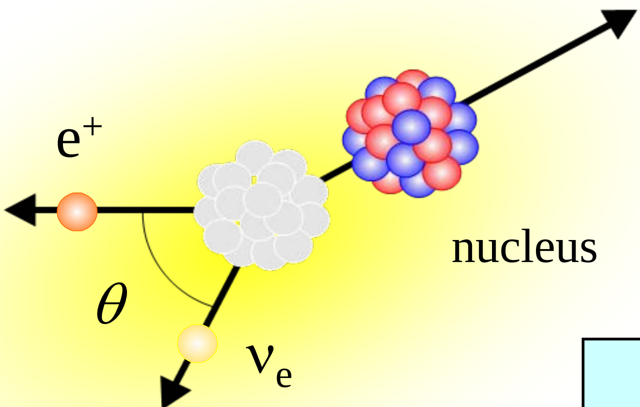
A new determination of the $|V_{ud}|$ quark mixing matrix element



ISOLDE Workshop

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in collaboration with

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M. Tandecki, T. Phalet – K.U. Leuven

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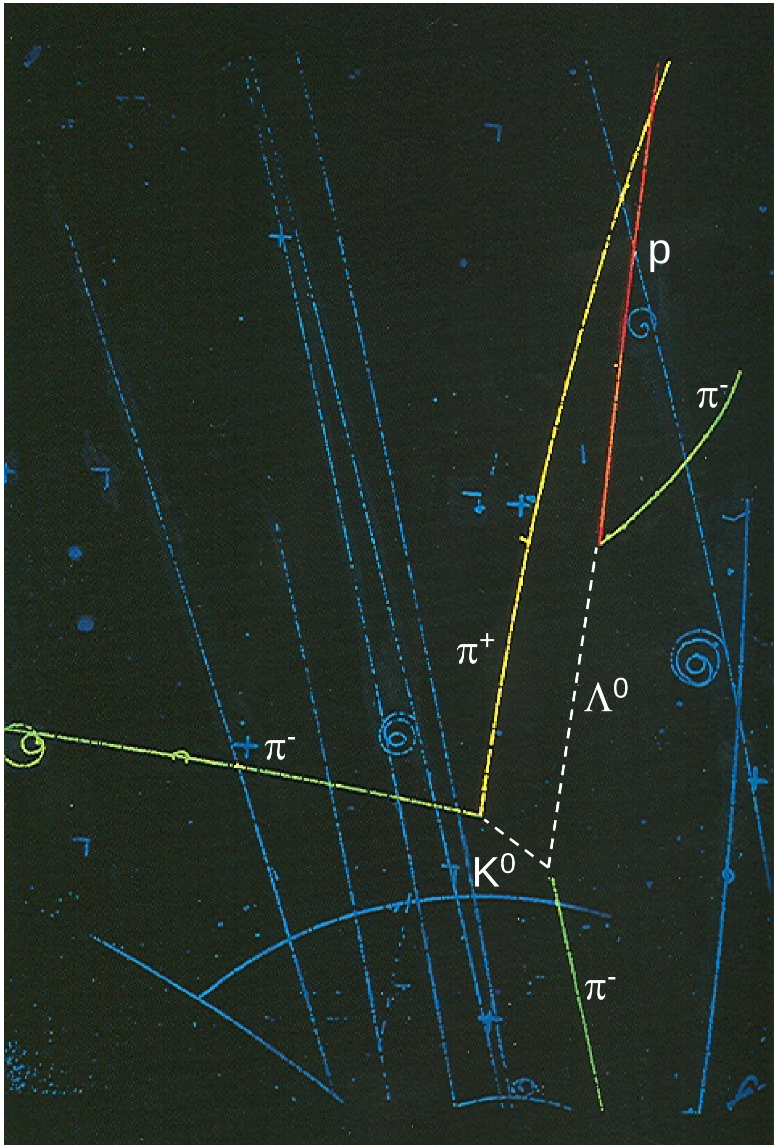
1. Cabibbo angle



“strange” particles

Since late 40’s :

- particles that often create V-type tracks
- easy to produce (always in pairs)
- slow in their decay

→ ????

