

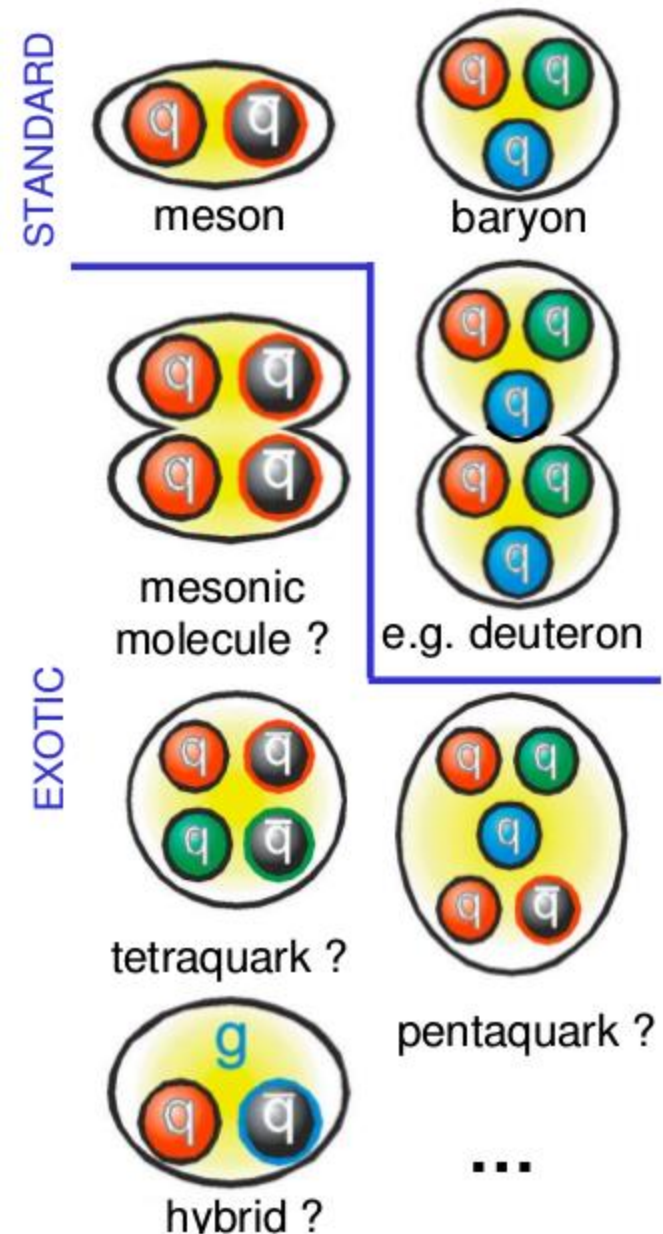
Exotic Hadron Spectroscopy in LHCb

Nathan Jurik
Syracuse University

On behalf of the LHCb Collaboration

Exotic Hadrons

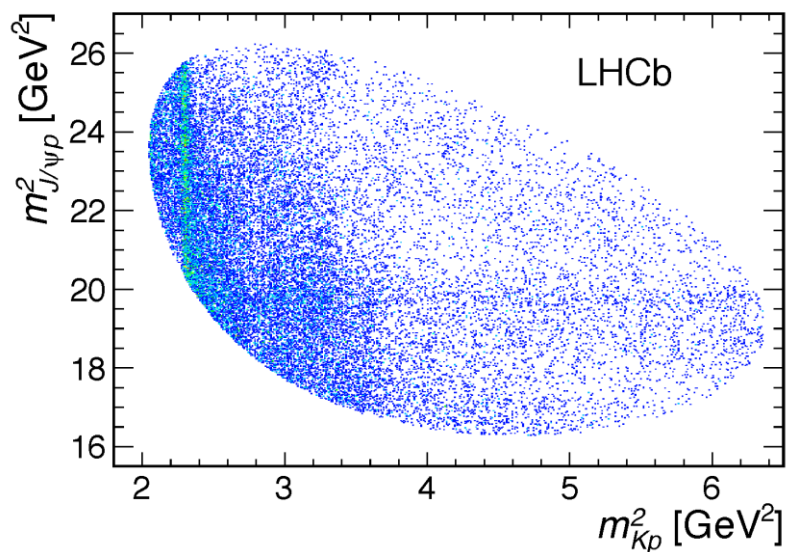
- Since the inception of the quark model in the 60's with Gell-Mann (64) and Zweig (64), hadrons beyond the $q\bar{q}$ mesons and qqq baryons have been hypothesized.
- These could be molecular bound states with mesons, tetraquarks, pentaquarks, or hybrids.



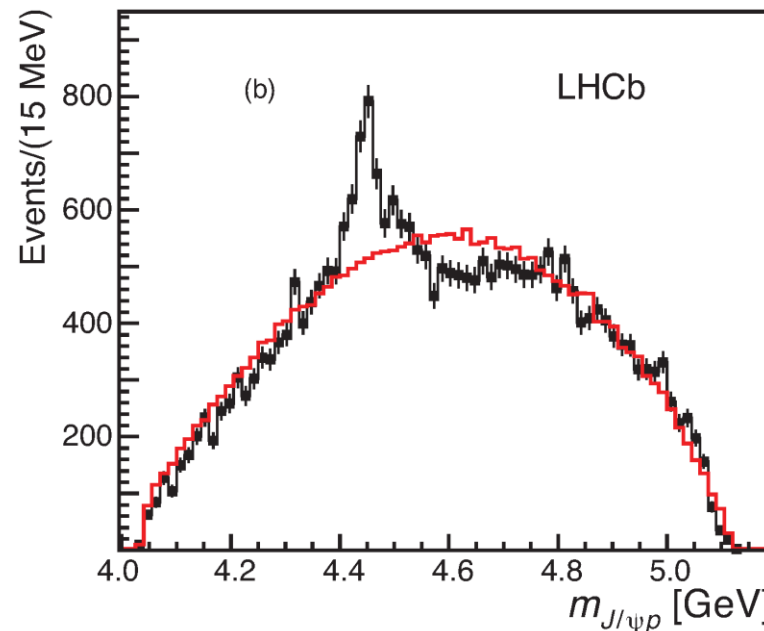
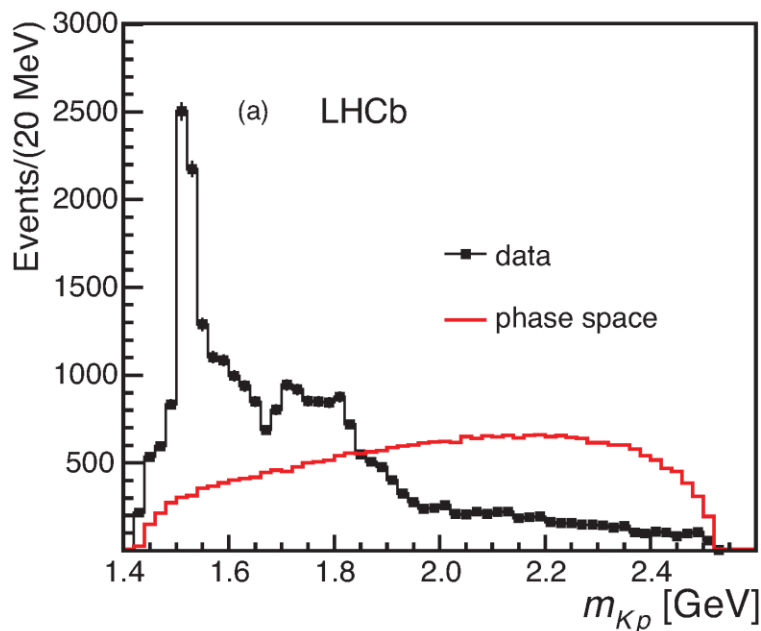
Outline

- In recent years strong candidates for these exotic hadrons been observed with contributions from multiple experiments.
- In this talk I will cover new results from LHCb for:
 - $P_c(4450)$, $P_c(4380)$: Recently observed pentaquark candidates [LHCb-PAPER-2015-029](#), [arXiv:1507.03414](#)
 - $X(3872)$: mesonic molecule, conventional charmonium, or a mixture. [Phys Rev D 92, 011102 \(2015\)](#)
 - $Z(4430)^-$: tetraquark candidate [LHCb-PAPER-2015-038](#), in preparation. Confirmed by LHCb in [Phys. Rev. Lett. 112, 222002 \(2014\)](#),

Pentaquarks In $\Lambda_b \rightarrow J/\psi p K$

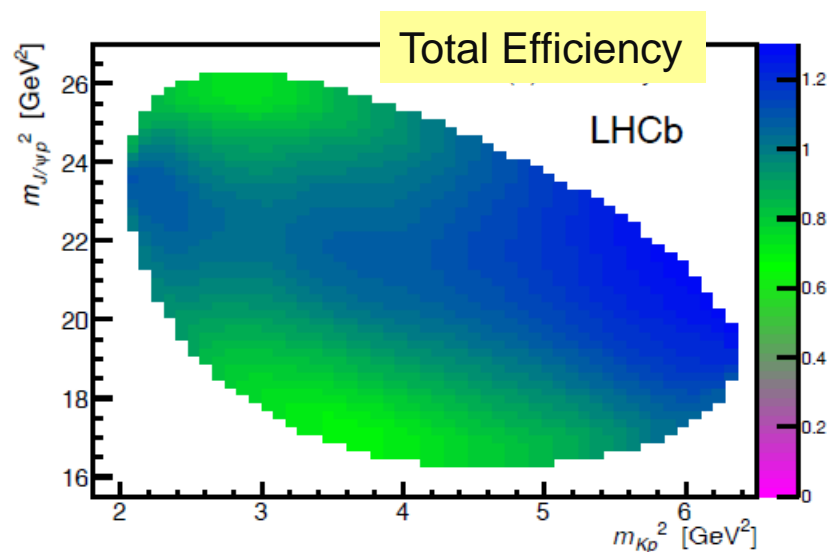


- In studying this decay, the expected Kp resonances were found. But also a surprising feature was found in the $J/\psi p$ system.



Is the peak “an artifact”?

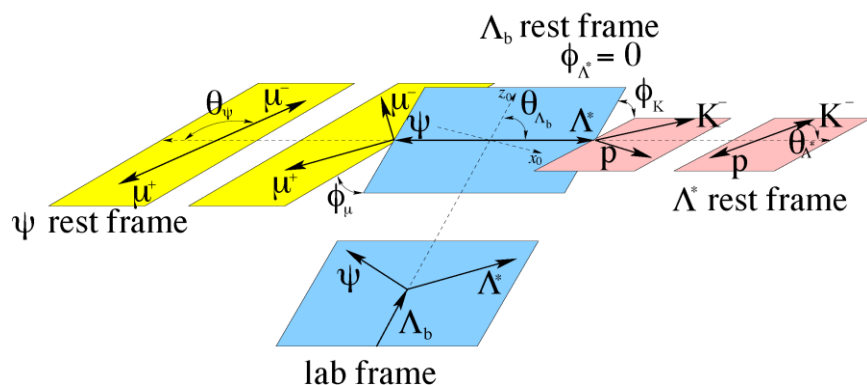
- Many checks done that show this is not the case:
 - Reflections of B^0 and B_s are vetoed
 - Ξ_b decays checked
 - Efficiency is smooth, can't create a peak
 - Λ_b sideband background doesn't peak
 - Clones & ghost tracks eliminated
- Can interferences between Λ^* resonances generate a peak in the $J/\psi p$ mass spectrum? If not can the data be described in all relevant kinematic variables with the addition of $J/\psi p$ resonances?
 - To answer these a full amplitude analysis is performed.



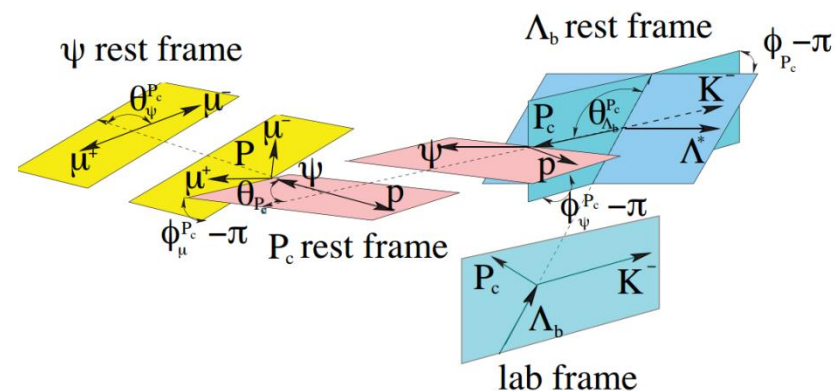
Amplitude Analysis Formalism

- The matrix element for $\Lambda_b \rightarrow J/\psi p K$, $J/\psi \rightarrow \mu^+ \mu^-$ was written down in the helicity formalism. Allows for the conventional $\Lambda^* \rightarrow p K$ resonances to interfere with pentaquark states $P_c \rightarrow J/\psi p$.
- Dynamics of the Λ^* decay chain depend on 1 mass and 5 angles. For the P_c chain, they depend on 1 mass and 6 angles, which are not independent from the Λ^* variables.
 - We perform a 6-dimensional unbinned maximum likelihood fit to maximize sensitivity to underlying decay dynamics.

