

# N<sup>3</sup>LO threshold corrections for top-pair and single-top production

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- Higher-order corrections
- Total  $t\bar{t}$  cross sections
- Top  $p_T$  distributions
- Top rapidity distributions
- Top forward-backward asymmetry
- Single-top production cross sections
- Single-top  $p_T$  distributions

## Higher-order corrections

QCD corrections significant for  $t\bar{t}$  and single-top production

Soft-gluon corrections are important

**Soft terms:**  $\left[ \frac{\ln^k(s_4/m_t^2)}{s_4} \right]_+$  with  $k \leq 2n - 1$ ,  $s_4$  distance from threshold

Resum these soft corrections - factorization and RGE

**NNLL accuracy**—two-loop soft anomalous dimensions

Approximate  $N^3\text{LO}$  (aN<sup>3</sup>LO) differential cross sections from expansion of resummed expressions

Calculation is for **partonic threshold for the double-differential cross section** using the standard moment-space resummation in perturbative QCD

**Latest results for  $t\bar{t}$ :**

**total cross section:** Phys. Rev. D 90, 014006 (2014) [arXiv:1405.7046 [hep-ph]]

**$p_T$  and  $y$  distributions:** Phys. Rev. D 91, 031501 (2015) [arXiv:1411.2633 [hep-ph]]

**$A_{FB}$ :** Phys. Rev. D 91, 071502 (2015) [arXiv:1501.01581 [hep-ph]]

## Partonic threshold approximation

Approximation works very well for LHC and Tevatron energies  
less than 1% difference between approximate and exact cross sections  
at both NLO and NNLO

also true for  $p_T$  and rapidity distributions and  $A_{FB}$

NK, PRD 82, 114030 (2010) and PRD84, 011504 (2011)

fixed-order expansion - no prescription is used

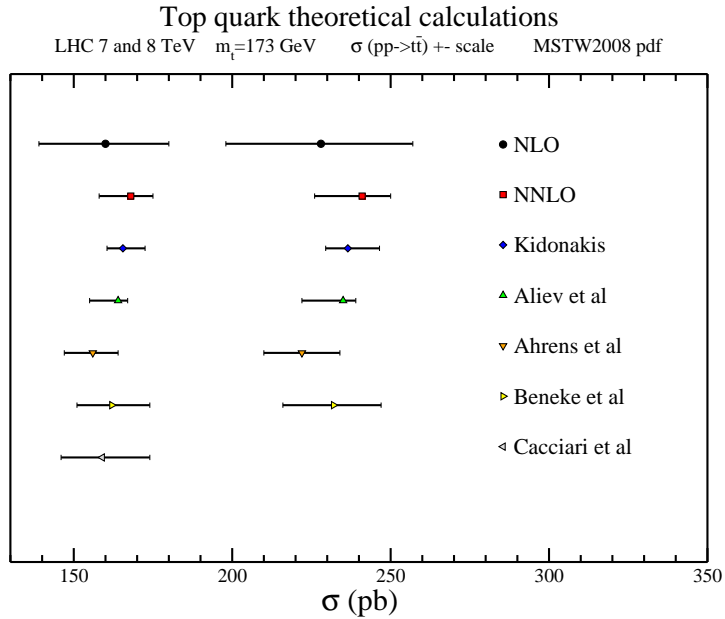
stability of the theoretical NNLO approximate result in this  
double-differential pQCD resummation approach over the past decade

the reliability of the NNLO approximate result and near-identical  
value to exact NNLO is very important for several reasons

- provides confidence of application to other processes
- used as background for many analyses (Higgs, etc)

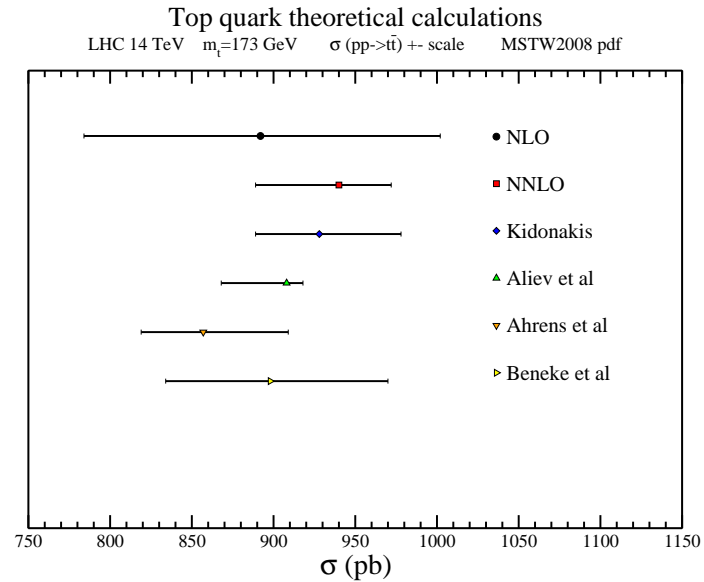
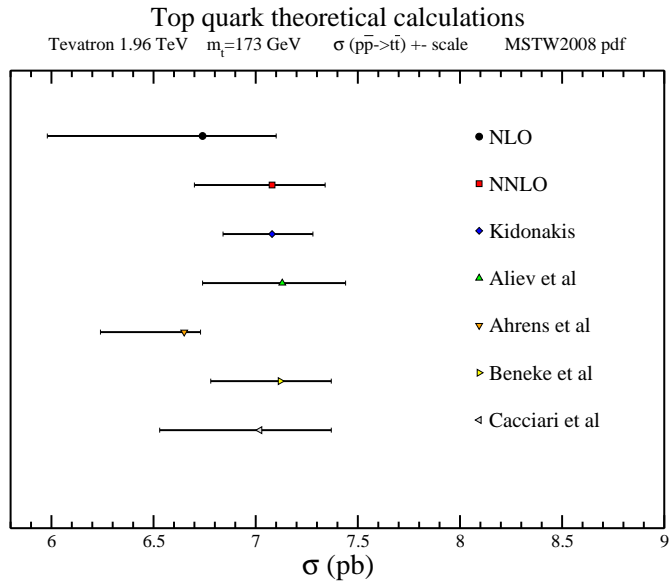
add  $aN^3LO$  corrections for best result