$$\pi^- + p \rightarrow K^0 + \Lambda^0$$

$$K^0 \rightarrow \pi^- + \pi^+$$

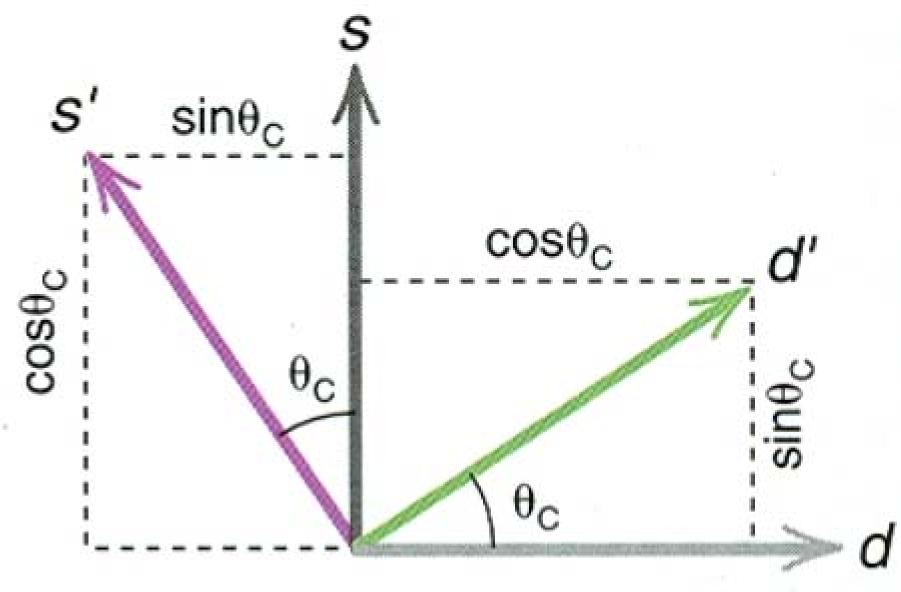
$$\Lambda^0 \rightarrow \pi^- + p$$

1963 **Cabibbo** strong interaction (mass) eigenstates (*d* and *s*)

$\neq$

weak interaction quark eigenstates (*d* ' and *s* ')

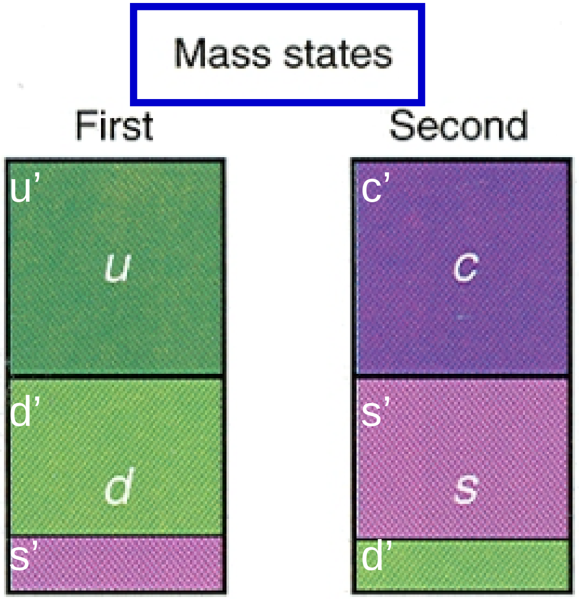
(two sets behave like two alternative sets of unit vectors)