Λ^* Decay Chain



P_c Decay Chain



Amplitude Analysis Formalism II

- The matrix element for the Λ^* decay chain is:

$$\mathcal{M}_{\lambda_{\Lambda_b^0}, \lambda_p, \Delta\lambda_\mu}^{\Lambda^*} \equiv \sum_n \sum_{\lambda_{\Lambda^*}} \sum_{\lambda_\psi} \mathcal{H}_{\lambda_{\Lambda^*}, \lambda_\psi}^{\Lambda_b^0 \rightarrow \Lambda_n^* \psi} D_{\lambda_{\Lambda_b^0}, \lambda_{\Lambda^*} - \lambda_\psi}^{\frac{1}{2}}(0, \theta_{\Lambda_b^0}, 0)^* \\ \mathcal{H}_{\lambda_p, 0}^{\Lambda_n^* \rightarrow K p} D_{\lambda_{\Lambda^*}, \lambda_p}^{J_{\Lambda_n^*}}(\phi_K, \theta_{\Lambda^*}, 0)^* R_n(m_{Kp}) D_{\lambda_\psi, \Delta\lambda_\mu}^1(\phi_\mu, \theta_\psi, 0)^*$$

- And for the P_c chain:

$$\mathcal{M}_{\lambda_{\Lambda_b^0}, \lambda_{P_c}, \Delta\lambda_\mu}^{P_c} \equiv \sum_j \sum_{\lambda_{P_c}} \sum_{\lambda_\psi^{P_c}} \mathcal{H}_{\lambda_{P_c}, 0}^{\Lambda_b^0 \rightarrow P_{cj} K} D_{\lambda_{\Lambda_b^0}, \lambda_{P_c}}^{\frac{1}{2}}(\phi_{P_c}, \theta_{\Lambda_b^0}^{P_c}, 0)^* \\ \mathcal{H}_{\lambda_\psi^{P_c}, \lambda_p^{P_c}}^{P_{cj} \rightarrow \psi p} D_{\lambda_{P_c}, \lambda_\psi^{P_c} - \lambda_p^{P_c}}^{J_{P_{cj}}}(\phi_\psi, \theta_{P_c}, 0)^* R_j(m_{\psi p}) D_{\lambda_\psi^{P_c}, \Delta\lambda_\mu^{P_c}}^1(\phi_\mu^{P_c}, \theta_\psi^{P_c}, 0)^*$$

Amplitude Analysis Formalism II

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- $R(m)$ are resonance parametrizations, which are generally described by a Breit-Wigner amplitude.

Amplitude Analysis Formalism II

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- \mathcal{H} are complex helicity couplings determined from the fit to the data.

Amplitude Analysis Formalism II

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- Wigner D-matrix arguments are Euler angles corresponding to the fitted angles.

Amplitude Analysis Formalism III

- They are added together as:

$$|\mathcal{M}|^2 = \sum_{\lambda_{\Lambda_b^0}} \sum_{\lambda_p} \sum_{\Delta\lambda_\mu} \left| \mathcal{M}_{\lambda_{\Lambda_b^0}, \lambda_p, \Delta\lambda_\mu}^{A*} + e^{i\Delta\lambda_\mu} \alpha_\mu \sum_{\lambda_p^{P_c}} d_{\lambda_p^{P_c}, \lambda_p}^{\frac{1}{2}}(\theta_p) \mathcal{M}_{\lambda_{\Lambda_b^0}, \lambda_p^{P_c}, \Delta\lambda_\mu}^{P_c} \right|^2$$

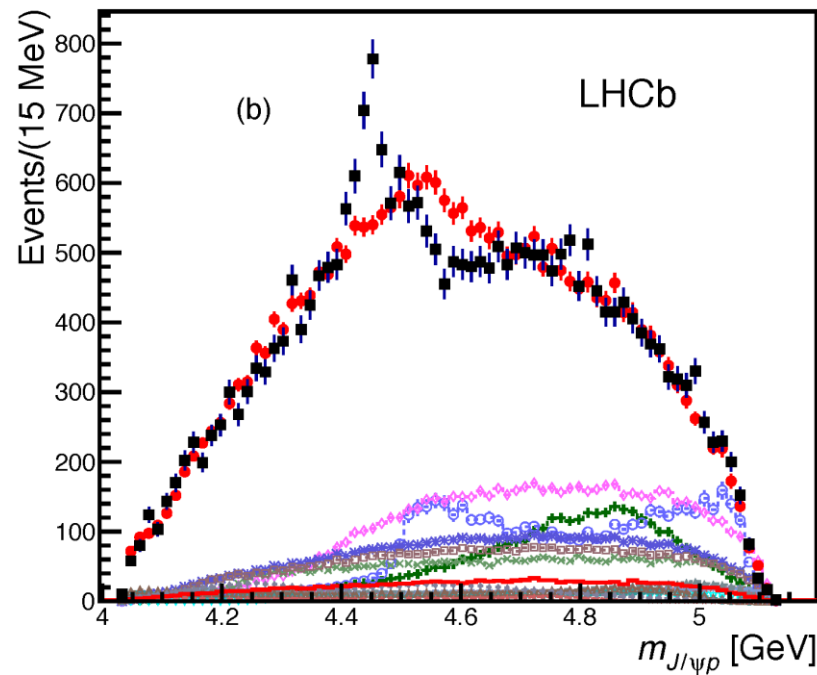
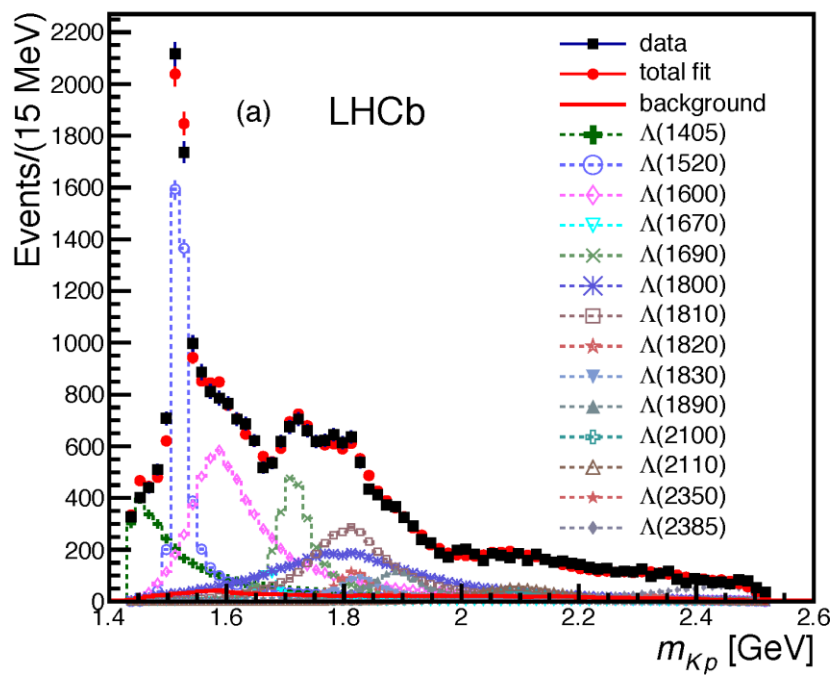
- α_μ and θ_p are further rotation angles to align the final state helicity axes of the μ and p , as helicity frames used are different for the two decay chains.
- Helicity couplings $\mathcal{H} \Rightarrow$ LS amplitudes B via:

$$\mathcal{H}_{\lambda_B, \lambda_C}^{A \rightarrow BC} = \sum_L \sum_S \sqrt{\frac{2L+1}{2J_A+1}} B_{L,S} \left(\begin{array}{cc|c} J_B & J_C & S \\ \lambda_B & -\lambda_C & \lambda_B - \lambda_C \end{array} \right) \times \left(\begin{array}{cc|c} L & S & J_A \\ 0 & \lambda_B - \lambda_C & \lambda_B - \lambda_C \end{array} \right)$$

- Convenient way to enforce parity conservation in the strong decays via: $P_A = P_B P_C (-1)^L$

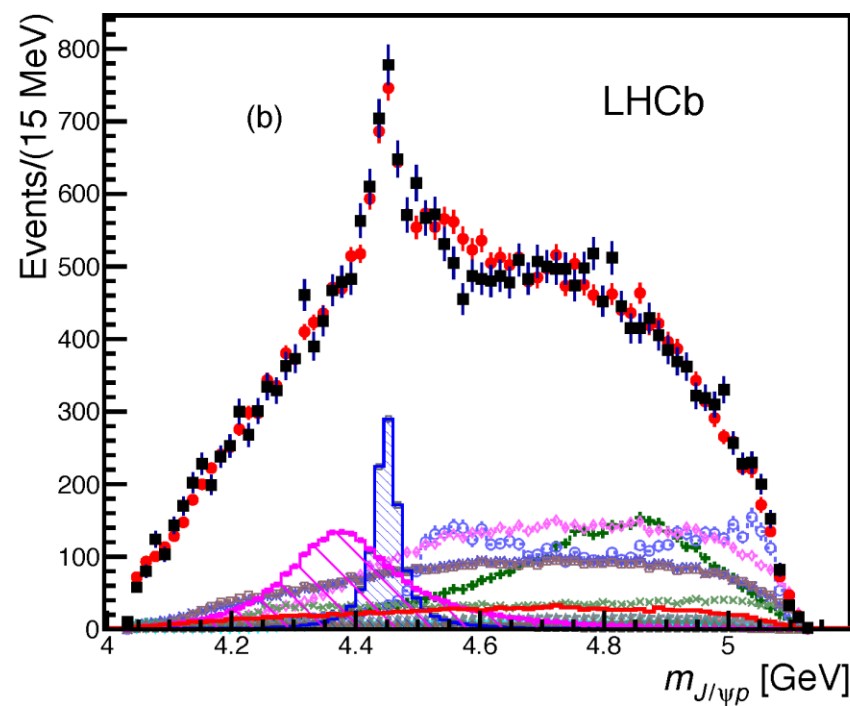
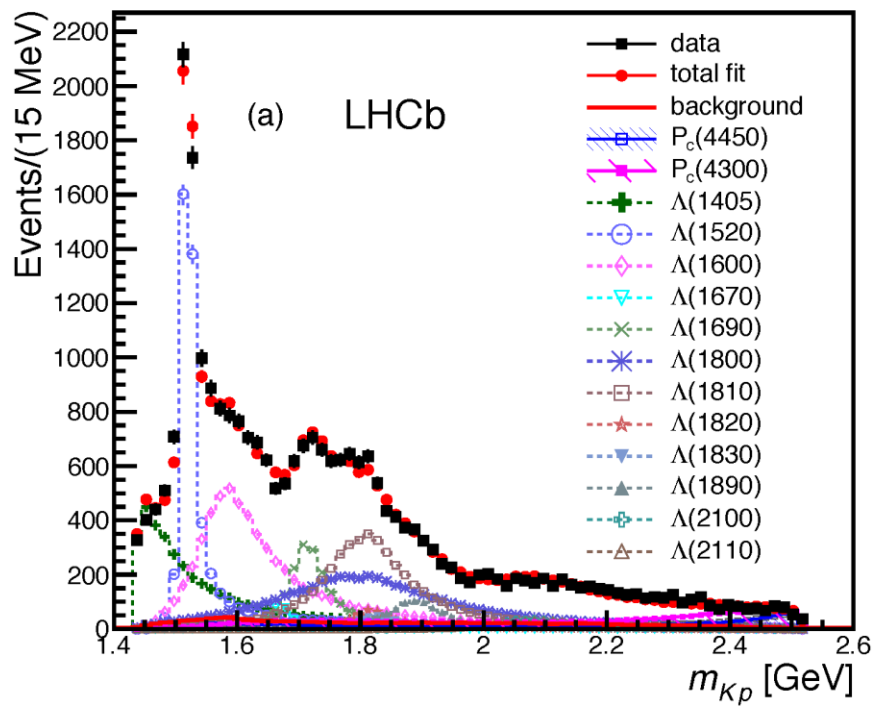
Fits With All Known Λ^* Resonances

