

Correlations between nuclear masses, radii and E0 transitions

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Simple nuclear mass formulas
(Correlations between masses and radii)

with A.E.L. Dieperink

Correlations between radii and E0 transitions

with S. Zerguine, A. Bouldjadri, S. Heinze

Liquid-drop mass formula

Relation between mass and binding energy:

$$M(N,Z)c^2 = N m_n c^2 + Z m_p c^2 - B(N,Z)$$

Liquid-drop mass formula:

$$B_{\text{LDM}}(N,Z) = a_{\text{vol}} A - a_{\text{sur}} A^{2/3} - a_{\text{cou}} \frac{Z(Z-1)}{A^{1/3}} - a_{\text{sym}} \frac{(N-Z)^2}{A} + a_{\text{pai}} \frac{\delta(N,Z)}{A^{1/2}}$$

Fit to nuclear masses in AME03: $\sigma_{\text{rms}} \approx 3.0 \text{ MeV}$.

C.F. von Weizsäcker, Z. Phys. **96** (1935) 431

ISOLDE workshop, CERN, November 2008

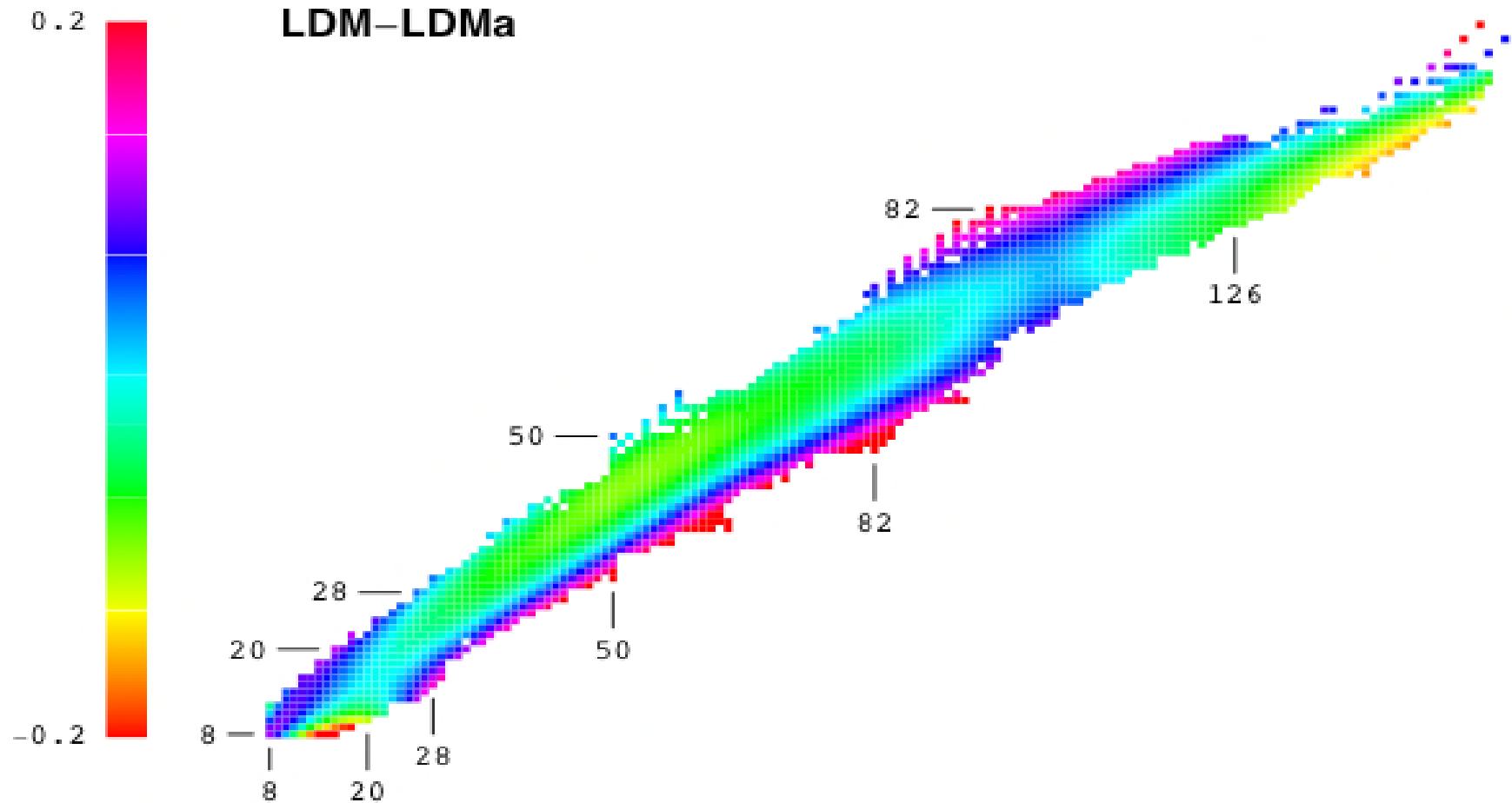
Deficiencies of LDM formula

Consistency of the Weizsäcker mass formula requires a *surface-symmetry* term. Derivation relies on the thermodynamics of a two-component system:

$$\frac{S_v}{1 + S_v A^{-1/3} / S_s} \frac{(N - Z)^2}{A} \approx -a_{\text{vsym}} \frac{(N - Z)^2}{4A} + a_{\text{ssym}} \frac{(N - Z)^2}{4A^{4/3}}$$

W.D. Myers & W.J. Swiatecki, Ann. Phys. **55** (1969) 395
A. Bohr & B.R. Mottelson, *Nuclear Structure II* (1975)
A.W. Steiner *et al.*, Phys. Reports **411** (2005) 325
P. Danielewicz, Nucl. Phys. A **727** (2003) 233

LDM versus LD_Ma



Quantal effects & Wigner cusp

The $(N-Z)^2$ dependence of the symmetry term arises in a macroscopic approximation.

Quantal theories gives rise to

$T(T+1)$: *isospin* $SU(2)$.

$T(T+4)$: *supermultiplet* $SU(4)$.

This suggests a generalization of the form $T(T+r)$, with r a parameter.