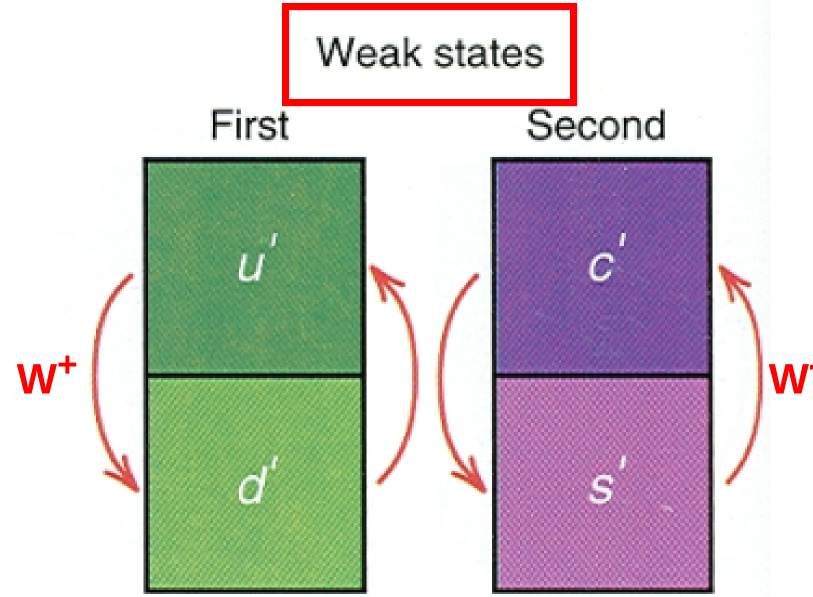


$$\begin{pmatrix} d \\ s \end{pmatrix} = \begin{pmatrix} \cos\theta_c & -\sin\theta_c \\ \sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d' \\ s' \end{pmatrix}$$

mass states	mixing matrix	weak states

$\theta_c \approx 13°$ Cabibbo angle





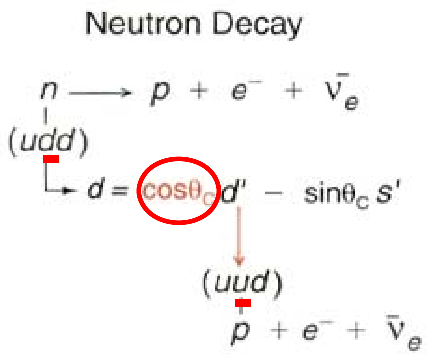
$$\begin{pmatrix} d \\ s \end{pmatrix} = \begin{pmatrix} \cos\theta_c & -\sin\theta_c \\ \sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d' \\ s' \end{pmatrix}$$

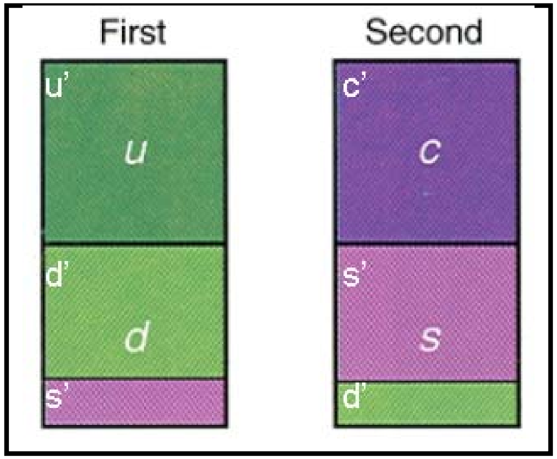
mass states — mixing matrix — weak states

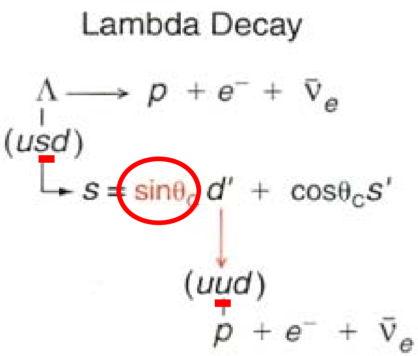
$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

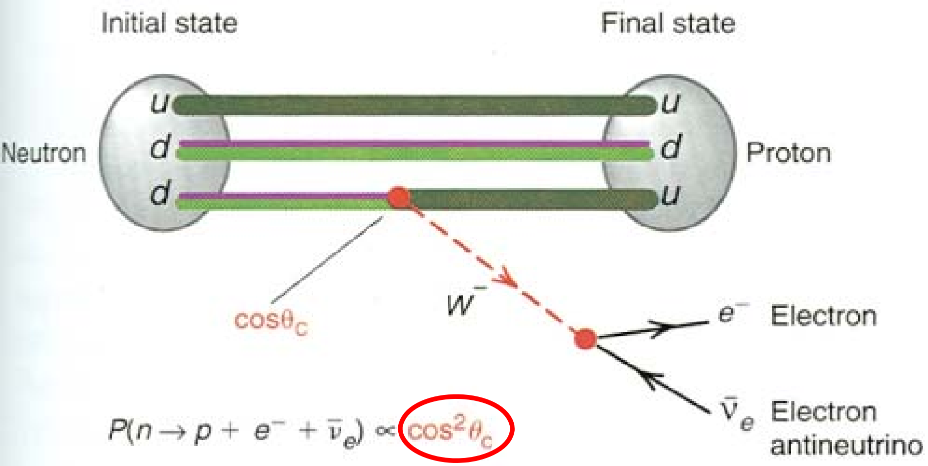
weak states — mixing matrix — mass states

