The LPM Effect in Sequential Bremsstrahlung

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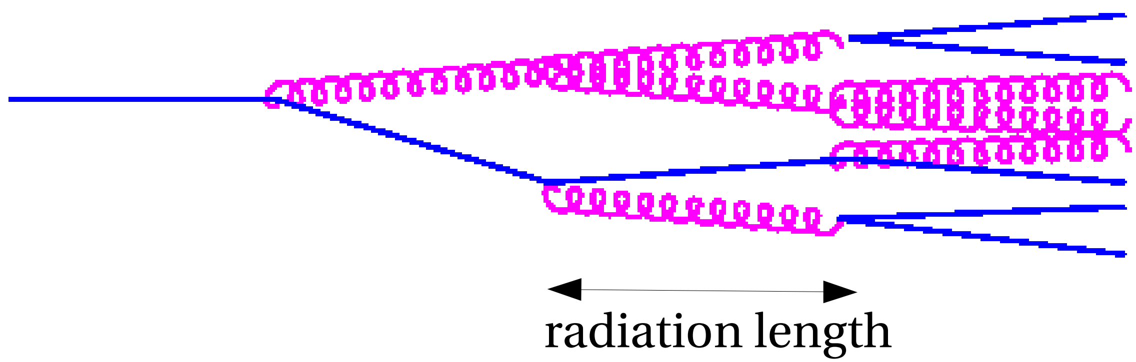
- What's the LPM effect?
- How does one compute it?
- What's new since 1950s (QED LPM) and 1990s (QCD LPM)

Reporting on work with Shahin Iqbal and Han-Chih Chang:

JHEP 04 (2015) 070, arXiV:1501.04964 + work in progress

At high energy, energy loss is dominated by nearly-collinear bremsstrahlung and pair production.

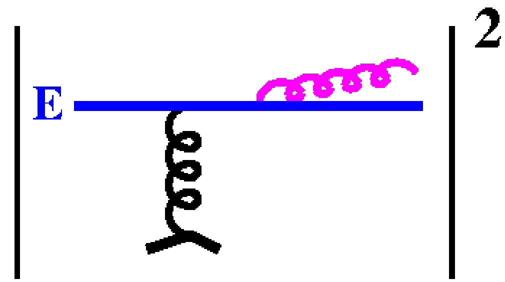


Think
- cosmic ray shower
- shower chamber

[QCD version: high-energy partons traversing a quark-gluon plasma.]

Naively

brem rate ~ $n\sigma v$ ~ (density of scatterers) $\times$



$\times 1$

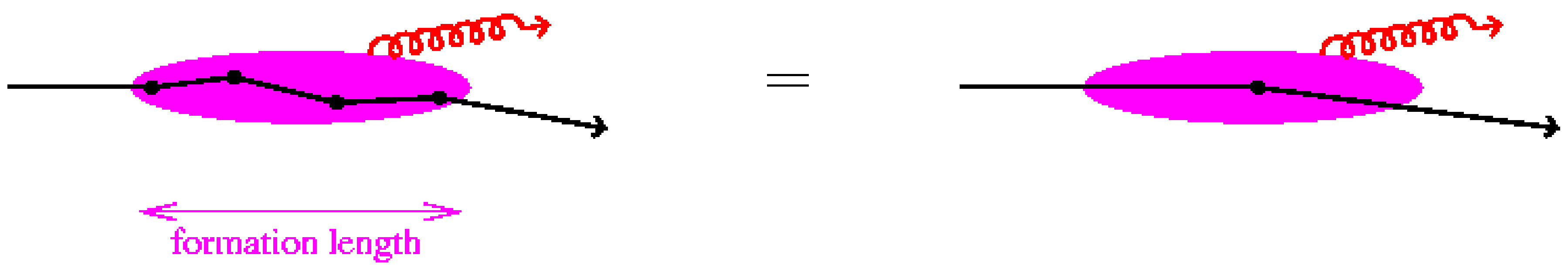
$$\Gamma_{\mathbf{brem}} \propto E^0 \quad \text{(up to logs)}$$

Complication: The Landau-Pomeranchuk-Migdal (LPM) Effect

At very high energy,

probabilities of brem from successive scatterings no longer independent;

brem from several successive (small angle) collisions not very different from brem from one collision.