NK, 2014 and 2015

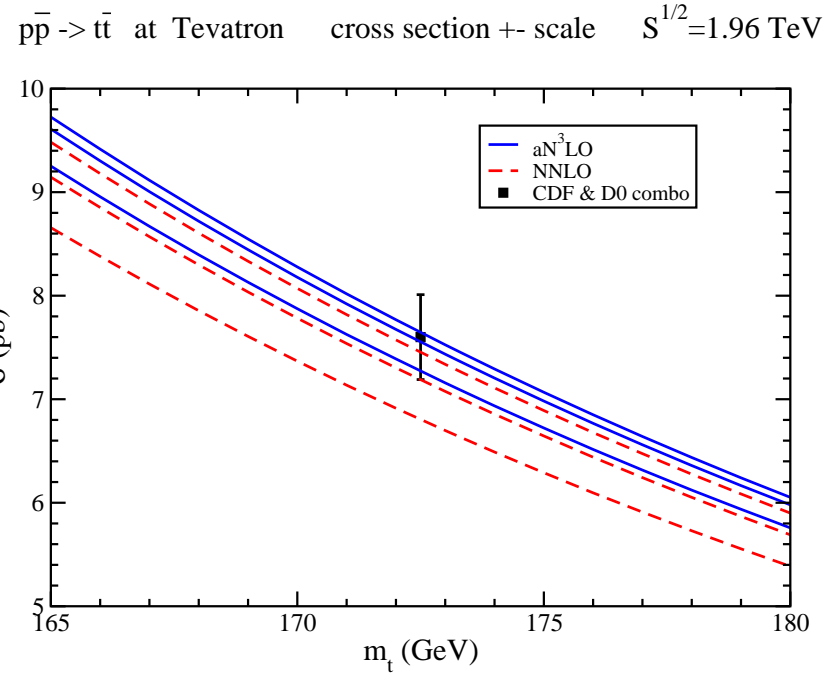
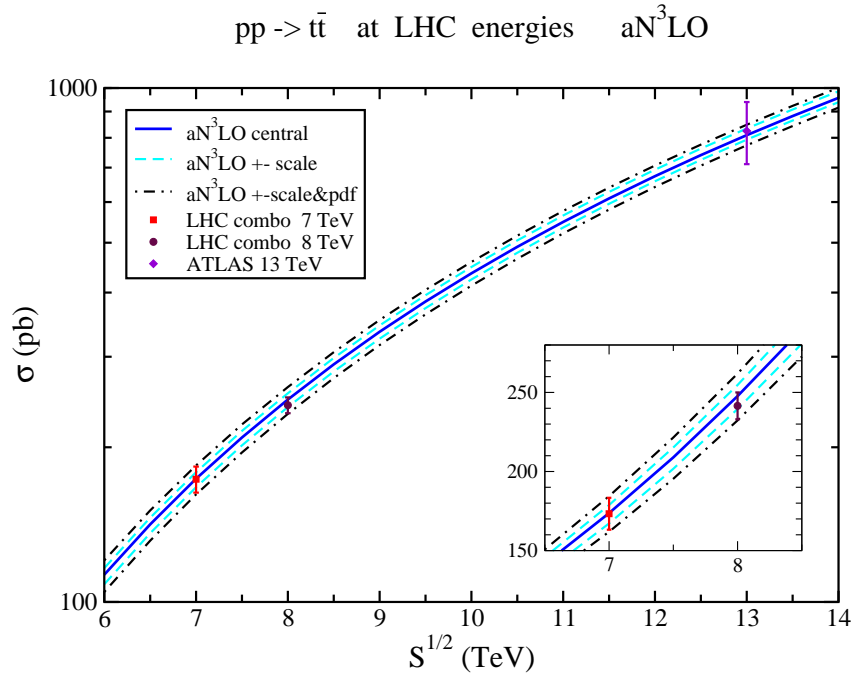


**Comparison of various NNLO approx approaches  
all with the same choice of parameters**

**Kidonakis, PRD 82, 114030 (2010) differential-pQCD**  
**Aliev et al, CPC 182, 1034 (2011) total-pQCD**  
**Ahrens et al, PLB 703, 135 (2011) differential -SCET**  
**Beneke et al, NPB 855, 695 (2012) total-SCET**  
**Cacciari et al, PLB 710, 612 (2012) total-pQCD**



# Top-pair cross sections at the LHC and the Tevatron



aN<sup>3</sup>LO total  $t\bar{t}$  cross sections with  $m_t = 173.3$  GeV

**Tevatron 1.96 TeV:  $7.37^{+0.09+0.38}_{-0.27-0.28}$  pb**

**LHC 7 TeV:  $174^{+5+9}_{-7-10}$  pb**

**LHC 8 TeV:  $248^{+7+12}_{-8-13}$  pb**

**LHC 13 TeV:  $810^{+24+30}_{-16-32}$  pb**

**LHC 14 TeV:  $957^{+28+34}_{-19-36}$  pb**

## Relative size of perturbative corrections

$$\sigma^{\text{aN}^3\text{LO}} = \sigma^{(0)} \left[ 1 + \frac{\sigma^{(1)}}{\sigma^{(0)}} + \frac{\sigma^{(2)}}{\sigma^{(0)}} + \frac{\sigma^{(3)}}{\sigma^{(0)}} \right]$$

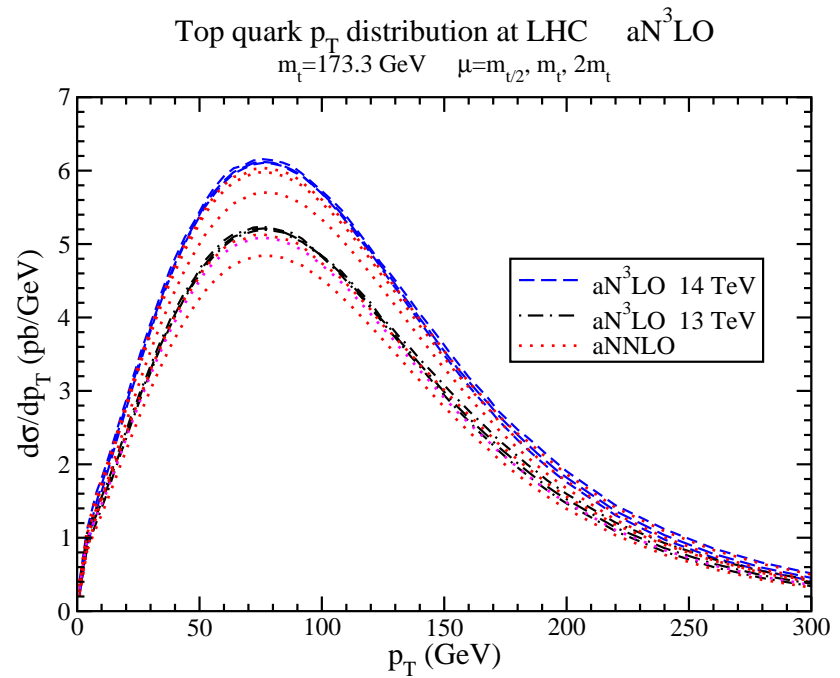
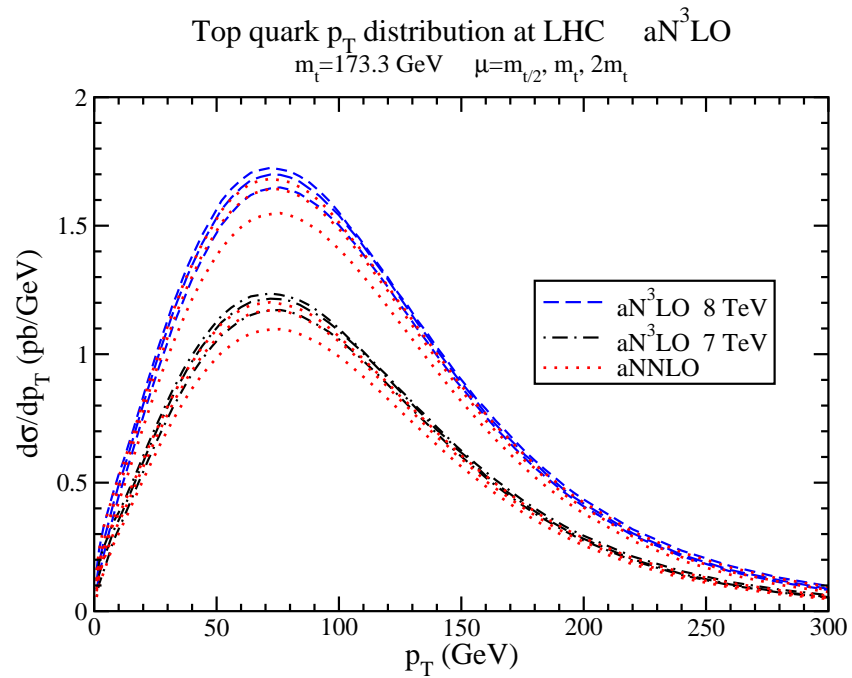
Fractional contributions to the perturbative series for the $t\bar{t}$ cross section					
corrections	Tevatron	LHC 7 TeV	LHC 8 TeV	LHC 13 TeV	LHC 14 TeV
$\sigma^{(1)}/\sigma^{(0)}$	0.236	0.470	0.476	0.493	0.496
$\sigma^{(2)}/\sigma^{(0)}$	0.106	0.178	0.177	0.172	0.170
$\sigma^{(3)}/\sigma^{(0)}$	0.068	0.066	0.059	0.045	0.043

