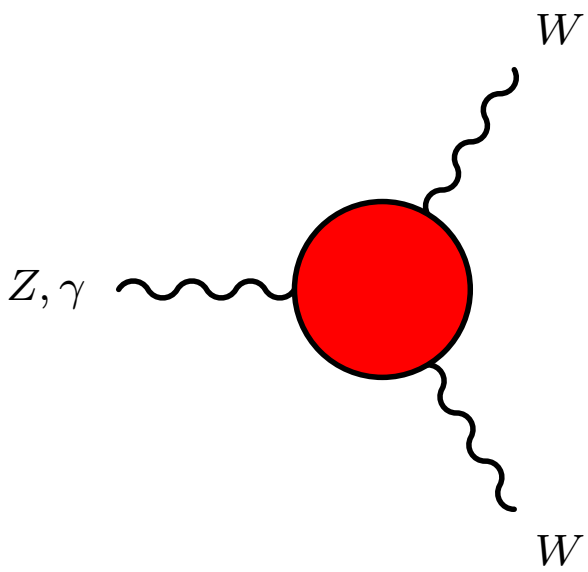
Then we write the chiral EFT to 4 derivatives as:

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \sum_n c_n \mathcal{P}_n$$

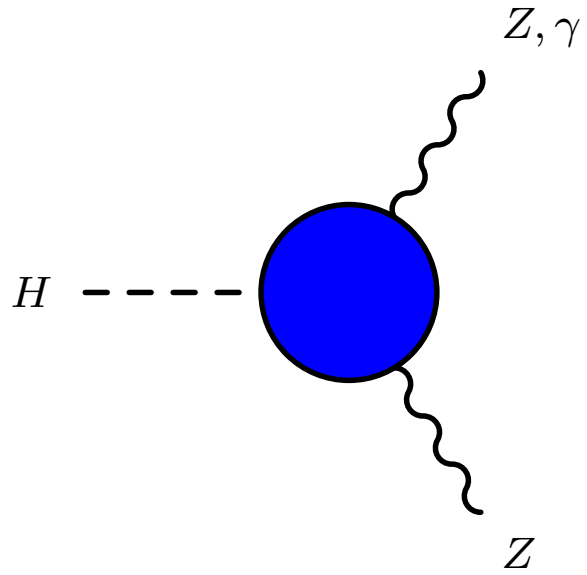
# Chiral Expansion to 4–Derivatives

Less symmetry → more operators,
Defining: $\boldsymbol{U}(x) = e^{i\sigma_a \pi^a(x)/v}$, $\boldsymbol{V}_\mu \equiv (\boldsymbol{D}_\mu \boldsymbol{U})\boldsymbol{U}^\dagger$, $\boldsymbol{T} \equiv \boldsymbol{U}\sigma_3\boldsymbol{U}^\dagger$.

$\mathcal{P}_2(h) = ig' B_{\mu\nu} \mathrm{Tr}(\boldsymbol{T}[\boldsymbol{V}^\mu, \boldsymbol{V}^\nu])\mathcal{F}_2$




$\mathcal{P}_4 = ig' B_{\mu\nu} \mathrm{Tr}(\boldsymbol{T}\boldsymbol{V}^\mu)\partial^\nu \mathcal{F}_4$




Where, for $\mathcal{O}_B = \frac{g'}{2}(D_\mu \Phi)^\dagger B^{\mu\nu}(D_\nu \Phi)$, these were correlated.

⇒ Potential for discrimination between expansions!

