T-duality off shell in 3D Type II superspace

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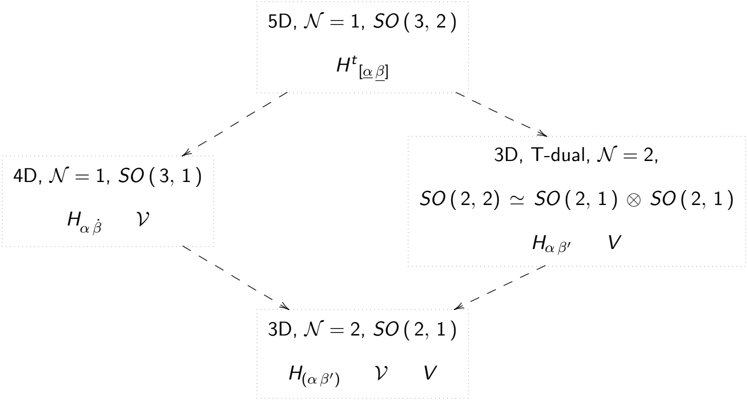
Brief introduction

- In the **arXiv:1308.6350** paper we obtained the curvature tensor (previously discovered in Warren Siegel paper) in a way manifestly covariant under O(D,D) T-duality
- The aim of this paper is to extend the techniques of the T-dually extended spaces from the bosonic case to the supersymmetric case
- We give the manifestly T-dual formulation of the massless sector of the classical 3D Type II superstring in off-shell 3D $\mathcal{N} = 2$ superspace.
- We want to motivate the natural identification between 4D $\mathcal{N} = 1$ supergravity, further compactified to 3D $\mathcal{N} = 2$ and T-dual 3D $\mathcal{N} = 2$ string theory
- Both can be thought to have an origin in higher dimensional F-theory


5D, $\mathcal{N} = 1$, $SO(3, 2)$
${H^t}_{[\underline{\alpha}\,\underline{\beta}]}$
4D, $\mathcal{N} = 1$, $SO(3, 1)$
$H_{\alpha\,\dot{\beta}}$ $\mathcal{V}$
3D, T-dual, $\mathcal{N} = 2$,
$SO(2, 2) \simeq SO(2, 1) \otimes SO(2, 1)$
$H_{\alpha\,\beta'}$ V
3D, $\mathcal{N} = 2$, $SO(2, 1)$
$H_{(\alpha\,\beta')}$ $\mathcal{V}$ V

Calculations

- Gauge fixing:

$$\gamma_{\mathbf{a}}{}^{\alpha\beta} E^{(1)}{}_{\alpha\beta} = 0 \quad \Rightarrow \quad \lambda_{\mathbf{a}} \propto \gamma_{\mathbf{a}}{}^{\alpha\beta} D_\alpha \lambda_\beta$$
$$\gamma^{\mathbf{a}\,\alpha\beta} E^{(1)}{}_{\alpha\mathbf{a}} = 0 \quad \Rightarrow \quad \lambda^\alpha \propto \gamma^{\mathbf{a}\,\alpha\beta} D_{[\mathbf{a}} \lambda_{\beta]}$$
$$E^{(1)}{}_{\mathbf{a}\mathbf{b}} = 0 \quad \Rightarrow \quad \lambda_{\mathbf{a}\mathbf{b}} \propto D_{[\mathbf{a}} \lambda_{\mathbf{b}]}$$

Same for Left $\rightarrow$ Right

- Fixes $\lambda_P, \lambda_\Omega, \lambda_\Sigma \propto \lambda_D$. Because of the coset constraints $\lambda_S = 0$.
- Gauge fixing constraints give the constraints on vielbeins:

$$E^{(1)}{}_{DD} = E^{(1)}{}_{\alpha\beta} = 0, \quad E^{(1)}{}_{PP} = E^{(1)}{}_{\mathbf{a}\mathbf{b}} = 0, \quad E^{(1)\alpha\beta}{}_{\beta} = 0$$

(part of $E^{(1)P}{}_{D}$)

- Later (by dimension $-\frac{1}{2}$ constraints) one can see that $E^{(1)}{}_{PD} = 0$.

