Extraction of the proton radius from electron-proton scattering data

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based on 1505.01489, with Gabriel Lee, John Arrington

DPF meeting
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Regardless of the existence of the “proton radius puzzle”:

- serious issues to confront in the precision era of lepton-nucleon scattering data
- addressing these issues will be critical to discovery potential of the accelerator neutrino program

Solving the simpler e-p problem prerequisite to more challenging neutrino processes

The applications, the problems, and the theoretical tools are central to HEP
Some facts about the Rydberg constant puzzle (a.k.a. proton radius puzzle)

1) It has generated a lot of attention and controversy

2) The most mundane resolution necessitates:
   • 5σ shift in fundamental Rydberg constant
   • discarding or revising decades of results in e-p scattering and hydrogen spectroscopy
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   This is HEP’s problem:

3) Systematic effects in electron-proton scattering impact neutrino-nucleus scattering, at a level large compared to precision requirements for oscillation measurements

“The good news is that it’s not my problem”
Recall hydrogen spectrum:

\[ E_n \sim \frac{R_\infty}{n^2} + \frac{r_E^2}{n^3} \]

\[ h c R_\infty = \frac{m_e c^2 \alpha^2}{2} \approx 13.6 \text{ eV} \]

Disentangle 2 unknowns, \( R_\infty \) and \( r_E \), using well-measured 1S-2S hydrogen transition and
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\[
hcR_\infty = \frac{m_e c^2 \alpha^2}{2} \approx 13.6 \text{ eV}
\]

proton charge radius

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(2) electron-proton scattering determination of \(r_E\)
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5σ discrepancy in Rydberg constant from (1+2) versus (3)
this talk: new extraction of proton charge and magnetic radii from electron scattering data
preliminaries
What is the proton charge radius?

recall scattering from extended classical charge distribution:

\[ \frac{d\sigma}{d\Omega} = \left( \frac{d\sigma}{d\Omega} \right)_{\text{pointlike}} |F(q^2)|^2 \]

\[ F(q^2) = \int d^3r \, e^{i \mathbf{q} \cdot \mathbf{r}} \rho(r) \]

\[ = \int d^3r \left[ 1 + i \mathbf{q} \cdot \mathbf{r} - \frac{1}{2} (\mathbf{q} \cdot \mathbf{r})^2 + \ldots \right] \rho(r) \]

\[ = 1 - \frac{1}{6} \langle r^2 \rangle q^2 + \ldots \]

for the relativistic, QM, case, define radius as slope of form factor

\[ r_E^2 \equiv 6 \frac{d}{dq^2} G_E(q^2) \bigg|_{q^2=0} \]

\[ G_E = F_1 + \frac{q^2}{4m_p^2} F_2 \]

\[ G_M = F_1 + F_2 \]

similarly for \( r_M \) from \( G_M \)

Richard Hill                    University of Chicago
Consider separately two datasets

- “Mainz”: high statistics 2010 Mainz A1 collaboration data (1422 datapoints)

- “world”: global cross section and polarization data excluding Mainz (406 datapoints below $Q^2=1\text{GeV}^2$)

Focus first on $r_E$ and the Mainz dataset, addressing in succession:

- Form factor shape

- Radiative corrections

- Uncorrelated systematic errors

- Correlated systematic errors

After fixing procedures, present final results for $r_E$ and $r_M$, for Mainz and world datasets
form factor shape
Radius defined as slope. Requires data over finite $Q^2$ range

[sensitivity studies based on bounded $z$ expansion fit]

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University of Chicago  
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Radius defined as slope. Requires data over finite $Q^2$ range.

Radius error

Size of $r_E$ anomaly (hydrogen)

Convergence radius for simple Taylor expansion

Maximum $Q^2$ $[\text{GeV}^2]$
Unfortunately, for the proton form factors, a simple Taylor expansion has finite (small) radius of convergence.

Fortunately, the analytic structure of amplitudes allows us to “resum” by change of variables into expansion covering the entire physical region.

\[ z(t, t_{\text{cut}}, t_0) = \frac{\sqrt{t_{\text{cut}} - t} - \sqrt{t_{\text{cut}} - t_0}}{\sqrt{t_{\text{cut}} - t} + \sqrt{t_{\text{cut}} - t_0}} \]

\[ 4m_{\pi}^2 \text{ (isoscalar channel)} \]

Point mapping to \( z = 0 \) (scheme choice)

\[ G_E(q^2) = \sum_k a_k [z(q^2)]^k \]

Fit for undetermined order unity coefficients \( a_k \)

* RJH, G. Paz, 1008.4619
Require form factors to lie within QCD-constrained class of curves: *larger (7σ) discrepancy with μ-Hydrogen!*
Besides $7\sigma$ discrepancy with $\mu H$, now $3\sigma$ tension with $H$, $3\sigma$ with A1 analysis of same dataset.