Two-family mixing among quarks

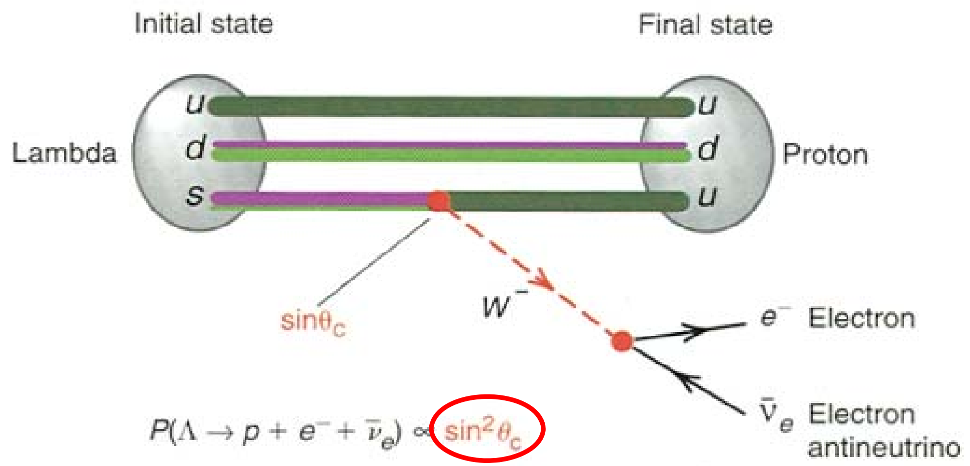








The neutron decay amplitude is proportional to $\cos\theta_c$, the amplitude of the state d' in the d quark. The decay probability is proportional to the square of the amplitude:



The lambda decay amplitude is proportional to $\sin\theta_c$, the amplitude of the state d' in the s quark. The lambda decay probability is proportional to the square of the amplitude.

$$\begin{pmatrix} d \\ s \end{pmatrix} = \begin{pmatrix} \cos\theta_c & -\sin\theta_c \\ \sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d' \\ s' \end{pmatrix}$$

2. Cabibbo-Kobayashi-Maskawa matrix

1973 Kobayashi & Maskawa : **- extension of Cabibbo matrix**

- 3rd generation of quarks, before discovery of *charm* !!

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

where e.g. $\boldsymbol{V_{uj}}$ = amplitude of quark mass eigenstate ***j (= d, s, b)***

into the quark weak eigenstate ***d'***

[i.e. $V_{ud} = \cos\theta_c$, $V_{us} = \sin\theta_c$]

parameterization: - 3 angles θ_{ij} with i, j = 1, 2, 3 (the generations of quarks)

- one phase δ_{13} ($e^{i\delta_{13}}$)accounting for the observed CP-violation

Unitarity test for all rows and columns of the CKM matrix

from β decays ⇑ **from K decays** ⇑

$$\begin{pmatrix} \boxed{0.97418(27)} & \boxed{0.2255(19)} & 0.00393(36) \\ 0.230(11) & 1.04(6) & 0.0412(11) \\ 0.0081(6) & 0.0387(23) & 0.77^{+0.18}_{-0.24} \end{pmatrix} \begin{matrix} \rightarrow 0.9999(10) \\ \rightarrow 1.14(13) \\ \rightarrow 0.59(32) \end{matrix}$$

↓ 1.0020(60) ↓ 1.13 (13) ↓ 0.59(32)

[C. Amsler et al. (Particle Data Group) Physics Letters B 667 (2008) 1]