- Fitting with all known Λ^* does not reproduce the $M(J/\psi p)$ peaking structures!
- Also added the following, without being able to describe the peaks.
 - Σ^* contributions ($\Delta I = 1$ in Λ_b decays; expected to be small)
 - Additional Λ^* states with free mass and width
 - 4 non resonant Λ^* states with $J^P = 1/2^\pm, 3/2^\pm$

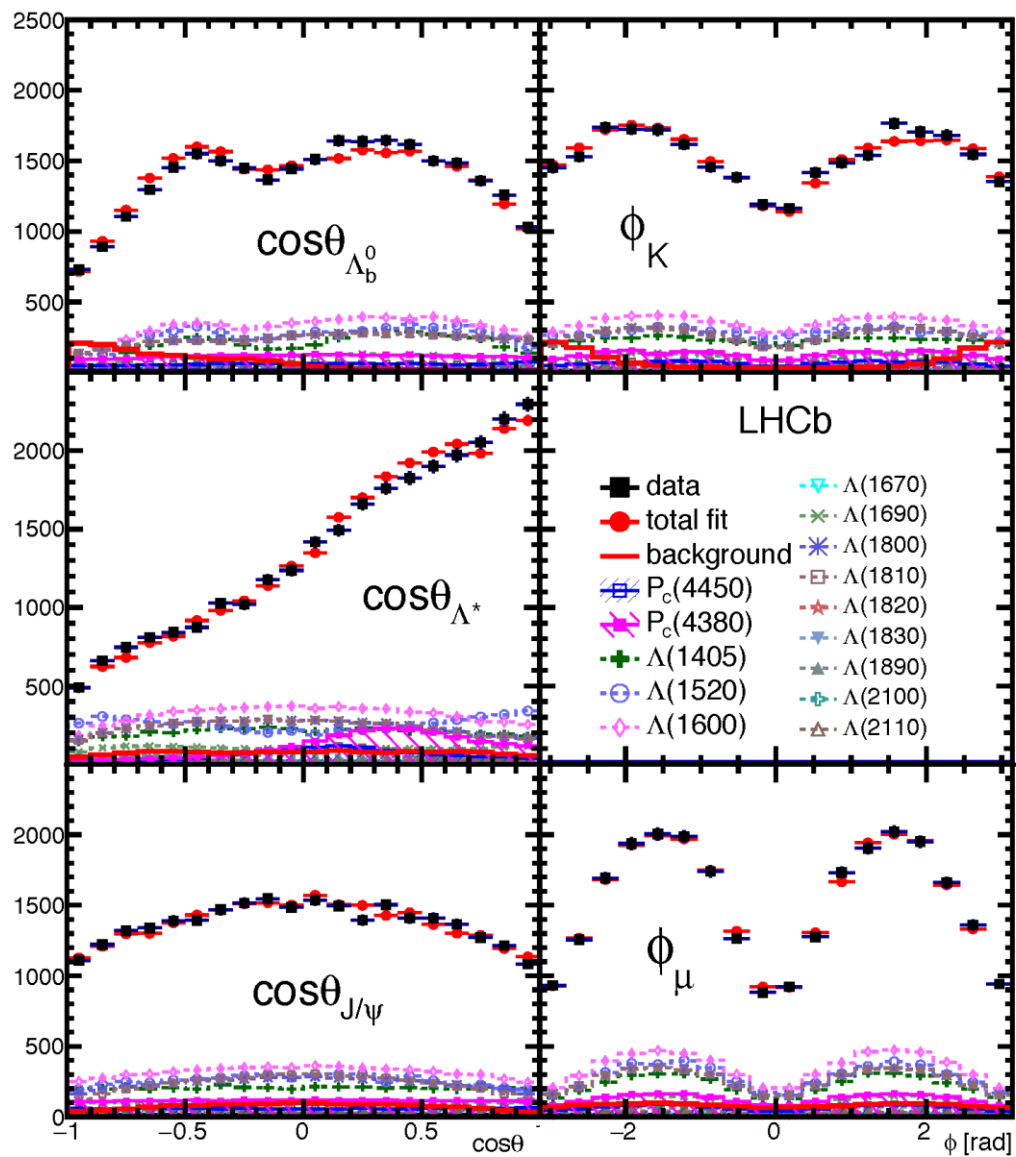


Fits With Two P_c Resonances

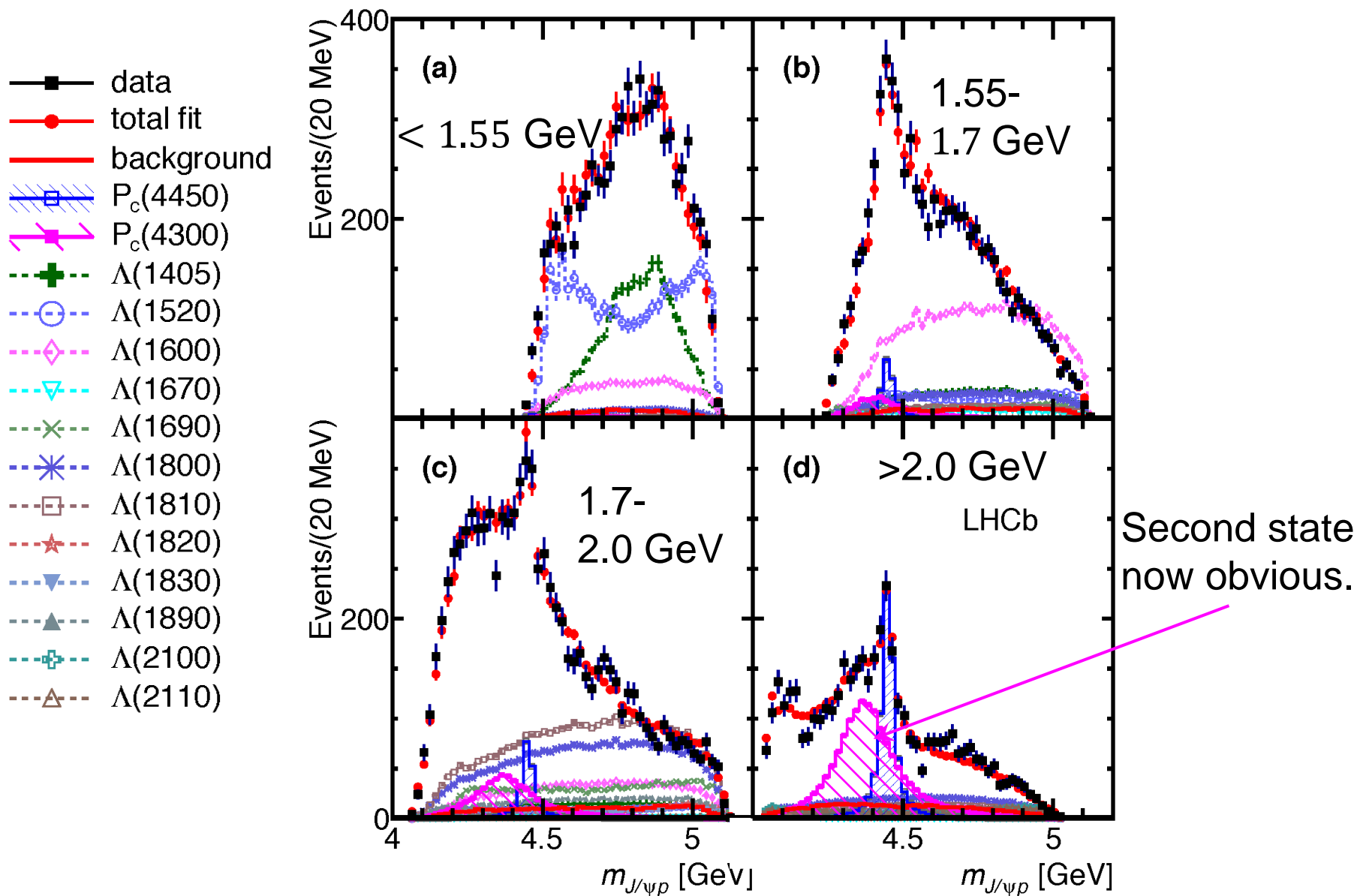
- With two P_c resonances, a good description of the data in all 6 dimensions is obtained (see next slide).
- Default model uses only well motivated Λ^* resonances.
- We fit all J^P combinations up to spin $7/2_-$.
- Best fit is with $J^P = (3/2^-, 5/2^+)$, but $(3/2^+, 5/2^-)$ and $(5/2^+, 3/2^-)$ not ruled out.



Angular Projections



$M(J/\psi p)$ in $M(Kp)$ Slices



Fit Results

- The observed states have masses, widths, and fit fractions of:

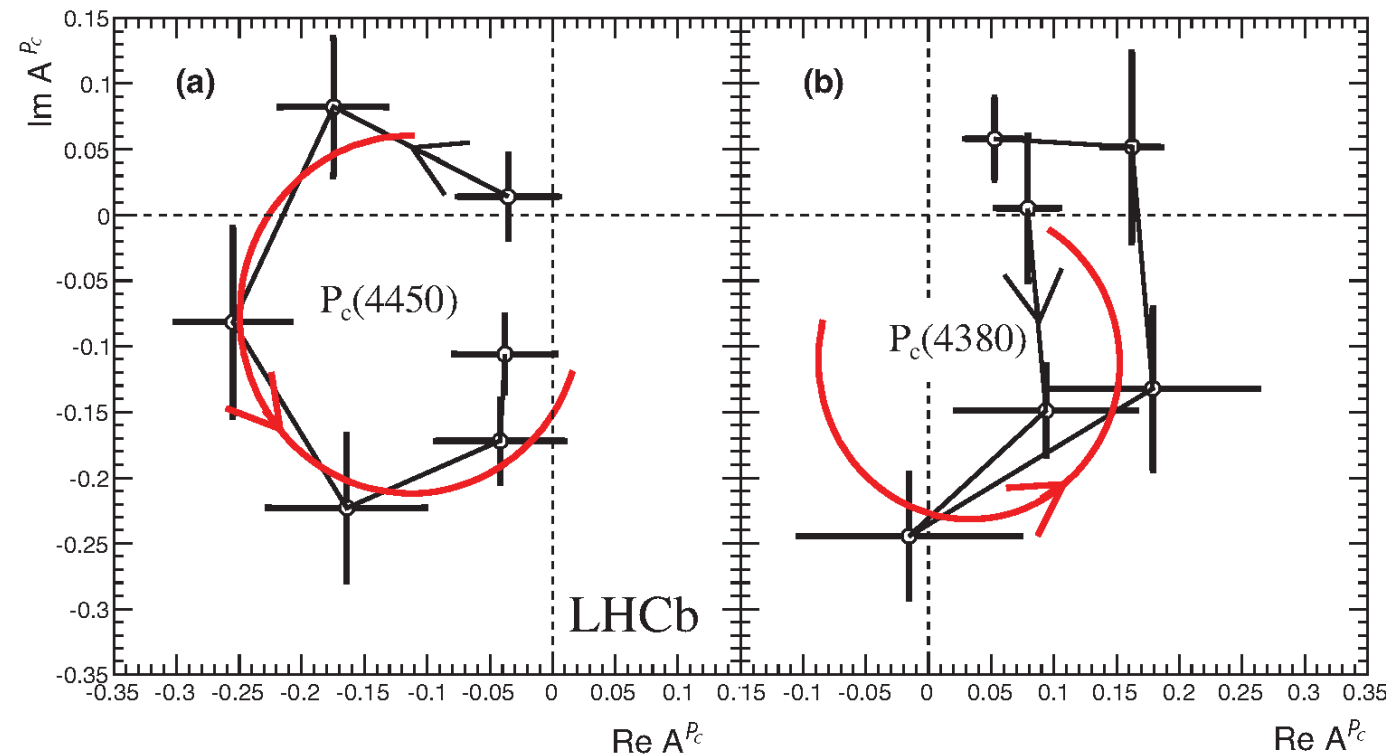
Resonance	Mass (MeV)	Width (MeV)	Fit Fraction (%)
$P_c(4380)^+$	$4380 \pm 8 \pm 29$	$205 \pm 18 \pm 86$	$8.4 \pm 0.7 \pm 4.2$
$P_c(4450)^+$	$4449.8 \pm 1.7 \pm 2.5$	$39 \pm 5 \pm 19$	$4.1 \pm 0.5 \pm 1.1$
$\Lambda(1405)$			$15 \pm 1 \pm 6$
$\Lambda(1520)$			$19 \pm 1 \pm 4$

where the fit fractions of the more prominent Λ^* states are also listed

- The dominant systematic uncertainties come from the Λ^* model.