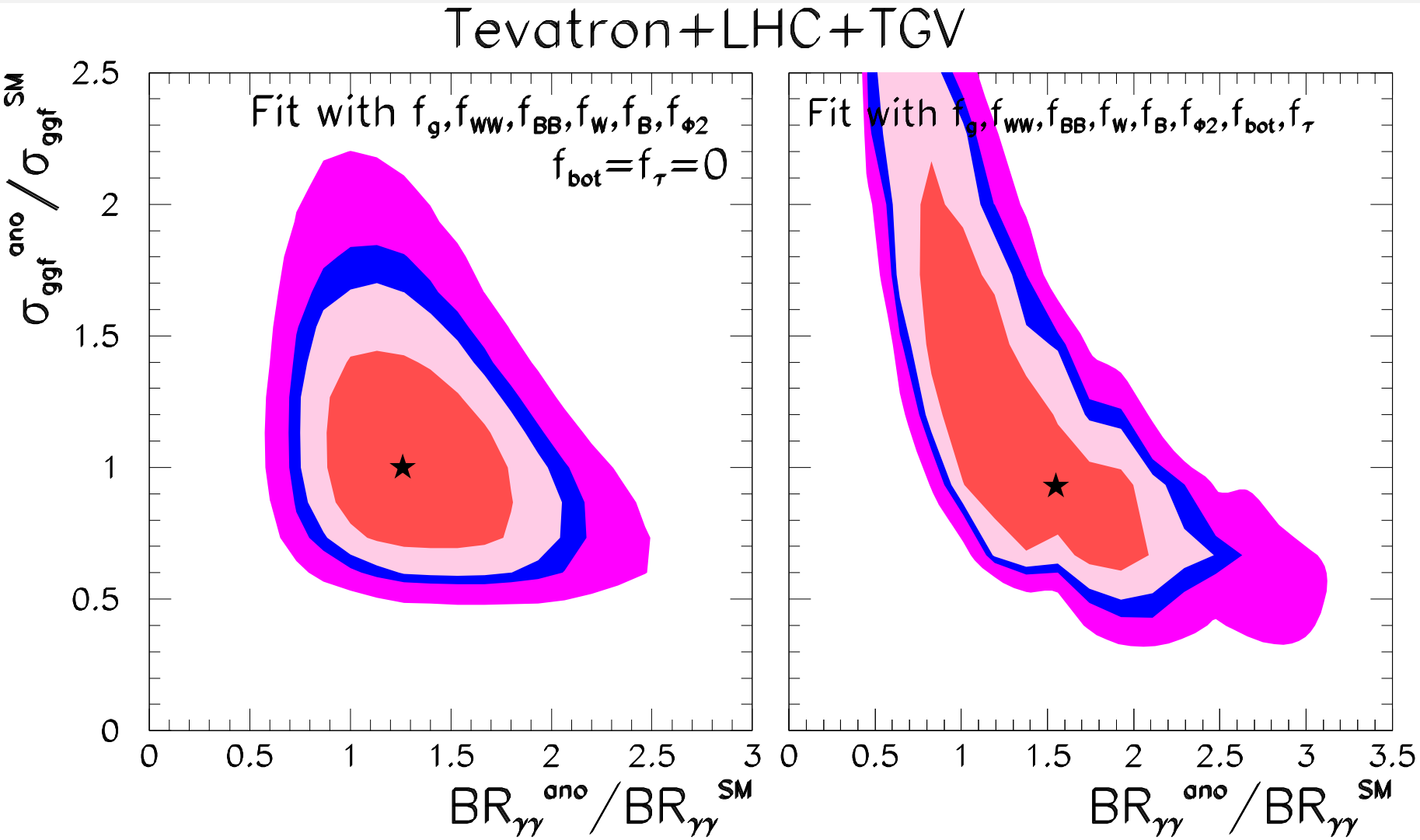
⇒ *A lot more* in arXiv:1311.1823.

# Conclusions

- SM Higgs mechanism completely determines the Higgs properties given $M_H$
- To test deviations model independently $\rightarrow$ EFT
- We have explored the linear EFT expansion (at dim=6), arXiv:1211.4580 & 1411.5026
  $\rightarrow$ Relevant for an elementary Higgs
  $\rightarrow$ Data driven approach, choose basis such that it is best constrained by data
  $\rightarrow$ Compatible within $1\sigma$ with SM predictions, still room for NP
  $\rightarrow$ $HVV$ and TGC correlation
  $\rightarrow$ Indirect TGC bounds becoming competitive with direct TGC measurements
  $\rightarrow$ Lowest energies at which perturbative unitarity is violated: $\sim 2-3$ TeV
- Briefly mentioned the chiral expansion (at $p^4$), more in arXiv:1311.1823
  $\rightarrow$ Relevant for a light composite Higgs
  $\rightarrow$ $HVV$ and TGC uncorrelated: possible discrimination between expansions
- Most up to date fit is in 1505.05516, includes more than presented:
  $\rightarrow$ Anomalous $Ht\bar{t}$
  $\rightarrow$ Kinematic distributions
  $\rightarrow$ Future impact of off shell measurements