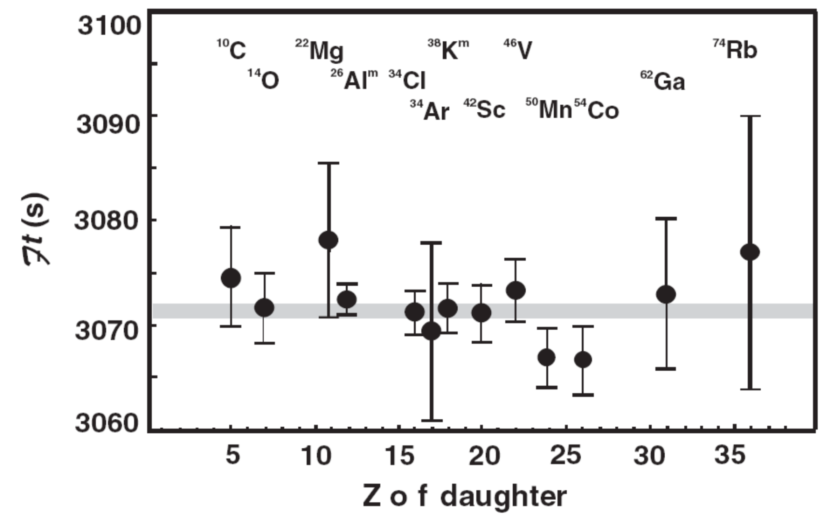
3. The V_{ud} matrix element

1. superallowed $0^+ \rightarrow 0^+$ transitions

$t_{1/2}$, Q_{EC} , BR

$$|V_{ud}| = 0.97418(26)$$

(Towner & Hardy, PR C 77 (2008) 025501)



2. free neutron decay

τ_n , $A_n \rightarrow \lambda = g_A/g_V$

$$|V_{ud}| = 0.9743(19)$$

(e.g. Severijns, Beck & Naviliat, Rev. Mod. Phys. 78 (2006) 991)

$$|V_{ud}|^2 = \frac{4905.5(18)}{\tau_n(1+3\lambda^2)}$$

3. pion decay $\pi^+ \rightarrow \pi^0 e^+ \nu_e$: τ_π , BR

$$|V_{ud}| = 0.9728(30)$$

(D. Pocanic et al., PRL 93 (2004) 181803)

$$\tau_\pi = (2.6033 \pm 0.0005) \times 10^{-8} \text{ s}$$

$$BR = (1.036 \pm 0.004_{stat} \pm 0.005_{syst}) \times 10^{-8}$$

4. Ft-value of $0^+ \rightarrow 0^+$ superallowed beta transitions

$$\mathcal{F}t^{0^+\rightarrow 0^+} \equiv ft^{0^+\rightarrow 0^+}\left(1+\delta^V_{NS}-\delta^V_C\right)\left(1+\delta_R\right) = \frac{K}{2G_F^2\, V_{ud}^2\, C_V^2\left(1+\Delta^V_R\right)}$$

- **radiative** correction $\delta_R{}' = \delta_1 + \delta_2 + \delta_3$ (order α, $Z\alpha^2$, $Z^2\alpha^3$)
  leading order α: exchange of γ or Z-boson between p and e^-
- **nuclear structure** dependent radiative correction δ^V_{NS}
- Coulomb (**isospin**) correction $\delta^V_c = \delta^V_{c1} + \delta^V_{c2}$
  - difference in configuration mixing
  - difference in radial part of wave functions
- nucleus independent **radiative** correction $\Delta^V_R = 0.02361(38)$

(Towner & Hardy, PR C 77 (2008) 025501 - Marciano & Sirlin, PRL 96 (2006) 032002)