Also: tension between fit to entire dataset and fit to data subsets

$\Rightarrow$ Revisit theoretical and experimental systematics
systematics: radiative corrections
In order to isolate the proton vertex defining form factors and radius

must subtract off radiative corrections that are part of the experimental measurement:

Through one-loop order, only essential difficulty is with Two-Photon Exchange: beyond present technology to compute from first principles, insufficient data to fully constrain
Consider a range of one-loop Two-Photon Exchange (TPE) corrections.

Model dependence in TPE, but appears small for $r_E$

Take Blunden et al. hadronic model as default  

PRC 72, 034612
Return later to log-enhanced higher-order effects
systematics: uncorrelated errors
In the A1 dataset, kinematically uncorrelated systematic errors are deduced by examining subset fluctuations around initial fit

- perform initial fit to entire dataset
- for each beam/spectrometer data subset, rescale statistical errors to account for systematics

Potential concerns:

- inferred systematic can be extremely small (as low as 0.05%)
- repeated measurements at identical kinematics drive systematic uncertainties to zero

Address these concerns:

- combine ("rebin") data taken at identical kinematics
- include constant systematic error independent of statistics (0.3-0.4% based on confidence level analysis)
Same fit to rebinned dataset:

- A1 analysis (spline fit)
- $z$ expansion
- + hadronic TPE
- rebin, + uncorr. syst.
systematics: correlated errors
In the A1 dataset, correlated systematic errors are estimated by considering modifications to each data subset:

\[ d\sigma \rightarrow (1 + \delta) d\sigma \]

where \( \delta \) depends on kinematics. e.g.:

\[
\delta \propto \frac{\theta - \theta_{\text{min}}}{\theta_{\text{max}} - \theta_{\text{min}}}
\]

We performed a more general analysis with a variety of functional forms and different subset groupings.

**Observations:**

- especially for \( r_M \), significant cancellation between effects of corrections applied to different spectrometers

\[
\delta r_M = 0.016 \text{ (spec.A)} - 0.008 \text{ (spec.B)} + 0.002 \text{ (specC)} = 0.010 \text{ fm}
\]

\[
\delta r_M = 0.016 \text{ (spec.A)} + 0.008 \text{ (spec.B)} + 0.002 \text{ (specC)} = 0.026 \text{ fm}
\]

- take 0.4% angular correction (vs. A1’s 0.2%) applied uniformly to beam/spectrometer groupings as consistent with known uncertainties
Same fit, including correlated systematic error:

- A1 analysis (spline fit)
- $z$ expansion
- + hadronic TPE
- rebin, + 0.3% uncorr. syst.
- + 0.4% corr. syst.

larger systematic shift would require:

- greater than 0.4% variation over subsets
- more extreme functional form
- conspiracy between shifts applied to different subsets
What could such a shift look like?

Large logarithms spoil QED perturbation theory when $Q^2 \sim \text{GeV}^2$

$$ |F(q^2)|^2 \rightarrow |F(q^2)|^2 \left( 1 - \frac{\alpha}{\pi} \log \frac{Q^2}{m_e^2} \log \frac{E^2}{(\Delta E)^2} + \ldots \right) \approx 0.5 $$

(electrons really like to radiate)

A standard ansatz sums leading logarithms by exponentiating 1st order:

$$ |F(q^2)|^2 \left( 1 - \frac{\alpha}{\pi} \log \frac{Q^2}{m_e^2} \log \frac{E^2}{(\Delta E)^2} + \ldots \right) \rightarrow |F(q^2)|^2 \exp \left[ - \frac{\alpha}{\pi} \log \frac{Q^2}{m_e^2} \log \frac{E^2}{(\Delta E)^2} \right] $$

Yennie, Frautschi, Suura, 1961

Captures leading logarithms when

$$ Q \sim E, \quad \Delta E \sim m_e $$

As consistency check, should find the same result for resumming:

$$ \log^2 \frac{Q^2}{m_e^2} \quad \text{vs.} \quad \log \frac{Q^2}{m_e^2} \log \frac{E^2}{(\Delta E)^2} $$
More detailed analysis of subleading radiative corrections required and in progress. Will present results using standard radiative correction models.
final results
Maximize radius sensitivity, minimize possible high-$Q^2$ systematics:

\[
\begin{align*}
\text{error on radius} \quad [\text{fm}] & \\
\text{maximum } Q^2 \quad [\text{GeV}^2] & \\
\end{align*}
\]

![Graph showing error on radius vs. maximum $Q^2$](image-url)
Proton charge radius

\[ r_E \text{ [fm]} \]