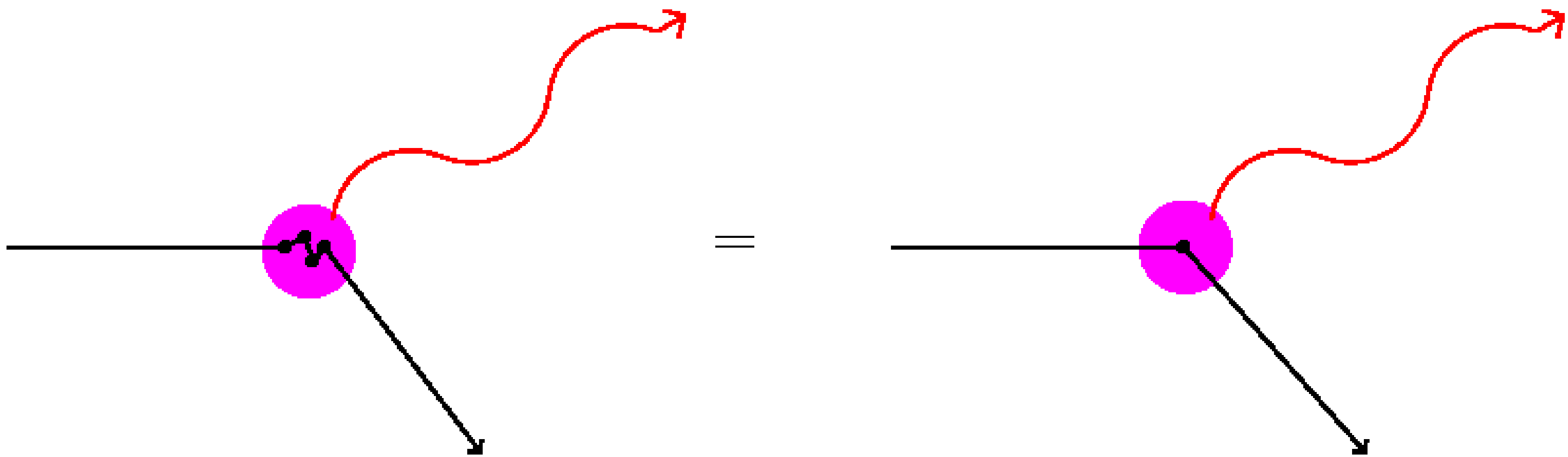
$$\text{formation length} \propto \sqrt{E}$$

Result: a reduction of the naive brem rate.

$$\boxed{\Gamma_{\mathbf{brem}} \propto E^{-1/2}}$$

The LPM Effect (QED)

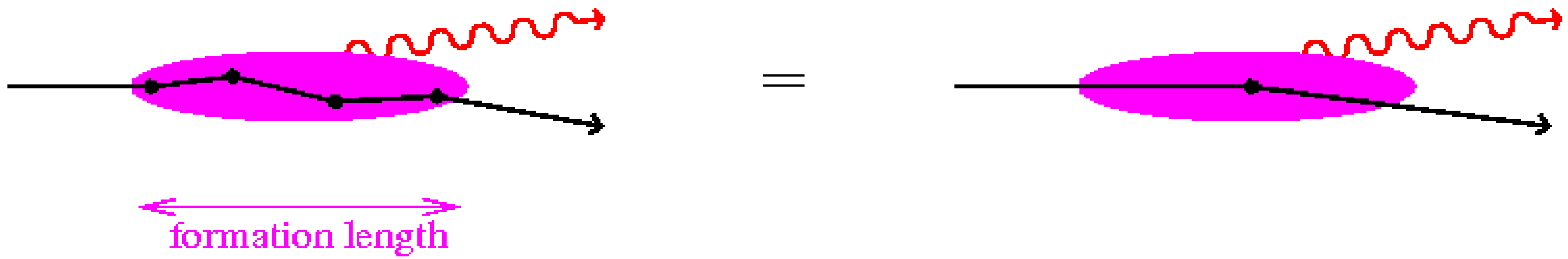
__Warm-up__: Recall that light cannot resolve details smaller than its wavelength.



