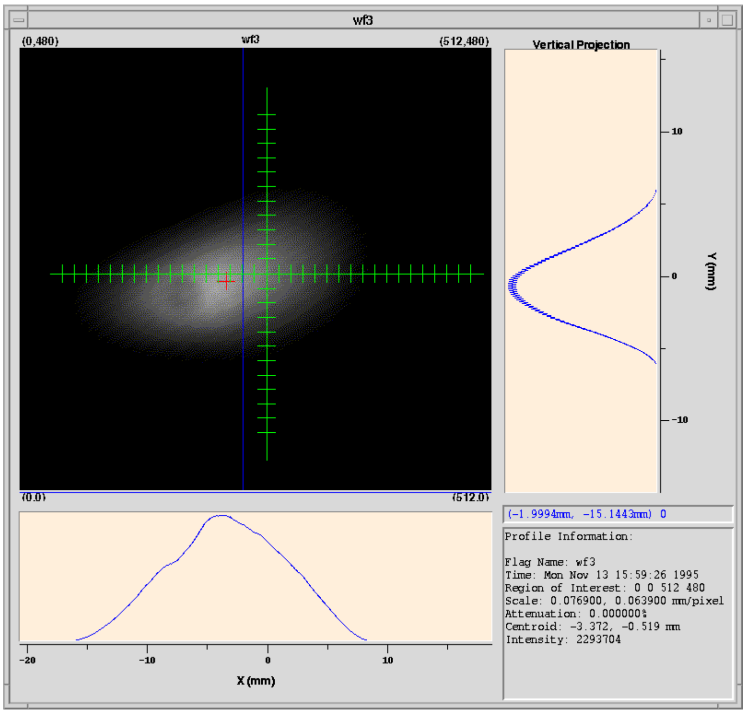
Solutions: LINACS/Transport Lines Emittance Measurement



by Kay Wittenburg, -DESY-

Solutions: LINACS/Transport Lines Emittance Measurement

1) <u>Explain ways of measuring the emittance of a charged particle beam in a Linear accelerator or a transport line without knowing the beam optic parameters α, β, γ.</u>

a) Exercise L1: Which one is the preferable method for a high energy proton transport line (p >5 GeV/c)?

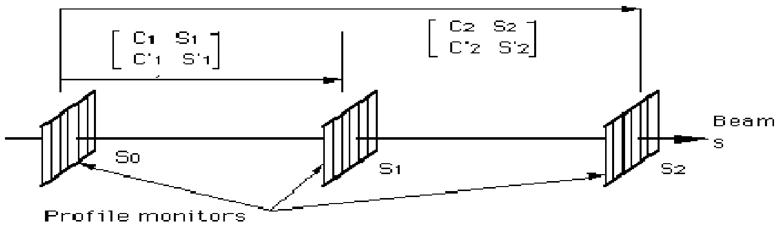
Solution: 3 (thin) screens/SEM grids or varying quadrupole which measure the different beam widths σ. For pepper pot or slits one needs a full absorbing aperture.

b) Exercise L2: Assuming that the geometry between the measurement stations and the transport matrices M of the transport line are well defined (including magnetic elements), describe a way to get the emittance using 3 screens and the σ-matrix.

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = \begin{pmatrix} \sigma_y^2 & \sigma_{yy'} \\ \sigma_{yy'} & \sigma_{y'}^2 \end{pmatrix} = \varepsilon_{rms} \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix} = \sigma\, matrix$$

Write down the equation how the beam width σ_y is transferred from s_0 to the next location s_1.

If β is known unambiguously as in a circular machine, then a single profile measurement determines ε by $\sigma_y^2 = \varepsilon\beta$. But it is not easy to be sure in a transfer line which β to use, or rather, whether the beam that has been measured is matched to the β-values used for the line. This problem can be resolved by using three monitors (see Fig. 1), i.e. the three width measurements determine the three unknown α, β and ε of the incoming beam.



σ elements at first Screen or Quadrupole (Ref. 1).

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = \begin{pmatrix} \sigma_y^2 & \sigma_{yy'} \\ \sigma_{yy'} & \sigma_{y'}^2 \end{pmatrix} = \varepsilon_{rms} \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix} = \sigma\, matrix$$

Beam width_{rms} of measured profile = $\sigma_y = \sqrt{\beta(s)\cdot\varepsilon}$,

L_1, L_2 = distances between screens or from Quadrupole to screen and Quadrupole field strength are given, therefore the transport matrix M is known.

Employing transfer matrix gives: $M \cdot \sigma \cdot M^t$

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \cdot \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} \cdot \begin{pmatrix} M_{11} & M_{21} \\ M_{12} & M_{22} \end{pmatrix} = \sigma^{measured} = \begin{pmatrix} \sigma_y^2 & \sigma_{yy'} \\ \sigma_{y'y} & \sigma_{y'}^2 \end{pmatrix}^{measured} = \varepsilon_{rms} \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

$$= \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \cdot \begin{pmatrix} \sigma_{11}M_{11} + \sigma_{12}M_{12} & \sigma_{11}M_{21} + \sigma_{12}M_{22} \\ \sigma_{21}M_{11} + \sigma_{22}M_{12} & \sigma_{12}M_{21} + \sigma_{22}M_{22} \end{pmatrix}$$

$$= \begin{pmatrix} M_{11}(\sigma_{11}M_{11} + \sigma_{12}M_{12}) + M_{12}(\sigma_{21}M_{11} + \sigma_{22}M_{12}) & \cdots \\ \cdots & \cdots \end{pmatrix}$$

$\sigma_{11}^{measured} = M_{11}^2\sigma_{11} + 2M_{11}\,M_{12}\sigma_{12} + M_{12}^2\,\sigma_{22}$ $\quad(\sigma_{12} = \sigma_{21})$ (1)

Solving σ_{11} σ_{12} and σ_{22} while Matrix elements are known: Needs minimum of three different measurements, either three screens or three different Quadrupole settings with different field strength.

$\varepsilon_{rms} = \sqrt{\det \sigma} = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2}$ (from $\beta\gamma - \alpha^2 = 1$) (2)

c) Exercise L3: In a transport line for p = 7.5 GeV/c protons are two measurement stations. The first is located exactly in the waist of the beam and shows a beam width of σ_y = 3 mm, the second at a distance of s = 10 m shows a width of σ_y = 9 mm. Assuming no optical elements in this part, calculate the emittance and the normalized emittance of the beam.

No optical elements => $M = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}$ (3)

Waist => $\alpha = \sigma_{12} = \sigma_{21} = 0$ => $\varepsilon_{rms} = \sqrt{\sigma_{11}\sigma_{22}}$ (4)

Momentum p = 7.5 GeV/c => relativistic $\gamma\beta \approx 7.5$

Measured width at s = 0 => $(3\text{ mm})^2 = \sigma_y^2\,(s{=}0) = \sigma_{11}$ (5)

Calculate σ_{22} with width measured at s = 10 m and with (1, 4) =>
$(9\text{ mm})^2 = \sigma_y^2\,(s{=}10) = M_{11}^2 \cdot \sigma_{11} + M_{12}^2 \cdot \sigma_{22} = \sigma_{11} + s^2 \cdot \sigma_{22}$ (σ_{11}, σ_{22} at s=0) (6)

with (5) => $\sigma_{22} = \dfrac{\sigma_y^2(10) - \sigma_y^2(0)}{s^2}$ (7)

With (4) and (7) =>

$$\varepsilon_{rms} = \sqrt{\sigma_{11}\sigma_{22}} = \sqrt{\sigma_y^2(0) \cdot \frac{\sigma_y^2(10) - \sigma_y^2(0)}{s^2}} = \frac{\sigma_y(0)}{s}\sqrt{\sigma_y^2(10) - \sigma_y^2(0)}$$

$= 2.5 \cdot 10^{-6}$ m rad

$\varepsilon^{normalized} = \varepsilon_{rms}\,\gamma\,\beta = 19 \cdot 10^{-6}$ m rad = 19 mm mrad

Additional exercise: Calculate β(s=0 and s=10m)

Beam width $\sigma_{rms} = \sqrt{\beta(s)\cdot\varepsilon}$
At s=10 m: $\sigma^2 = \beta\varepsilon$ => $\beta = 32.4$ m
At s= 0 m : $\beta = 3.6$ m

What is the influence on the emittance ε assuming at s = 10m this b, a dispersion of D = 1 m and a momentum spread of $\Delta p/p = 10^{-3}$?

$$\varepsilon = \frac{\sigma^2 - \left(D\cdot\frac{\Delta p}{p}\right)^2}{\beta} = \frac{81\cdot 10^{-6} - 1\cdot 10^{-6}}{32.4} = 2.469\,\pi\, mm\, mrad$$

or ≈1% which is less than the typical accuracy of a profile measurement

References

For solutions attached:

