Magnetic Catalysis in Graphene

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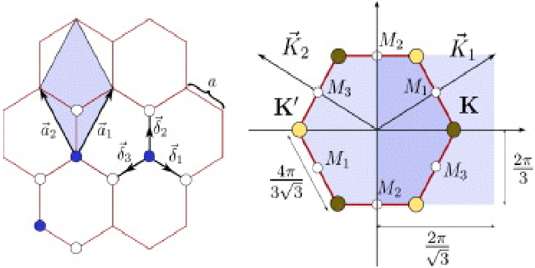
Outline

- Brief introduction of graphene and it's low-energy excitations
- Introduction to magnetic catalysis
- Lattice field theory studies of magnetic catalysis in graphene EFT

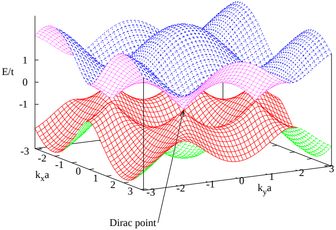
- Carbon atoms arranged in a hexagonal lattice, two atoms per unit cell [Castro Neto et al., Rev. Mod. Phys. (2009)]
- Predicted to be a semimetal from band structure [Wallace, Phys. Rev. (1947)]
- Points where two bands touch referred to as Dirac points κ, κ' (Corners of BZ)

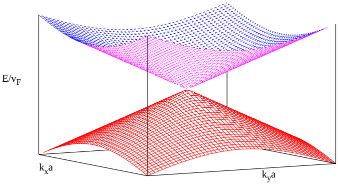


- Dispersion linear near Dirac points
- Ignoring hopping between sites of the same sublattice, excitations near these points described by massless Dirac-like quasiparticles

$$\mathcal{H}_{\kappa,\kappa'}(\vec{k}) = \hbar v_F \vec{\sigma} \cdot \vec{k}$$

- 2 sublattices + 2 Dirac points + 2 spin projections = two identical four-component Dirac spinors


E/t
1
0
-1
-3
-2
-1
0
1
2
3
$k_x a$
-3
-2
-1
0
1
2
3
$k_y a$
Dirac point


E/v_F
$k_x a$
$k_y a$

- Quasiparticle interactions are Coulombic ($v_F/c \approx 1/300$)

$$U_C(r-r') = \int dr \int dr' \hat{\rho}(\vec{r}) \frac{e^2}{\epsilon|\vec{r}-\vec{r}'|} \hat{\rho}(\vec{r}'),$$

$$\hat{\rho}(r) = \Psi^\dagger(r)\Psi(r),$$

$$\Psi_\sigma^T = (\psi_{\kappa A\sigma}, \psi_{\kappa B\sigma}, \psi_{\kappa' B\sigma}, \psi_{\kappa' A\sigma})$$

- Low-energy theory has $U(4)$ symmetry
- Effective coupling between quasiparticles described by graphene fine-structure constant

$$\alpha_g \equiv \frac{e^2}{\epsilon v_F 4\pi} > 1$$

- Introduced by Miransky and collaborators to describe dynamical symmetry breaking in the presence of a magnetic field [Gusynin et. al., PRL (1994)]
- Studied in several $(2+1)$ and $(3+1)$-dimensional field theories (NJL model, QED) as well as graphene [I. Shovkovy, arXiv:1207:5081]

$$(2+1): \quad m_{dyn} \propto \alpha\sqrt{|eB|}$$

$$(3+1): \quad m_{dyn} \propto \sqrt{|eB|}e^{\pi/[\alpha \log(\alpha N_f)]}$$

- Fermion-antifermion pairing leads to spontaneous breaking of $U(4)$ symmetry in graphene
- Symmetry breaking pattern $U(4) \rightarrow U(2) \otimes U(2)$

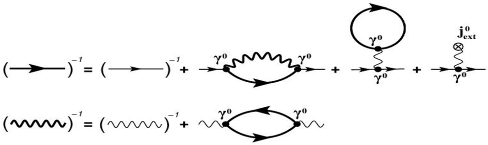
- Underlying physics described by a dimensional reduction $D \to D - 2$ [Miransky, arXiv:hep-ph/9509320]
- Free, $(3+1)$, Dirac fermions in an external magnetic field $\vec{B} = B\hat{z}$ have the following spectrum

$$E_n^{(3+1)}(p_z) = \pm\sqrt{m^2 + 2|eB|n + (p_z)^2} \qquad (1)$$

- Lowest Landau Level corresponds to $n = 0$. Spin polarized (only contains $s_z = +1/2$)
- Degeneracy associated with s_z and p_y (in Landau gauge)
- $m \ll \sqrt{|eB|}$, low-energy regime completely determined by LLL

$$E^{(3+1)}(p_z) = \pm\sqrt{m^2 + (p_z)^2} \qquad (2)$$

- Previous approaches used Schwinger-Dyson equations in Hartree-Fock approximation [Gorbar et. al., arXiv:1105.1360]