At 14 TeV  $\sigma^{\text{aN}^3\text{LO}} = 1.709 \sigma^{(0)}$

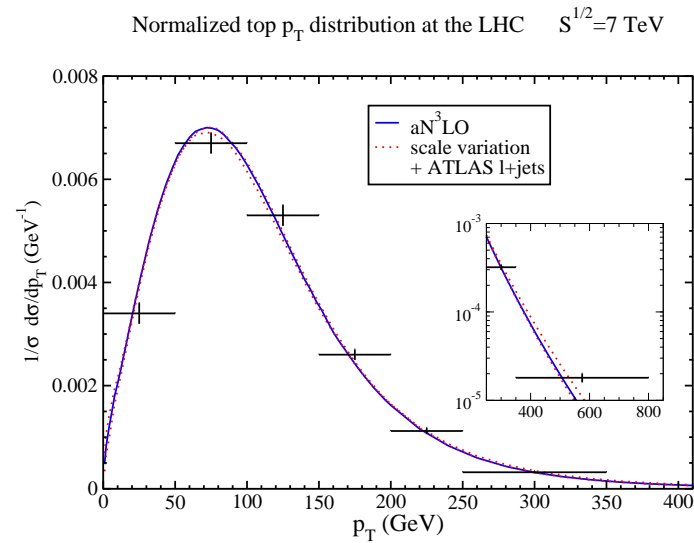
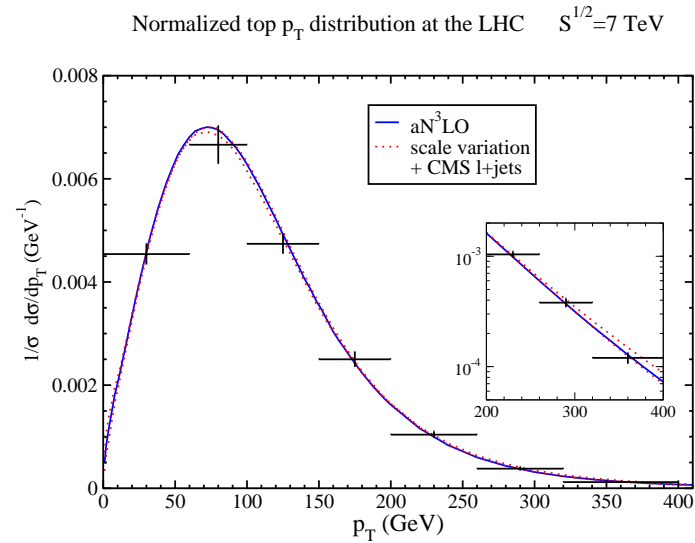
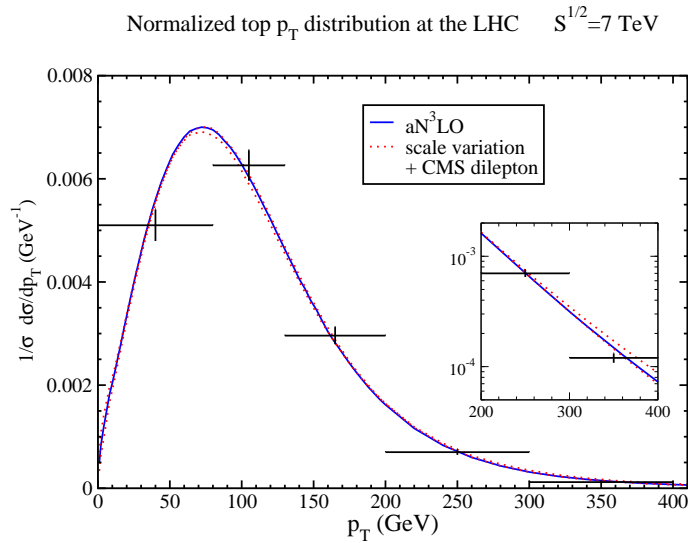
Series approximated well by  $\sum_{n=1}^4 1/n! = 1.708 \dots$

It is amusing to note that  $\sum_{n=1}^{\infty} 1/n! = e - 1 = 1.718 \dots$

# Top quark $p_T$ distribution at the LHC



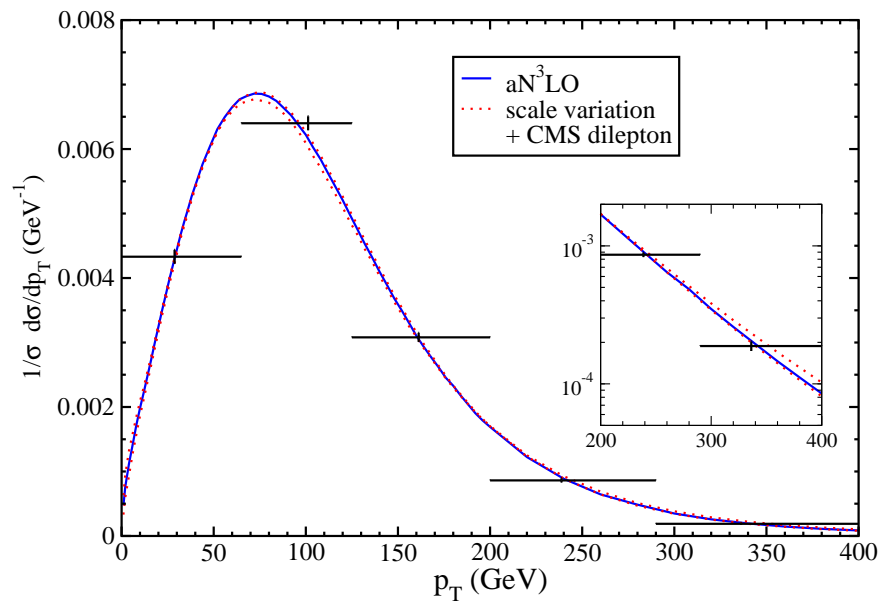
# Normalized top quark $p_T$ distribution at 7 TeV LHC