5. Ft-value of T = 1/2 mirror beta transitions

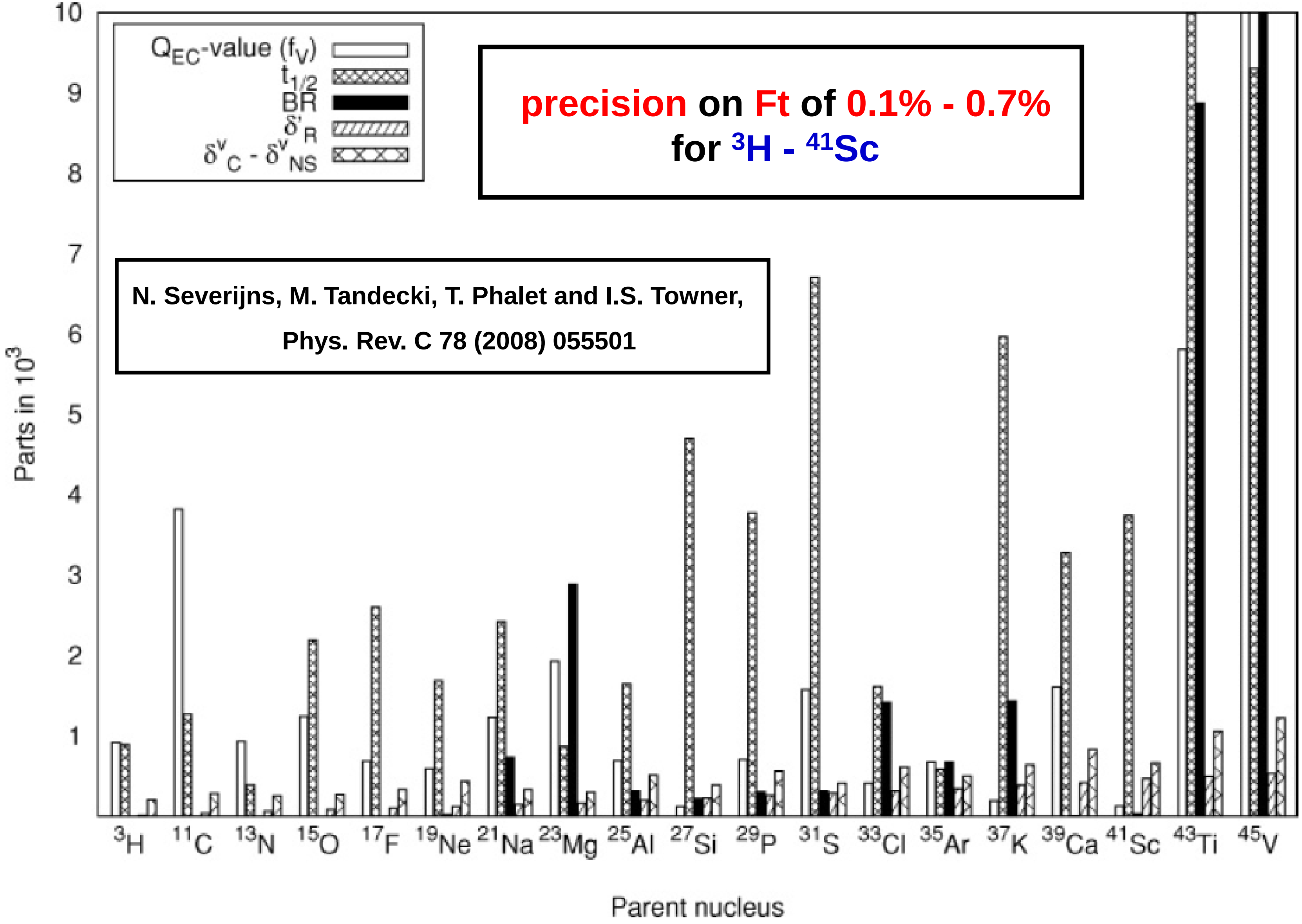
$$\mathcal{F}t^{mir} \equiv f_V t^{mir}(1+\delta_{NS}^V - \delta_C^V)(1+\delta_R') = \frac{K}{G_F^2 V_{ud}^2 C_V^2 (1+\Delta_R^V)\left(1+\frac{f_A}{f_V}\rho^2\right)}$$

$t_{1/2}$, Q_{EC}, BR (→ $f_V t^{mir}$)

calculated (→ δ_{NS}^V, δ_C^V, δ_R')

$$\rho = \frac{C_A M_{GT}}{C_V M_F}$$

N. Severijns, M. Tandecki, T. Phalet and I.S. Towner,
Phys. Rev. C 78 (2008) 055501



QEC-value (fV)
t1/2
BR
δ'R
δvC - δvNS
Parts in 10^3
Parent nucleus
3H 11C 13N 15O 17F 19Ne 21Na 23Mg 25Al 27Si 29P 31S 33Cl 35Ar 37K 39Ca 41Sc 43Ti 45V
precision on Ft of 0.1% - 0.7%
for 3H - 41Sc
N. Severijns, M. Tandecki, T. Phalet and I.S. Towner,
Phys. Rev. C 78 (2008) 055501

$$\mathcal{F}t^{mir} \equiv f_V t^{mir}(1+\delta_{NS}^V-\delta_C^V)(1+\delta_R') = \frac{K}{G_F^2 V_{ud}^2 C_V^2 (1+\Delta_R^V)\left(1+\frac{f_A}{f_V}\rho^2\right)}$$