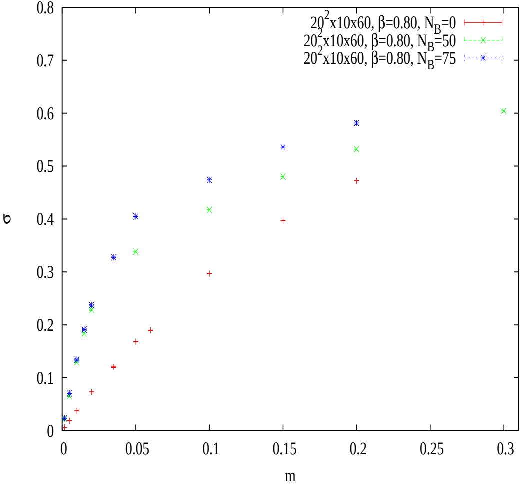
- Work in static approximation $\Pi(k,\omega) \rightarrow \Pi(k,0)$
- One can use a general ansatz of the form

$$iG_F = \left[(i\partial_t + \mu_s + \tilde{\mu}_s\gamma^3\gamma^5)\gamma^0 - v_F(\gamma \cdot \pi) - \tilde{\Delta}_s + \Delta_s\gamma^3\gamma^5\right]$$

- For filling factor $\nu = 0$, Miransky and collaborators find a triplet state with respect to both $U(2)$'s, where the dynamical Dirac mass, $\tilde{\Delta}$, is the order parameter

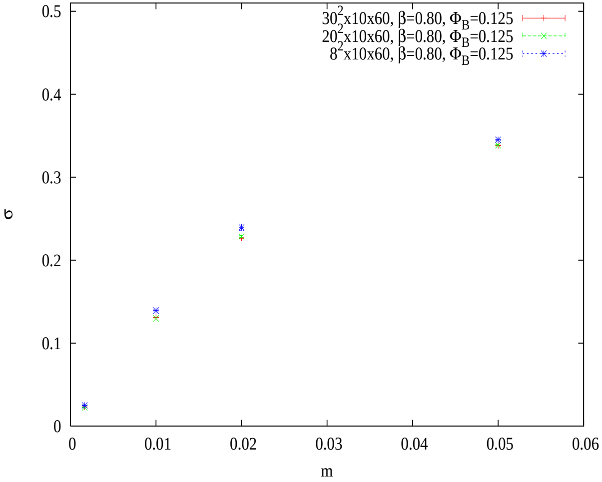
- Motivation of current study is need for nonperturbative calculation
- Discretize continuum EFT on a cubic lattice
- Tried and tested lattice methods used for chiral symmetry breaking in QCD
- Two identical, massless species of Dirac fermions reproduced by staggered fermion formulation
- Lattice action retains residual $U(1)$ chiral symmetry
- Early studies did not find support for SSB at arbitrarily weak coupling [Boyda et. al., arXiv:1308.2814]

- Identify subcritical region in coupling constant $\beta \equiv \frac{1}{g^2} > \beta_c$
- In this region, $\langle\bar{\psi}\psi\rangle$ has linear dependence on mass
- Introduce external magnetic field and try to observe the catalysis



20^2x10x60, β=0.80, N_B=0
20^2x10x60, β=0.80, N_B=50
20^2x10x60, β=0.80, N_B=75
σ
m
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.05
0.15
0.25

- Infinite volume limit needs to be taken before symmetry breaking parameter taken to zero
- Take zero-temperature limit, $T \to 0$
- Chiral limit, $m \to 0$

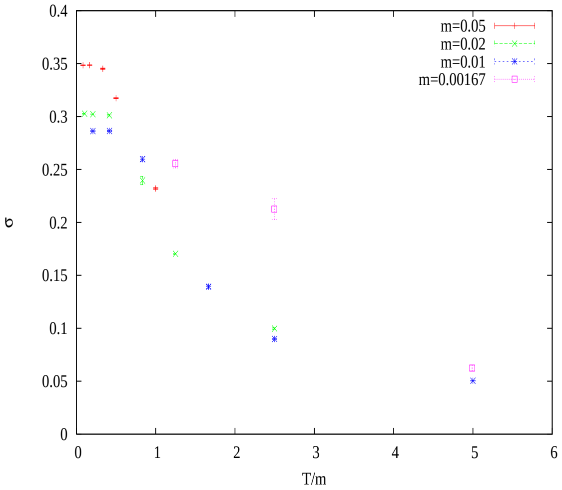


30^2x10x60, β=0.80, Φ_B=0.125
20^2x10x60, β=0.80, Φ_B=0.125
8^2x10x60, β=0.80, Φ_B=0.125
σ
m
0
0.1
0.2
0.3
0.4
0.5
0.01
0.02
0.03
0.04
0.05
0.06