Although consistent with the SM there is still large room for deviations which will be/are being probed at LHC run II

# Backup: Gluon Fusion – $\mathrm{BR}_{\gamma\gamma}$ Correlation




Inclusion of $f_{\mathrm{bot}} \rightarrow$ loosen bounds on GGF $\rightarrow$ more parameter space for $\mathrm{BR}_{\gamma\gamma} < \mathrm{BR}_{\gamma\gamma}^{\mathrm{SM}}$

# Backup: Optical Theorem

The optical theorem states:

$$\mathrm{Im}T^J(12 \to 34) = \sum_{12 \to 1'2'} \frac{|\vec{p}_{1'2'}|}{\sqrt{s}} T^{J*}(12 \to 1'2') T^J(1'2' \to 34),$$

with:

$$|\vec{p}_{ij}| = \frac{\sqrt{[s-(m_i+m_j)^2][s-(m_i-m_j)^2]}}{2\sqrt{s}}.$$

Which we may rewrite as:

$$\mathrm{Im}T^J(12 \to 12) = \frac{|\vec{p}_{12}|}{\sqrt{s}} \left|T^J(12 \to 12)\right|^2 + \sum_{1'2' \neq 12} \frac{|\vec{p}_{1'2'}|}{\sqrt{s}} \left|T^J(12 \to 1'2')\right|^2.$$

For only one intermediate channel we obtain:

$$\mathrm{Im}T^J(12 \to 12) = \frac{|\vec{p}_{12}|}{\sqrt{s}} |T^J(12 \to 12)|^2.$$

Rewriting $T^J$ as,

$$T^J(12 \to 12) = \frac{\sqrt{s}}{|\vec{p}_{12}|} e^{i\delta} \sin\delta,$$

Gives the result for elastic scattering:

$$|T^J(12 \to 12)| \leq \frac{\sqrt{s}}{|\vec{p}_{12}|} \to 2.$$

# Backup: Optical Theorem (II)

Alternatively considering fermion scattering we obtain from the optical theorem:

$$\begin{aligned} 2\mathrm{Im}[T^J(f_{1\sigma_1}\bar{f}_{2\sigma_2} \to f_{1\sigma_1}\bar{f}_{2\sigma_2})] &= \left|T^J(f_{1\sigma_1}\bar{f}_{2\sigma_2} \to f_{1\sigma_1}\bar{f}_{2\sigma_2})\right|^2 \\ &+ \sum_{V_{3\lambda_3}, V_{4\lambda_4}} \left|T^J(f_{1\sigma_1}\bar{f}_{2\sigma_2} \to V_{3\lambda_3}V_{4\lambda_4})\right|^2 \\ &+ \sum_N \left|T^J(f_{1\sigma_1}\bar{f}_{2\sigma_2} \to N)\right|^2 , \end{aligned}$$

Defining,

$$T^J(12 \to 12) \equiv y + ix, \qquad d \equiv \sum_{1'2' \neq 12} \frac{|\vec{p}_{1'2'}|}{\sqrt{s}} \left|T^J(12 \to 1'2')\right|^2 ,$$

Allows us to rewrite the above as:

$$x = \frac{|\vec{p}_{12}|}{\sqrt{s}}(x^2 + y^2) + d.$$

Finally in order for the assumption that x is real to hold, the quadratic equation requires:

$$2 \sum_{1'2' \neq 12} \frac{|\vec{p}_{1'2'}|}{\sqrt{s}} \left|T^J(12 \to 1'2')\right|^2 \leq 1 \to \sum_{V_{3\lambda_3}, V_{4\lambda_4}} \left|T^J(f_{1\sigma_1}\bar{f}_{2\sigma_2} \to V_{3\lambda_3}V_{4\lambda_4})\right|^2 \leq 1$$