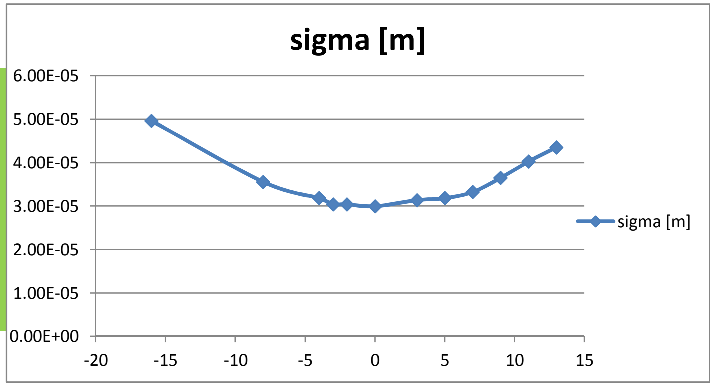
Criegee, PLIN note 88-04

Another method: (P.J. Bryant, 5th CAS, Finnland)

Emittance of a Laser Beam, solutions

s [mm]	left[pixels]	right[pixels]	FWHM [pixel]	sigma [m]	emittance
-16	73	126	53	4.96E-05	7.41E-08
-8	70	108	38	3.56E-05	7.18E-08
-4	51	85	34	3.18E-05	8.06E-08
-3	38.5	71	32.5	3.04E-05	5.31E-08
-2	45.5	78	32.5	3.04E-05	7.96E-08
0	75	107	32	3.00E-05	
3	23	56.5	33.5	3.14E-05	9.27E-08
5	54	88	34	3.18E-05	6.44E-08
7	30	65.5	35.5	3.32E-05	6.16E-08
9	92	131	39	3.65E-05	6.95E-08
11	49	92	43	4.03E-05	7.32E-08
13	59	105.5	46.5	4.35E-05	7.28E-08
					mean:
					7.21E-08

From pictures



-12
-10
-8
-6
-4
0
2
4
6
8
10
12

From Solutions: LINACS/Transport Lines
Emittance Measurement

$$\varepsilon_{rms} = \sqrt{\sigma_{11}\sigma_{22}} = \sqrt{\sigma_y^2(0)\cdot\frac{\sigma_y^2(s)-\sigma_y^2(0)}{s^2}} = \frac{\sigma_y(0)}{s}\sqrt{\sigma_y^2(s)-\sigma_y^2(0)}$$

at s=0mm: FWHM = 32 pixels			From picture at s=0
pixel =	2.20E-06	m	From camera manual, pixel size
1 mm =	1.00E-03	m	

PLIN - Note 88-04
L. Criegee
August 18, 1988

Emittance Measurements for Linac III

Principle

The measurement of the *Courant-Snyder* beam parameters $\alpha, \beta, \gamma, \epsilon$ is based on the evolution of the beam matrix σ. If the transport is described by the 2 x 2 matrix

$$R = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} ,$$

with $R_{11}R_{22} - R_{12}R_{21} = 1$, the beam evolution is given by[1]

$$\sigma = \begin{pmatrix} \epsilon\beta & -\epsilon\alpha \\ -\epsilon\alpha & \epsilon\gamma \end{pmatrix} = R \begin{pmatrix} \epsilon\beta_0 & -\epsilon\alpha_0 \\ -\epsilon\alpha_0 & \epsilon\gamma_0 \end{pmatrix} R^T .$$

$\epsilon\beta$ is the measurable squared beam envelope $\hat{x}^2$ at the end of the transport. It depends linearly on the initial beam matrix elements:

$$\hat{x}^2 = \epsilon\beta = R_{11}^2 \cdot \epsilon\beta_0 - 2R_{11}R_{12} \cdot \epsilon\alpha_0 + R_{12}^2 \cdot \epsilon\gamma_0$$

To obtain $\epsilon\alpha_0$, $\epsilon\beta_0$, $\epsilon\gamma_0$, and $\epsilon^2 = \epsilon\beta_0 \cdot \epsilon\gamma_0 - \epsilon\alpha_0 \cdot \epsilon\alpha_0$, one has set R to three (sufficiently different) values, measure the resulting $\hat{x}^2$, and solve three linear equations.

3-Position Method

One simple example is the simultaneous measurement with three profile harps as forseen for the HEBT. Here the transport matrix

$$R(S) = \begin{pmatrix} 1 & S \\ 0 & 1 \end{pmatrix}$$

describes just a variable drift space. The three settings $S_1 = -L$, $S_2 = 0$, $S_3 = +L$ give the beam transport from a reference position S_2 to two symmetric ones up- and downstream. They lead to the equations

$$\left.\begin{aligned} \hat{x}_1^2 &= \epsilon\beta_2 - 2L \cdot \epsilon\alpha_2 + L^2 \cdot \epsilon\gamma_2 \\ \hat{x}_2^2 &= \epsilon\beta_2 \\ \hat{x}_3^2 &= \epsilon\beta_2 + 2L \cdot \epsilon\alpha_2 + L^2 \cdot \epsilon\gamma_2 \end{aligned}\right\} \text{ with the solution } \left\{\begin{aligned} \epsilon\beta_2 &= \hat{x}_2^2 \\ \epsilon\alpha_2 &= (\hat{x}_3^2 - \hat{x}_1^2)/(4L) \\ \epsilon\gamma_2 &= (\hat{x}_1^2 - 2\hat{x}_2^2 + \hat{x}_3^2)/(2L^2) \\ \epsilon^2 &= \epsilon\beta_2 \cdot \epsilon\gamma_2 - (\epsilon\alpha_2)^2 \end{aligned}\right.$$

The measurements should be done in the vicinity of a beam waist where the curvature due to $\epsilon\gamma$ is most noticeable. If the beam waist is exactly at S_2, we have $\hat{x}_3 = \hat{x}_1$, $\alpha_2 = 0$. The emittance formula is then reduced to

$$\epsilon = \hat{x}_2\sqrt{\hat{x}_3^2 - \hat{x}_2^2}/L \qquad (\text{for } \hat{x}_2 = minimum)$$

[1] see K.L.Brown et al., CERN 80-04 (TRANSPORT manual)

$\beta \cdot \gamma - \alpha^2 = 1$

3-Gradient Method

If the 3-Position method cannot be used, the beam parameters can be measured by varying the gradient of a quadrupole. The system of the quadrupole and a subsequent transport section with constant elements C, S, C' and S' ($CS' - C'S = 1$) is described by

$$R(k) = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix} \begin{pmatrix} \cos kl & k^{-1}\sin kl \\ -k\sin kl & \cos kl \end{pmatrix} \quad \text{with} \quad \begin{cases} R_{11}(k) = & C\cos kl & -Sk\sin kl \\ R_{12}(k) = & Ck^{-1}\sin kl & +S\cos kl \end{cases}$$

The measurement of $\hat{x}^2$, as defined above, for three different quadrupole settings k yields the linear equations to determine all beam parameters.

Simplified 3-Gradient Method

The function $\hat{x}^2(k)$ is greatly simplified[2] if the quadrupole can be described as a thin lense ($kl \to 0$, $k^2 l \to q = f^{-1}$):

$$\begin{aligned} R_{11}(k) &\to R_{11}(q) = C - q \cdot S \\ R_{12}(k) &\to R_{12}(q) = S \end{aligned}$$

This leads to

$$\begin{aligned} \hat{x}^2(q) &= (C - q \cdot S)^2 \cdot \epsilon\beta_0 - 2S(C - qS) \cdot \epsilon\alpha_0 + S^2 \cdot \epsilon\gamma_0 \\ &= S^2\epsilon^2/\hat{x}_0^2 + \hat{x}_0^2 \cdot (q - q_{min})^2 \ . \end{aligned}$$

with $q_{min} = C/S - \alpha_0/\beta_0$ being the setting for minimal $\hat{x}^2$, and $\hat{x}_0^2 = \epsilon\beta_0$ the squared envelope before the quadrupole. The emittance is then given by

$$\epsilon = \frac{\hat{x}(q_{min})\ \sqrt{\hat{x}^2(q) - \hat{x}^2(q_{min})}}{S^2|q - q_{min}|} \ .$$

The other beam parameters (at the entrance of the quadrupole) are

$$\begin{aligned} \hat{x}_0 &= \sqrt{\hat{x}^2(q) - \hat{x}^2(q_{min})}/(S|q - q_{min}|) \\ \beta_0 &= \hat{x}_0^2/\epsilon \\ \alpha_0 &= \beta_0\ (C/S - q_{min}) \ . \end{aligned}$$

Some comments may be in order:

1. The formula for the emittance is simple enough to be coded in POCAL.

2. The emittance measurement after linac tank I forsees the variation of quad No 52, while No 53 is turned off[3]. The above formalism allows also measurements with No 53 powered (fixed $C' \neq 0$) and also to account for the defocussing by (linear) space charge and by RF acceleration.

3. Suitable quadrupole settings and the quality of the approximations can be investigated by beam transport calculations.

[2] G. Jacobs, private communication
[3] S.H. Wang, PLIN - Note 88-01 (June 24, 1988)

An other method: (P.J. Bryant, 5th CAS, Finnland)

By definition, Eq. (4),

$$\varepsilon = \pi \frac{\sigma_0^2}{\beta_0} = \pi \frac{\sigma_1^2}{\beta_1} = \pi \frac{\sigma_2^2}{\beta_2} \tag{5}$$

where β_0, β_1 and β_2 are the β-values corresponding to the beam and are therefore uncertain. Although we may not know β and α, we do know the transfer matrices and how β and α propagate through the structure (see lectures by K. Steffen in these proceedings).