Resonance Phase Motion

- The Breit-Wigner shape for individual P_c 's is replaced with 6 independent amplitudes in $M_0 \pm \Gamma_0$
- $P_c(4450)$: shows resonance behavior: a rapid counter-clockwise change of phase across the pole mass
- $P_c(4380)$: does show large phase change, but is not conclusive



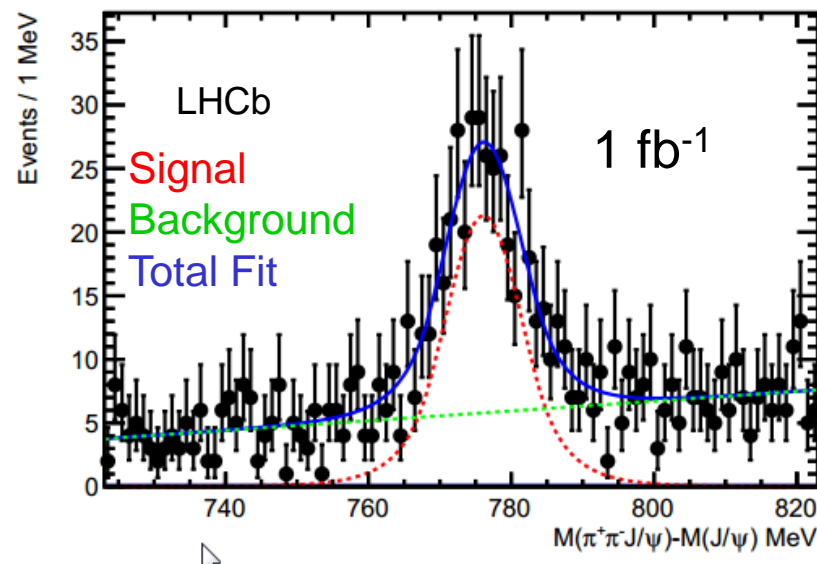
Plot fitted values for amplitudes in an Argand diagram

Breit-Wigner
Prediction
Fitted Values

X(3872) quantum numbers

Phys. Rev. Lett. 110, 222001 (2013)

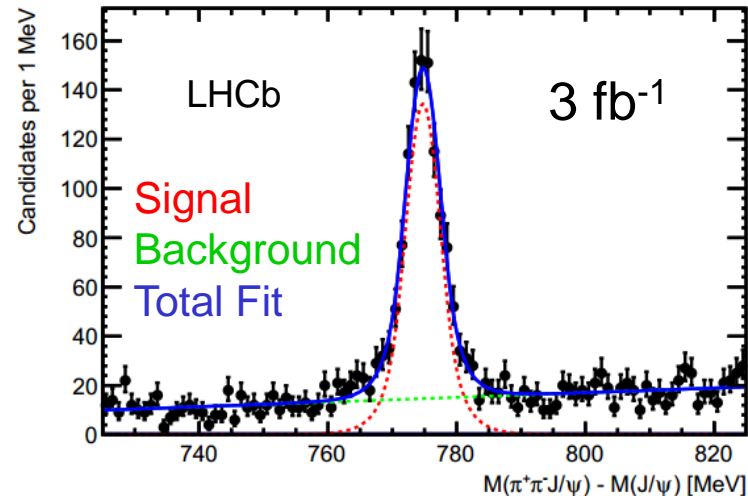
- X(3872) is a long-lived state ($\Gamma < 1.2$ MeV) with a mass within the errors off $m(D^0) + m(D^{0*})$.
- Currently unknown if it is a conventional charmonium state, something exotic ($D^0 D^{0*}$ molecule, tetraquark), or a mixture. Determination of its J^{PC} is important for narrowing down possible interpretations.
- Quantum numbers were narrowed down by CDF to be 1^{++} or 2^{-+} .
- Using 1 fb^{-1} of 2011 data, LHCb used $B^+ \rightarrow X(3872)K^+$, with $X(3872) \rightarrow J/\psi \pi^+ \pi^-$, and $J/\psi \rightarrow \mu^+ \mu^-$ to pin down the J^{PC} to be 1^{++} .



X(3872) quantum numbers

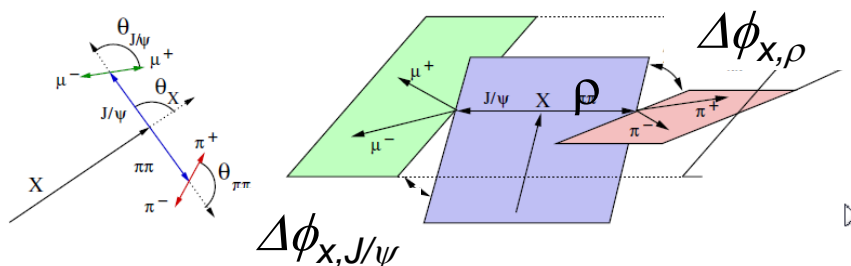
Phys Rev D 92, 011102 (2015)

- All previous analyses had assumed the presence of only the lowest possible orbital angular momentum in the $X(3872) \rightarrow J/\psi \pi^+ \pi^-$ decays.
- However a significant $L > L_{\min}$ could invalidate $J^P=1^+$ assignment and hint at molecular structure of X(3872)
 - It is important to check this!
 - Knowing the contribution of higher angular momentum is also interesting for better understanding these states.
- Using improved statistics from the full 3 fb^{-1} of Run 1 data, the analysis was done without any assumptions on orbital angular momentum present.



Determination of X(3872) J^{PC}: helicity formalism

$B^+ \rightarrow X(3872) K^+$,
 $X(3872) \rightarrow J/\psi \pi^+ \pi^-$,
 $J/\psi \rightarrow \mu^+ \mu^-$



5 independent angles describing the decay in helicity formalism

Ω – all 5 angles

$$|\mathcal{M}(\Omega|J_X)|^2 = \sum_{\Delta\lambda_\mu=-1,+1} \left| \sum_{\lambda_{J/\psi}, \lambda_\rho=-1,0,+1} A_{\lambda_{J/\psi}, \lambda_\rho} \times D_{0, \lambda_{J/\psi}-\lambda_\rho}^{J_X}(0, \theta_X, 0)^* \times D_{\lambda_\rho, 0}^1(\Delta\phi_{X, \rho}, \theta_\rho, 0)^* \times D_{\lambda_{J/\psi}, \Delta\lambda_\mu}^1(\Delta\phi_{X, J/\psi}, \theta_{J/\psi}, 0)^* \right|^2$$

Matrix element

$$A_{\lambda_{J/\psi}, \lambda_\rho} = \sum_L \sum_S B_{LS} \times \left(\begin{array}{cc} J_{J/\psi} & J_X \\ \lambda_{J/\psi} & -\lambda_\rho \end{array} \middle| \begin{array}{c} S \\ \lambda_{J/\psi} - \lambda_\rho \end{array} \right) \times \left(\begin{array}{cc} L & S \\ 0 & \lambda_{J/\psi} - \lambda_\rho \end{array} \middle| \begin{array}{c} J_X \\ \lambda_{J/\psi} - \lambda_\rho \end{array} \right)$$

Relation of the helicity couplings to LS amplitudes

$$P_X = P_{J/\psi} P_\rho (-1)^L = (-1)^L$$

Parity conservation

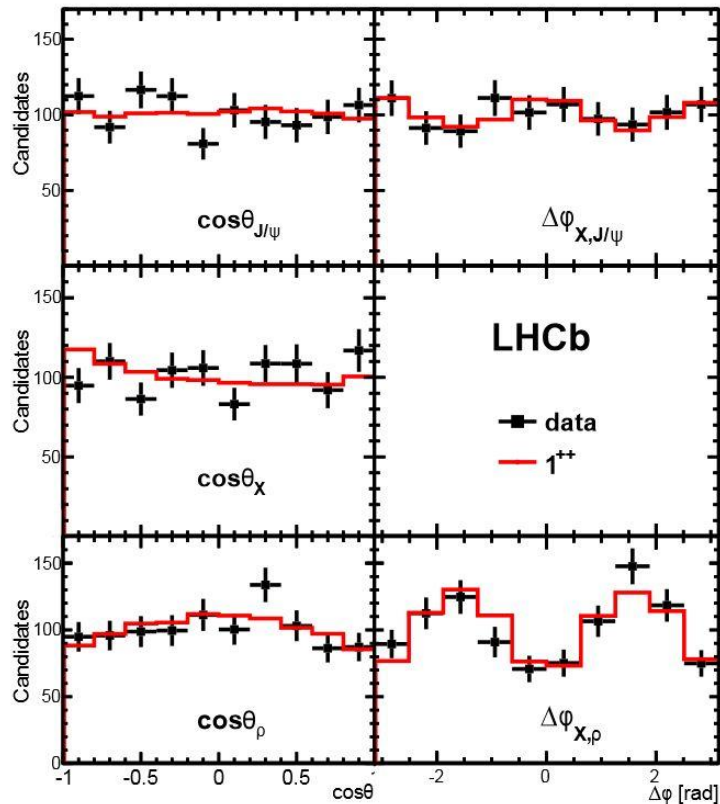
	B_{LS}	
	J^{PC} all L	minimal L
LHCb 2015	0^{-+} B_{11}	B_{11}
	0^{++} B_{00}, B_{22}	B_{00}
	1^{-+} $B_{10}, B_{11}, B_{12}, B_{32}$	B_{10}, B_{11}, B_{12}
	1^{++} B_{01}, B_{21}, B_{22}	B_{01}
	2^{-+} $B_{11}, B_{12}, B_{31}, B_{32}$	B_{11}, B_{12}
	2^{++} $B_{02}, B_{20}, B_{21}, B_{22}, B_{42}$	B_{02}
	3^{-+} $B_{12}, B_{30}, B_{31}, B_{32}, B_{52}$	B_{12}
	3^{++} $B_{21}, B_{22}, B_{41}, B_{42}$	B_{21}, B_{22}
	4^{-+} $B_{31}, B_{32}, B_{51}, B_{52}$	B_{31}, B_{32}
	4^{++} $B_{22}, B_{40}, B_{41}, B_{42}, B_{62}$	B_{22}
Many more amplitudes to fit		
		CDF 2007
		LHCb 2013