[Photon emission from different scatterings have same phase → coherent.]

Now: Just Lorentz boost above picture by a lot!

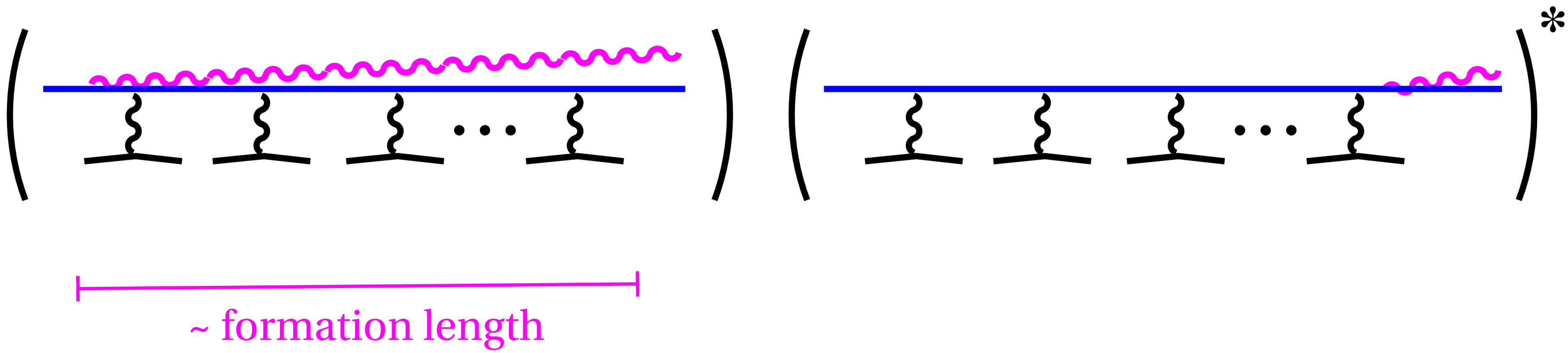
The LPM Effect (QED)



Note: (1) bigger E requires bigger boost → more time dilation → longer formation length

(2) big boost → this process is **very collinear.**

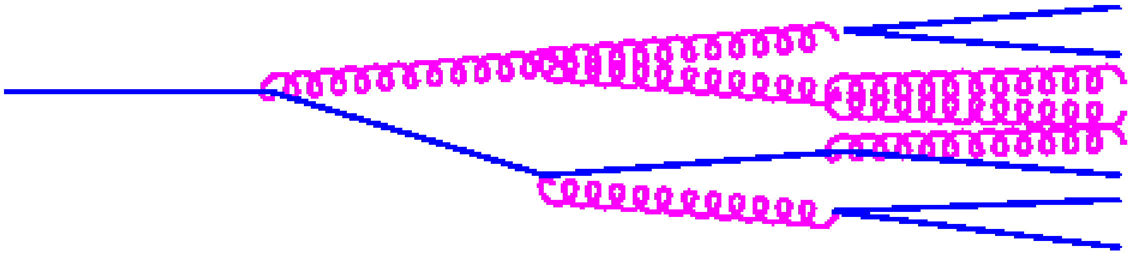
In Feynman diagrams, LPM manifests as having to compute interference contributions



Long standing problem concerning LPM effect:

Are splittings independent?