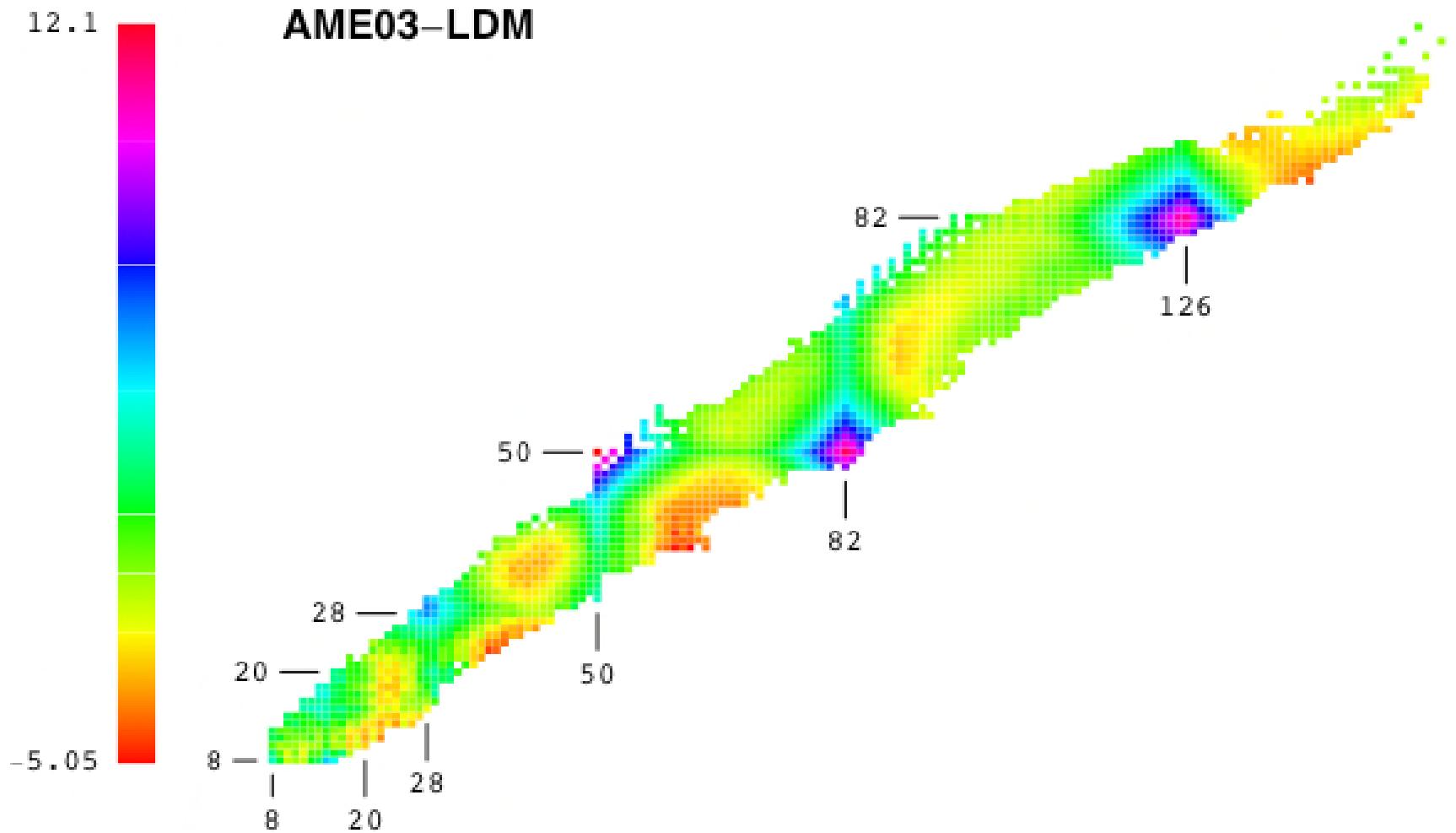
Modified nuclear mass formula

Add Wigner and surface-symmetry energy:

$$B_{\text{LDM}}(N, Z) = a_{\text{vol}} A - a_{\text{sur}} A^{2/3} - a_{\text{cou}} \frac{Z(Z-1)}{A^{1/3}} + a_{\text{pai}} \frac{\delta(N, Z)}{A^{1/2}}$$
$$- \frac{S_v}{1 + S_v A^{-1/3} / S_s} \frac{T(T+r)}{A}$$

Fit to AME03: $\sigma_{\text{rms}} \approx 2.4 \text{ MeV.}$

The ‘unfolding’ of the mass surface



Shell corrections

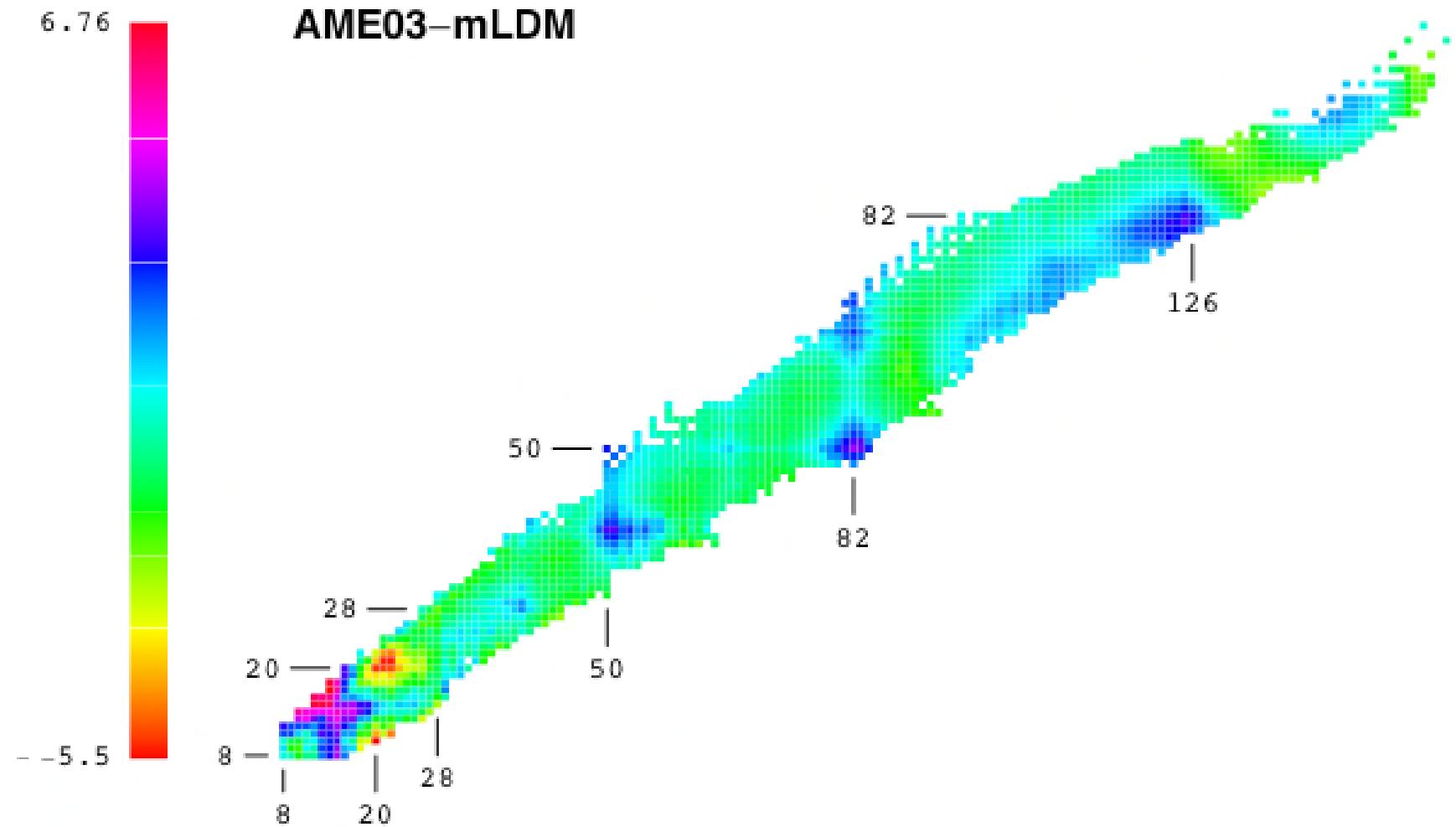
Observed deviations suggest shell corrections depending on $n+z$, the total number of valence neutrons + protons (particles or holes).