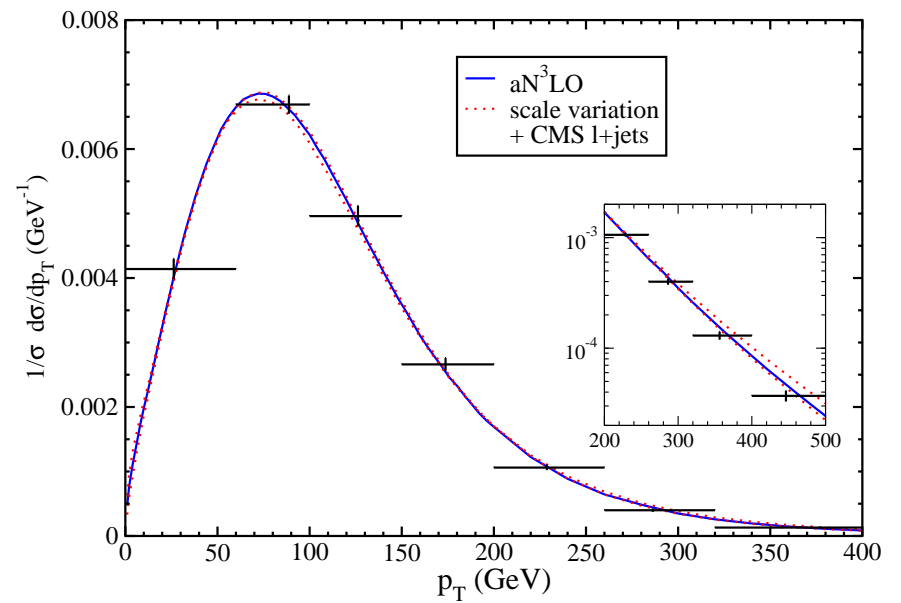


# Normalized top quark $p_T$ distribution at 8 TeV LHC

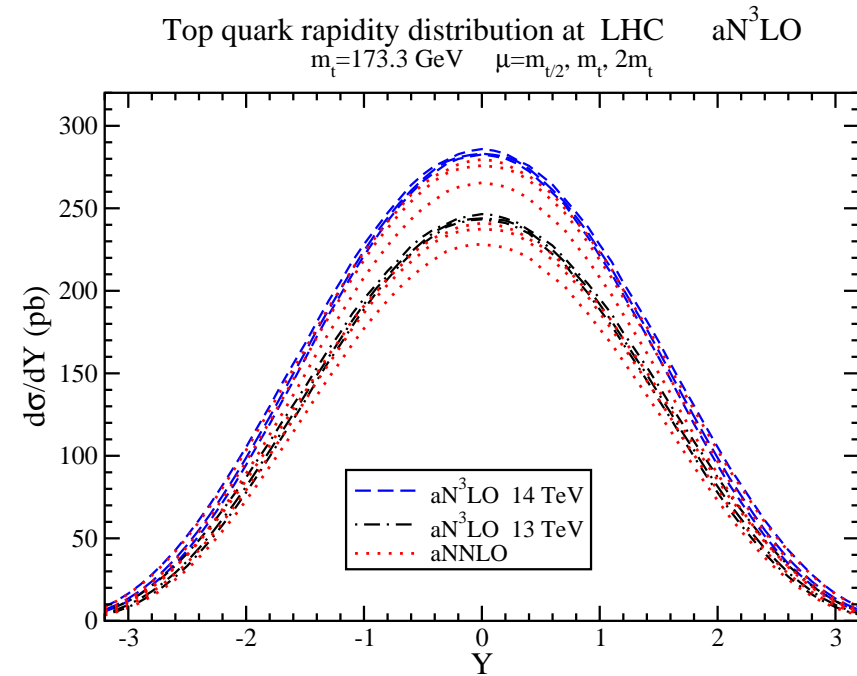
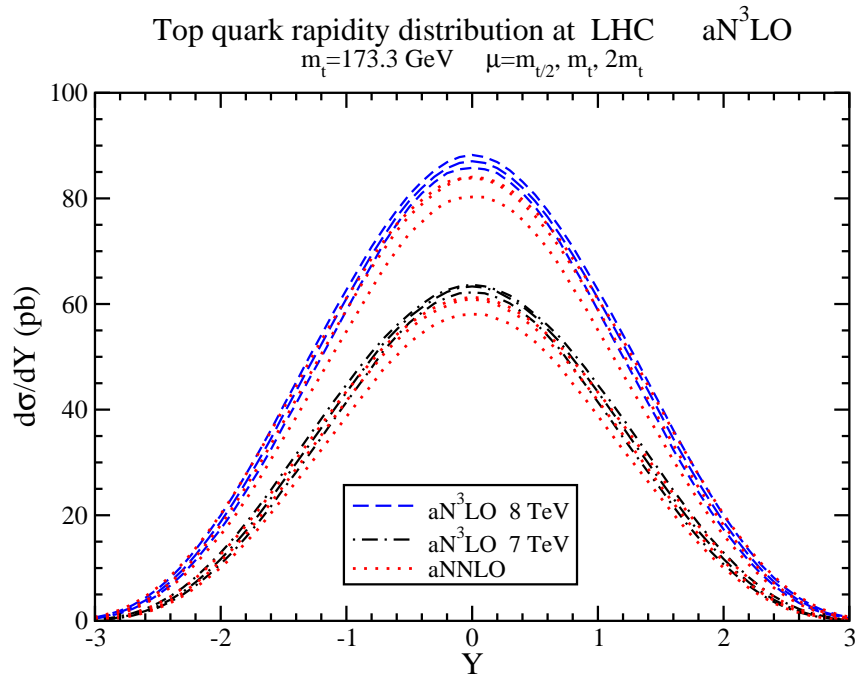
Normalized top  $p_T$  distribution at the LHC  $S^{1/2}=8$  TeV



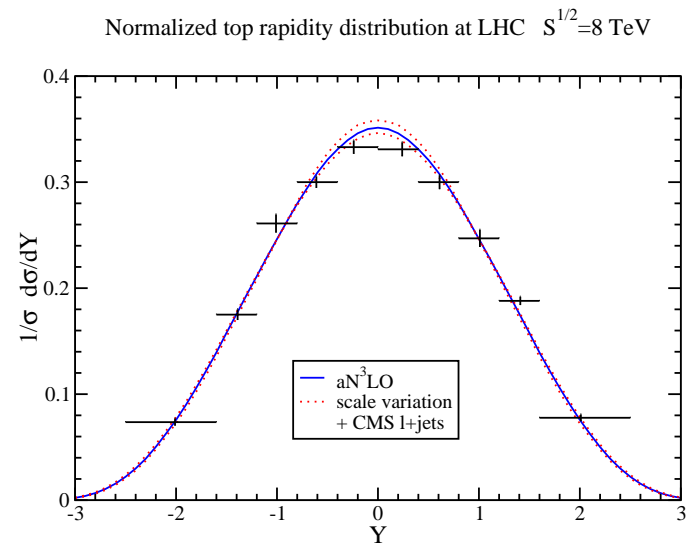
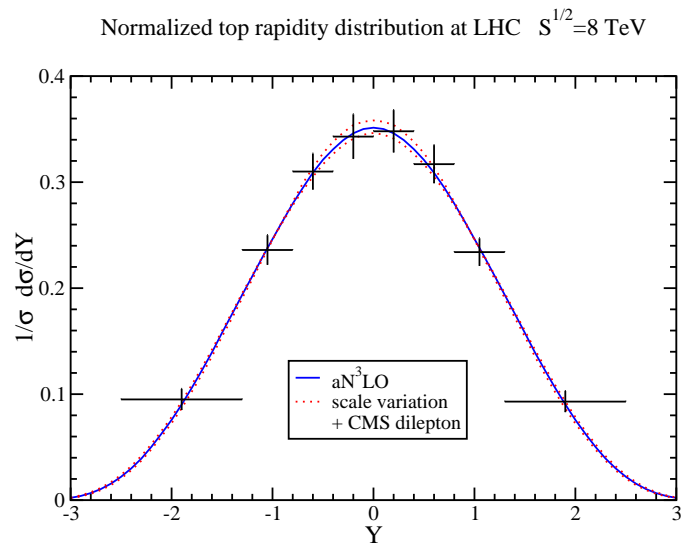
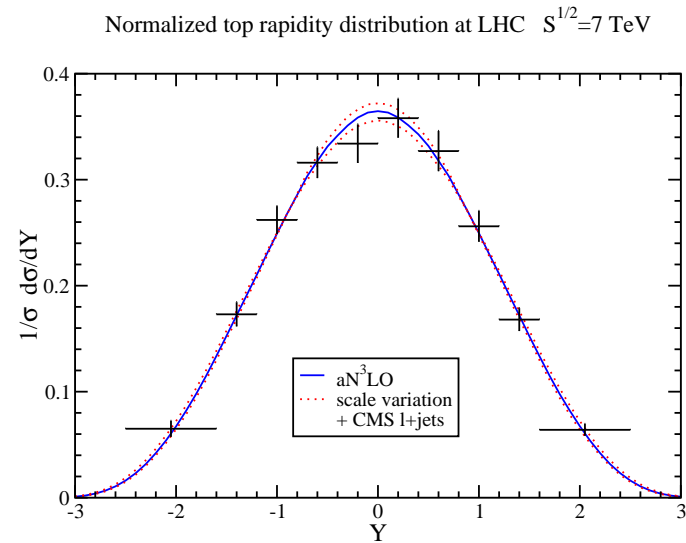
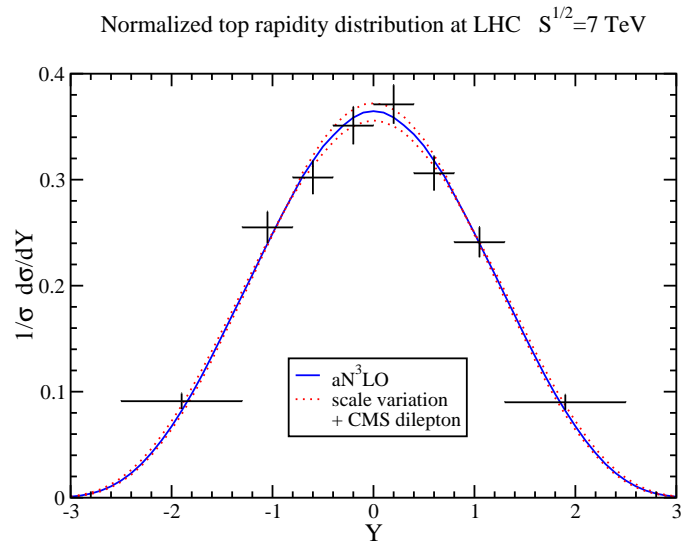
Normalized top  $p_T$  distribution at the LHC  $S^{1/2}=8$  TeV