LS amplitudes to be determined from the data

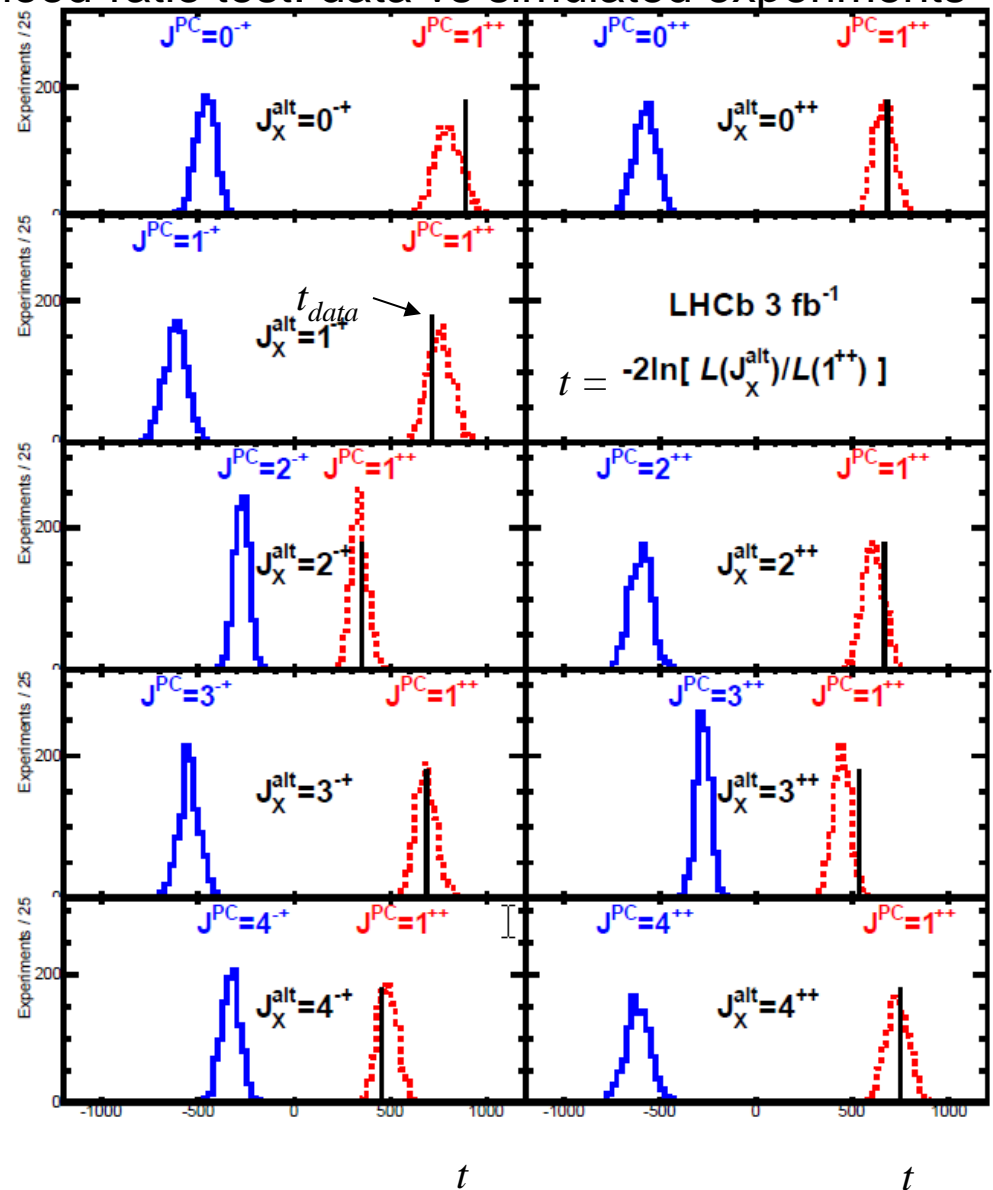
Determination of $X(3872)$ J^{PC}

Likelihood-ratio test: data vs simulated experiments

Projection of all angles with 1^{++} hypothesis



- Data unambiguously prefers 1^{++} hypothesis (new: no assumptions about L)



D-wave fraction in $X(3872) \rightarrow \rho^0 J/\psi$ for $J^{PC}=1^{++}$

- Fit to the real data:

$$\frac{|B_{21}|^2}{|B_{10}|^2} = 0.0018 \pm 0.0042 \quad \frac{|B_{22}|^2}{|B_{10}|^2} = 0.0066 \pm 0.0081$$

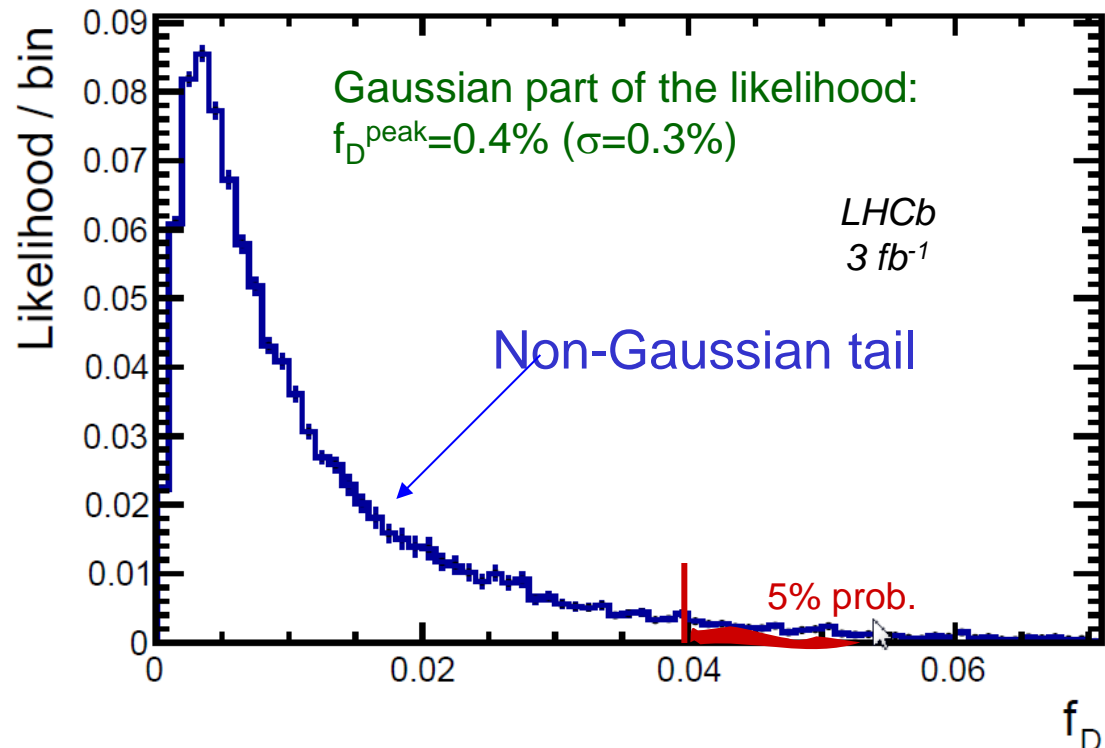
D-wave significance using Wilk's theorem applied to the likelihood ratio with/without D wave amplitudes: 0.8σ

D-wave amplitudes are consistent with zero

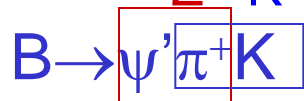
The likelihood was profiled as a function of the D-wave fraction:

$$f_D = \frac{\int |\mathcal{M}(\Omega)_D|^2 d\Omega}{\int |\mathcal{M}(\Omega)_{S+D}|^2 d\Omega}$$

$f_D < 4\%$ at 95% CL



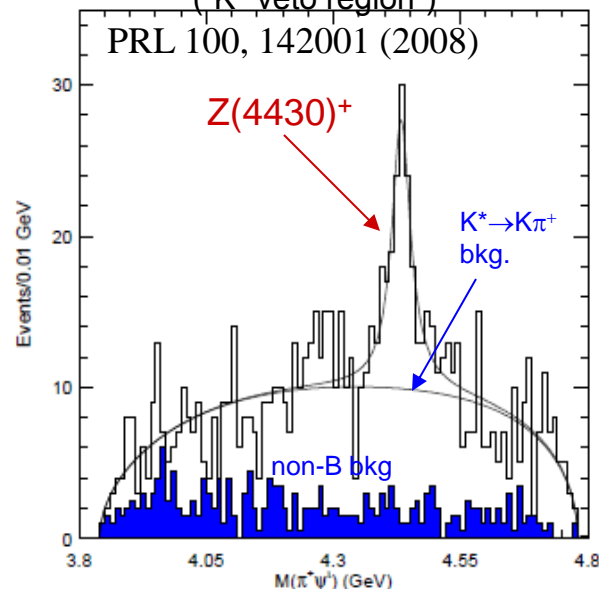
$Z^+ K^*$ $Z(4430)^+$ previous measurements



Belle 2008

1D $M(\psi' \pi^-)$ mass fit

("K* veto region")



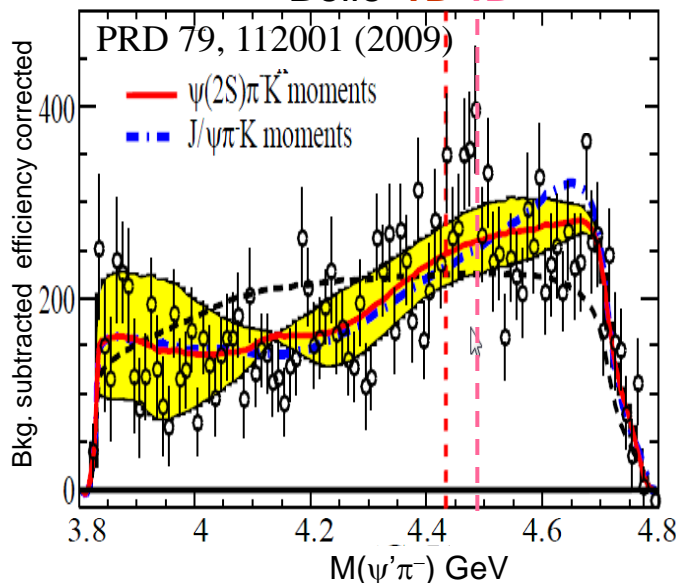
$$M(Z) = 4433 \pm 4 \pm 2 \text{ MeV}$$

$$\Gamma(Z) = 45^{+18}_{-13} {}^{+30}_{-13} \text{ MeV}$$

significance 6.5σ

BaBar 2009
Harmonic moments of K^* s (2D)
reflected to $M(\psi' \pi^+)$

Belle 1D4D



BaBar did not confirm $Z(4430)^+$
in B sample comparable to Belle.

*Did not numerically contradict the
Belle results.*

Almost **model independent**
approach to $K^* \rightarrow K\pi^+$
backgrounds.

Belle 2013

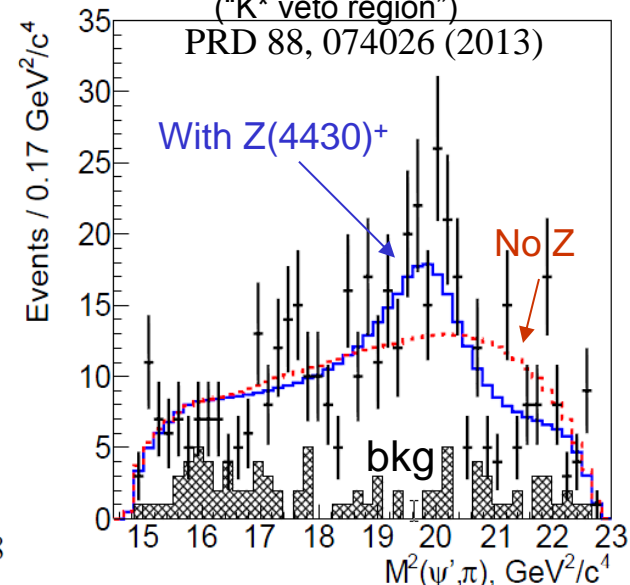
(2D amplitude fit in 2009)

4D amplitude fit

(subsample with $\psi' \rightarrow l^+ l^-$)

$0.996 \text{ GeV}/c^2 < M(K, \pi) < 1.332 \text{ GeV}/c^2$

("K* veto region")



$$M(Z) = 4485^{+22}_{-22} {}^{+28}_{-11} \text{ MeV}$$

$$\Gamma(Z) = 200^{+41}_{-46} {}^{+26}_{-35} \text{ MeV}$$

6.4σ (5.6σ with sys.)