A simple parametrisation consists of two terms, linear and quadratic in $n+z$.

$$B_{\text{mLDM}}(N, Z) = B_{\text{LDM}}(N, Z) + a_1(n + z) + a_2(n + z)^2$$

Fit to AME03: $\sigma_{\text{rms}} \approx 1.2 \text{ MeV}$.

Shell-corrected LDM

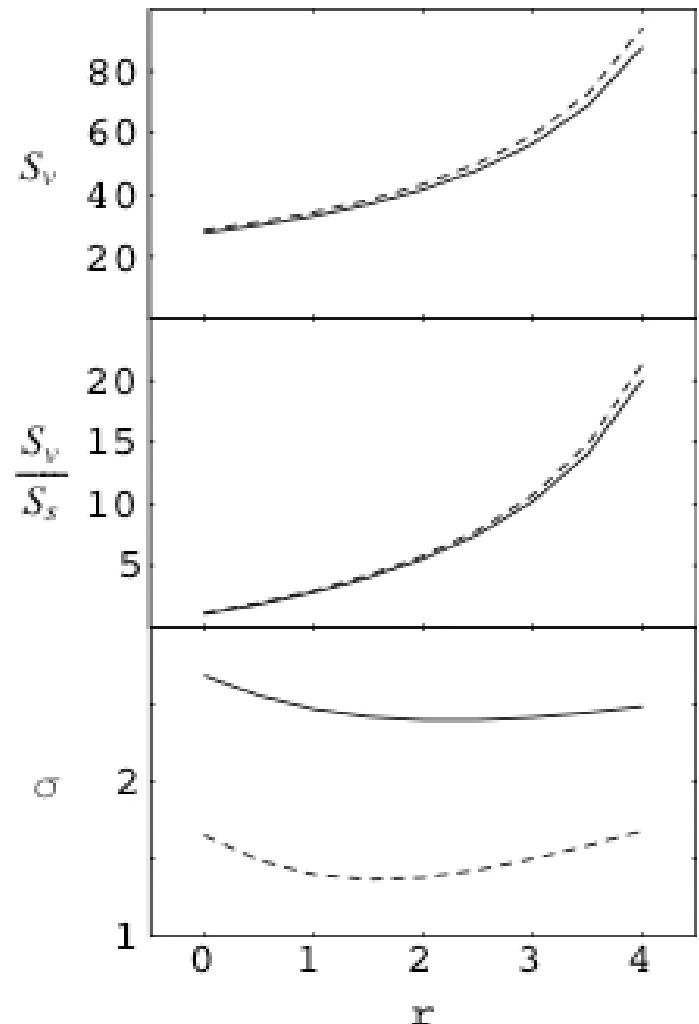


Dependence on r

Deviation σ_{rms} decreases significantly with shell corrections.

Deviation σ_{rms} has shallow minimum in r .

Both S_v and S_v/S_s are ill determined.



Better shell corrections

Midshell cusp behaviour of $n+z$ is not realistic.

Best: $n \rightarrow \sin(\pi n/\Omega_n)$ & $z \rightarrow \sin(\pi z/\Omega_z)$.

Shell-model inspired corrections involve shell size:

$$B_{\text{shell}}(N, Z) = a_2 S_2(n, z) + a_3 S_3(n, z) + a_4 S_2(n, z)^2 + a_{\text{np}} S_{\text{np}}(n, z)$$