- Dimensional constraints
- Put the torsions of negative (engineering) dimensions to 0
- We also put the (unfixed) torsions of zero dimension to 0
- We will also put the dimension $\frac{1}{2}$ (unfixed) torsions to 0
- Doing that we produce just algebraic constraints on veilbeins.
- The nontrivial dimensional constraints are:

$$T_{D\,D}{}^{\Omega} = 0,\ T_{D\,D}{}^{P} = f_{D\,D}{}^{P},\ T_{D\,D}{}^{D} = 0,\ T_{P\,P}{}^{\Omega} = 0$$

- Dimensional constraints: unmixed solution

${T_{DD}}^{\Omega} = 0$ and $\gamma^{\mathbf{a}\,\alpha\beta} {E^{(1)}}_{\alpha\,\mathbf{a}} = 0$	$\Rightarrow$	${E^{(1)}}_{PD} = 0$
${T_{DD}}^{P} = {f_{DD}}^{P}$	$\Rightarrow$	${E^{(1)}}_{D\Omega} = 0$
${T_{PP}}^{\Omega} = 0$ or ${T_{DD}}^{D} = 0$	$\Rightarrow$	${E^{(1)}}_{\Sigma D} = {E^{(1)\mathbf{ab}}}_{\alpha} = -2\,{\gamma^{[\mathbf{a}}}_{\alpha\rho} E^{(1)\rho\,\|\,\mathbf{b}]} \equiv \gamma\,.\,{{E^{(1)}}_{\Omega}}^{P}$

- Dimensional constraints: mixed solution

${T_{DD}}^{\tilde{\Omega}} = 0$	$\Rightarrow$	${E^{(1)}}_{P\tilde{D}} \equiv {E^{(1)}}_{\mathbf{a}\tilde{\alpha}} = -\frac{1}{2}\,{\gamma_{\mathbf{a}}}^{\beta\epsilon} D_{\beta}\,{E^{(1)}}_{\epsilon\tilde{\alpha}} \equiv -\gamma\,.\,D_D\,.\,{E^{(1)}}_{D\tilde{D}}$
${T_{DD}}^{\tilde{P}} = 0$	$\Rightarrow$	${E^{(1)}}_{P\tilde{P}} \equiv {E^{(1)}}_{\mathbf{a}\tilde{\mathbf{b}}} = -\frac{1}{2}\,{\gamma_{\mathbf{a}}}^{\beta\epsilon} D_{\beta}\,{E^{(1)}}_{\epsilon\tilde{\mathbf{b}}} \equiv -\gamma\,.\,D_D\,.\,{E^{(1)}}_{D\tilde{P}}$
${T_{D\tilde{D}}}^{P} = 0$	$\Rightarrow$	${E^{(1)}}_{\Omega\tilde{D}} \equiv {E^{(1)\alpha}}_{\tilde{\beta}} = -\frac{1}{6}\,\gamma^{\mathbf{a}\,\epsilon\alpha} D_{[\epsilon}\,{E^{(1)}}_{\mathbf{a}]\tilde{\beta}} \equiv -\gamma\,.\,D_{[D}\,.\,{E^{(1)}}_{P]\tilde{D}}$
${T_{P\tilde{P}}}^{\Omega} = 0$	$\Rightarrow$	${E^{(1)}}_{\Omega\tilde{P}} \equiv {E^{(1)\alpha}}_{\tilde{\mathbf{a}}} = -\frac{1}{6}\,\gamma^{\mathbf{b}\,\epsilon\alpha} D_{[\epsilon}\,{E^{(1)}}_{\mathbf{b}]\tilde{\mathbf{a}}} \equiv -\gamma\,.\,D_{[D}\,.\,{E^{(1)}}_{P]\tilde{P}}$
${T_{PP}}^{\tilde{\Omega}} = 0$	$\Rightarrow$	${E^{(1)}}_{\Sigma\tilde{D}} \equiv {E^{(1)\mathbf{ab}}}_{\tilde{\alpha}} = \eta^{\mathbf{ac}}\,\eta^{\mathbf{bd}} D_{[\mathbf{c}}\,{E^{(1)}}_{\mathbf{d}]\tilde{\alpha}} \equiv \eta\,\eta\,.\,D_{[P}\,.\,{E^{(1)}}_{P]\tilde{D}}$

- The net result of dimension 1 unmixed algebraic constraints is that everything can be expressed in terms of $E^{(1)}{}_{D\,\Sigma}$:

$$B = -\frac{1}{\vartheta + 6\,\zeta}\,\gamma^{\mathbf{a}}{}_{\beta}^{\ \alpha}\,D_\alpha\,E^{(1)\,\beta}{}_{\mathbf{a}}$$

$$E^{(1)\,\alpha\,\beta} = \frac{1}{12}\,\gamma^{\mathbf{a}\,(\alpha|\,\epsilon}\,D_\epsilon\,E^{(1)\,\beta)}{}_{\mathbf{a}} + \frac{1}{12}\,\gamma_{\mathbf{a}}{}^{\alpha\,\beta}E^{(1)\mathbf{ab}}{}_{\mathbf{b}}$$

$$E^{(1)}{}_{\mathbf{c}}{}^{\mathbf{ab}} = -\frac{1}{2}\,\gamma_{\mathbf{c}}{}^{\alpha\,\beta}\,D_\alpha\,E^{(1)}{}_{\beta}{}^{\mathbf{ab}} + (\,\vartheta + 4\,\zeta\,)\,\eta_{\mathbf{c\,e}}\,\varepsilon^{\mathbf{e\,a\,b}}\,B$$