# Inclusion of EWPD ($S$, $T$, and $U$)

Additionally we include the effects of $f_{WW}$, $f_{BB}$, $f_W$, $f_B$, and $f_{\Phi,2}$ at 1–loop:
(Hagiwara et al.; Alam, Dawson, Szalapski)

$$
\begin{aligned}
\alpha_{\rm EM}\Delta S &= \frac{1}{6}\frac{e^2}{16\pi^2}\left\{3(f_W+f_B)\frac{m_H^2}{\Lambda^2}\log\left(\frac{\mu^2}{m_H^2}\right)\right. \\
&\quad + 2\left[\left(5c_{\rm W}^2-2\right)f_W-\left(5c_{\rm W}^2-3\right)f_B\right]\frac{m_Z^2}{\Lambda^2}\log\left(\frac{\mu^2}{m_H^2}\right) \\
&\quad - \left[\left(22c_{\rm W}^2-1\right)f_W-\left(30c_{\rm W}^2+1\right)f_B\right]\frac{m_Z^2}{\Lambda^2}\log\left(\frac{\mu^2}{m_Z^2}\right) \\
&\quad \left.-24\left(c_{\rm W}^2 f_{WW}+s_{\rm W}^2 f_{BB}\right)\frac{m_Z^2}{\Lambda^2}\log\left(\frac{\mu^2}{m_H^2}\right)\right\} \\
\alpha_{\rm EM}\Delta T &= \frac{3}{4c_{\rm W}^2}\frac{e^2}{16\pi^2}\left\{f_B\frac{m_H^2}{\Lambda^2}\log\left(\frac{\mu^2}{m_H^2}\right)+\left(c_{\rm W}^2 f_W+f_B\right)\frac{m_Z^2}{\Lambda^2}\log\left(\frac{\mu^2}{m_H^2}\right)\right. \\
&\quad + \left[2c_{\rm W}^2 f_W+\left(3c_{\rm W}^2-1\right)f_B\right]\frac{m_Z^2}{\Lambda^2}\log\left(\frac{\mu^2}{m_Z^2}\right) \\
&\quad \left.- f_{\Phi,2}\frac{v^2}{\Lambda^2}\log\left(\frac{\mu^2}{m_H^2}\right)\right\} \\
\alpha_{\rm EM}\Delta U &= -\frac{1}{3}\frac{e^2 s_{\rm W}^2}{16\pi^2}\left\{(-4f_W+5f_B)\frac{m_Z^2}{\Lambda^2}\log\left(\frac{\mu^2}{m_H^2}\right)+(2f_W-5f_B)\frac{m_Z^2}{\Lambda^2}\log\left(\frac{\mu^2}{m_Z^2}\right)\right\}
\end{aligned}
$$

Some issues - finite part? renormalization scale $\mu$ =?

# $\mathcal{L}_{\text{eff}}$, to Four–Derivatives

$$\begin{aligned}\mathcal{L}_{\text{eff}} &= \xi\left[c_B\mathcal{P}_B(h)+c_W\mathcal{P}_W(h)+c_G\mathcal{P}_G(h)+c_C\mathcal{P}_C(h)+c_T\mathcal{P}_T(h)+c_H\mathcal{P}_H(h)\right]\\ &\quad+\xi\sum_{i=1}^{10}c_i\mathcal{P}_i(h)+\xi^2\sum_{i=11}^{25}c_i\mathcal{P}_i(h)+\xi^4 c_{26}\mathcal{P}_{26}(h)+\textstyle\sum_i \xi^{n_i}c^i_{HH}\mathcal{P}^i_{HH}(h)\end{aligned}$$

Define: $\boldsymbol{U}(x)=e^{i\sigma_a\pi^a(x)/v}$, $\boldsymbol{V}_\mu\equiv(\boldsymbol{D}_\mu\boldsymbol{U})\boldsymbol{U}^\dagger$, $\boldsymbol{T}\equiv\boldsymbol{U}\sigma_3\boldsymbol{U}^\dagger$, $\xi\equiv\frac{v^2}{f^2}$.

$\xi$ helps us to compare between the linear and chiral expansions.