$$S_2(n, z) = n(\Omega_n - n) + z(\Omega_z - z)$$

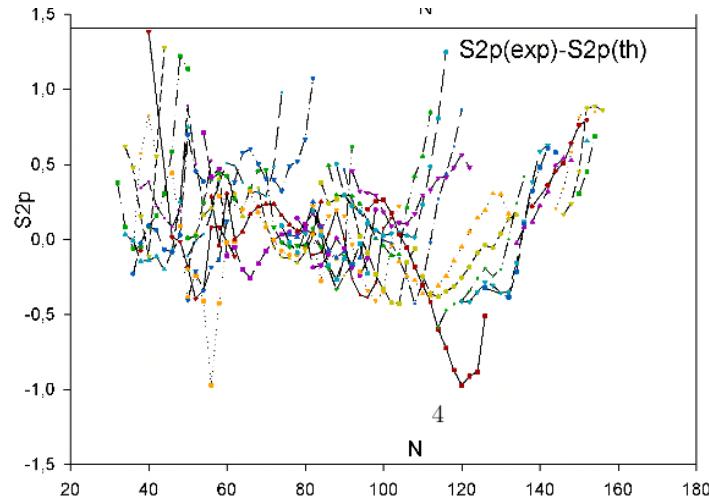
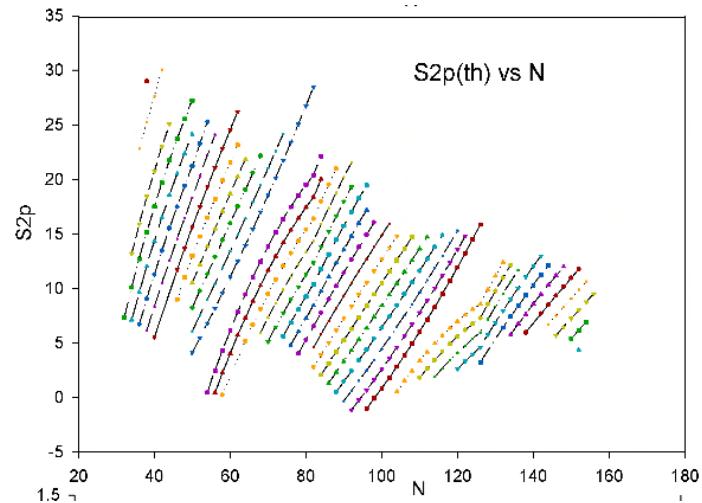
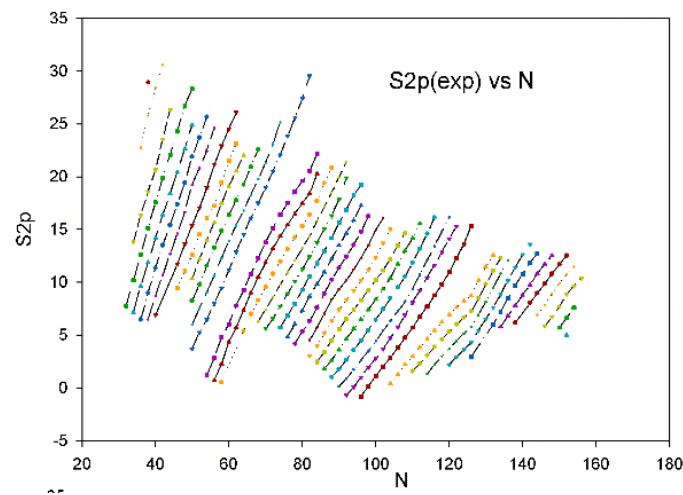
$$S_3(n, z) = n(\Omega_n - n)(\Omega_n - 2n) + z(\Omega_z - z)(\Omega_z - 2z)$$

$$S_{\text{np}}(n, z) = n(\Omega_n - n)z(\Omega_z - z)$$

Fit to AME03: $\sigma_{\text{rms}} \approx 0.8 \text{ MeV.}$

J. Duflo & A. Zuker, Phys. Rev. C **52** (1995) 23R.
A.E.L. Dieperink & P. Van Isacker, to be published.

Two-nucleon separation energies



Correlation between mass & radius

Aim: Use parameters of LDM in expression for radius or *vice versa*.

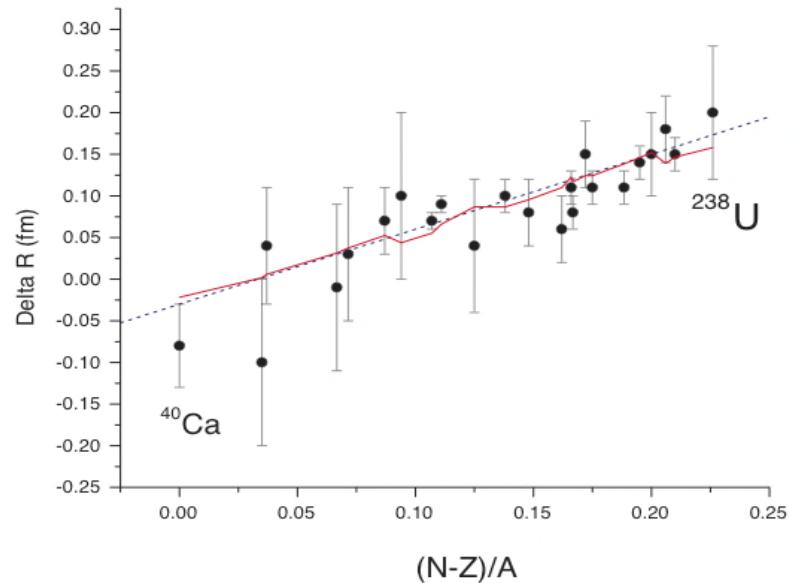
Example: Expression for neutron skin.

Work in progress: Consistent treatment of shell and deformation effects.

Relation to neutron skins

Dependence of neutron skin
on S_v and S_s (hard-sphere
approximation):

$$\frac{R_n - R_p}{R} = \frac{A}{6NZ} \frac{N-Z}{1 + S_s A^{1/3} / S_v} + \Delta E_{\text{cou}}$$



P. Danielewicz, Nucl. Phys. A 727 (2003) 233
A.E.L. Dieperink & P. Van Isacker, Eur. Phys. J. A 32 (2007) 11

Electric monopole transitions

E0 transitions cannot occur via single-photon emission.

Three possible processes:

Internal conversion.

Pair creation (if $E > 2m_e c^2$).

Two-phonon emission (rare).

The probability for an E0 transition to occur is given by $P = \Omega \rho^2$ with Ω and ρ^2 electronic and nuclear factors.

E0 matrix element

The nuclear factor is the matrix element

$$\rho = \sum_{k \in \text{protons}}^Z \left\langle f \left| \left(\frac{r_k}{R} \right)^2 - \sigma \left(\frac{r_k}{R} \right)^4 + \dots \right| i \right\rangle \quad (R = r_0 A^{1/3}, r_0 = 1.2 \text{ fm})$$

Higher-order terms are usually not considered,
 $\sigma=0$, (cfr. Church & Wenner) and hence contact
is made with the charge radius:

$$\langle r^2 \rangle_s = \frac{1}{Z} \sum_{k \in \text{protons}}^Z \langle s | r_k^2 | s \rangle$$

E.L. Church & J. Wenner, Phys. Rev. **103** (1956) 1035

ISOLDE workshop, CERN, November 2008

E0 and radius operators

Definition of a ‘charge radius operator’:

$$\langle s | \hat{T}(r^2) | s \rangle \equiv \langle r^2 \rangle_s = \frac{1}{Z} \sum_{k \in \text{protons}}^Z \langle s | r_k^2 | s \rangle \Rightarrow \hat{T}(r^2) = \frac{1}{Z} \sum_{k \in \text{protons}}^Z r_k^2$$

Definition of an ‘E0 transition operator’ (for $\sigma=0$):

$$\rho \equiv \frac{\langle f | \hat{T}(\text{E0}) | i \rangle}{eR^2} \Rightarrow \hat{T}(\text{E0}) = e \sum_{k \in \text{protons}}^Z r_k^2$$

Hence we find the following (standard) relation:

$$\hat{T}(\text{E0}) = eZ \hat{T}(r^2)$$

Effective charges

Addition of neutrons produces a change in the charge radius \Rightarrow need for effective charges.