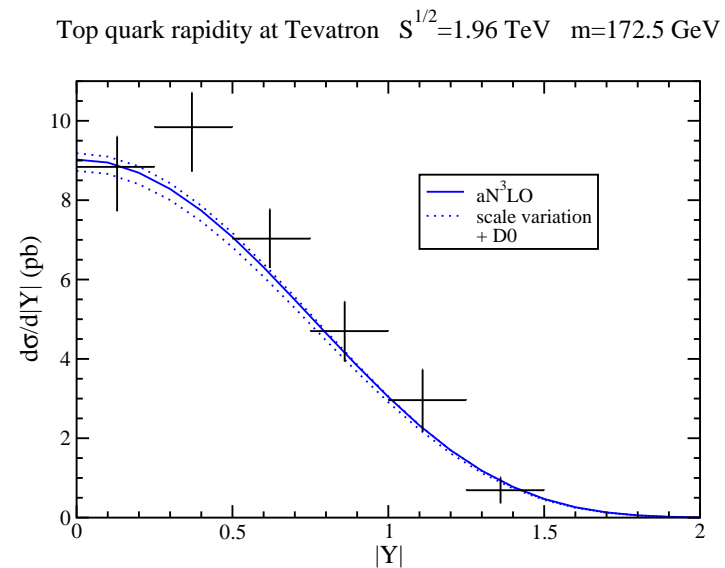
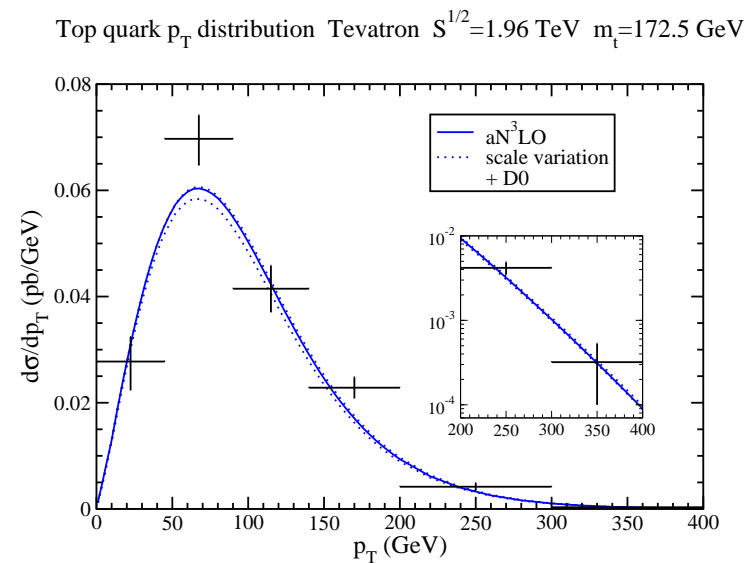
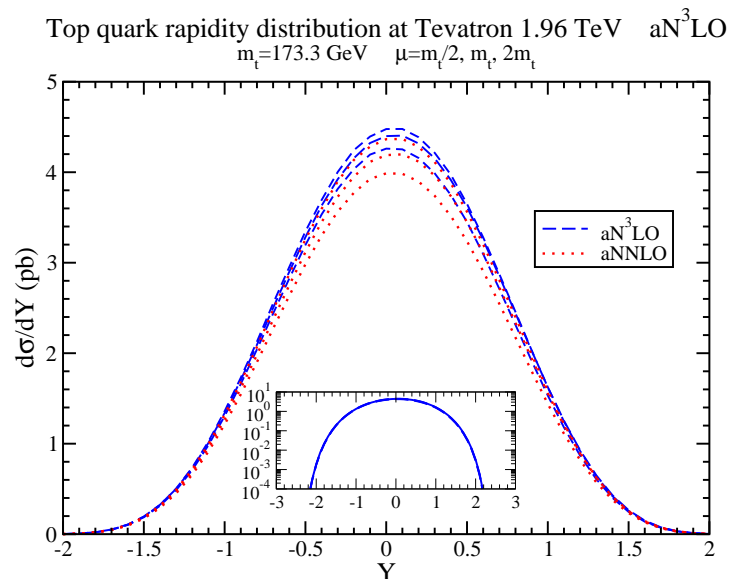
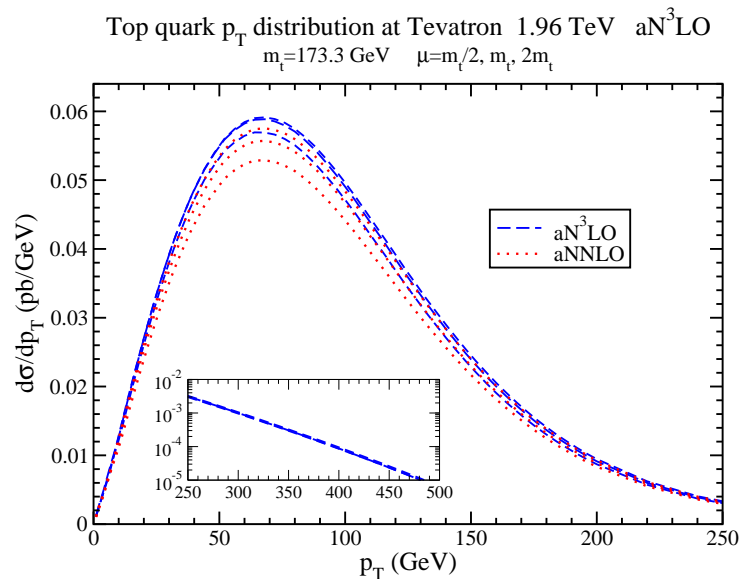
# Top quark rapidity distribution at the LHC