**Weighted by $\xi$:**

$$\begin{aligned}
\mathcal{P}_C(h) &= -\tfrac{v^2}{4}\mathrm{Tr}(\boldsymbol{V}^\mu\boldsymbol{V}_\mu)\mathcal{F}_C & \mathcal{P}_4 &= ig'B_{\mu\nu}\mathrm{Tr}(\boldsymbol{T}\boldsymbol{V}^\mu)\partial^\nu\mathcal{F}_4\\
\mathcal{P}_T(h) &= \tfrac{v^2}{4}\mathrm{Tr}(\boldsymbol{T}\boldsymbol{V}_\mu)\mathrm{Tr}(\boldsymbol{T}\boldsymbol{V}^\mu)\mathcal{F}_T & \mathcal{P}_5 &= ig\mathrm{Tr}(W_{\mu\nu}\boldsymbol{V}^\mu)\partial^\nu\mathcal{F}_5\\
\mathcal{P}_B(h) &= -\tfrac{g'^2}{4}B_{\mu\nu}B^{\mu\nu}\mathcal{F}_B & \mathcal{P}_6 &= (\mathrm{Tr}(\boldsymbol{V}_\mu\boldsymbol{V}^\mu))^2\mathcal{F}_6\\
\mathcal{P}_W(h) &= -\tfrac{g^2}{4}W^a_{\mu\nu}W^{a\mu\nu}\mathcal{F}_W & \mathcal{P}_7 &= \mathrm{Tr}(\boldsymbol{V}_\mu\boldsymbol{V}^\mu)^2\partial_\nu\partial^\nu\mathcal{F}_7\\
\mathcal{P}_G(h) &= -\tfrac{g_s^2}{4}G^a_{\mu\nu}G^{a\mu\nu}\mathcal{F}_G & \mathcal{P}_8 &= \mathrm{Tr}(\boldsymbol{V}_\mu\boldsymbol{V}^\mu)\partial^\nu\mathcal{F}_8(h)\partial_\nu\mathcal{F}'_8\\
\mathcal{P}_1(h) &= gg'B_{\mu\nu}\mathrm{Tr}(\boldsymbol{T}W^{\mu\nu})\mathcal{F}_1 & \mathcal{P}_9 &= \mathrm{Tr}((\mathcal{D}_\mu\boldsymbol{V}^\mu)^2)\partial^\nu\mathcal{F}_9\\
\mathcal{P}_2(h) &= ig'B_{\mu\nu}\mathrm{Tr}(\boldsymbol{T}[\boldsymbol{V}^\mu,\boldsymbol{V}^\nu])\mathcal{F}_2 & \mathcal{P}_{10} &= \mathrm{Tr}(\boldsymbol{V}_\nu\mathcal{D}_\mu\boldsymbol{V}^\mu)\mathcal{F}_{10}\\
\mathcal{P}_3(h) &= ig\mathrm{Tr}(W_{\mu\nu}[\boldsymbol{V}^\mu,\boldsymbol{V}^\nu])\mathcal{F}_3
\end{aligned}$$

# (Very) Brief Comparison of Bases

Order in $\xi$ of an operator $\rightarrow$ minimum canonical dimension to which one must expand in the linear expansion necessary to generate the corresponding effects of the operator in the chiral expansion.

For small $\xi$ we can relate the Linear basis to the Chiral (more operators):