- \( r_E^{\text{Mainz}} = 0.895(14) \) (Mainz final, \( Q^2_{\text{max}} = 0.5 \text{ GeV}^2 \))
- \( r_E^{\text{world}} = 0.918(24) \) (World data, \( Q^2_{\text{max}} = 0.6 \text{ GeV}^2 \))

\[ \overline{r}_E^{\text{avg.}} = 0.904(15) \text{ (simple average)} \]
Proton magnetic radius

A1 analysis (spline fit)
z expansion
+ hadronic TPE
rebin, + 0.3% uncorr. syst.
+ 0.4% corr. syst.
Mainz final ($Q_{\text{max}}^2 = 0.5$ GeV$^2$)

world data ($Q_{\text{max}}^2 = 0.6$ GeV$^2$)

Mainz + world average

$\frac{r}{M}_{\text{Mainz}} = 0.777(34)(17)$

$\frac{r}{M}_{\text{world}} = 0.913(37)$

simple average: $\frac{r}{M}_{\text{avg.}} = 0.847(27)$
summary
Performed the most comprehensive analysis of global electron-proton scattering data

\textbf{r_E summary}
Employing standard models for radiative corrections, and reasonable experimental systematics: Mainz and world values consistent. Combination is 4\sigma from muonic hydrogen

\textbf{r_M summary}
Mainz and world values differ by 2.5\sigma.
Implications:

*most* mundane resolution involves $5\sigma$ shift in Rydberg, and discarding/revising large body of results in both electron scattering and hydrogen spectroscopy.

Tension in low- and high-$Q^2$ data may point to underestimated systematic. Identified naively subheading radiative corrections as a concern.

The same issues facing electron-proton scattering are critical for the HEP accelerator neutrino program.
thanks for your attention (!)
back up
Mainz data rebinning

- one set of points ($E_{\text{beam}}=315$ MeV, $\theta=30.01^\circ$) inconsistent with statistical scatter. Excluded.

- 657 independent cross section measurements (from original 1422)

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Constant 0.25% uncorrelated systematic

Outlier
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<td>27.2</td>
<td>0.78</td>
<td>76.8</td>
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<td>25</td>
<td>1.53</td>
<td>4.3</td>
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<td>1.11</td>
<td>30.2</td>
<td>0.90</td>
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<td>21</td>
<td>0.79</td>
<td>73.7</td>
<td>0.62</td>
<td>90.5</td>
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</table>

Constant 0.3% uncorrelated systematic

Constant 0.4% uncorrelated systematic
Mainz correlated systematics

In the A1 analysis, correlated systematic errors are estimated by considering modifications to each data subset:

$$d\sigma \rightarrow (1 + \delta)d\sigma$$

where $\delta$ depends on kinematics

Since the normalizations of individual data subsets are free parameters, only variations in $\delta$ over subsets relevant. Simple ansatz:

$$1 + \delta_{\text{corr}} = 1 + a \frac{x - x_{\text{min}}}{x_{\text{max}} - x_{\text{min}}}$$

A1 analysis:

- $x = \theta$
- $a \approx 0.2\%$, equal in sign and magnitude for all beam/spectrometer subsets
We performed a more general analysis with different functional forms and different subset groupings,

- $x=\theta, 1/\theta, Q^2, 1/Q^2, E', 1/E', \varepsilon, \sin^4(\theta/2)$

- data groupings: beam/spectrometer (18 subsets)
spectrometer (3 subsets); normalization (34 subsets)

Observations:

- especially for $r_M$, significant cancellation between corrections applied to three spectrometers when $a=$constant

- take results for $x=\theta, a=0.4\%$, applied to beam/spectrometer groupings as “minimum” consistent with known uncertainties
Comparing muonic hydrogen to the individual measurements makes the conflict seem not as big: all but one agree with $\mu_p$ to within 2 s.d.

We need more measurements in hydrogen.

• no straightforward systematic explanation identified, but $\sim 5\sigma$ deviation results from summing many $\sim 2\sigma$ effects
Experimental landscape: historical e-p extractions

Figure 1: Proton radius determinations over time. Electronic measurements seem to settle around $r_p = 0.88$ fm, whereas the muonic hydrogen value [1,2] is at 0.84 fm. Values are (from left to right): Orsay [10], Stanford [11], Saskatoon [12, 13], Mainz [14] (all in blue) are early electron scattering measurements. Recent new scattering measurements are from MAMI [4] and Jlab [15]. The green and cyan points denote various reanalyses of the world electron scattering data [16–21]. The red symbols originate from laser spectroscopy of atomic hydrogen and advances in hydrogen QED theory (see [3] and references therein). The green and red points in the year 2003 denote the reanalysis of the world electron scattering data [19] and the world data from hydrogen and deuterium spectroscopy which have determined the value of $r_p$ in the CODATA adjustments [3, 22] since the 2002 edition.

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