$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_1 = \begin{pmatrix} C^2 & -2CS & S^2 \\ -CC' & CS' + SC' & -SS' \\ C'^2 & -2C'S' & S'^2 \end{pmatrix} \begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_0 \tag{6}$$

where $\gamma = (1 + \alpha^2)/\beta$. Thus, from Eq. (6)

$$\beta_1 = C_1^2 \beta_0 - 2C_1 S_1 \alpha_0 + \frac{S_1^2}{\beta_0}\left(1 + \alpha_0^2\right) \tag{7}$$

$$\beta_2 = C_2^2 \beta_0 - 2C_2 S_2 \alpha_0 + \frac{S_2^2}{\beta_0}\left(1 + \alpha_0^2\right) \tag{8}$$

and from Eq. (5),

$$\beta_0 = \pi \frac{\sigma_0^2}{\varepsilon} \tag{9}$$

$$\beta_1 = \left(\frac{\sigma_1}{\sigma_0}\right)^2 \beta_0 \tag{10}$$

$$\beta_2 = \left(\frac{\sigma_2}{\sigma_0}\right)^2 \beta_0 \tag{11}$$

From Eqs. (7) and (8), we can find α_0 and using Eqs. (10) and (11), we can express α_0 as,

$$\alpha_0 = \frac{1}{2}\beta_0 \Gamma \tag{12}$$

where

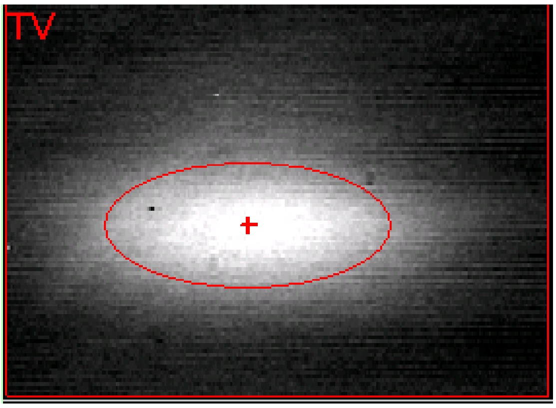
$$\Gamma = \frac{\left(\sigma_2 / \sigma_0\right)^2 / S_2^2 - \left(\sigma_1 / \sigma_0\right)^2 / S_1^2 - \left(C_2 / S_2\right)^2 + \left(C_1 / S_1\right)^2}{\left(C_1 / S_1\right) - \left(C_2 / S_2\right)} \; . \tag{13}$$

Since Γ is fully determined, direct substitution back into Eq. (7) or Eq. (8), using Eq. (10) or Eq. (11) to re-express β_1 or β_2, yields β_0 which via Eq. (9) gives the emittance,

$$\beta_0 = 1 \Big/ \left| \sqrt{\left(\sigma_{\substack{1\\2}} / \sigma_0\right)^2 / S_{\substack{1\\2}}^2 - \left(C_{\substack{1\\2}} / S_{\substack{1\\2}}\right)^2 + \left(C_{\substack{1\\2}} / S_{\substack{1\\2}}\right)\Gamma - \Gamma^2 / 4} \right| \tag{14A}$$

$$\varepsilon = \left(\pi \sigma_0^2\right) \left| \sqrt{\left(\sigma_{\substack{1\\2}} / \sigma_0\right)^2 / S_{\substack{1\\2}}^2 - \left(C_{\substack{1\\2}} / S_{\substack{1\\2}}\right)^2 + \left(C_{\substack{1\\2}} / S_{\substack{1\\2}}\right)\Gamma - \Gamma^2 / 4} \right| \; . \tag{14B}$$

Solutions: Synchrotron Light Profile Monitor

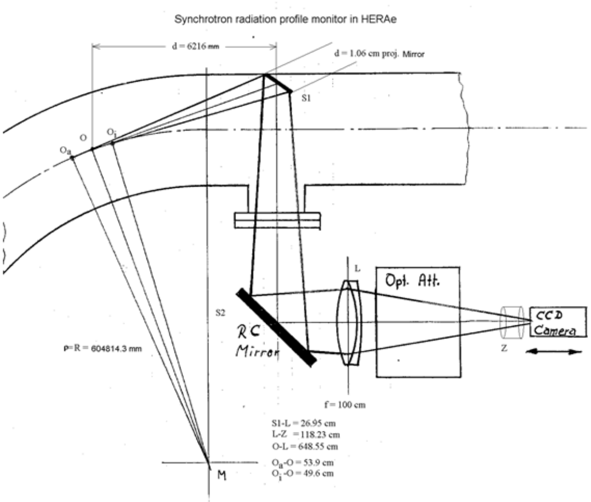


by. K. Wittenburg, G. Kube

DESY

Solutions: Synchrotron light profile monitor

An Example: HERAe



R = ρ = 604814.3 mm
G = O-L = 6485.5 mm
B = L-Z = 1182.3 mm
O-S1 = 6216 mm
L = O_a-O_i = 1035 mm
opening angle (horizontal): $\tan\theta/2 = d/2/6216 \Rightarrow \theta/2 = \arctan d/2/6216$ = <u>0.85 mrad</u>
opening angle (vertikal): $\Psi(\lambda) = 1/\gamma\, (\lambda/\lambda_c)^{1/3}$
with
$\gamma = E/m_0c^2 = E\,[\text{MeV}]/0.511$ = 23483 at 12 GeV and 52838 at 27 GeV
and

$$\lambda_c = {}^{4\pi\rho}\!\big/_{3\gamma^3} = 0.017\,nm < \lambda_c < 0.19\,nm$$

<u>Exercise SR1: Which problems with the setup can be expected?:</u>
Heating of mirror => total emitted Power per electron:

$$P = \frac{e^2 c\gamma^4}{6\pi\varepsilon_0\rho^2}$$

total Power of 46 mA circulating electrons at 27 GeV (Number of electrons $N_e = 6 \cdot 10^{12}$)
$P_{tot} = 6 \cdot 10^6$ W
The mirror will get $P_{tot} * \Theta / (2\pi)$ = 1600 W (Integral over all wavelength!!!)
=> mirror is moveable, mirror has to be cooled
=> Material with low Z is nearly not visible for short wavelength => Beryllium

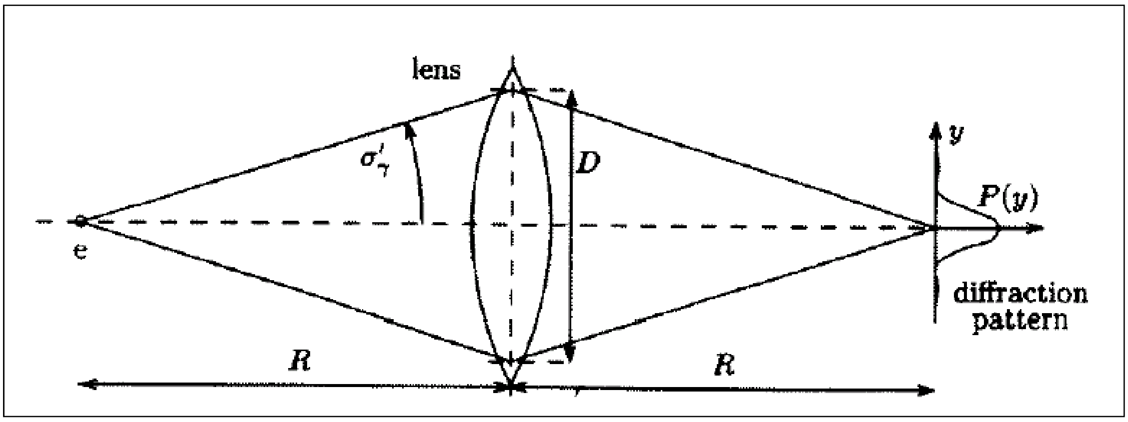
<u>Exercise SR2: What limits the spatial resolution?</u>
Diffraction, depth of field, arc, camera are physical reasons
Mirror and lenses, vibrations and alignment has to be made very precisely, => technical solutions

How to calibrate the optics?