Can I just put LPM result for $\Gamma_{\rm brem}$ etc. into a Monte Carlo to get

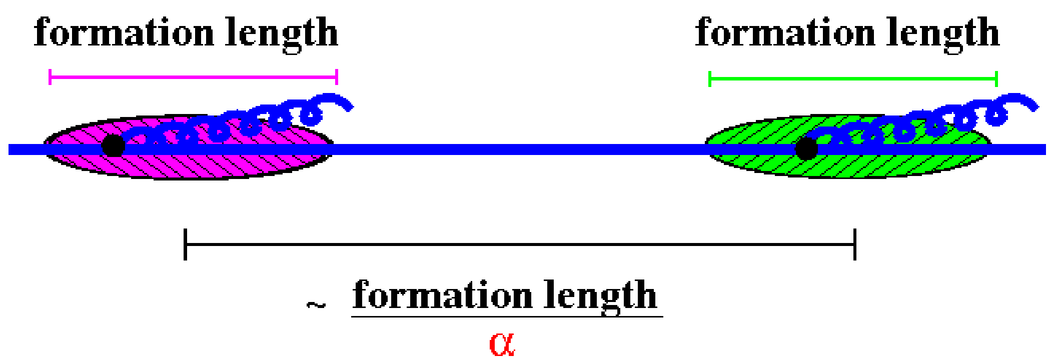


?

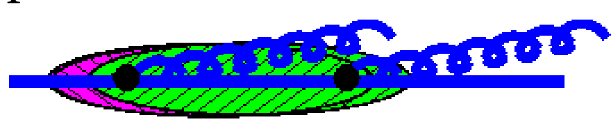
Crudely (sweeping some interesting physics under the rug),

Chance of brem ~ α per formation time

So two consecutive splittings will typically look like



Chance of overlap



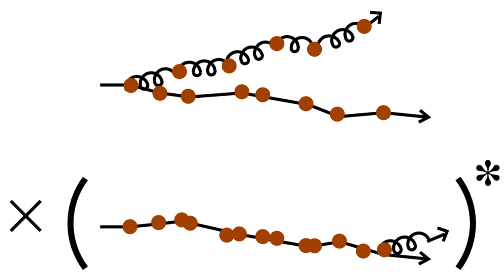
$\propto \alpha$

Scale for QGP applications: $\alpha_s(Q_\perp)$ where $Q_\perp \sim (\hat{q}E)^{1/4}$ $[\hat{q} \sim \mathrm{GeV}^3]$

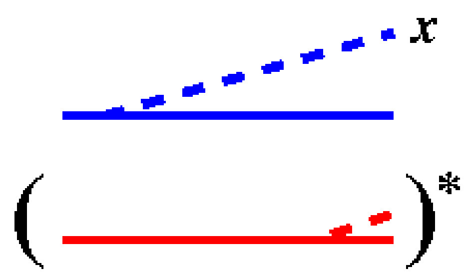
Moral: It's interesting to figure out how to calculate correction due to overlap.

Formalism for LPM: single brem

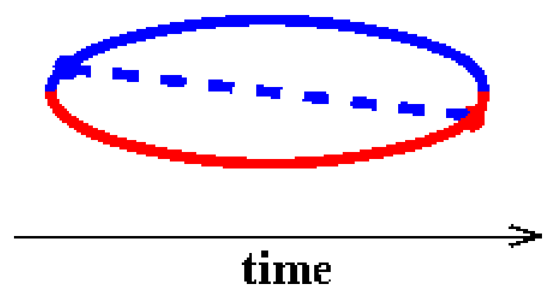
Shorthand henceforth: Draw



as simply

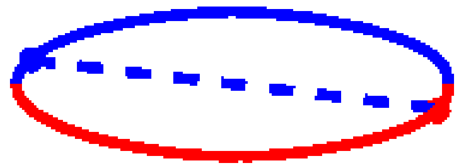


But will be even more convenient to draw as



Can (formally) interpret this as 3 particles moving forward in time [Zakharov 1990's]:

2 particles from the amplitude (evolving with e^{-iHt})
1 particle from the conjugate amplitude (evolving with e^{+iHt})

Will show that evolution in  can be described by

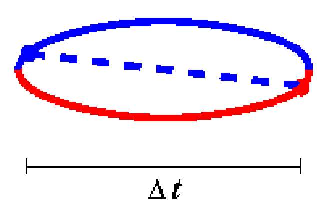
3-particle non-relativistic Quantum Mechanics in 2 dimensions

$$\mathcal{H}_{\text{eff}} = \frac{p_{\perp 1}^2}{2m_1} + \frac{p_{\perp 2}^2}{2m_2} + \frac{p_{\perp 3}^2}{2m_3} + V(b_1, b_2, b_3)$$

with weird properties:

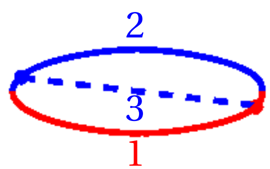
- $m_1 + m_2 + m_3 = 0$
- $V \propto -i$ (i.e. $\mathcal{H}$ is non-Hermitian)

⇨ interference vanishes as $\Delta t \to \infty$, as it must!