- Behavior in approach to chiral limit appears not to be affected by finite spatial volume
- Confirmed from calculation of spatial screening masses $M_s L > 1$
- Magnetic length sets scale for problem

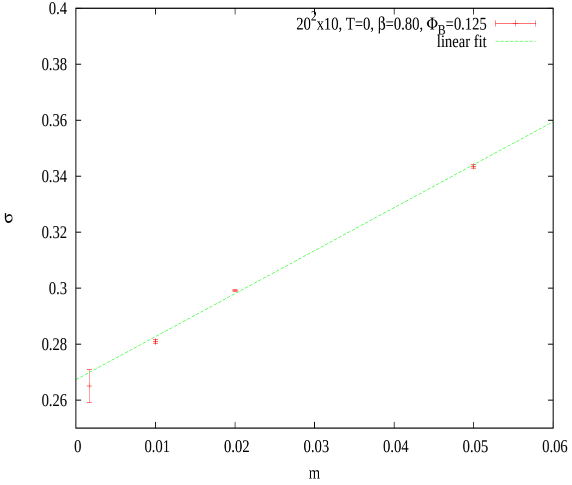
$$a_s < l_B < L, \;\; l_B \equiv \sqrt{\frac{\hbar c}{|e|B}} \qquad (3)$$

Finite Temperature Effects



- Temperature effects dominating behavior at small mass
- Restoration of symmetry at finite T
- Rescaled axis shows that ratio $m/T = mN_\tau$ important in determining where these effects become significant
- Separation of LLL with next LL another scale that is important ($\Delta E = E_{n=1} - E_{n=0}$)

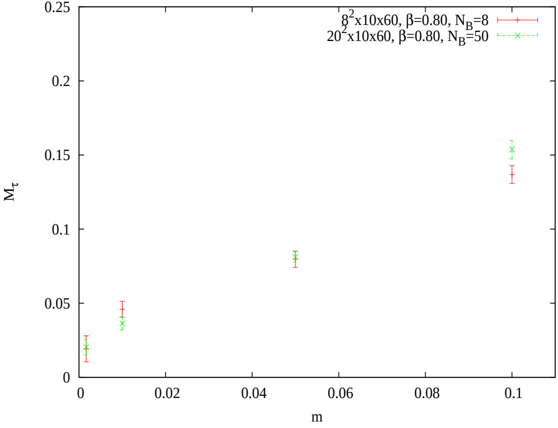
$T = 0$ extrapolated points



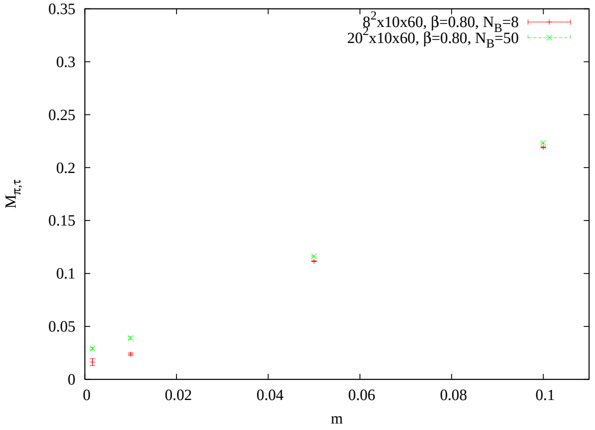
Other Observables

- Dirac quasiparticle mass determined from propagator $G_F(\tau) = \langle \psi(\tau)\bar{\psi}(0) \rangle$. Expect these excitations to be gapped in the chiral and zero-temperature limits.
- Goldstone mode due to spontaneous breaking of residual $U(1)$ symmetry ($J^P = 0^-$)
- Expect this excitation to obey a Gell-Mann-Oakes-Renner relation, $m_{PS}^2 \sim m$.

Dirac Quasiparticle Mass



Pseudoscalar Mass



Conclusion

- Univeral phenomenon
- Evidence favorable for existence in graphene EFT
- Similar extrapolation for pseudoscalar and fermion masses
- Several magnetic fluxes to determine behavior of observables as a function of B

Collaborators

- Carleton DeTar (U. of Utah)
- Savvas Zafeiropoulos (U. of Frankfurt)