$J^P = 1^+$ preferred by $>3.4\sigma$

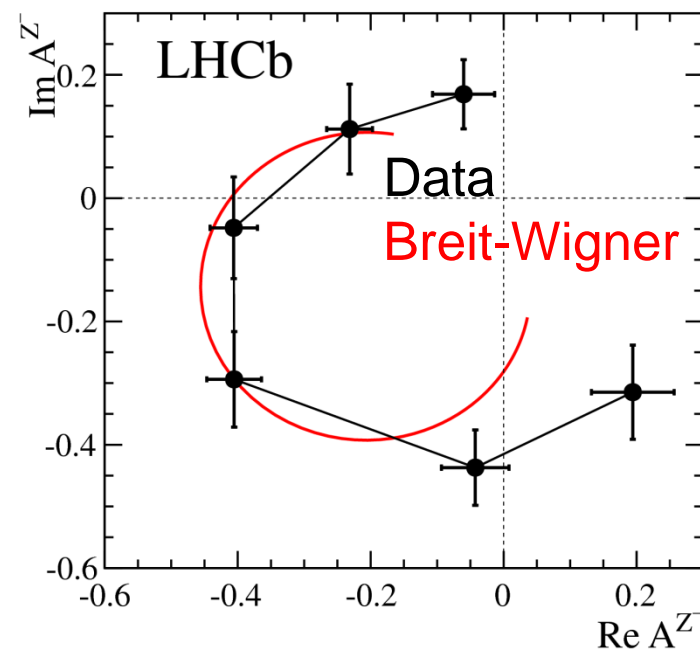
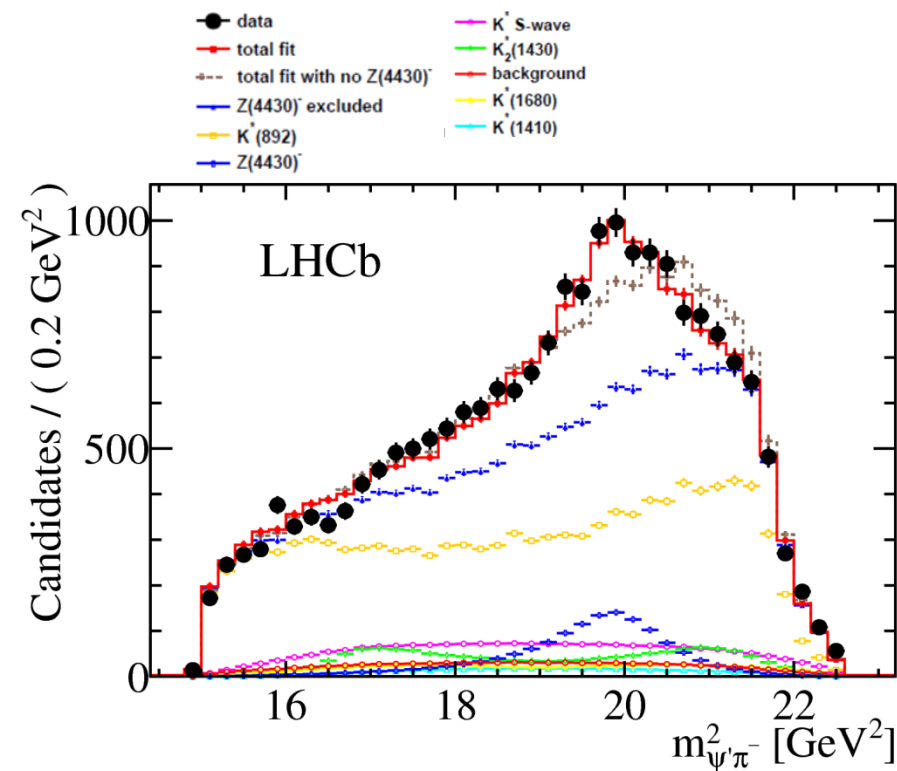
Model dependent approach
to $K^* \rightarrow K\pi^+$ backgrounds.
Higher statistical sensitivity.

Ad hoc assumption about
the $K^* \rightarrow K\pi^+$ background
shape.

Z(4430)⁺ 4D Amplitude Analysis in LHCb

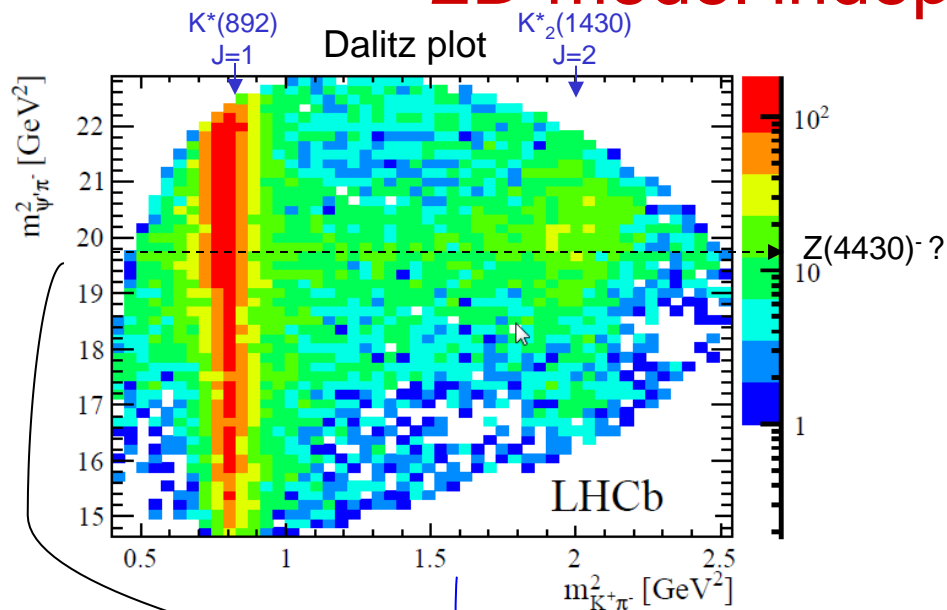
Phys Rev Lett 112, 222002 (2014)

- With an order of magnitude larger statistics and smaller backgrounds, LHCb was well suited to settle the matter.



- In 2014 LHCb overwhelmingly confirmed the Z(4430)⁺ and demonstrated its resonance phase motion.
- The quantum numbers were measured to be $J^P = 1^+$

2D model independent analysis

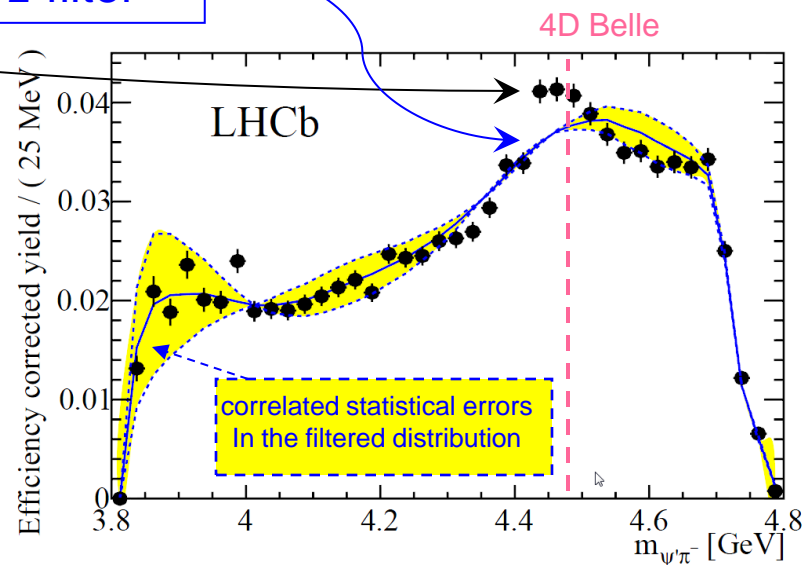


This qualitative analysis was included in the 2014 paper. In the new analysis in preparation it is made quantitative.

actual distribution

$K^* J \leq 2$ filter

Excess of events over
the $K^* J_{\max}=2$ filtered
distribution
in the $Z(4430)^-$ region
is apparent !

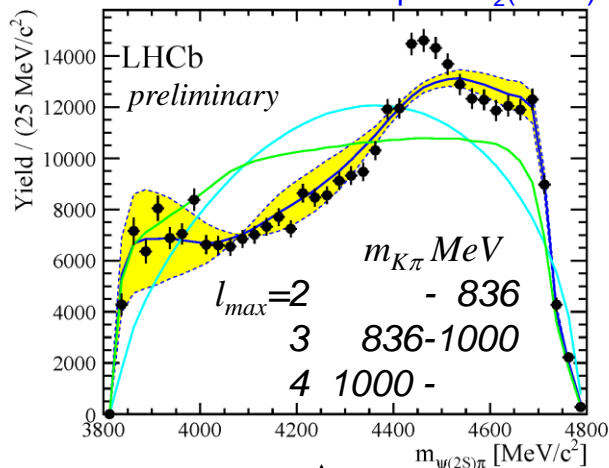


Quantitative results from the model independent approach

LHCb-PAPER-2015-038, in preparation

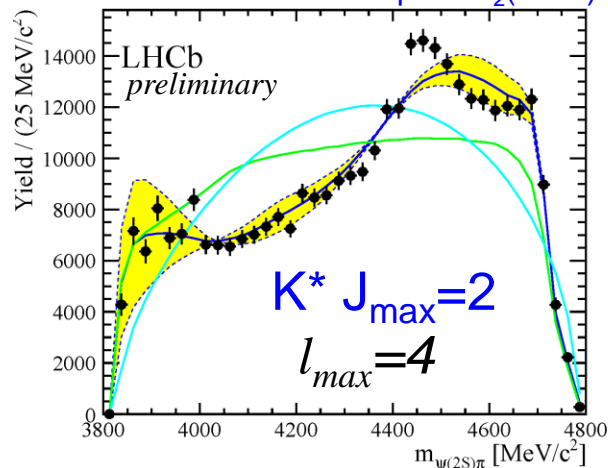
- The procedure on the previous slide is repeated for different J_{max} contributions.

Allows for K^* states up to $K^*_2(1430)$



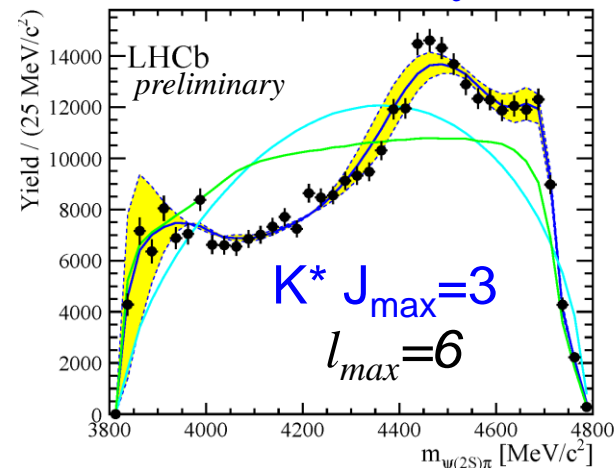
J_{max} is K^* mass-dependent.

Allows for K^* states up to $K^*_2(1430)$

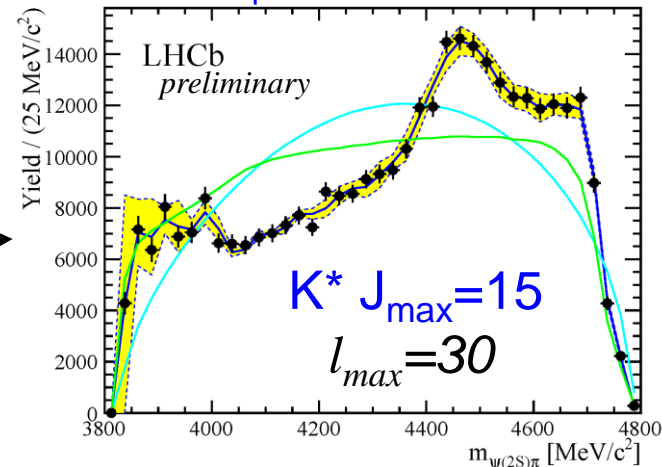


High moments wouldn't contribute except in presence of a Z

Allows for a tail of $K^*_3(1780)$



Allows implausible K^* contributions



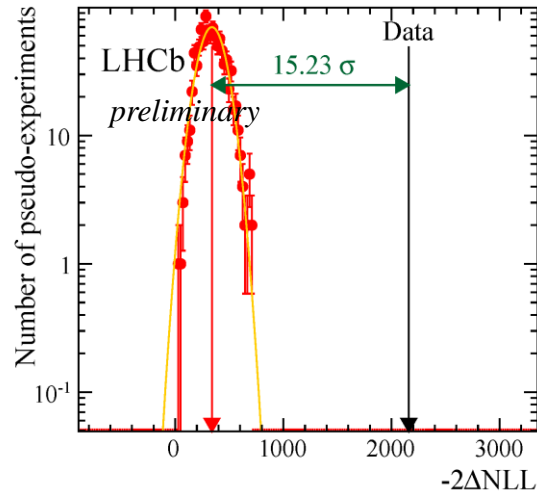
Quantitative results from the model independent approach

Test significance of implausible $l_{max} < l < 30 \cos(\theta_{K^*})$ moments using the log-likelihood ratio:

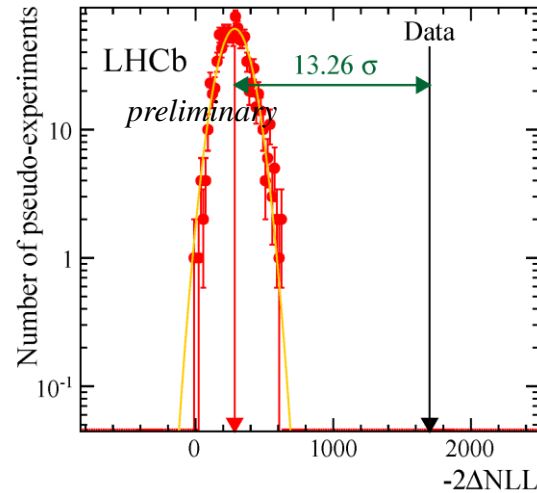
$$\Delta(-2\text{NLL}) = -2 \log \frac{\mathcal{L}_{l_{max}}}{\mathcal{L}_{30}} = -2 \log \frac{\prod_i \mathcal{F}_{l_{max}}(m_{\psi'\pi}^i)}{\prod_i \mathcal{F}_{30}(m_{\psi'\pi}^i)}$$

Statistical simulations of pseudo-experiments were generated from the $l < l_{max}$ hypotheses, and $\Delta(-2\text{NLL})$ calculated:

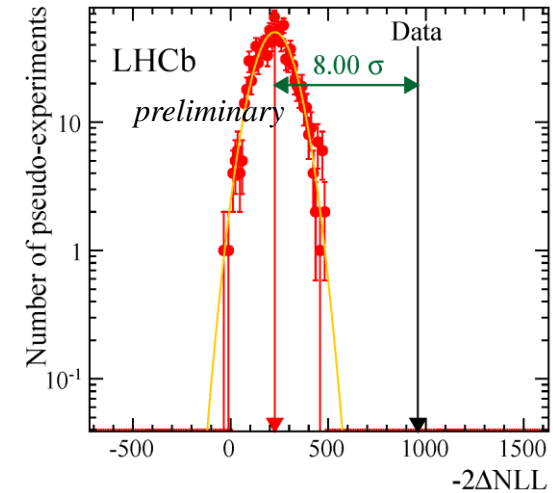
K^* mass-dependent $J_{max} < 2$



$J_{max} < 2$



$J_{max} < 3$



Explanation of the data with plausible K^* contributions is ruled out at high significance without assuming anything about K^* resonance shapes or their interference patterns!