Kinetic terms:

Energy of a high-p_z particle: $$\epsilon_p = \sqrt{p_z^2 + p_\perp^2} \simeq p_z + \frac{p_\perp^2}{2p_z}$$

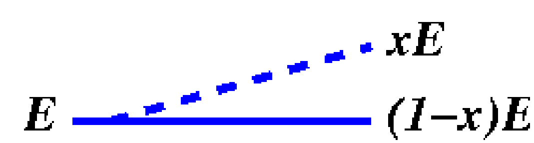
Evolution of 

is $e^{-i\mathcal{H}t}$ with

$$\mathcal{H}_{\rm kin} = -\epsilon_{p_1} + \epsilon_{p_2} + \epsilon_{p_3} \simeq -\frac{p_{\perp 1}^2}{2p_{z1}} + \frac{p_{\perp 2}^2}{2p_{z2}} + \frac{p_{\perp 3}^2}{2p_{z3}}$$

$$\simeq -\frac{p_{\perp 1}^2}{2E} + \frac{p_{\perp 2}^2}{2(1-x)E} + \frac{p_{\perp 3}^2}{2xE}$$

conjugate evolves with $e^{+i\mathcal{H}t}$



This is 2-dimensional non-relativistic QM with

$$(m_1, m_2, m_3) = (-E, (1-x)E, xE)$$

As promised,

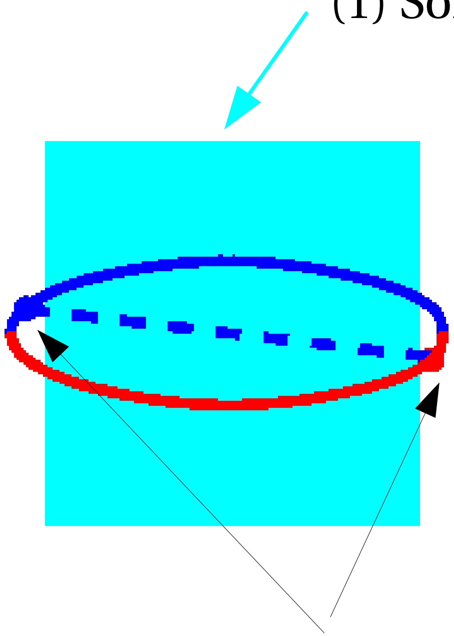
$$m_1 + m_2 + m_3 = 0$$

Potential term:

$V(b_1, b_2, b_3)$ incorporates (statistically averaged) effect of collisions with the medium.

How to put the calculation together:

(1) Solve for propagation in 3-particle QM in shaded region.



(2) Tie together with QFT matrix elements for vertices

$$\propto \sqrt{\textbf{DGLAP splitting functions}}$$

$$\propto \sqrt{P_{i\to j}(x)}$$

Simplifcation: 3-particle QM → 1-particle QM

Can use various symmetries of problem to get rid of 2 d.o.f.

$$\mathcal{H} = \frac{P_B^2}{2M} + V(B)$$

[BDMPS-Z (1990's)]

Method 1. Can solve numerically. [Zakharov (2004+); Caron-Huot & Gale (2010)]

Simplifcation: Harmonic Oscillator

Method 2. High energies → very collinear → b's small.