Grid (yardstick) at point of emission, orbit bumps, …

Diffraction:

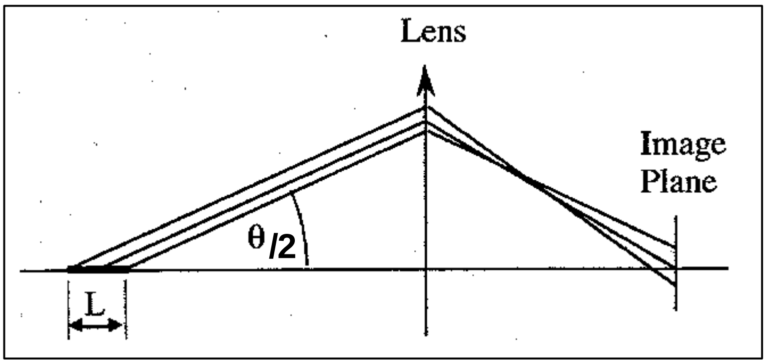
EQ 1:
Diffraction limit (for Object):
For normal slit:
$\sigma_{Diff} = 0.47 * \lambda/\theta/2$ (horizontal, mirror defines opening angle θ)
$\sigma_{Diff} \approx 0.47 * \lambda/\Psi$ (vertikal)



Depth of field:

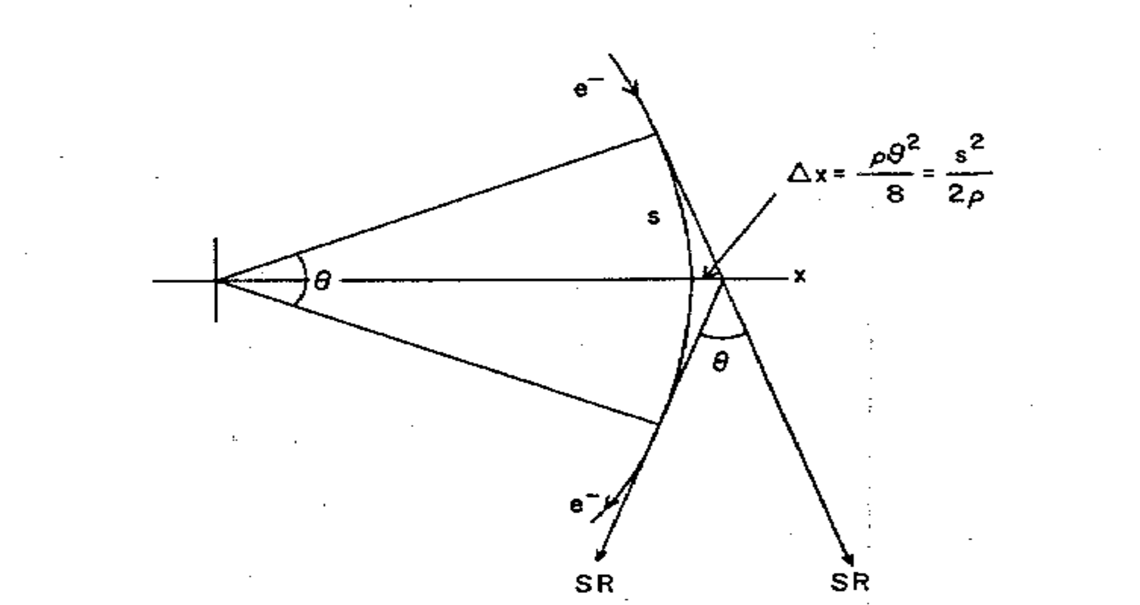
EQ 2:
depth of field:
Vertical: $\Delta_{depth} \approx L/2 * \Psi = \sigma_{depth}$
Horizontal: $\Delta_{depth} \approx L/2 * \theta/2 = \sigma_{depth}$ (mirror defines opening angle θ)

$L \approx \rho \tan\theta$ or $2\rho (\theta/2 + \Psi)$



Arc:

EQ 3:
Arc (horizontal):
Observation of the beam in the horizontal plane is complicated by the fact that the light is emitted by all points along the arc. The horizontal width of the apparent source is related to the observation angle as:
$\Delta x_{arc} = \rho\, \theta^2/8 = \sigma_{arc}$ (mirror defines opening angle θ)



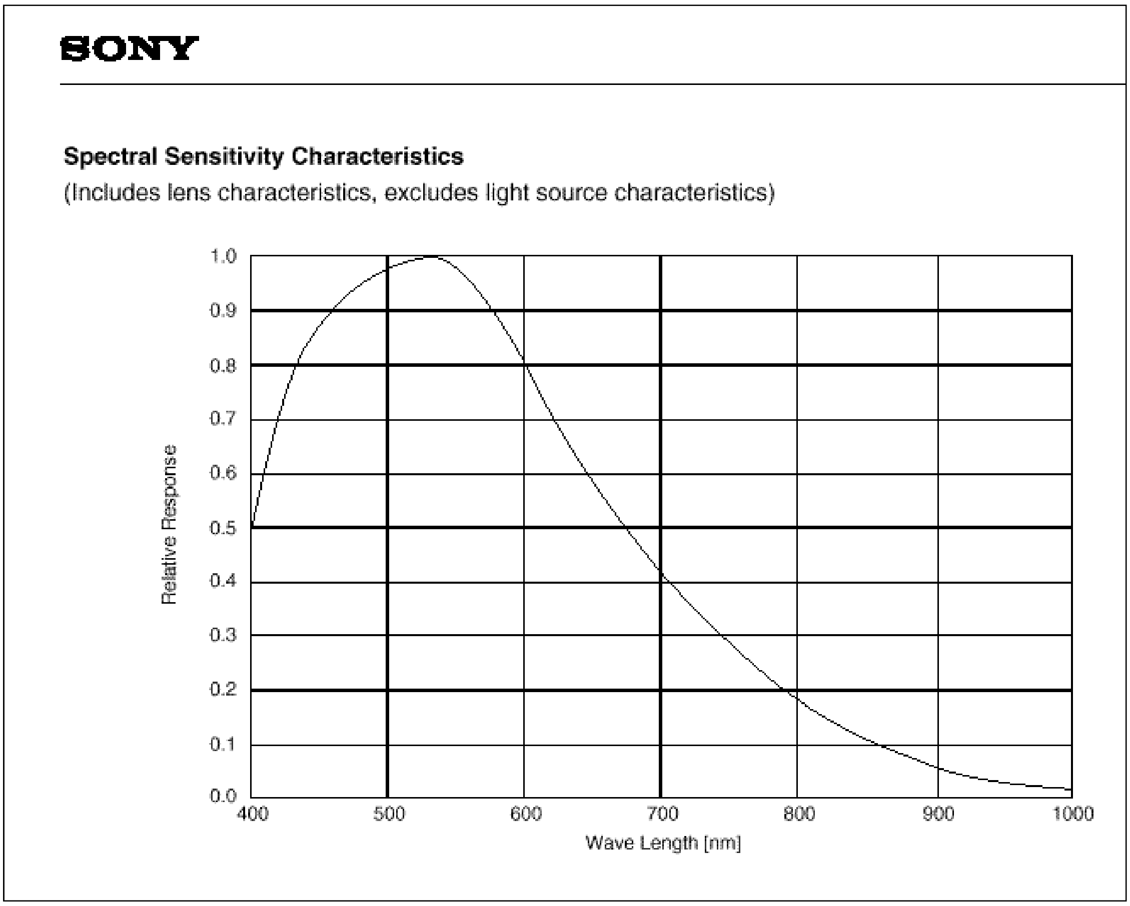
Camera:

EQ 4:
Camera:
image gain = G/B = 5.485
typical resolution of camera CCD chip: $\sigma_{chip} = 6.7\ \mu m$
$\sigma_{camera} = \sigma_{chip} * G/B = 37\ \mu m$

λ not monochromatic !

$\sigma_{Diff} = 0.47 * \lambda/\theta$ (horizontal)	= ???
$\sigma_{Diff} = 0.47 * \lambda/\Psi$ (vertikal)	= ???
$\sigma_{depth} = L/2 * \theta/2$	= 440 µm
$\sigma_{arc} = \rho\, \theta^2/8$	= 219 µm (horizontal)
$\sigma_{camera} = \sigma_{chip} * G/B$	= 37 µm

typical spectral sensitivities from CCD Sensors:



Assume: λ = 550 nm;

($\gamma = E/m_0c^2$)

$\gamma_{12} = 2.35 * 10^4$ (E = 12 GeV)

$\gamma_{35} = 6.85 * 10^4$ (E = 35 GeV)

$\lambda_{c,12} = (4\pi\rho)/(3\gamma^3) = 0.195$ nm at 12 GeV

$\lambda_{c,35} = (4\pi\rho)/(3\gamma^3) = 0.008$ nm at 35 GeV

opening angle (horizontal): $\tan\theta/2 = d/2/6216 \Rightarrow \theta/2 = \arctan d/2/6216$ = 0.85 mrad

opening angle (vertikal): $\Psi(\lambda) = 1/\gamma\ (\lambda/\lambda_c)^{1/3} = [(3\lambda)/(4\pi\rho)]^{1/3}$ = 0.6 mrad (indep. on γ for $\lambda >> \lambda_c$)

$\sigma_{diff} = 0.47 * \lambda/\theta/2$ = 304 µm (horizontal)

$\sigma_{diff} = 0.47 * \lambda/\Psi$ = 431 µm (vertical, mirror has to be larger than spot on mirror)