Generalized operators:

$$\langle r^2 \rangle_s = \frac{1}{e_n N + e_p Z} \sum_{k=1}^A \langle s | e_k r_k^2 | s \rangle \Rightarrow \hat{T}(r^2) = \frac{1}{e_n N + e_p Z} \sum_{k=1}^A e_k r_k^2$$

$$\hat{T}(E0) = \sum_{k=1}^A e_k r_k^2$$

Generalized (non-standard) relation:

$$\underline{\underline{\hat{T}(E0) = (e_n N + e_p Z) \hat{T}(r^2)}}$$

Effective charges from radii

Estimate with harmonic-oscillator wave functions:

$$\begin{aligned}\langle r^2 \rangle_s &= \frac{1}{e_n N + e_p Z} \sum_{k=1}^A \langle s | e_k r_k^2 | s \rangle \\ &= \frac{3^{4/3}}{4} \frac{b^2}{e_n N + e_p Z} (e_n N^{4/3} + e_p Z^{4/3}) \\ &= \frac{3\sqrt[3]{2}}{5} r_0^2 \frac{A^{1/3} (e_n N^{4/3} + e_p Z^{4/3})}{e_n N + e_p Z}\end{aligned}$$

Fit for rare-earth nuclei ($Z=58$ to 74) gives: $r_0=1.25$ fm, $e_n=0.26e$ and $e_p=1.12e$.

E0 transitions in nuclear models

Nuclear shell model: E0 transitions between states in a single oscillator shell vanish.

Geometric collective model: Strong E0 transitions occur between β - and ground-state band.

Interacting boson model (intermediate between shell model and collective model): The IBM can be used to test the relation between radii and E0 transitions.

Radii and E0s in IBM

The charge radius and E0 operators in IBM:

$$\hat{T}(r^2) = \left\langle r^2 \right\rangle_{\text{core}} + \alpha N_b + \eta \frac{\hat{n}_d}{N_b}, \quad \hat{T}(\text{E0}) = \eta \frac{e_n N + e_p Z}{N_b} \hat{n}_d$$

Estimates of parameters:

$$\left\langle r^2 \right\rangle_{\text{av}} \approx \frac{3}{5} r_0^2 A^{2/3} \Rightarrow |\alpha| \approx \frac{4}{5} r_0^2 A^{-1/3} \sim 0.2 \text{ fm}^2$$

$$\left\langle r^2 \right\rangle_{\text{def}} \approx \frac{3}{4\pi} r_0^2 A^{2/3} \beta^2 \Rightarrow \eta \approx \frac{4}{3} r_0^2 A^{-4/3} N_b^2 (1 + \bar{\beta}^2)$$

$$\sim 0.25 - 0.75 \text{ fm}^2$$

Application to rare-earth nuclei

Application to even-even nuclei with $Z=58-74$.

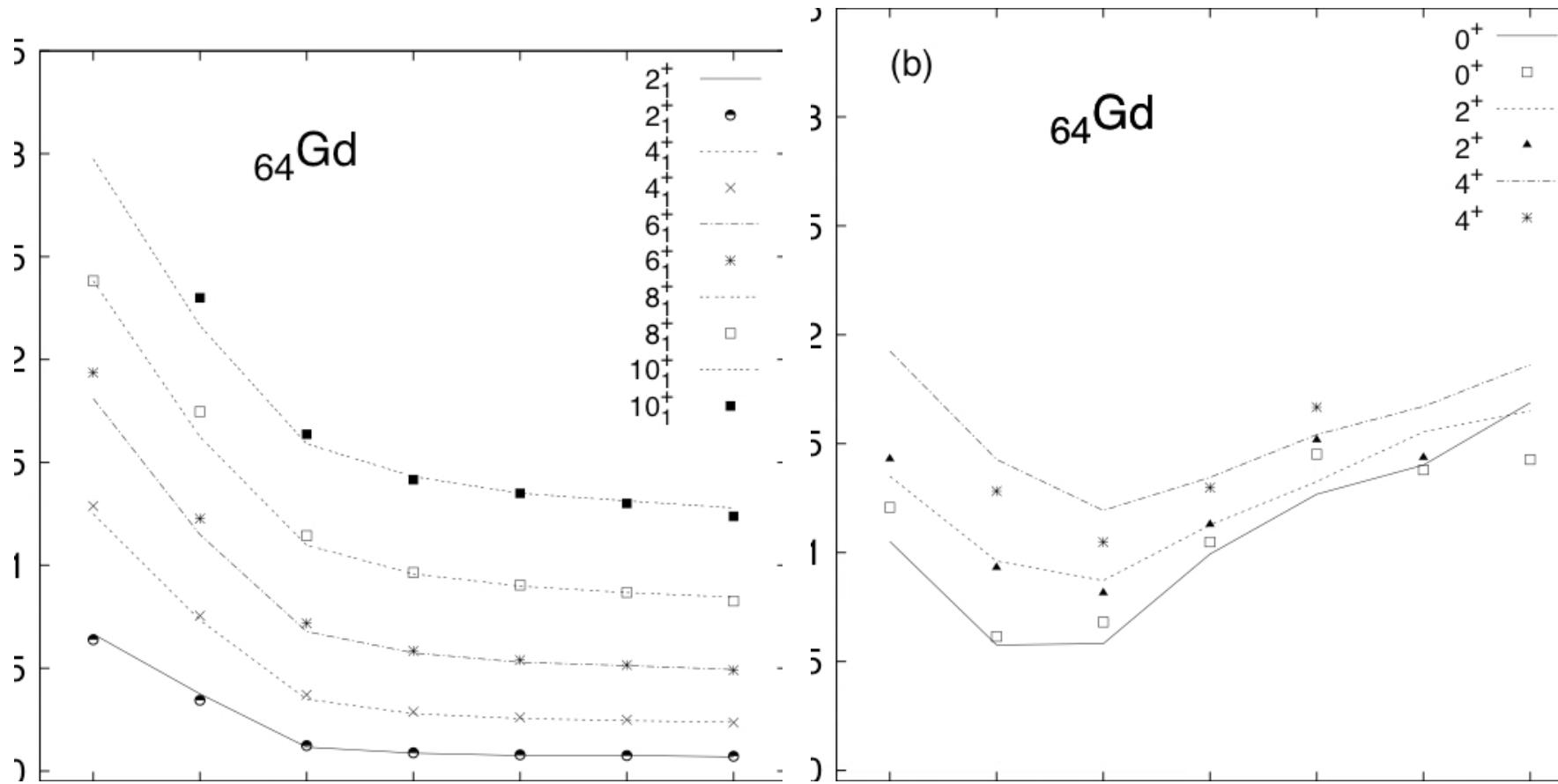
Procedure:

Fix IBM hamiltonian parameters from spectra with special care to the spherical-to-deformed transitional region.