- The dimension 1 mixed constraints give:

$$E^{(1)}{}_{\Sigma\,\tilde{P}} \equiv E^{(1)\mathbf{bc}}{}_{\tilde{\mathbf{a}}} = \eta^{\mathbf{b\,d}}\,\eta^{\mathbf{c\,e}}\,D_{[\mathbf{d}}\,E^{(1)}{}_{\mathbf{e}]\,\tilde{\mathbf{a}}} \equiv \eta\eta\,.\,D_{[P}\,E^{(1)}{}_{P]\,\tilde{P}}$$

$$E^{(1)}{}_{\Omega\,\tilde{\Omega}} \equiv E^{(1)\alpha\,\tilde{\beta}} = \frac{1}{6}\,\gamma^{\mathbf{a}\ \alpha\,\epsilon}\,D_{[\mathbf{a}}\,E^{(1)}{}_{\epsilon]}{}^{\tilde{\beta}} \equiv \gamma\,.\,D_{[P}\,E^{(1)}{}_{D]\tilde{\Omega}}$$

- $\tilde{T}^{(1)}{}_{D} = 0$ constraints gives:

$$B = \frac{-1}{\vartheta + 6\,\zeta}\,\varepsilon^{\nu\,\alpha}\,D_\nu\left[\gamma^{\tilde{\mathbf{a}}\ \tilde{\beta}\,\tilde{\epsilon}}\left(-\ \frac{1}{6}[\,D_{\tilde{\mathbf{a}}},\,D_{\tilde{\beta}}] + \frac{1}{4}\,D_{\tilde{\mathbf{a}}}\,D_{\tilde{\beta}}\right)E^{(1)}{}_{\tilde{\epsilon}\,\alpha} - D_\alpha\,\phi^{(1)}\right]$$

- We found the equations of motion:

$$B + \tilde{B} = 0 \text{ and } B - \tilde{B} = 0$$

- Simplify the structure of B and $\tilde{B}$, the structure of e.o.m.:

$$0 = (D^2 + \tilde{D}^2)(-\tfrac{1}{8}\,\varepsilon^{\alpha\nu}\,\varepsilon^{\tilde{\epsilon}\tilde{\sigma}}\,D_\nu\,D_{\tilde{\sigma}}\,E^{(1)}{}_{\tilde{\epsilon}\alpha} + \phi^{(1)})$$

- Rewrite it using a new field V:

$$(D^2 - \tilde{D}^2)\,V =: (-\tfrac{1}{8}\,\varepsilon^{\alpha\nu}\,\varepsilon^{\tilde{\epsilon}\tilde{\sigma}}\,D_\nu\,D_{\tilde{\sigma}}\,E^{(1)}{}_{\tilde{\epsilon}\alpha} + \phi^{(1)})$$

- Use previous definition:

$$0 = (D^2 + \tilde{D}^2)(D^2 - \tilde{D}^2)\,V$$

- Operator $(D^2 + \tilde{D}^2)(D^2 - \tilde{D}^2)$ acts on the scalar field V, get a nicer form:

$$(D^2 + \tilde{D}^2)(D^2 - \tilde{D}^2)\,V = 4\,D^{\mathcal{A}}\,D_{\mathcal{A}}\,V$$

- The second equation of becomes the e.o.m. for the V field:

$$(D^2 - \tilde{D}^2)^2\,V = 0$$

Conclusion and Further development

- We started with T-dual $\mathcal{N}=2$ string theory, i.e. effective $\mathcal{N}=2$ supergravity in 3 dimensions
- We first obtained the dimension -1 prepotential as the vielbein component $E^{(1)}{}_{D\tilde{D}} \equiv E^{(1)}{}_{\alpha\tilde{\beta}}$ and the dimension $-\frac{3}{2}$ unconstrained gauge parameter $\Lambda_D \equiv \Lambda_\alpha$ (also $\Lambda_{\tilde{D}}$) without solving any differential constraints
- In particular the structure of the linear dilaton ϕ was derived
- It matches the structure obtained from 4D $\mathcal{N}=1$ and its compactification
- This suggests that the T-dually extended superspace approach can be extended also to higher dimensional cases (and also to non flat backgrounds like $AdS_5 \otimes S_5$), as is examined in the current (unfinished) work.

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