# Normalized top quark rapidity distribution at the LHC



# Top quark $p_T$ and rapidity distributions at the Tevatron



## Top forward-backward asymmetry at the Tevatron

$$A_{\text{FB}} = \frac{\sigma(y_t > 0) - \sigma(y_t < 0)}{\sigma(y_t > 0) + \sigma(y_t < 0)} \equiv \frac{\Delta\sigma}{\sigma} \quad (1)$$

$$A_{\text{FB}} = \frac{\Delta\sigma^{\text{EW}} + \alpha_s^3 \Delta\sigma^{(1)} + \alpha_s^4 \Delta\sigma^{(2)} + \alpha_s^5 \Delta\sigma^{(3)} + \dots}{\alpha_s^2 \sigma^{(0)} + \alpha_s^3 \sigma^{(1)} + \alpha_s^4 \sigma^{(2)} + \alpha_s^5 \sigma^{(3)} + \dots} \quad (2)$$

$$A_{\text{FB}} = \frac{\Delta\sigma^{\text{EW}}}{\alpha_s^2 \sigma^{(0)}} + \alpha_s \frac{\Delta\sigma^{(1)}}{\sigma^{(0)}} - \frac{\Delta\sigma^{\text{EW}} \sigma^{(1)}}{\alpha_s (\sigma^{(0)})^2} + \alpha_s^2 \left[ \frac{\Delta\sigma^{(2)}}{\sigma^{(0)}} - \frac{\Delta\sigma^{(1)} \sigma^{(1)}}{(\sigma^{(0)})^2} \right] + \frac{\Delta\sigma^{\text{EW}}}{(\sigma^{(0)})^3} \left[ (\sigma^{(1)})^2 - \sigma^{(0)} \sigma^{(2)} \right] + \alpha_s^3 \left[ \frac{\Delta\sigma^{(3)}}{\sigma^{(0)}} - \frac{\Delta\sigma^{(2)} \sigma^{(1)}}{(\sigma^{(0)})^2} + \frac{\Delta\sigma^{(1)} (\sigma^{(1)})^2}{(\sigma^{(0)})^3} - \frac{\Delta\sigma^{(1)} \sigma^{(2)}}{(\sigma^{(0)})^2} \right] + \dots \quad (3)$$

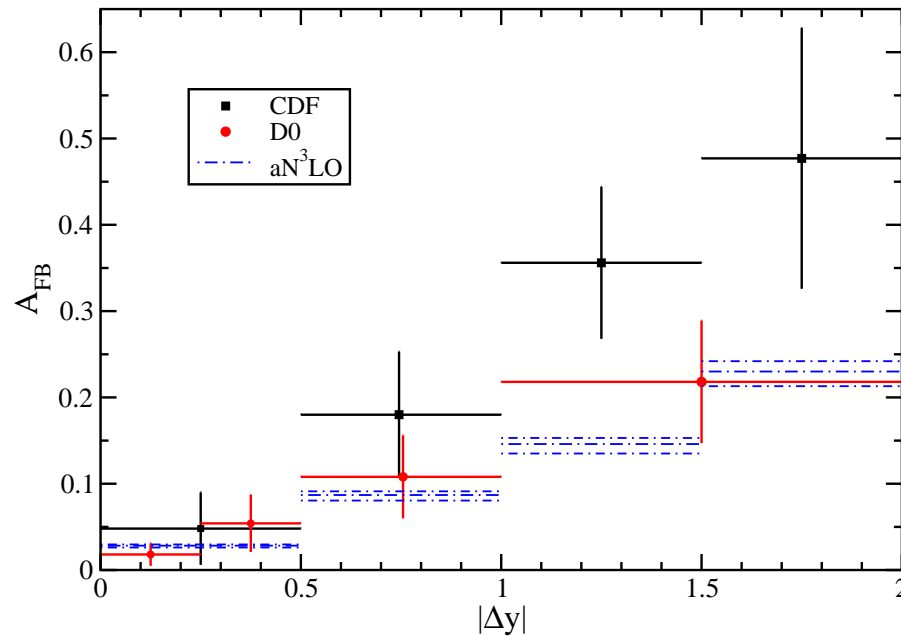
Top-quark asymmetry at the Tevatron		
aN <sup>3</sup> LO $A_{FB}$ %	$p\bar{p}$ frame	$t\bar{t}$ frame
QCD only Eq. (2)	$5.6^{+0.3}_{-0.4}$	$8.1^{+0.4}_{-0.6}$
QCD only Eq. (3)	$6.0 \pm 0.1$	$8.7 \pm 0.2$
QCD+EW Eq. (2)	$6.4^{+0.5}_{-0.6}$	$9.4^{+0.7}_{-0.9}$
QCD+EW Eq. (3)	$6.8 \pm 0.3$	$10.0 \pm 0.6$

large corrections: aN<sup>3</sup>LO/NNLO ratio is 1.08 [Eq. (2)] or 1.05 [Eq. (3)]

## Top differential $A_{\text{FB}}$ at the Tevatron

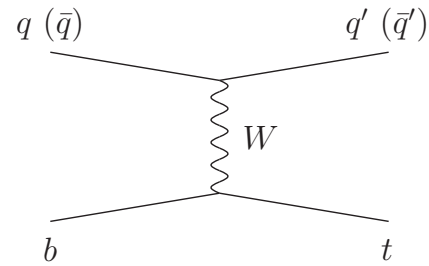
$$A_{\text{FB}}^{\text{bin}} = \frac{\sigma_{\text{bin}}^+(\Delta y) - \sigma_{\text{bin}}^-(\Delta y)}{\sigma_{\text{bin}}^+(\Delta y) + \sigma_{\text{bin}}^-(\Delta y)} \quad \text{with} \quad \Delta y = y_t - y_{\bar{t}}$$

$A_{\text{FB}}$  at Tevatron  $S^{1/2}=1.96 \text{ TeV}$   $m_t=173.3 \text{ GeV}$

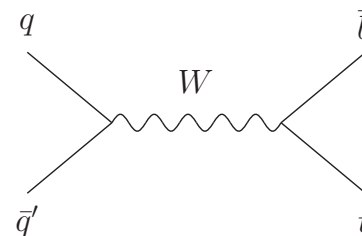


## Single-top partonic processes at LO

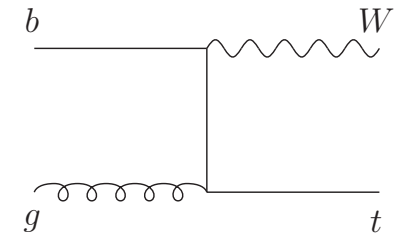
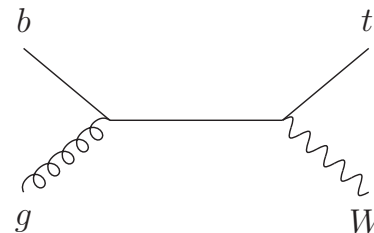
- $t$  channel:  $qb \rightarrow q't$  and  $\bar{q}b \rightarrow \bar{q}'t$   
dominant at Tevatron and LHC



- $s$  channel:  $q\bar{q}' \rightarrow \bar{b}t$   
small at Tevatron and LHC



- associated  $tW$  production:  $bg \rightarrow tW^-$   
very small at Tevatron, significant at LHC



## Single top $t$ -channel production at aNNLO at the LHC

$m_t = 173.3 \text{ GeV}$

LHC	$t$	$\bar{t}$	Total (pb)
8 TeV	$55.9^{+2.1}_{-0.3} \pm 1.1$	$30.6 \pm 0.7^{+0.9}_{-1.1}$	$86.5^{+2.8+2.0}_{-1.0-2.2}$
13 TeV	$136^{+3}_{-1} \pm 3$	$82^{+2}_{-1} \pm 2$	$218^{+5}_{-2} \pm 5$
14 TeV	$154^{+4}_{-1} \pm 3$	$94^{+2+2}_{-1-3}$	$248^{+6+5}_{-2-6}$