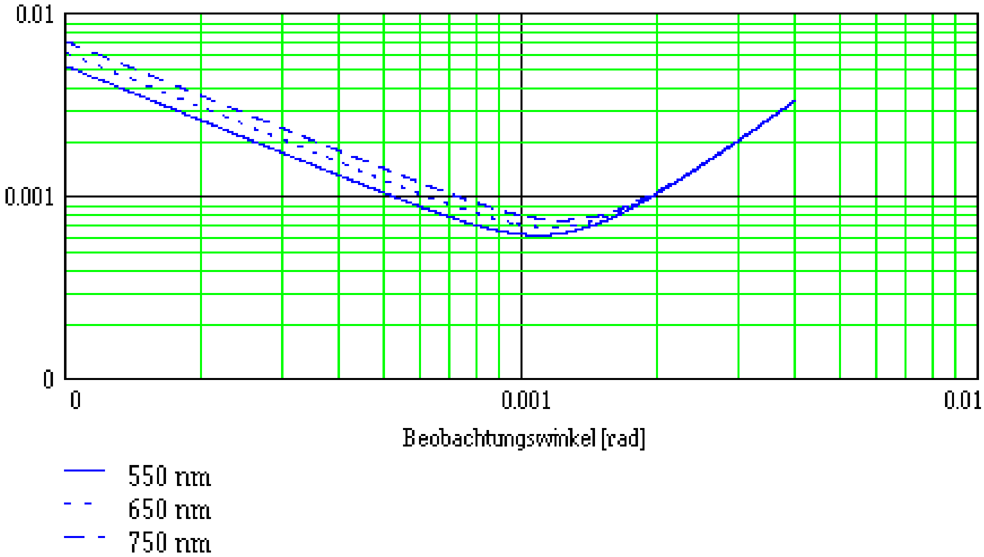
$\sigma_{depth} = L/2 * \theta/2$ = 440 µm

$\sigma_{arc} = \rho\ \theta^2/8$ = 219 µm (horizontal)

$\sigma_{camera} = \sigma_{chip} * G/B$ = 37 µm

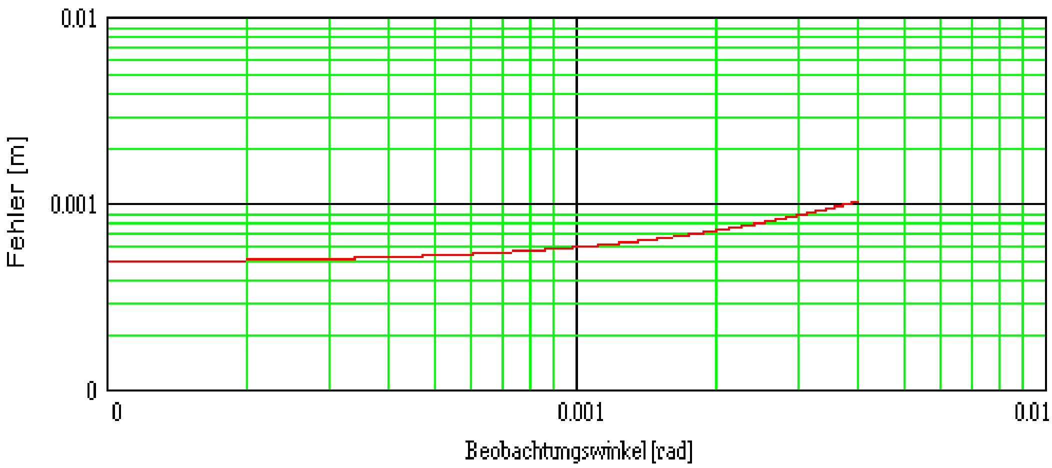
$\sigma_{cor} = (\sigma_{diff}^2 + \sigma_{depth}^2 + \sigma_{arc}^2 + \sigma_{camera}^2)^{1/2}$ = 579 µm ; (horizontal)
$\sigma_{cor} = (\sigma_{diff}^2 + \sigma_{depth}^2 + \sigma_{camera}^2)^{1/2}$ = 617 µm ; (vertical)

Horizontal:
resolution = $[(\rho\, \theta^2/8)^2 + (L/2 * \theta/2)^2 + (0.47 * \lambda/\theta/2)^2]^{1/2}$ with $L \approx \rho \tan\theta \approx \rho\, \theta$



Minimum at θ =1.05 rad

Vertical:
Resolution = $[(L/2 * \Psi)^2 + (0.47 * \lambda/\Psi)^2]^{1/2}$ with $L \approx \rho \tan\theta \approx \rho\, \theta$



Not the whole truth:

1) Diffraction:

a) Ψ_{exact} is larger than the Gauss approximation (e.g. 0.79 → 1.08 mrad at Tristan)

b) For a gaussian beam the diffraction width is $\sigma_{diff} \approx 1/\pi * \lambda/\Psi$

(Ref: **ON OPTICAL RESOLUTION OF BEAM SIZE MEASUREMENTS BY MEANS OF SYNCHROTRON RADIATION**. By A. Ogata (KEK, Tsukuba). 1991. Published in Nucl.Instrum.Meth.A301:596-598,1991)

=> $\sigma_{diff} \approx 1/\pi * \lambda/\Psi_{exact}$ = <u>218 µm</u> (Ψ_{exact} = 0.8 mrad, λ = 550 nm) vertical

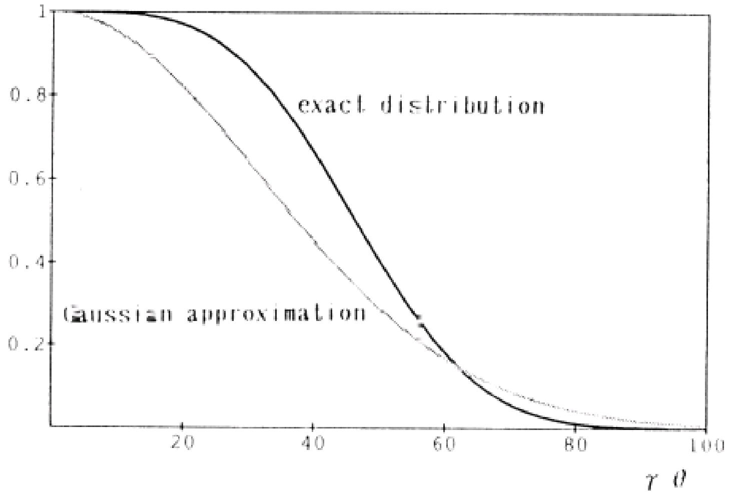
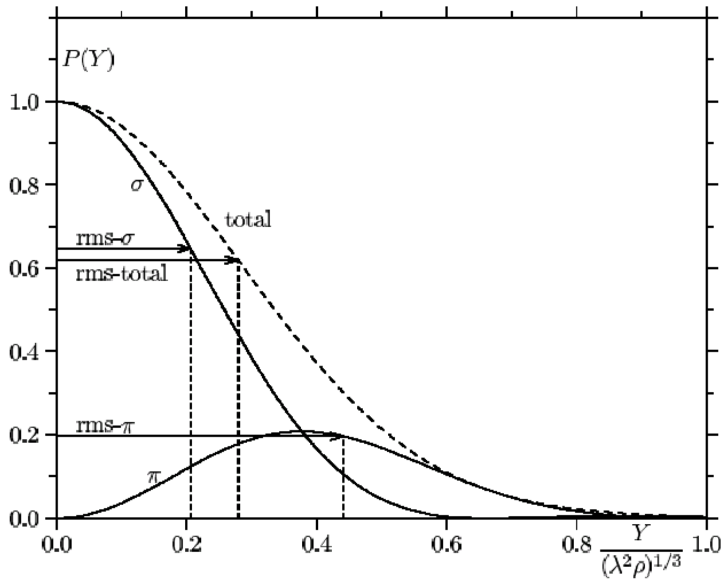


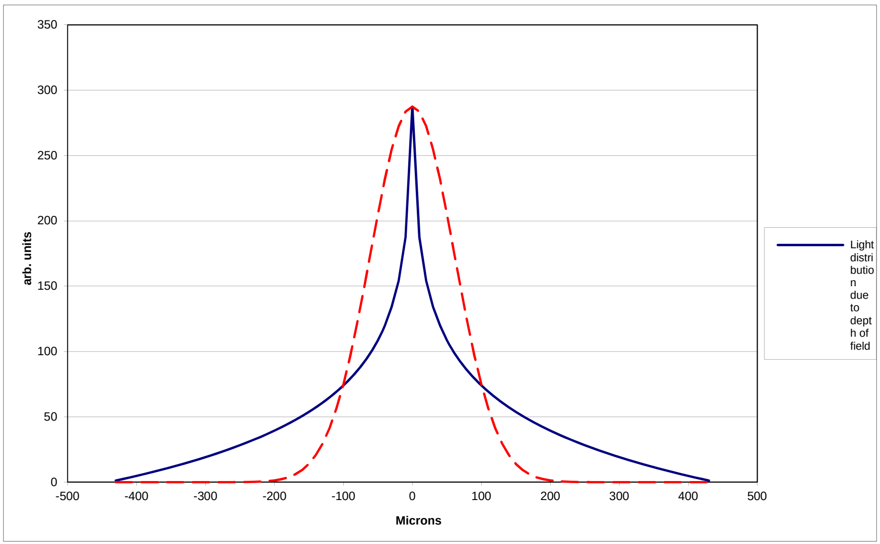
Fig. 1 Angular distribution of the 500 nm component of the synchrotron radiation from the TRISTAN MR bending magnet (246 m bending radius) operated at 30 GeV, and its Gaussian approx mation.



8: Fraunhofer diffraction for synchrotron radiation from long magnets

2) Depth of field:

The formula $R_{depth} = L/2 * \theta/2$ describes the radius of the distribution due to the depth of field effect. It is not gaussian and has long tails. The resolution of an image is probably much better than the formula above. A gaussian approximation with the same integral is shown in the figure below resulting in a width of $\sigma_{depth} = 61$ µm.