Determine α and η from measured isotope shifts.

Calculate ρ^2 (depends only on η).

Example: Gd isotopes

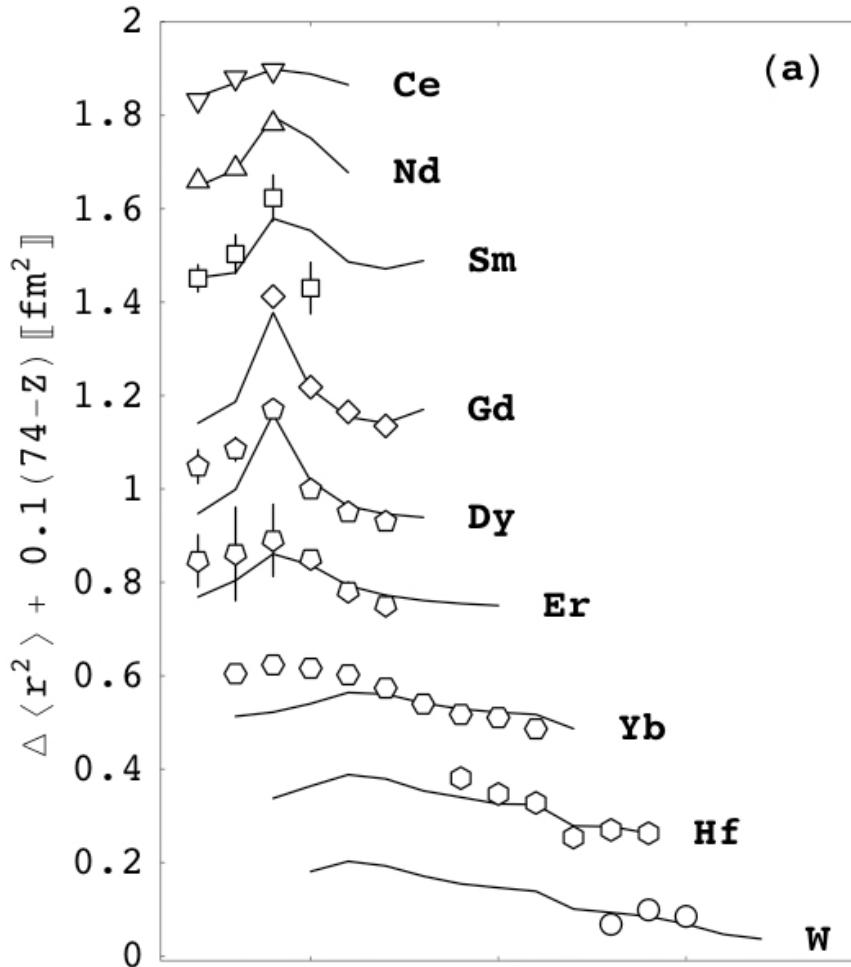


Isotope shifts

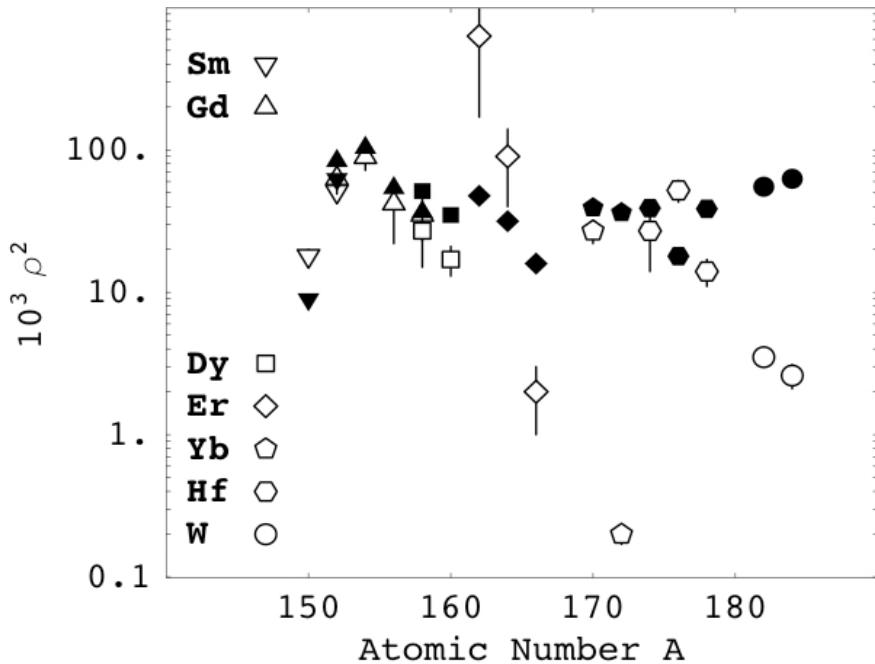
Isotopes shifts depend on the parameters α and η :

α (linear slope) varies between 0.10 and 0.25 fm^2 ;

η (deformation dependence) equals 0.6 fm^2 (constant for all nuclei).



ρ^2 values



Isotope	Transition		J	$\rho^2 \times 10^3$	
	E_i	E_f		Calc	Expt
^{150}Sm	740	\rightarrow 0	0	9	18 2
	1046	\rightarrow 334	2	20	100 40
^{152}Sm	685	\rightarrow 0	0	62	51 5
	811	\rightarrow 122	2	49	69 6
^{154}Sm	1023	\rightarrow 366	4	34	88 14
	1083	\rightarrow 0	0	3	0.7 4
^{152}Gd	1083	\rightarrow 685	0	56	22 9
	1099	\rightarrow 0	0	49	96 42
^{154}Gd	615	\rightarrow 0	0	84	63 14
	931	\rightarrow 344	2	95	35 3
^{156}Gd	681	\rightarrow 0	0	104	89 17
	815	\rightarrow 123	2	81	74 9
^{158}Gd	1049	\rightarrow 0	0	54	42 20
	1129	\rightarrow 89	2	50	55 5
^{158}Dy	1452	\rightarrow 0	0	37	35 12
	1517	\rightarrow 79	2	33	17 3
^{160}Dy	1086	\rightarrow 99	2	51	27 12
^{162}Er	1350	\rightarrow 87	2	35	17 4
^{164}Er	1171	\rightarrow 102	2	48	630 460
^{166}Er	1484	\rightarrow 91	2	32	90 50
^{168}Er	1460	\rightarrow 0	0	16	2 1
^{170}Yb	1229	\rightarrow 0	0	39	27 5
^{172}Yb	1405	\rightarrow 0	0	36	0.20 3
^{174}Hf	900	\rightarrow 91	2	39	27 13
^{176}Hf	1227	\rightarrow 89	2	18	52 9
^{178}Hf	1496	\rightarrow 93	2	39	14 3
^{182}W	1257	\rightarrow 100	2	55	3.5 3
^{184}W	1121	\rightarrow 111	2	63	2.6 5

Conclusions

Inclusion of surface *and* Wigner corrections in the liquid-drop mass formula to determine the symmetry energy in nuclei.