$$
\begin{aligned}
\mathcal{O}_{WW} &= \frac{g^2}{4}\Phi^\dagger W^{\mu\nu} W_{\mu\nu}\Phi && \longrightarrow && \mathcal{P}_W &= -\frac{g^2}{2}\mathrm{Tr}(\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu})\mathcal{F}_W(h)\\
\mathcal{O}_{BB} &= \frac{g'^2}{4}\Phi^\dagger B^{\mu\nu} B_{\mu\nu}\Phi && \longrightarrow && \mathcal{P}_B &= -\frac{g^2}{4}\mathrm{Tr}(B_{\mu\nu}B^{\mu\nu})\mathcal{F}_B(h)\\
\mathcal{O}_{BW} &= \frac{gg'}{4}\Phi^\dagger B^{\mu\nu} W_{\mu\nu}\Phi && \longrightarrow && \mathcal{P}_1 &= gg' B_{\mu\nu}\mathrm{Tr}(\mathbf{T}W^{\mu\nu})\mathcal{F}_1\\
\mathcal{O}_B &= \frac{g'}{2}(D_\mu\Phi)^\dagger B^{\mu\nu}(D_\nu\Phi) && \longrightarrow && \mathcal{P}_2 &= ig' B_{\mu\nu}\mathrm{Tr}(\mathbf{T}[\mathbf{V}^\mu,\mathbf{V}^\nu])\mathcal{F}_2(h)\\
& && && \mathcal{P}_4 &= ig' B_{\mu\nu}\mathrm{Tr}(\mathbf{T}\mathbf{V}^\mu)\partial^\nu\mathcal{F}_4(h)\\
\mathcal{O}_W &= \frac{g}{2}(D_\mu\Phi)^\dagger W^{\mu\nu}(D_\nu\Phi) && \longrightarrow && \mathcal{P}_3 &= ig\mathrm{Tr}(W_{\mu\nu}[\mathbf{V}^\mu,\mathbf{V}^\nu])\mathcal{F}_3(h)\\
& && && \mathcal{P}_5 &= ig\mathrm{Tr}(W_{\mu\nu}\mathbf{V}^\mu)\partial^\nu\mathcal{F}_5(h)\\
\mathcal{O}_{\phi,1} &= (D_\mu\Phi)^\dagger\Phi\Phi^\dagger(D_\nu\Phi) && \longrightarrow && \mathcal{P}_T &= \frac{v^2}{4}\mathrm{Tr}(\mathbf{T}\mathbf{V}_\mu)\mathrm{Tr}(\mathbf{T}\mathbf{V}^\mu)\mathcal{F}_T(h)\\
\mathcal{O}_{\phi,2} &= \frac{1}{2}\partial_\mu(\Phi^\dagger\Phi)\partial^\mu(\Phi^\dagger\Phi) && \longrightarrow && \mathcal{P}_H &= \frac{1}{2}(\partial_\mu h)(\partial^\mu h)\mathcal{F}_H(h)\\
\mathcal{O}_{\phi,4} &= (D_\mu\Phi)^\dagger(D_\nu\Phi)(\Phi^\dagger\Phi) && \longrightarrow && \mathcal{P}_C &= -\frac{v^2}{4}\mathrm{Tr}(\mathbf{V}_\mu\mathbf{V}^\mu)\mathcal{F}_C
\end{aligned}
$$

Higgs–Gauge, Triple Gauge, and Quartic Gauge couplings are no longer correlated!

# (Very) Brief Comparison of Bases II

Each operator contributes to particular HVV and TGCs.
In particular[1]:

| | $\mathcal{O}_{\rm BB}$ | $\mathcal{O}_{\rm WW}$ | $\mathcal{O}_{\rm B}$ | $\mathcal{O}_{\rm W}$ | $\mathcal{P}_{\rm B}$ | $\mathcal{P}_{\rm W}$ | $\mathcal{P}_2$ | $\mathcal{P}_4$ | $\mathcal{P}_3$ | $\mathcal{P}_5$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $h\gamma\gamma$ | ✘ | ✘ | | | ✘ | ✘ | | | | |
| $hZ\gamma$ | ✘ | ✘ | ✘ | ✘ | ✘ | ✘ | | ✘ | | ✘ |
| $hZZ$ | ✘ | ✘ | ✘ | ✘ | ✘ | ✘ | | ✘ | | ✘ |
| $HW^+W^-$ | | ✘ | | ✘ | | ✘ | | | | ✘ |
| $\gamma W^+W^-$ | | | ✘ | ✘ | | | ✘ | | ✘ | |
| $ZW^+W^-$ | | | ✘ | ✘ | | | ✘ | | ✘ | |

Notice that green illustrates the decorrelation of TGC from Higgs data for

$$\mathcal{O}_{\rm B} = \frac{v^2}{16}\mathcal{P}_2 + \frac{v^2}{8}\mathcal{P}_4$$

And blue illustrates a similar decorrelation for

$$\mathcal{O}_{\rm W} = \frac{v^2}{8}\mathcal{P}_3 - \frac{v^2}{4}\mathcal{P}_5$$