So make small B approximation to $V(B)$ → a harmonic oscillator problem

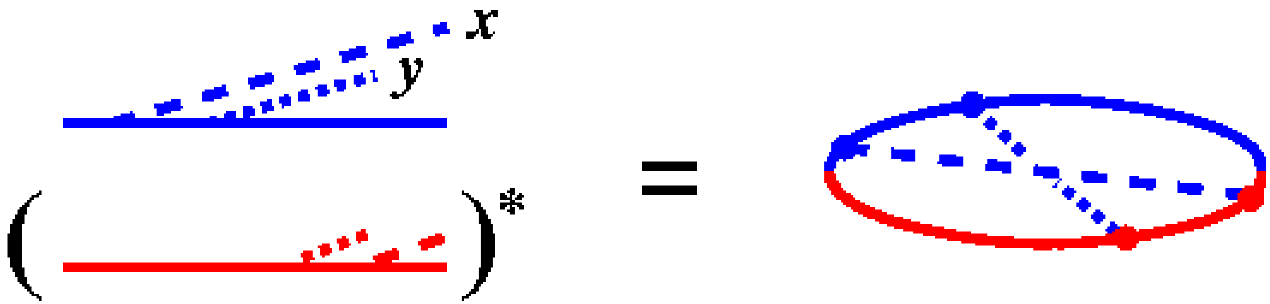
$$\mathcal{H} = \frac{P_B^2}{2M} + \frac{1}{2} M \Omega_0^2 B^2$$

[Baier *et al*. (1998)]

(a non-Hermitian one: $\Omega_0^2 \propto -i$)

What's been done for double brem

Example of an interference contribution:



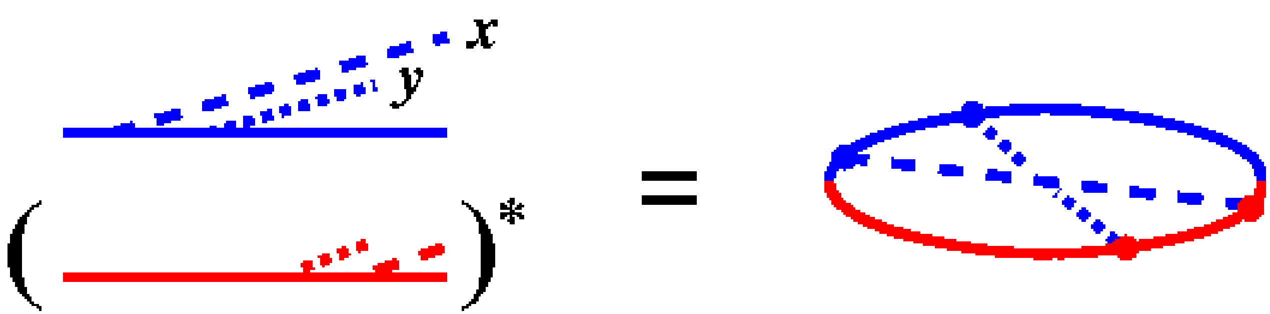
Previously **:** results in **soft** limit $y \ll x \ll 1$ for QCD.

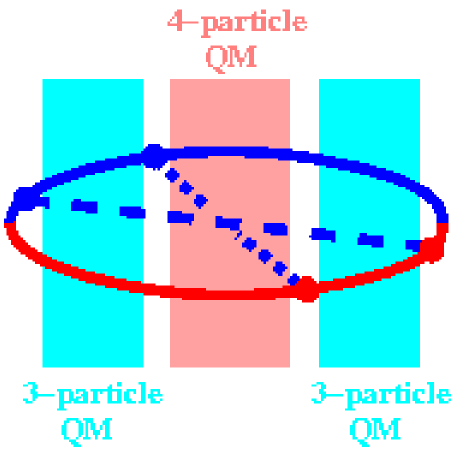
Blaizot & Mehtar-Tani; Iancu; Wu (2014)

Our goal **:** Compute effects of overlapping formation times for **any** x and y.

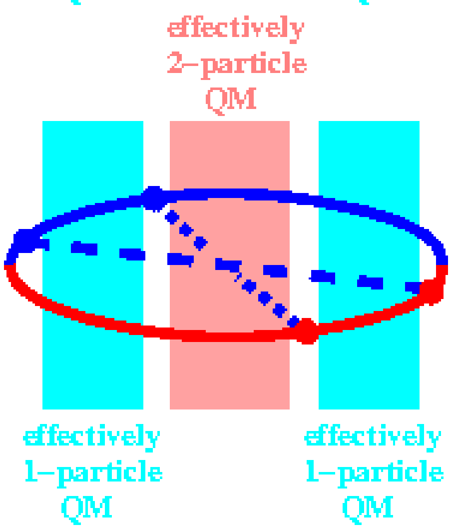
Formalism for LPM: double brem

Example of an interference contribution:





To compute : Sew together QFT matrix element for vertices with QM evolution in between.