$t_{1/2}$, Q_{EC} , BR — calculated

$$\rho = \frac{C_A M_{GT}}{C_V M_F}$$

N. Severijns, M. Tandecki, T. Phalet and I.S. Towner, Phys. Rev. C 78 (2008) 055501

$$\mathcal{F}t_0^{mir} \equiv f_V t^{mir}(1+\delta_{NS}^V-\delta_C^V)(1+\delta_R')\left(1+\frac{f_A}{f_V}\rho^2\right) = \frac{K}{G_F^2 V_{ud}^2 C_V^2 (1+\Delta_R^V)}$$

should be constant for all T=1/2 mirror decays !

(test of Conserved Vector Current hypothesis)

from experiment

$$\mathcal{F}t_0^{mirror} \equiv f_V t^{mirror}(1+\delta_R')(1+\delta_{NS}^V-\delta_C^V)\left(1+\frac{f_A}{f_V}\rho^2\right) = \frac{K}{G_F^2 V_{ud}^2 C_V^2 (1+\Delta_R^V)}$$

should be constant for all T=1/2 mirror decays !

from experiment

$$\Downarrow$$

$$V_{ud}^2 = \frac{K}{Ft_0^{mir}\ G_F^2\ C_V^2\ (1+\Delta_R^V)}$$

6. Ft_0-values for T = 1/2 mirror beta transitions

	^{19}Ne	^{21}Na	^{29}P	^{35}Ar	^{37}K
J	1/2	3/2	1/2	3/2	3/2
Ft (s) [1]	1718.4(32)	4085(12)	4809(19)	5688.6(72)	4562(28)
$a_{\beta\nu}$		0.5502(60) [2]			
A_β	-0.0391(14) [3]		0.681(86) [4]	0.430(22) [5]	
B_ν					-0.755(24) [6]
ρ [7,8]	1.5995(45)	-0.7136(72)	-0.593(104)	-0.279(16)	0.561(27)
Ft_0 (s) [7,8]	6177(30)	6203(47)	6536(606)	6128(49)	6004(146)

[1]) N. Severijns et al., PR C 78 (2008) 055501
[2]) F. Calaprice et al., PRL 35 (1975) 1566
[3]) P. Vetter et al., PR C 77 (2008) 035508
[4]) G.S. Masson, P.A. Quin, PR C 42 (1990) 1110
[5]) J.D. Garnett et al., PRL 60 (1998) 499
A. Converse et al., PL B 304 (1993) 60
[6]) D. Melconian et al., PL B 649 (2007) 3370
[7]) including weak magnetism (cf. ref. 8)