$\pm$  scale  $\pm$  pdf errors with MSTW2008 NNLO pdf 90% CL

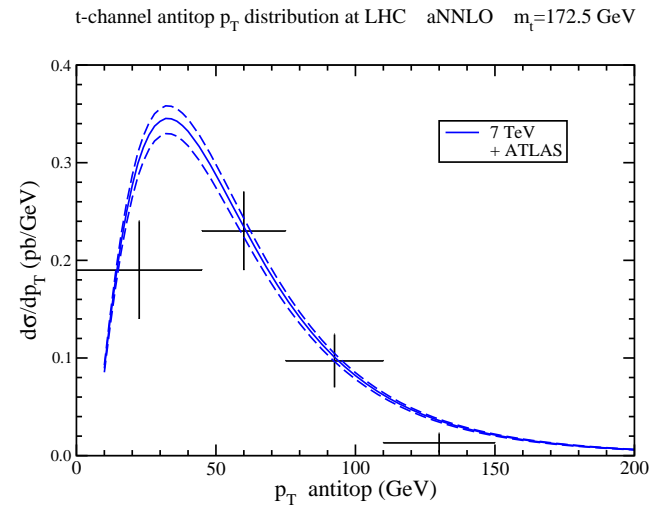
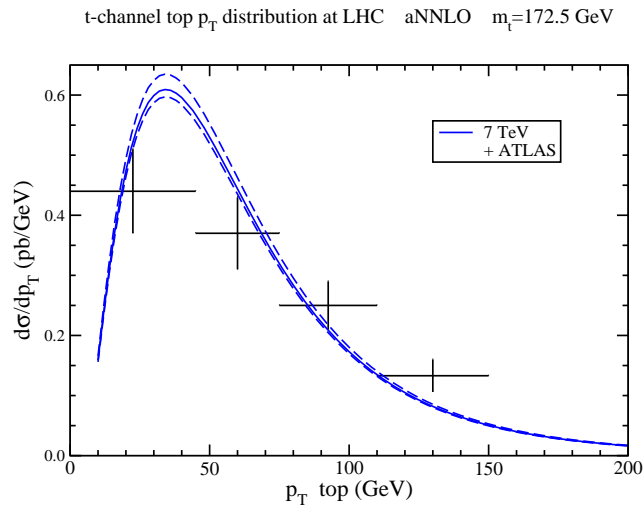
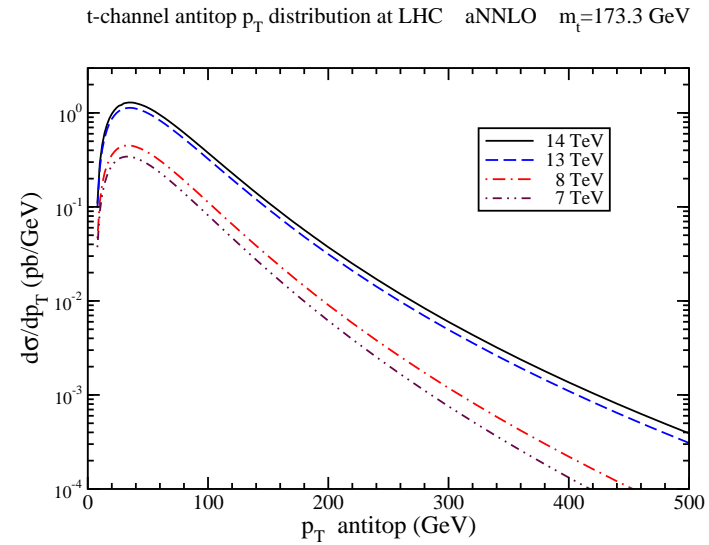
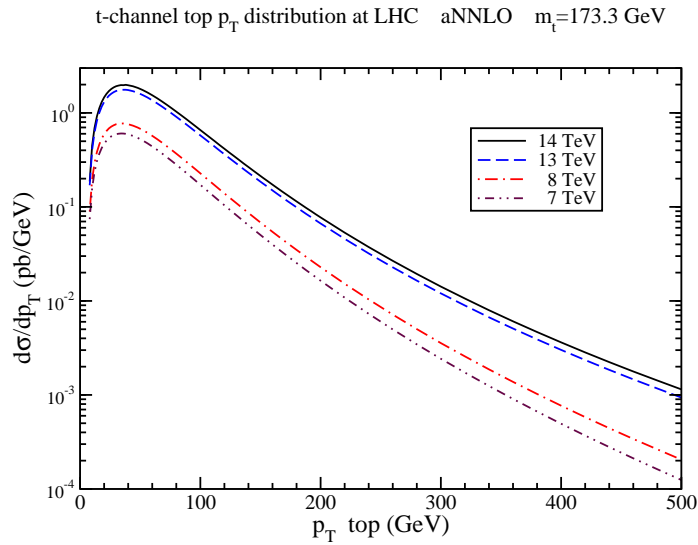
ratio  $\sigma(t)/\sigma(\bar{t}) = 1.82^{+0.10}_{-0.09}$  at 8 TeV

- compares well with CMS result  $1.95 \pm 0.10 \pm 0.19$

agrees with NNLO result in 1404.7116 [hep-ph]



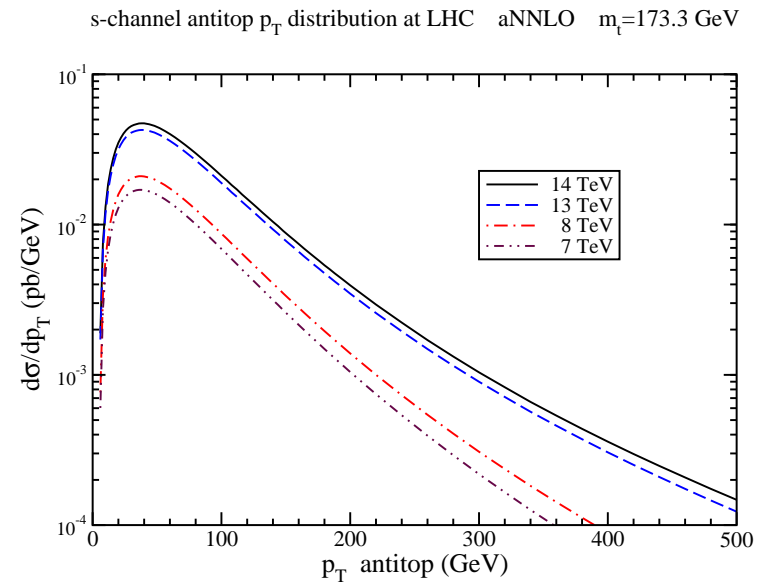
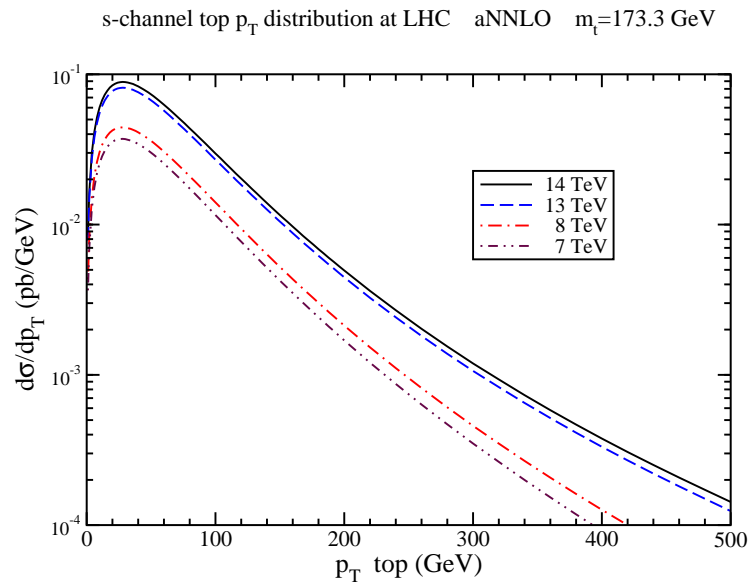
# Single top $t$ -channel aNNLO $p_T$ distributions at the LHC