Conclusions

- Two states have been observed which are consistent with the long sought after pentaquarks.
 - The two states are necessary to provide a good description of the data in all 6 dimensions.
- The need for an exotic tetraquark contribution in $B_0 \rightarrow \psi(2S)K^- \pi^+$ decays was demonstrated in a quantitative way using a model independent analysis, supplementing the model dependent amplitude analysis of $Z(4430)^-$ published last year.
- The quantum numbers of $X(3872)$ have been confirmed without any assumptions being made about the orbital angular momentum contributions present in the decay
 - The contribution of the higher waves was measured to be $<4\%$ at a 95% confidence level.
- We look forward to the discovery of more exotic hadrons and learning about their internal structure.

P_c Backup

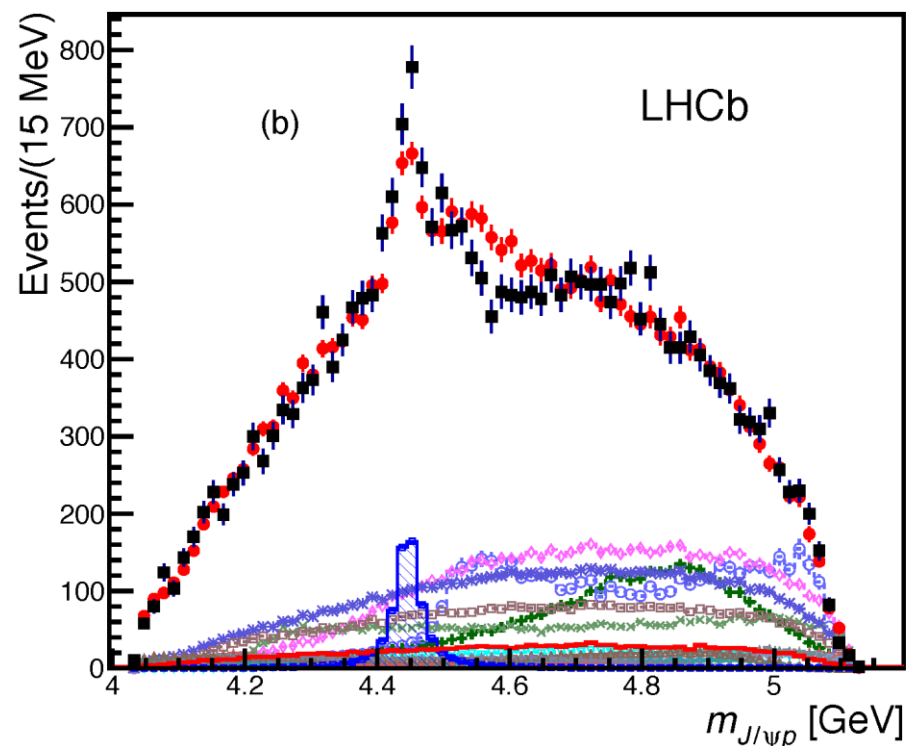
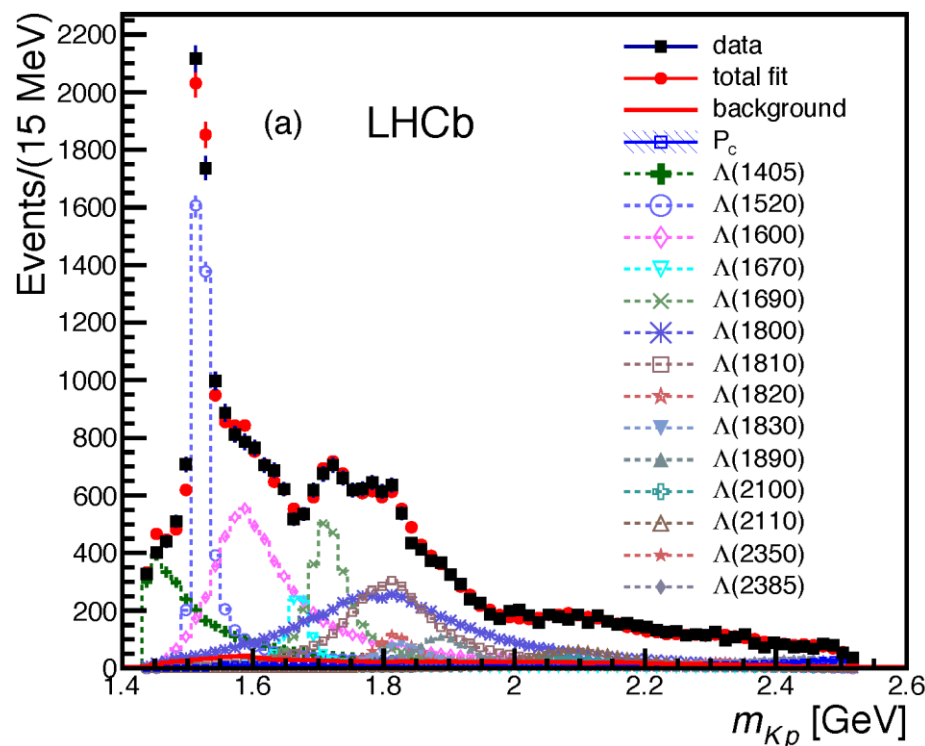
Λ^* Resonances

- Rich spectrum of possible resonances.
- We use two models, “reduced” and “extended” as part of tests on the dependence of the Λ^* model.
- Masses and widths are fixed to PDG values and varied in systematics.

State	J^P	M_0 (MeV)	Γ_0 (MeV)	# Amplitudes included	
				# Reduced	# Extended
$\Lambda(1405)$	$1/2^-$	$1405.1^{+1.3}_{-1.0}$	50.5 ± 2.0	3	4
$\Lambda(1520)$	$3/2^-$	1519.5 ± 1.0	15.6 ± 1.0	5	6
$\Lambda(1600)$	$1/2^+$	1600	150	3	4
$\Lambda(1670)$	$1/2^-$	1670	35	3	4
$\Lambda(1690)$	$3/2^-$	1690	60	5	6
$\Lambda(1800)$	$1/2^-$	1800	300	4	4
$\Lambda(1810)$	$1/2^+$	1810	150	3	4
$\Lambda(1820)$	$5/2^+$	1820	80	1	6
$\Lambda(1830)$	$5/2^-$	1830	95	1	6
$\Lambda(1890)$	$3/2^+$	1890	100	3	6
$\Lambda(2100)$	$7/2^-$	2100	200	1	6
$\Lambda(2110)$	$5/2^+$	2110	200	1	6
$\Lambda(2350)$	$9/2^+$	2350	150	0	6
$\Lambda(2585)$?	≈ 2585	200	0	6

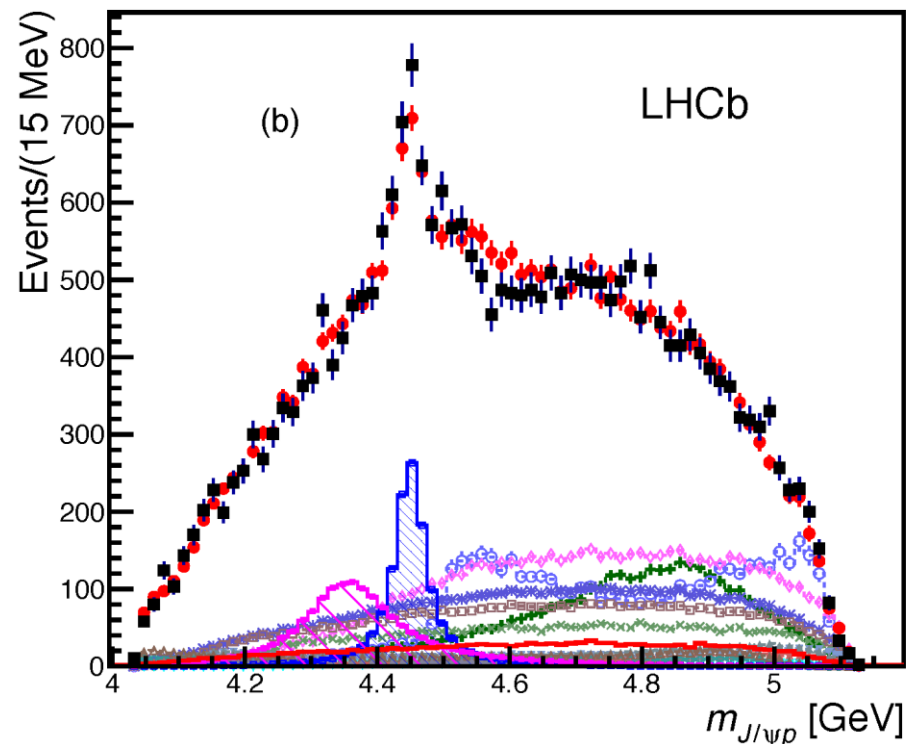
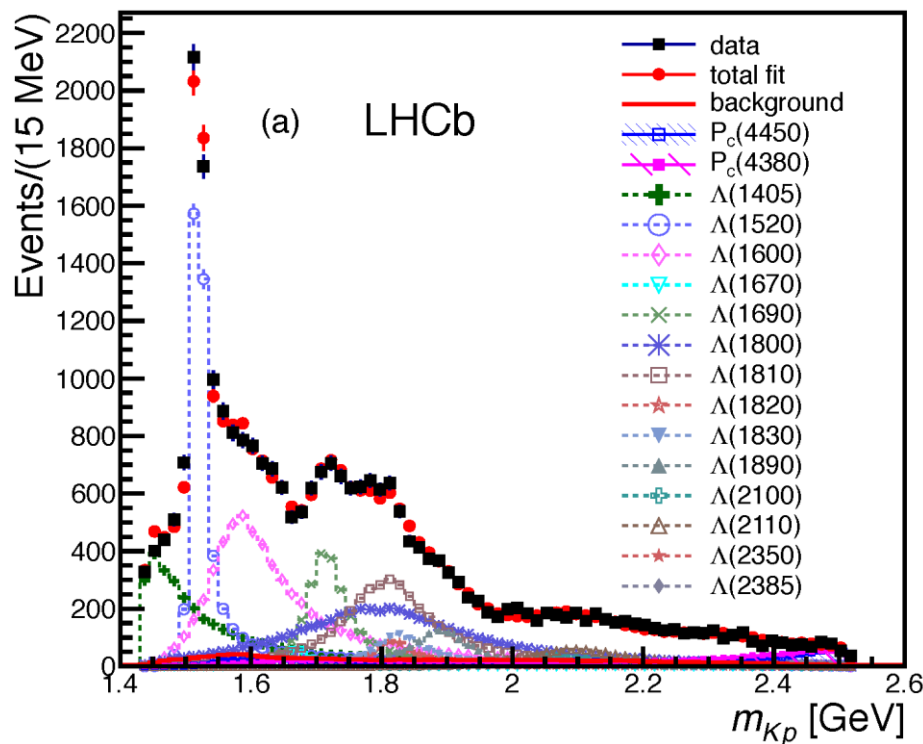
Extended Model with a P_c

- We try all J^P up to $7/2^\pm$
- None give good fits, the best fit with $J^P = 5/2^+$ is shown.