Use of information from masses for radii and *vice versa*.

Consistent treatment of charge radii and E0 transitions assuming the same effective charges.

Nuclear mass formulas

Global mass formulas:

Liquid-drop model (LDM): von Weizsäcker.

Macroscopic models with microscopic corrections: FRDM,

...

Microscopic models: HFBn, RMF, DZ, ...

Local mass formulas:

Extrapolations by Wapstra & Audi.

*IMME, Garvey-Kelson relations, Liran-Zeldes formula,
neural networks, ...*

D. Lunney *et al.*, Rev. Mod. Phys. **75** (2003) 1021
K. Blaum, Phys. Reports **425** (2006) 1

ISOLDE workshop, CERN, November 2008

Wigner energy

Wigner energy B_W is decomposed in two parts:

$$B_W = -W(A)|N - Z| - d(A)\delta_{N,Z}\pi_{np}$$

$W(A)$ and $d(A)$ can be fixed empirically from

$$\delta V_{np} = \frac{1}{4} [B(N, Z) - B(N-2, Z) - B(N, Z-2) + B(N-2, Z-2)]$$

...and similar expressions for odd-mass and odd-odd nuclei:

$$d(A) \leq W(A) \approx 47 A^{-1} \text{ MeV}$$

P. Möller & R. Nix, Nucl. Phys. A **536** (1992) 20

J.-Y. Zhang *et al.*, Phys. Lett. B **227** (1989) 1

W. Satula *et al.*, Phys. Lett. B **407** (1997) 103

Supermultiplet model

Wigner's explanation of the 'kinks in the mass defect curve' was based on SU(4) symmetry.

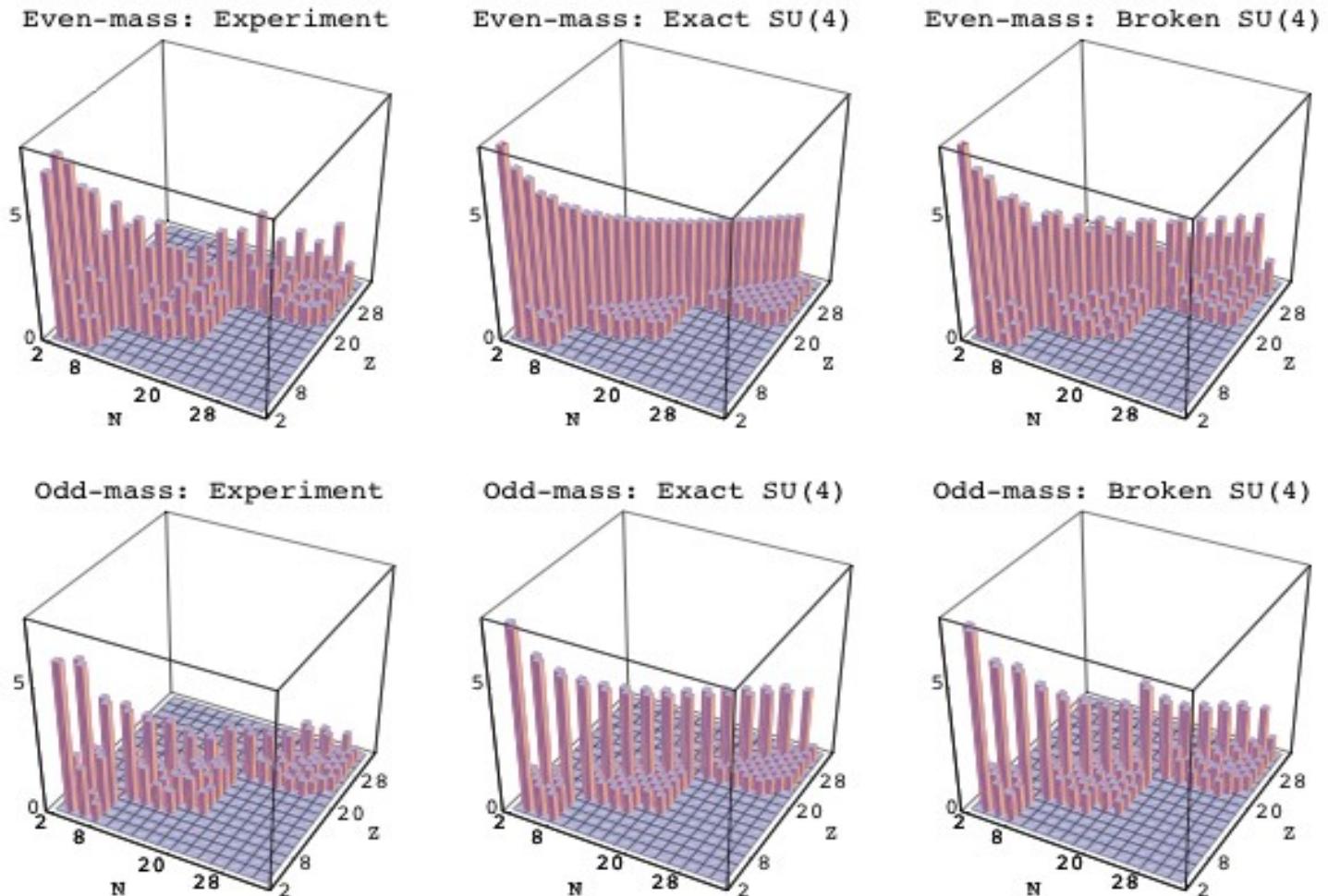
Symmetry contribution to the nuclear binding energy is

$$-K(A)g(\lambda, \mu, \nu) = K(A) \left[(N - Z)^2 + 8|N - Z| + 8\delta_{N,Z}\pi_{np} + 6\delta_{pairing} \right]$$

SU(4) symmetry is broken by spin-orbit term. Effects of SU(4) mixing must be included.