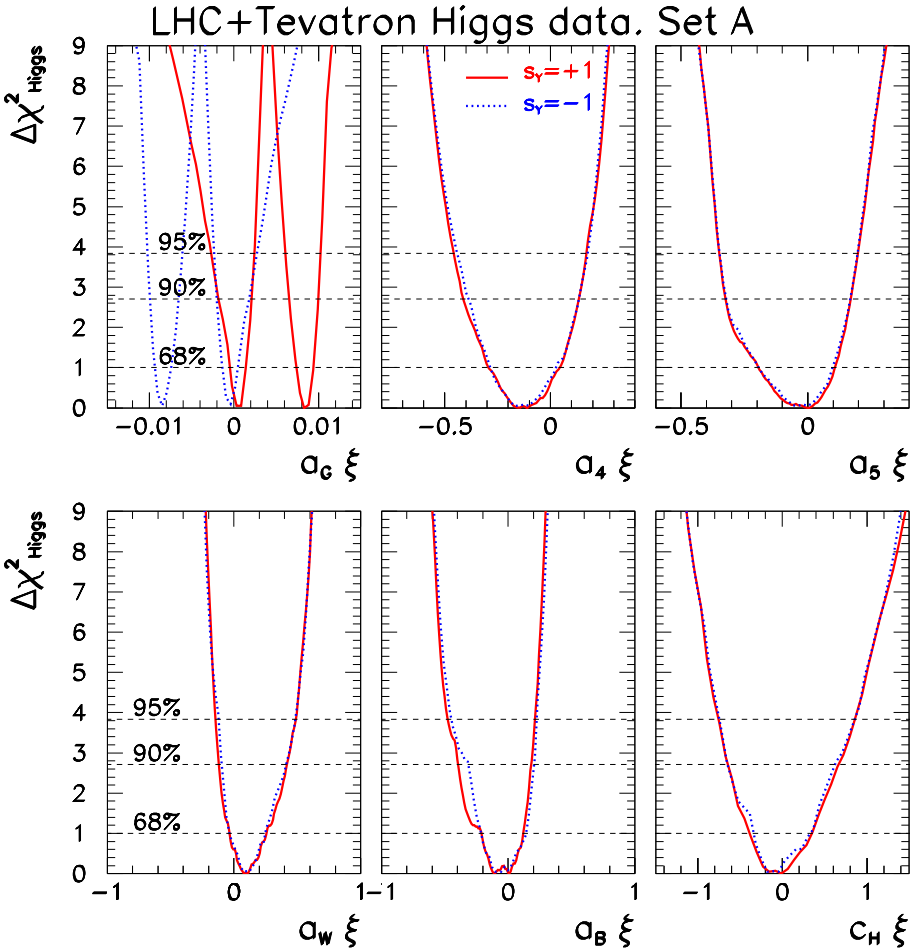
[1] $\mathcal{O}_{\phi 4,\phi 2}$ $(\mathcal{P}_{H,T})$ give shifts to all SM $HVV$ & $Hff$ vertices

# Higgs Phenomenology in Chiral Expansion

For comparison w/ linear expansion relevant ops. are: $\mathcal{P}_{\rm G}$ $\mathcal{P}_4$ $\mathcal{P}_5$ $\mathcal{P}_{\rm B}$ $\mathcal{P}_{\rm W}$ $\mathcal{P}_{\rm H}$ $\mathcal{P}_{\rm C}$

Taking: $c_i\mathcal{F}(h) = c_i + 2a_i\frac{h}{v} + b_i\frac{h^2}{v^2} + \cdots$

Bounds from Higgs Analysis (Set A $\rightarrow$ SM–like $HVV$ and $Hff$ shifted simultaneously):