σ_{diff}	$= 0.47 * \lambda/\theta/2$	= 304 µm (horizontal)	
σ_{diff}	$= 1/\pi * \lambda/\Psi$	= 218 µm (vertical)	(431 µm)
σ_{depth}	$= L/2 * \theta/2$	= 61 µm	(440 µm)
σ_{arc}	$= \rho\, \theta^2/8$	= 219 µm (horizontal)	
σ_{camera}	$= \sigma_{chip} * G/B$	= 37 µm	

$\sigma_{cor} = (\sigma_{diff}^2 + \sigma_{depth}^2 + \sigma_{arc}^2 + \sigma_{camera}^2)^{1/2}$ = 381 µm ; (horizontal)

$\sigma_{cor} = (\sigma_{diff}^2 + \sigma_{depth}^2 + \sigma_{camera}^2)^{1/2}$ = 229 µm ; (vertical)

Beam width $\sigma_{beam} = (\sigma_{fit\_measured}^2 - \sigma_{cor}^2)^{1/2}$

Exercise SR3: Discuss possible improvements:

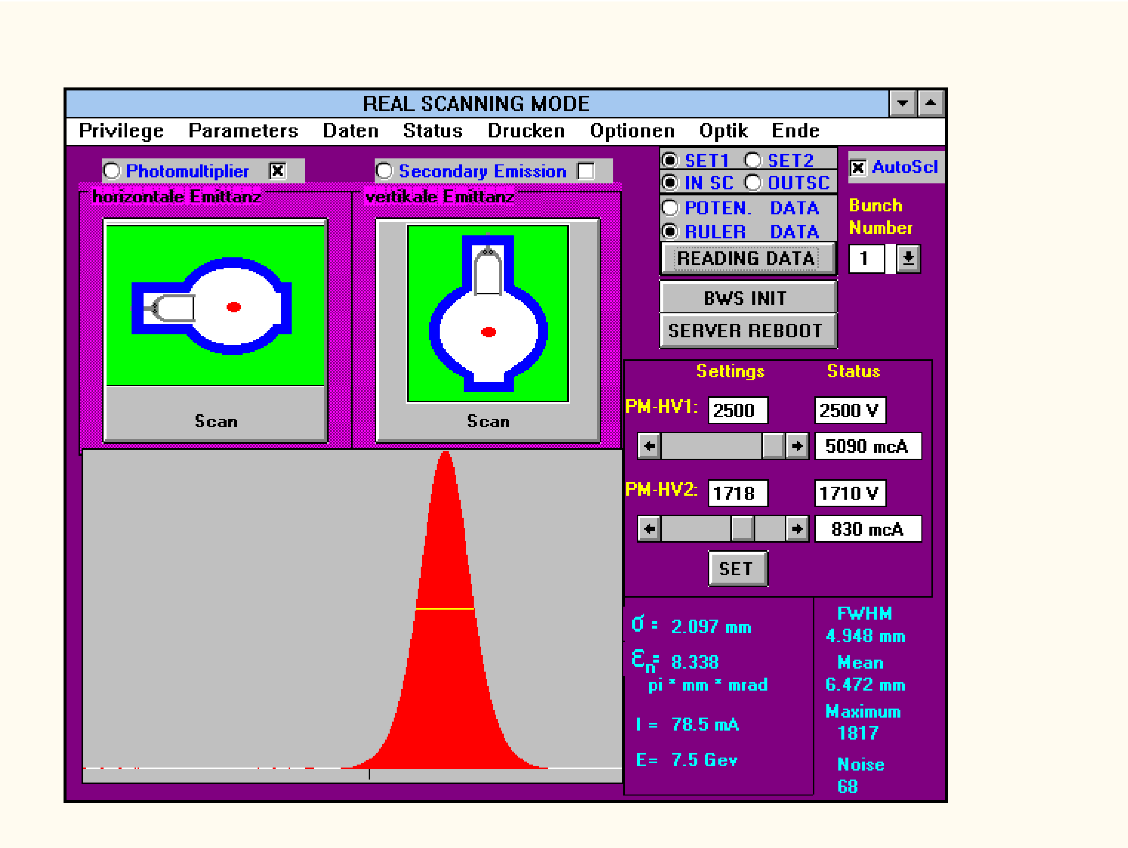
Monochromator at shorter wavelength
use optimum readout angle
Polarization - filter
use x-ray ($\lambda < 0.1$ nm)

More:

Interferometer

The principle of measurement of the profile of an object by means of spatial coherency was first proposed by H.Fizeau and is now known as the Van Cittert-Zernike theorem. It is well known that A.A. Michelson measured the angular dimension (extent) of a star with this method.

Solutions: Wire Scanners



by Kay Wittenburg, -DESY-

Solutions: Wire Scanner

Exercise WIRE: Discuss where one should locate the Scintillator in case of a proton and an electron accelerator?

Projected angular distribution could be approximated by Gaussian with a width given by:

$$\Theta_{mean} = \frac{0.014\,GeV}{pc} \cdot \sqrt{\frac{d'}{L_{rad}}} \cdot \left(1 + 1/9 \cdot \log_{10} \frac{d'}{L_{rad}}\right)$$

d' = 1.5×10^{-3} cm – the thickness of the target, X_0=12.3 cm – quartz-wire radiation length, $x/X_0 = 1.22\times10^{-4}$
Note that the angle depends on the momentum of the particle!
It is corresponding to:

$\Theta_{mean} \approx 3.0\times10^{-6}$ rad

for electron momentum of 30GeV/c.

Scattered particles will arrive a vacuum chamber of radius R = 2 cm at:

$$z \approx \frac{R}{\Theta_{mean}\sqrt{2}} \approx 4.9km!!!!$$

What to do?

Do Monte Carlo simulations of best location for scintillators. Simulation should include all magnetic fields as well as all elastic and inelastic scattering cross sections. For protons the inelastic cross section is very high, therefore one can locate the detector (scintillator) close to the scanner, while for electrons one has to calculate (simulate) the best location.

Known limitations of wire scanners are:

1. The smallest measurable beam size is limited by the finite wire diameter of a few microns.
2. Higher Order Modes may couple to conductive wires and can destroy them.
3. High beam intensities combined with small beam sizes will destroy the wire due to the high heat load.
4. Emittance blow up.

Higher Order Modes

<u>Exercise WIRE1: Discuss methods of proving this behavior. What are possible solutions against the RF coupling?</u>

Methods:

a) Measurement of wire resistivity
b) Measurement of thermo-ionic emission
c) Optical observation of glowing wire
d) Measurement of RF coupling in Laboratory with spectrum analyzer

a) <u>Measurement of wire resistivity</u>

The wire resistivity will change depending on the temperature of the wire, even without scanning.

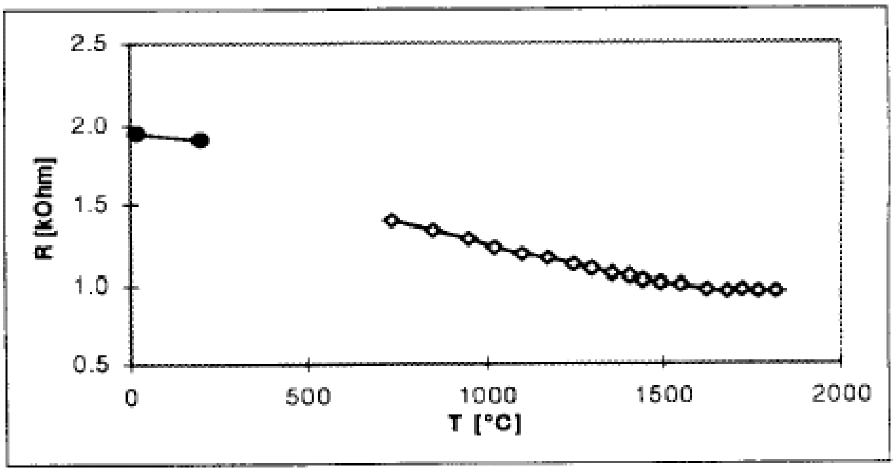


Fig 4: Measured wire resistance variations with temperature.

Here: 8 μm Carbon wire

(from <u>OBSERVATION OF THERMAL EFFECTS ON THE LEP WIRE SCANNERS</u>. By J. Camas, C. Fischer, J.J. Gras, R. Jung, J. Koopman (CERN). CERN-SL-95-20-BI, May 1995. 4pp. Presented at the 16th Particle Accelerator Conference - PAC 95, Dallas, TX, USA, 1 - 5 May 1995. Published in IEEE PAC 1995:2649-2651)

b) Measurement of thermoionic emission

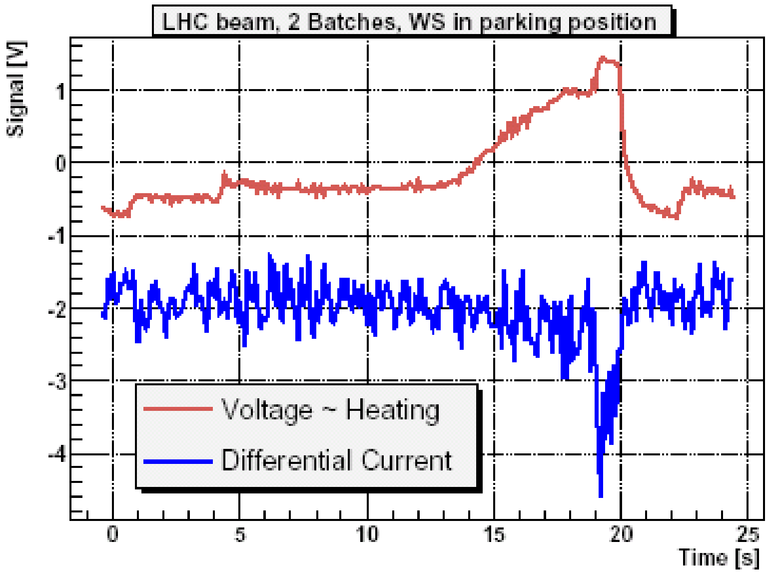


Figure WIRE5: Wire heating due to the LHC beam injection in the SPS (No scan, wire in parking position). The beam energy ramp/bunch length decreasing begin t=11 s.