E.P. Wigner, Phys. Rev. **51** (1937) 106, 947
D.D. Warner *et al.*, Nature, to be published

Evidence for the Wigner cusp



Symmetry energy

Energy per particle in nuclear matter:

$$E(\rho, x) = E\left(\rho, x = \frac{1}{2}\right) + S(\rho)(1 - 2x)^2, \quad x = Z/A$$

Symmetry energy $S(\rho)$ is density dependent:

$$S(\rho) = a_4 + p_0(\rho - \rho_0) + \Delta K(\rho - \rho_0)^2$$

In Thomas-Fermi approximation:

$$\frac{S_v}{S_s} = \frac{3}{R\rho_0} \int dr \rho(r) \left(\frac{S(\rho_0)}{S(\rho)} - 1 \right)$$

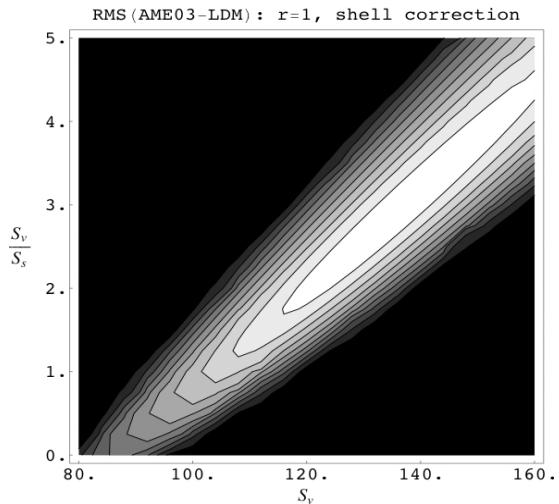
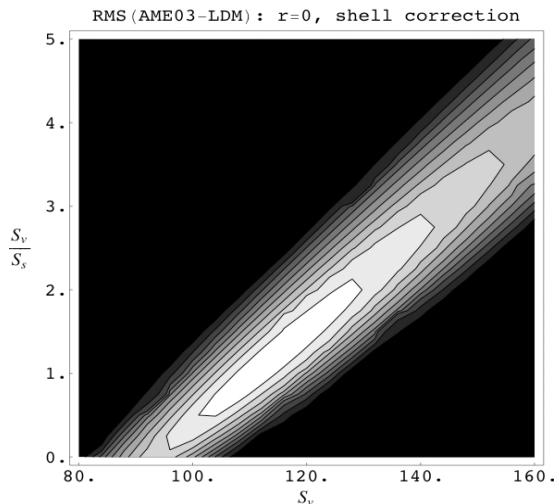
Correlations

Volume- and surface-symmetry terms are correlated.

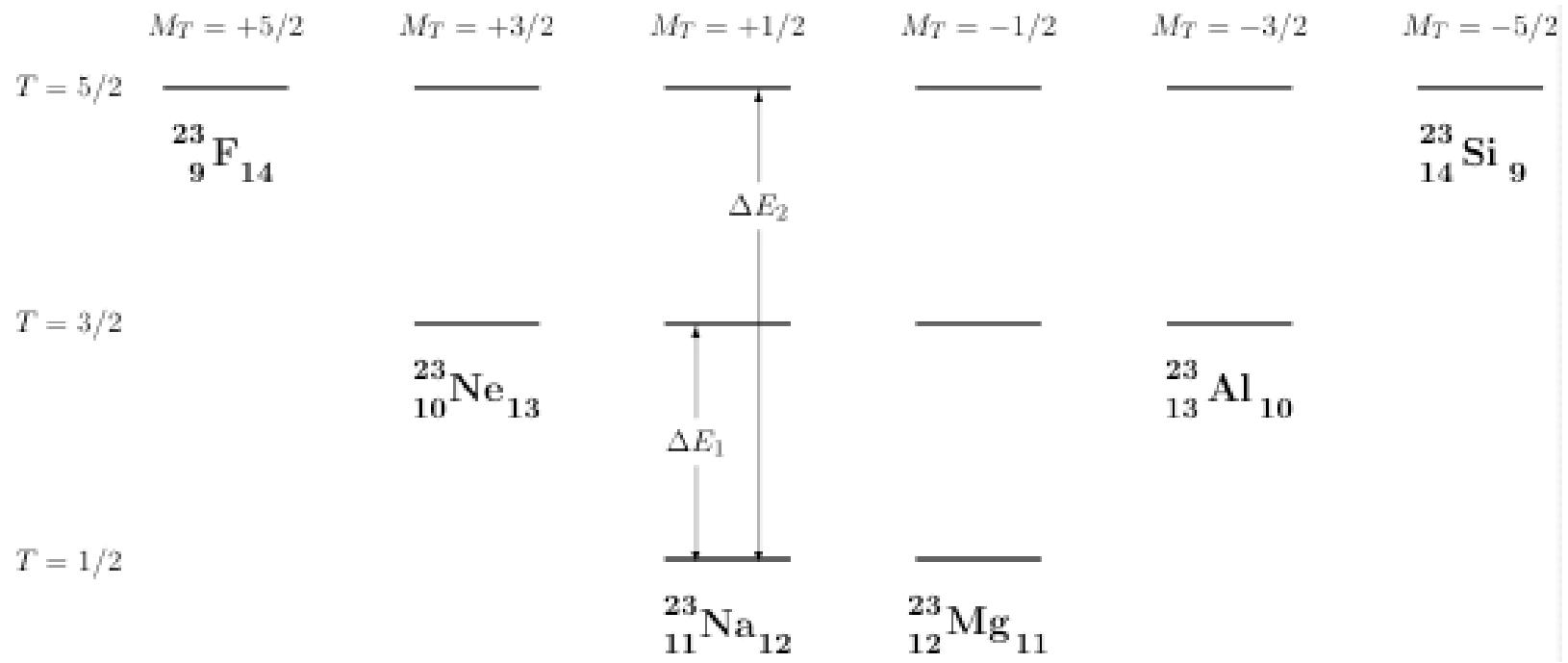
Correlation depends strongly on r .

What nuclear properties are needed to determine S_v and S_s ?

How can we fix r ?



r from isobaric multiplets



From $T=3/2$ and $T=5/2$ states in $M_T=\pm 1/2$ nuclei:

$$\frac{\Delta E_2}{\Delta E_1} = \frac{2(r+3)}{r+2} \Rightarrow r = \frac{6\Delta E_1 - 2\Delta E_2}{\Delta E_2 - 2\Delta E_1}$$

In ^{23}Na : $\Delta E_1 = 7.891$ & $\Delta E_2 = 19.586 \Rightarrow r = 2.15$

J. Jänecke & T.W. O'Donnell, Phys. Lett. B **605** (2005) 87