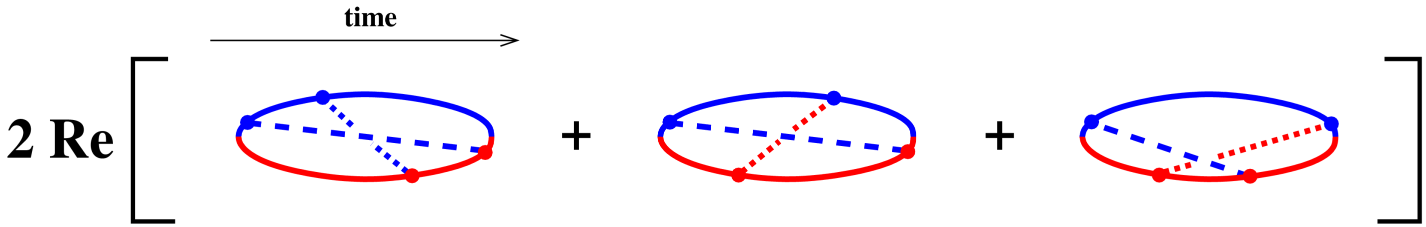


Simplify : Using symmetries, as before.

Published Work

[all for $g \to gg \to ggg$]

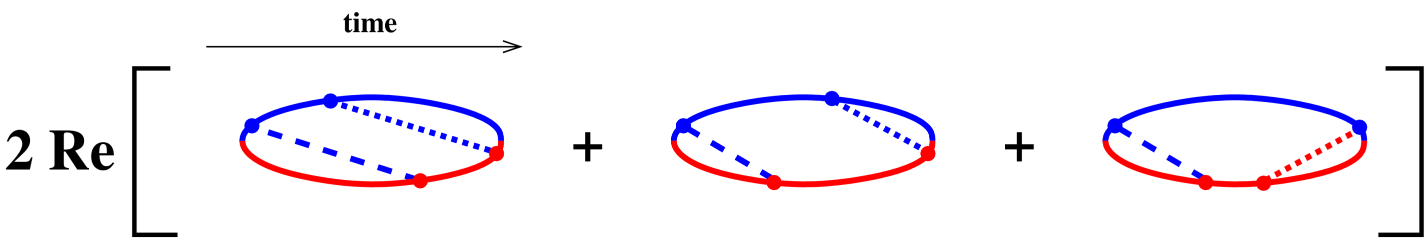
crossed diagrams:



+ permutations of $(x, y, 1\text{-}x\text{-}y)$

Forthcoming

sequential diagrams:

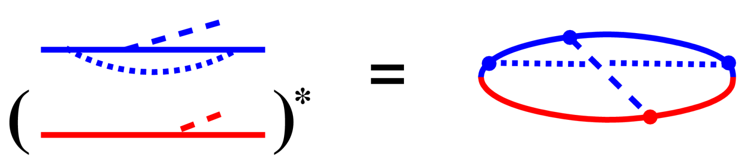


+ permutations of $(x, y, 1\text{-}x\text{-}y)$

Still in progress

4-gluon vertices, e.g.

virtual corrections, e.g.

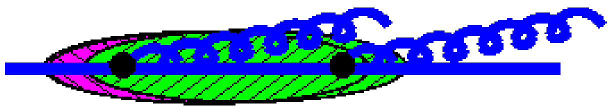


correct single brem rate

[parts of which included in $y \ll x \ll 1$ work of earlier refs.]

Summary

Subtle problems in the field theory description of very-high energy showering

e.g. 

can be reduced to problems in

2-dimensional non-relativistic non-Hermitian quantum mechanics

and even

2-dimensional non-relativistic non-Hermitian harmonic oscillators!

(Just when you thought you couldn't learn anything more from the harmonic oscillator...)