# Single top $s$ -channel production at aNNLO at the LHC

LHC	$t$	$\bar{t}$	Total (pb)
8 TeV	$3.75 \pm 0.07 \pm 0.13$	$1.90 \pm 0.01 \pm 0.08$	$5.65 \pm 0.08 \pm 0.21$
13 TeV	$7.07 \pm 0.13^{+0.24}_{-0.22}$	$4.10 \pm 0.05^{+0.14}_{-0.16}$	$11.17 \pm 0.18 \pm 0.38$
14 TeV	$7.79 \pm 0.14^{+0.31}_{-0.24}$	$4.57 \pm 0.05^{+0.18}_{-0.17}$	$12.35 \pm 0.19^{+0.49}_{-0.41}$

( $m_t = 173.3$  GeV)  $\pm$  scale  $\pm$  pdf errors with MSTW2008 NNLO pdf 90% CL



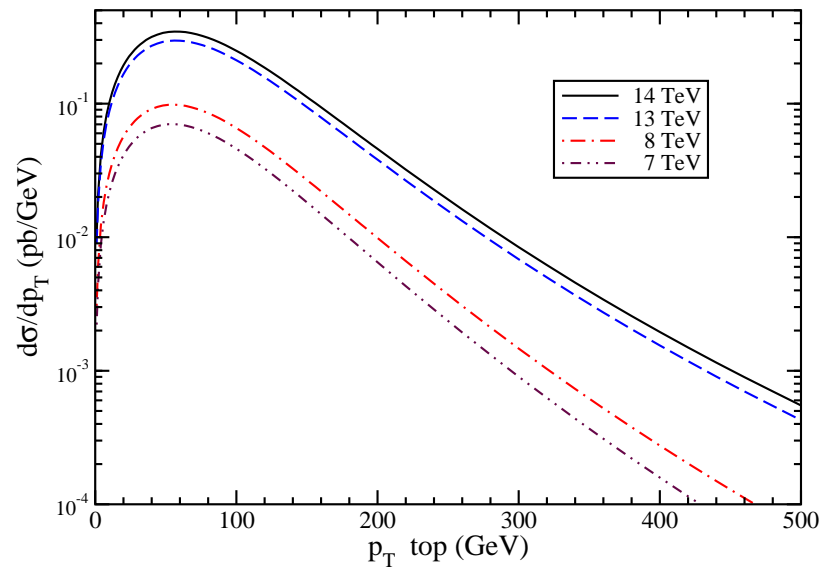
# Associated $tW^-$ production at aNNLO at the LHC

$m_t = 173.3 \text{ GeV}$

LHC	$tW^-$	$tW^- + \bar{t}W^+$ (pb)
8 TeV	$11.0 \pm 0.3 \pm 0.7$	$22.0 \pm 0.6 \pm 1.4$
13 TeV	$35.20 \pm 0.9^{+1.6}_{-1.7}$	$70.40 \pm 1.8^{+3.2}_{-3.4}$
14 TeV	$41.6 \pm 1.0^{+1.5}_{-2.3}$	$83.1 \pm 2.0^{+3.1}_{-4.6}$

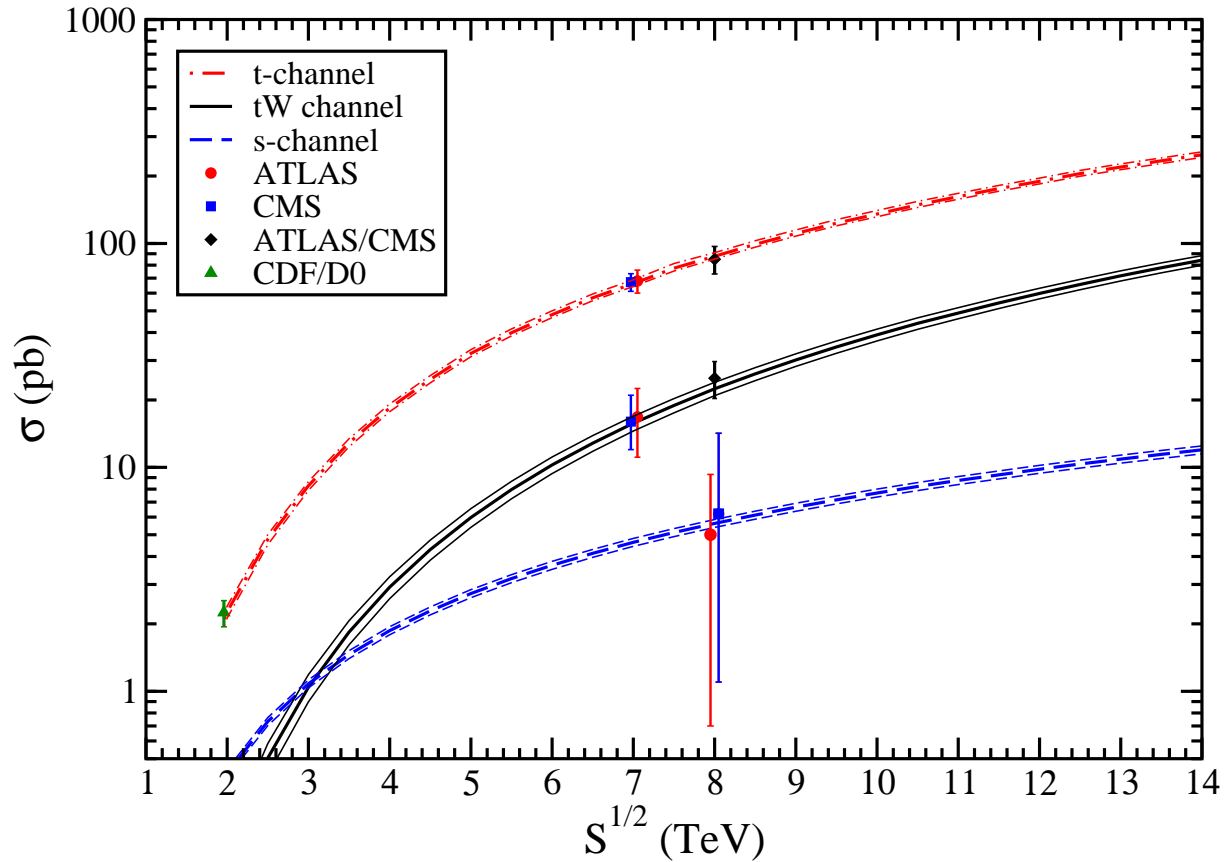
$\pm$  scale  $\pm$  pdf errors with MSTW2008 NNLO pdf 90% CL

Top  $p_T$  distribution in  $tW^-$  production at LHC aNNLO  $m_t=173.3 \text{ GeV}$



# Single-top cross sections

aNNLO single-top cross sections +-scale&pdf  $m_t=172.5$  GeV



Excellent agreement of theory with data for all three channels

## Summary

- soft-gluon corrections for  $t\bar{t}$  and single-top production through N<sup>3</sup>LO
- total cross sections
- top quark  $p_T$  and rapidity distributions
- top quark forward-backward asymmetry
- corrections are significant at the LHC and Tevatron
- excellent agreement with LHC and Tevatron data