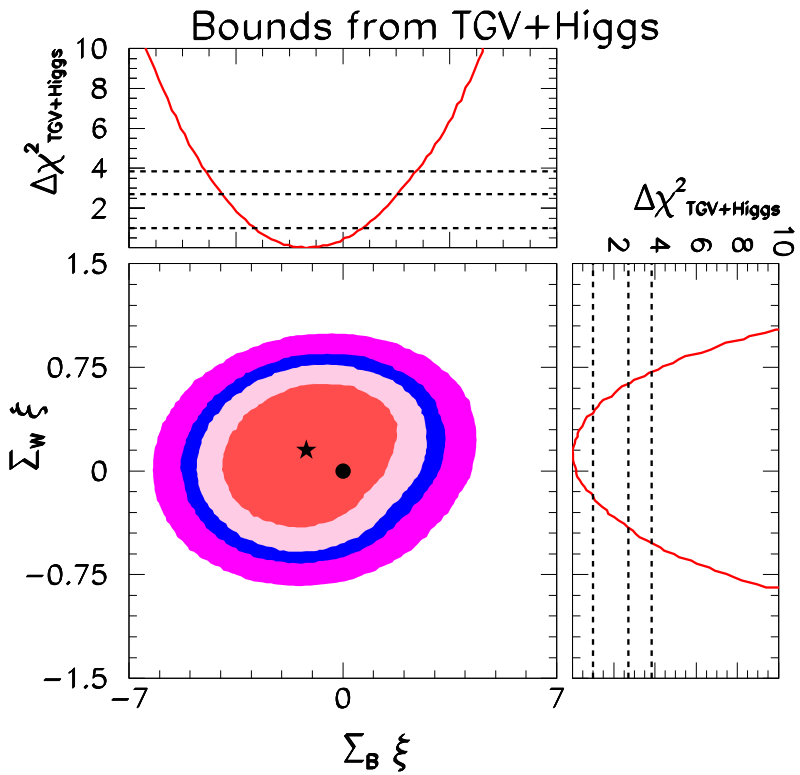
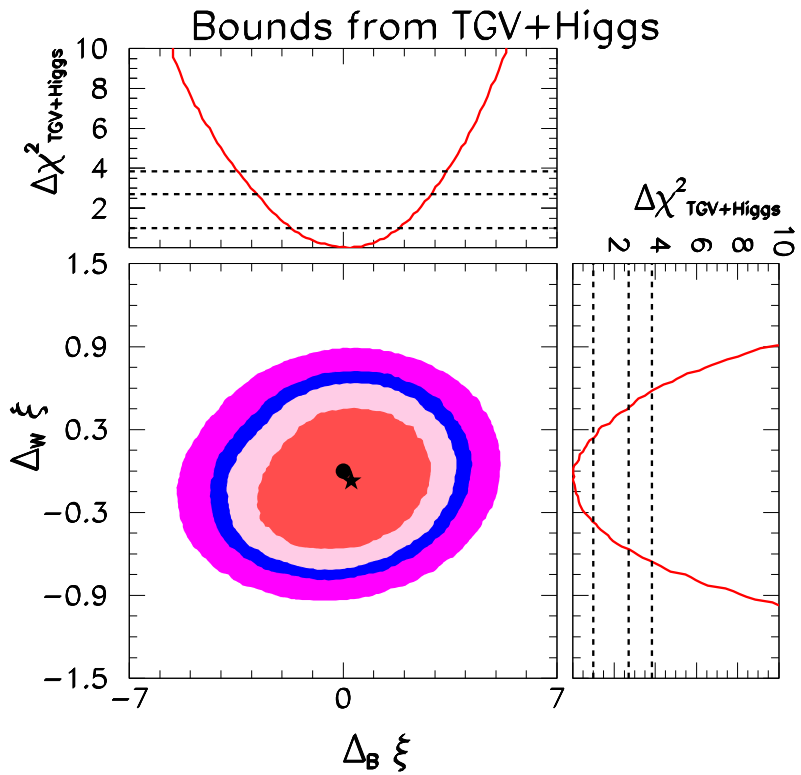



Higgs Only, *No TGC bounds*
Best fit near $c_i = 0$ (SM)
$Hgg$ degeneracy from $a_{\rm G} \sim -2$SM-loop
$H\gamma\gamma \rightarrow$ anticorrelation between $a_{\rm W}$ & $a_{\rm B}$

# Discriminating Linear from Chiral Realizations

Recalling the relation between $\mathcal{O}_{\rm B}$ and $\mathcal{P}_2, \mathcal{P}_4$ and $\mathcal{O}_{\rm W}$ and $\mathcal{P}_3, \mathcal{P}_5$:

$$\mathcal{O}_{\rm B} = \frac{v^2}{16}\mathcal{P}_2 + \frac{v^2}{8}\mathcal{P}_4 \quad \mathcal{O}_{\rm W} = \frac{v^2}{8}\mathcal{P}_3 - \frac{v^2}{4}\mathcal{P}_5$$

We define the discriminating quantities (using a similar relation for $\mathcal{O}_W$ and $\mathcal{P}_3, \mathcal{P}_5$):

$$\Sigma_{\rm B} \equiv 4(2c_2 + a_4) \quad \Sigma_{\rm W} \equiv 2(2c_3 - a_5), \quad \Sigma_{\rm B(W)} \to \frac{f_{\rm B(W)} v^2}{\Lambda^2}$$
$$\Delta_{\rm B} \equiv 4(2c_2 - a_4) \quad \Delta_{\rm W} \equiv 2(2c_3 + a_5), \quad \Delta_{\rm B} = \Delta_{\rm W} \to 0$$