A constant current was supplied to the wire and the voltage drop across it was fed to a digital scope together with the difference between the input and output currents. The differential current (I_{out}-I_{in}) grow up is due to the wire heating and consequent emission of electrons for thermionic effect. Fig. WIRE5 shows such voltage and differential current evolutions during the SPS cycle with LHC type beam. No scans were performed along this cycle. It is thus evident that the wire heating does not depend on the direct wire-beam interaction only.

(From **CAVITY MODE RELATED WIRE BREAKING OF THE SPS WIRE SCANNERS AND LOSS MEASUREMENTS OF WIRE MATERIALS**
F. Caspers, B. Dehning, E.Jensen, J. Koopman, J.F. Malo, CERN, Geneva, Switzerland
F. Roncarolo, CERN/University of Lausanne, Switzerland; DIPAC03)

c) Optical observation of glowing wire

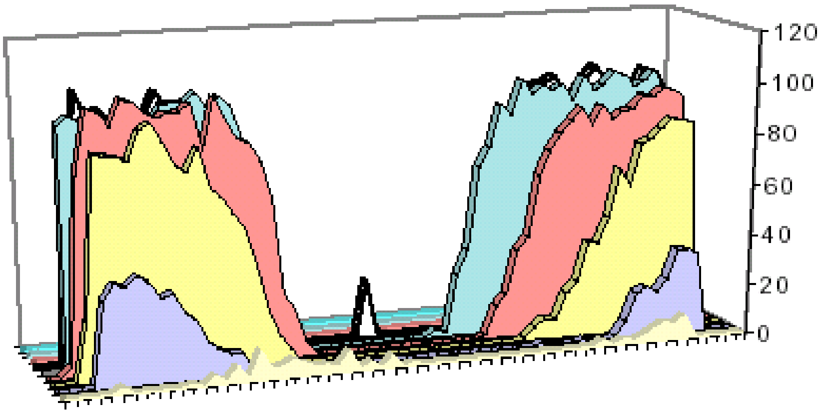


Figure WIRE6: Digitized video recording of an 8 μm carbon wire scanning a 0.8 mA beam. The wire is parallel to the horizontal axis, and the light intensity is plotted along the vertical axis (arbitrary units). Successive profiles are separated by 20 ms. The central spot corresponds to the passage of the wire through the beam. Thus, RF heating led to (huge) thermal glowing before the beam interacts with the wire.
(from: QUARTZ WIRES VERSUS CARBON FIBERS FOR IMPROVED BEAM HANDLING CAPACITY OF THE LEP WIRE SCANNERS.
By C. Fischer, R. Jung, J. Koopman (CERN). CERN-SL-96-09-BI, May 1996. 8pp. Talk given at 7th Beam Instrumentation Workshop (BIW 96), Argonne, IL, 6-9 May 1996.

d) Measurement of RF coupling with spectrum analyzer

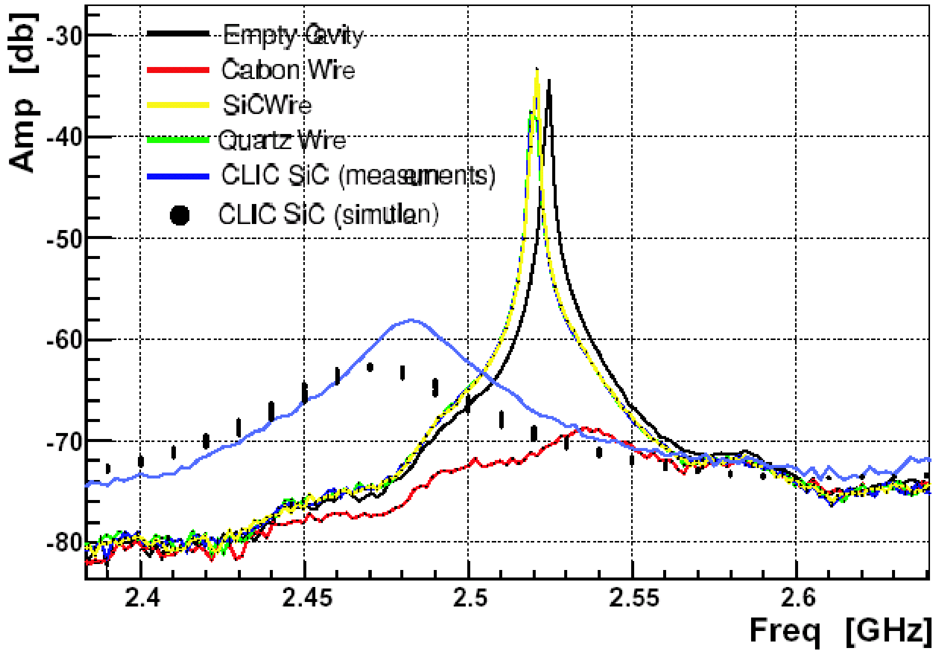


Figure WIRE7: Resonant cavity signal in presence of Carbon (36 μm), Silicon Carbide and Quartz wires

The plot qualitatively proves the RF power absorption of Carbon, and the non-absorption of Silicon Carbide and Quartz. Absorbed energy is mainly converted into heat.

Solutions:

- Damping of Higher Order Modes with Ferrites etc.
- Non conducting wires

Wire heat load

Estimation of the wire temperature after one scan with speed v (assume no cooling mechanisms):

$$T = C \cdot dE/dx_m \cdot d' \cdot N \cdot \frac{1}{c_p \cdot G} \quad [^0C]$$

Solving unknown N and G:

Solving G:
G [g] is the mass of the part of the wire interacting with the beam. The mass G is defined by the beam dimension in the direction of the wire (perpendicular to the measuring direction) and by the wire diameter d':

$$G = wire\ volume \cdot \rho = 2 \cdot \sigma_v \cdot d'^2 \cdot \rho \quad [g].$$

Solving N:
The number of particles N hitting the wire during one scan depends on the speed of the scan (~1/v), the revolution frequency (~f_{rev}), the wire diameter (~d') and the beam current (~NB · n_{bunch}):

$$N = \frac{d' \cdot f_{rev}}{v} \cdot (NB \cdot n_{bunch}).$$

Fig. WIRE8 shows the a graphical representation of the parameters. The quotient d·f/v is the ratio of the scanned beam area or, in other words, like a grid seen by one bunch, assuming that all bunches are equal. However, the ratio can exceed the value 1 (a foil) if the scanning distance between two bunches is smaller than the wire diameter. Note that N does not depend on the beam widths σ.

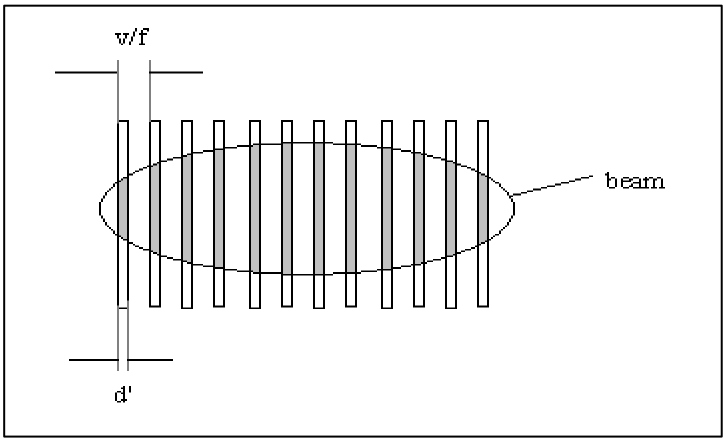


Figure WIRE8: Geometrical meaning of the parameters v/f and d'

Therefore, the temperature increase of the wire after one scan becomes:

$$T_h = C \cdot dE/dx_m \cdot d' \cdot \frac{d' \cdot f_{rev}}{v} \cdot (NB \cdot n_{bunch}) \cdot \frac{1}{c_p \cdot 2 \cdot \sigma_v \cdot d'^2 \cdot \rho} \cdot \alpha \quad [^0C]$$

with dE/dx_m [MeV/cm].

$$with \quad \frac{dE/dx_m}{\rho} = dE/dx \left[\frac{MeV \cdot cm^2}{g}\right] \quad and \quad f_{rev} \cdot NB = f_{bunch}$$

T becomes:

$$T_h = C \cdot dE/dx \cdot n_{bunch} \cdot \frac{f_{bunch}}{v} \cdot \frac{1}{c_p \cdot 2 \cdot \sigma_v} \cdot \alpha \quad [^0C]$$

Where $_{h,}$ denotes the horizontal (h) scanning direction. The cooling factor 'α' is described in the next section. Note that the temperature does not depend on the wire diameter and that it depends on the beam dimension perpendicular to the measuring direction. The temperature increase is inverse proportional to the scanning speed, therefore a faster scanner has a correspondingly smaller temperature increase.