Extended Model with Two P_c Resonances

- We add a second P_c and get a good fit!
- A good description of data is seen in all 6 dimensions.



Significance

- The significances are calculated using the extended model (the dominant source of systematic uncertainty).
- In the following scenarios, we see for: $\Delta(-2 \ln \mathcal{L})$:
 - One P_c hypothesis verse no P_c : 14.7^2
 - Two P_c hypothesis verse one P_c : 11.6^2
 - Two P_c hypothesis verse no P_c : 18.7^2
- Toy experiments were run to properly obtain the significances:
 - One P_c hypothesis verse no P_c null hypothesis: 12σ
 - Two P_c hypothesis verse one P_c null hypothesis: 9σ
 - Two P_c hypothesis verse no P_c null hypothesis: 15σ

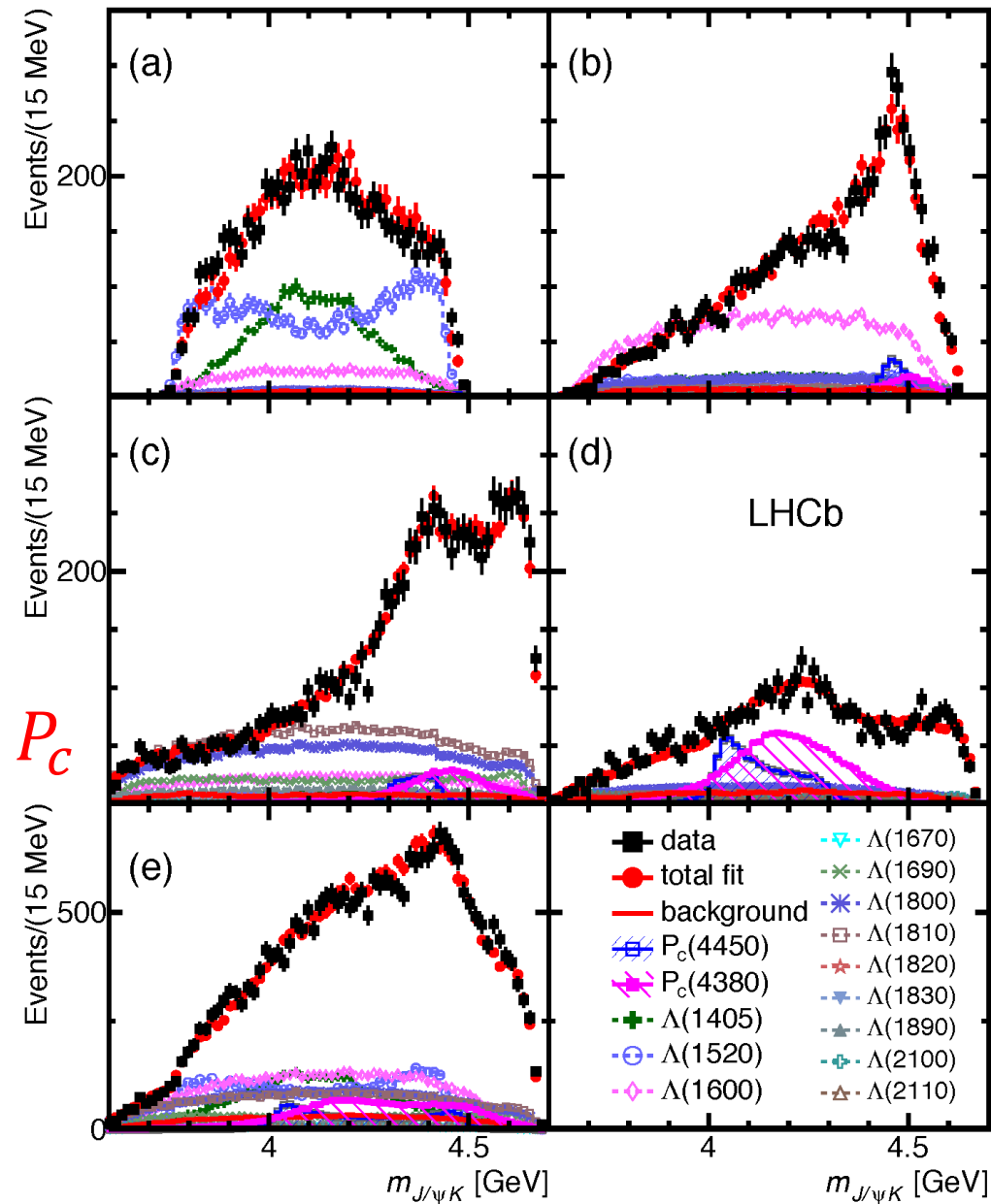
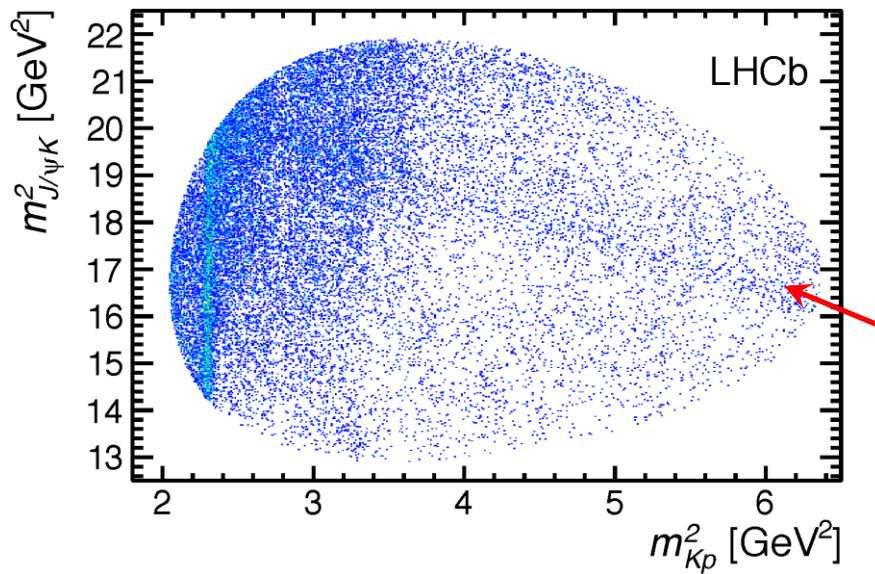
Systematic Uncertainties

Source	M_0 (MeV)		Γ_0 (MeV)		Fit fractions (%)			
	low	high	low	high	low	high	$\Lambda(1405)$	$\Lambda(1520)$
Extended vs. reduced	21	0.2	54	10	3.14	0.32	1.37	0.15
Λ^* masses & widths	7	0.7	20	4	0.58	0.37	2.49	2.45
Proton ID	2	0.3	1	2	0.27	0.14	0.20	0.05
$10 < p_p < 100$ GeV	0	1.2	1	1	0.09	0.03	0.31	0.01
Nonresonant	3	0.3	34	2	2.35	0.13	3.28	0.39
Separate sidebands	0	0	5	0	0.24	0.14	0.02	0.03
J^P ($3/2^+$, $5/2^-$) or ($5/2^+$, $3/2^-$)	10	1.2	34	10	0.76	0.44		
$d = 1.5 - 4.5$ GeV $^{-1}$	9	0.6	19	3	0.29	0.42	0.36	1.91
$L_{\Lambda_b^0}^{P_c} \Lambda_b^0 \rightarrow P_c^+ \text{ (low/high)} K^-$	6	0.7	4	8	0.37	0.16		
$L_{P_c} P_c^+ \text{ (low/high)} \rightarrow J/\psi p$	4	0.4	31	7	0.63	0.37		
$L_{\Lambda_b^0}^{\Lambda^*} \Lambda_b^0 \rightarrow J/\psi \Lambda^*$	11	0.3	20	2	0.81	0.53	3.34	2.31
Efficiencies	1	0.4	4	0	0.13	0.02	0.26	0.23
Change $\Lambda(1405)$ coupling	0	0	0	0	0	0	1.90	0
Overall	29	2.5	86	19	4.21	1.05	5.82	3.89
sFit/cFit cross check	5	1.0	11	3	0.46	0.01	0.45	0.13

sFit/cFit comparison is included only as a cross-check and does not contribute to overall systematic uncertainty

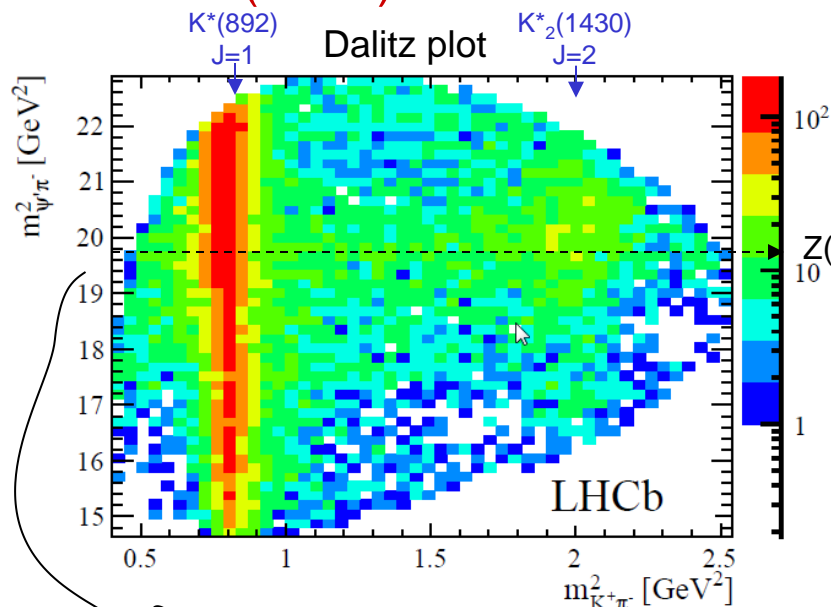
$J/\psi K$ System

- $J/\psi K$ system is well described by the Λ^* and P_c reflections.

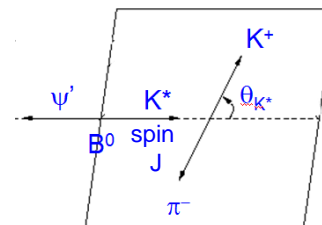


$Z(4430)^-$ Backup

Z(4430)⁺ in LHCb: 2D model independent analysis (a la BaBar)



“Rectangular Dalitz plot”



Decompose into
Legendre moments

$$\langle P_l^U \rangle = \frac{1}{N_{data}} \sum_{i=1}^{N_{data}} \frac{1}{\epsilon_i} P_l(\cos \theta_{K^*i})$$

vs. $m_{K+\pi-}$



Pass only moments with l
not more than $l_{max} = J_{max}/2$

“K* J_{max} filtered”

$\cos(\theta_{K^*})$

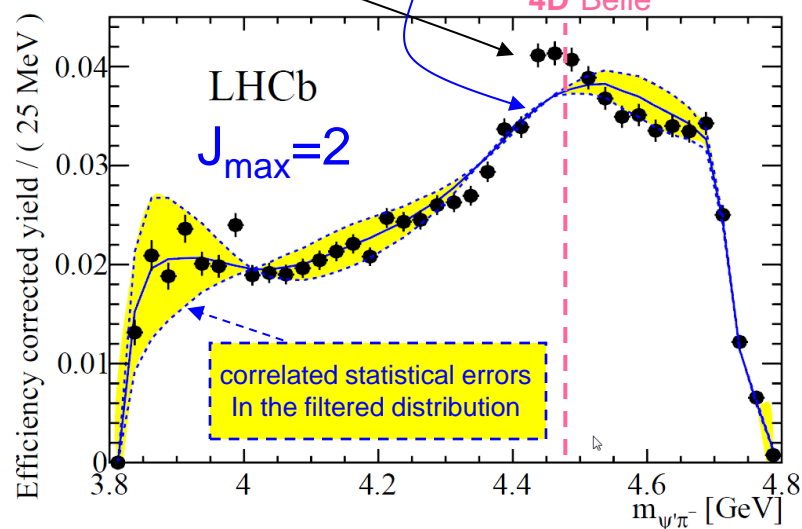
vs. $m_{K+\pi-}$

Excess of events over
the K* $J_{max}=2$ filtered distribution
in the Z(4430)⁺ region
is apparent !

“K* J_{max} filtered”

Dalitz plot

4D Belle



*This qualitative analysis was
included in the 2014 paper*