[8]) O. Naviliat-Cuncic and N. Severijns, arXiv:0809.0994 [nucl-ex] & submitted

7. V_{ud} from T = 1/2 mirror beta transitions

^{19}Ne, ^{21}Na, ^{29}P, ^{35}Ar, ^{37}K

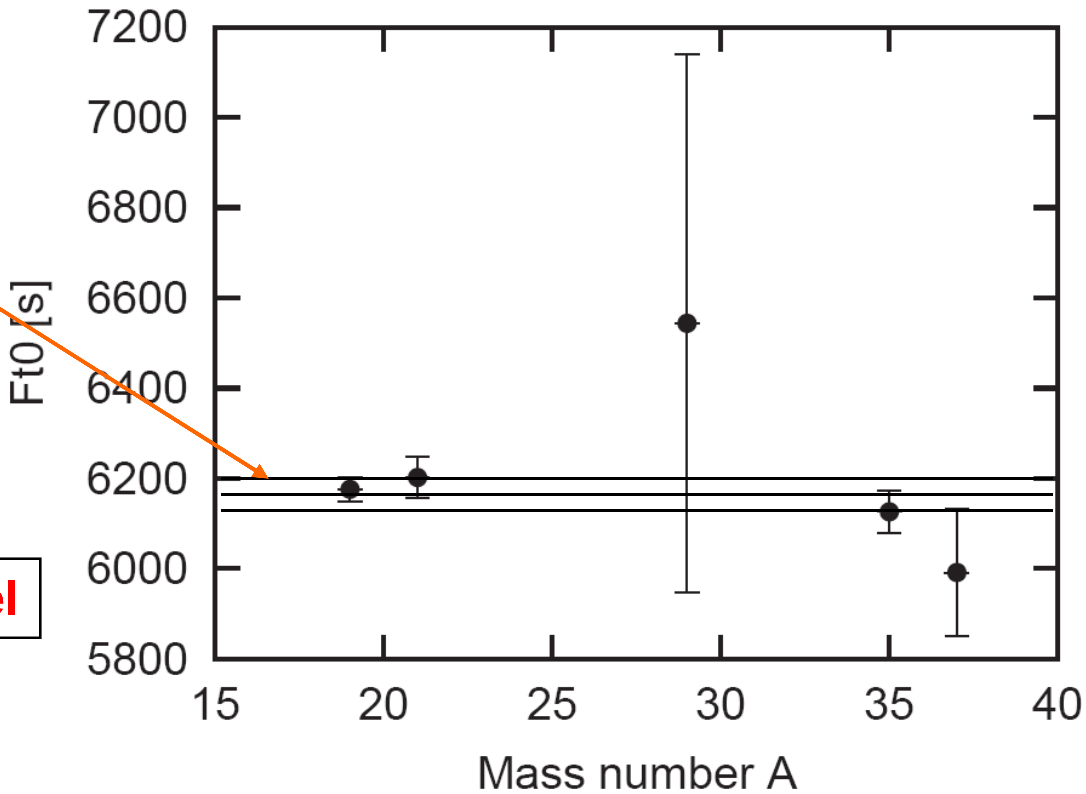
$\Downarrow$

$$(Ft_0^{mir})_{avg} = 6169 \pm 22 \text{ s}$$

$$(\chi^2/\nu = 0.73)$$

$\Downarrow$

confirms CVC at 3.5 permille level



&

$$V_{ud}^2 = \frac{K}{(Ft_0^{mir})_{avg}\ G_F^2\ C_V^2\ (1+\Delta_R^V)} \Rightarrow \left|V_{ud}\right| = 0.9722(17)$$

8. Comparison with other values for V_{ud}

1. superallowed $0^+ \rightarrow 0^+$ transitions

$$|V_{ud}| = 0.97418(26)$$

Towner & Hardy, PR C 77 (2008) 025501

2. T = 1/2 mirror beta transitions

$$|V_{ud}| = 0.9722(17)$$

Naviliat & Severijns, arXiv:0809.0994 [nucl-ex] & submitted

3. free neutron decay

$$|V_{ud}| = 0.9743(19)$$

e.g. Severijns, Beck & Naviliat, Rev. Mod. Phys. 78 (2006) 991 and Particle Data Group, Phys. Lett. B 667 (2008) 1

4. pion decay $\pi^+ \longrightarrow \pi^0 e^+ \nu_e$

$$|V_{ud}| = 0.9728(30)$$

Pocanic et al., PRL 93 (2004) 181803

9. Conclusion and outlook

Conclusion:

- new method to determine V_{ud} using T = 1/2 mirror beta transitions
- result has similar precision as value from neutron decay and is compatible with the other values for V_{ud}

Prospects for further improvement:

- new and more precise measurements (mostly $t_{1/2}$, Q_{EC}) leading to the Ft values of the T=1/2 mirror transitions are needed
- new precision measurements of $a_{\beta\nu}$, A_{β} , … in these transitions
- very sensitive transition: ^{19}Ne

1973 Kobayashi & Maskawa :

- extension of Cabibbo matrix

- 3rd generation of quarks !!

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

where e.g. V_{uj} = amplitude of quark mass eigenstate ***j (= d, s, b)*** into the quark weak eigenstate ***d'***

$$V \cong \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

with : 3 angles, $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$

(i, j = 1, 2, 3 [the generations of quarks])

and

δ_{13} **a phase accounting for the observed CP-violation**