Exercise WIRE2: Which kind of wire Material you will prefer for a wire scanner in this accelerator?

The wire parameters dE/dx / c_p and the Quotient T_h/T_m should be minimal for a choice of the material ($\alpha = 1$):

Material	dE/dx / c_p	T_h [^{0}C]	T_h/T_m
AL	7.7	$1.1 \cdot 10^4$	16.9
W	50.6	$7.1 \cdot 10^4$	20.9
C	5.4	$0.77 \cdot 10^4$	2.2
Be	4.1	$0.58 \cdot 10^4$	4.8
SiO2	12.9	$1.8 \cdot 10^4$	10.6

Table WIRE3: Temperature

From Table WIRE3 follows, that even the best material (Carbon) will be a Factor 2.2 above its melting temperature.

Exercise WIRE2a: Discuss cooling mechanisms which will cool the wire.

1) Secondary particles emitted from the wire
2) Heat transport along the wire
3) Black body radiation
4) Change of c_p with temperature

1) Secondaries: Some energy is lost from the wire by secondary particles. In the work in (J. Bosser et al.; The micron wire scanner at the SPS, CERN SPS/86-26 (MS) (1986)) about 70% is assumed. In DESY III (example above) no carbon wire was broken during more than 10 years of operation. At HERA, the theoretical temperature of the carbon wire (without secondaries) exceeds the melting temperature after a scan by far (T = 12 800 ^{0}C). Considering the loss by secondaries of 70%, the temperature reaches nearly the melting point. In practice, the wire breaks about once in 6 months. The observation is that the wire becomes thinner at the beam center. This may indicate, that during a scan some material of the wire is emitted because of nuclear interactions or is vaporized because it is very close to the melting temperature. This supports the estimate of the 70% loss and one has to multiply the factor $\alpha = 0.3$ in the equation above.

2) Heat transport: The transport of heat along the wire does not contribute to short time cooling of the wire (P. Lefevre; Measure tres peu destructive des distribution transversales dans le PS de 800 MeV a 26 GeV, CERN PS/DL/Note 78-8). However,

frequent use of the scanner heats up also the ends of the wire and its connection to the wire holders (fork).

3) Black body radiation: The temperature T_{bb} at which the radiated power is equal to the deposited power in the wire during one scan P_{dep} [MeV/s] can be calculated from the Stefan-Bolzmann-law:

$$T_{bb} = \sqrt{\sqrt{\frac{P_{dep}}{s \cdot A}}}$$

where s = 35.4 MeV / (s^1 cm^2 $^0K^4$) is the Stefan-Bolzmann-constant and A is the area of radiating surface. The surface of the heated wire portion A is 2 · σ_v · d · π [cm^2]. The power can be calculated by:

$$P_{dep\ h,v} = \alpha \cdot dE/dx \cdot d' \cdot n_{bunch} \cdot \frac{f_{bunch} \cdot d'}{v} \cdot \frac{1}{t_{scan}} \quad [MeV/s]$$

where t_{scan} = 2 · $\sigma_{h,v}$ / v is the time for a scan (in the assumpion of 2 σ it is neglected that only about 70% of the power is concentrated within 2 σ). α is the expected loss from secondaries.
For the example above T_{bb} = 3900 ^{0}C. Therefore the black body radiation is a fraction of cooling for fast scans.

4) c_p(T): The heat capacitance is a function of the temperature. Fig. 2 shows the increase of c_p for Carbon with T. The expected temperature after a scan is inversely proportional to c_p. Therefore one can expect a slightly smaller resulting temperature because of this dependence.

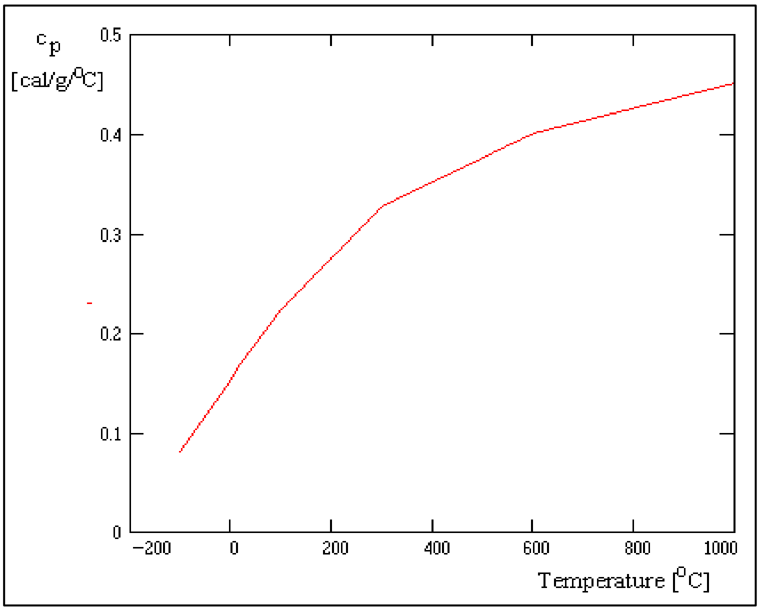


Figure WIRE9: The heat capacitance versus the temperature of Carbon.

Emittance blow up

Exercise WIRE3: Calculate the emittance blowup of the proton beam after one scan at a position with β = 11.8 m for p =0.3 and 7 GeV/c (Carbon wire):
Assume a measurement position close to a Quadrupole (α=0)

For small deflection angles a good approximation for average root mean square scattering angle is given by:

$$\delta\Theta = \frac{0.014 GeV}{pc} \cdot \sqrt{\frac{d'}{L_{rad}}} \cdot \left(1 + 1/9 \cdot \log_{10} \frac{d'}{L_{rad}}\right)$$

Remember:

$$\gamma(s)y^2 + 2\alpha(s)yy' + \beta(s)y'^2 = \varepsilon$$

A fraction Ψ of the circulating beam particles will hit the wire:

$\Psi = \frac{d' \cdot f_{rev}}{v}$ (see exercise WIRE2) ,

The resulting mean deflection angle is than:

$$\Theta = \delta\Theta \cdot \Psi$$

and the emittance blowup: from: $\gamma(s)y^2 + 2\alpha(s)yy' + \beta(s)y'^2 = \varepsilon$

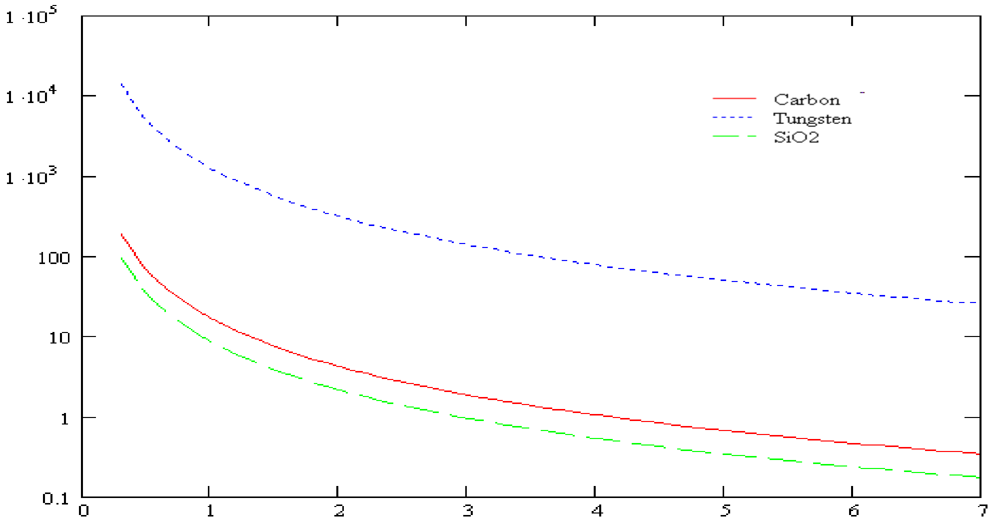
with α = 0 and $y'^2 = \sqrt{2\pi} \cdot \delta\Theta^2 \cdot \Psi^2$ (this angle adds to the angular spread of the beam)

$$\delta\varepsilon_{rms} = \sqrt{2\pi} \cdot \delta\Theta^2 \cdot \Psi^2 \cdot \beta = 5.1 \cdot 10^{-8} \pi rad = 5.1 \cdot 10^{-2} \pi\ mm\, mrad$$

=> ε_{rms} = 15.05 π mm mrad

$\sqrt{2\pi}$ *from* ???? ; π/2 amd 1/2 from D. Möhl's paper, $\delta\varepsilon = \pi \frac{1}{4} \cdot \Theta_{rms}{}^2 \cdot \beta$ from M. Giovannozzi (CAS 2005)

δε/ε [%/scan]



Momentum [GeV/c]