Improved formalism for superallowed Fermi β decay between analogs and half-lives of the rp-process waiting point A~70 nuclei

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Exotic nuclei near the N = Z line in the A ~ 70 mass region

characteristics

- **shape-coexistence and -mixing**
- **competition between proton-neutron and like-nucleon pairing correlations**
- **drastic changes in structure with particle number, angular momentum and excitation energy**

expectations

- ***large* isospin mixing effect on the *superallowed Fermi β* decay**
- ***Gamow-Teller β* decay of low-lying excited states in waiting-point nuclei *relevant* for the rp-process**

requirements for the self-consistent models

- **realistic effective interactions in large model spaces**
- **beyond mean-field approaches**

complex VAMPIR

- variational approaches with symmetry projection before variation

General teoretical tools

Model space

$$\{|i\rangle \equiv |\tau nljm\rangle\}$$

$$\{c_i^\dagger, c_k^\dagger, \dots\}_M$$

$$\{c_i, c_k, \dots\}_M$$

Effective many-body Hamiltonian

$$\hat{H} = \sum_{i=1}^{M} \varepsilon(i) c_i^\dagger c_i + \frac{1}{4} \sum_{i,k,r,s=1}^{M} v(ikrs) c_i^\dagger c_k^\dagger c_s c_r$$

Hartree-Fock-Bogoliubov transformation

$$\begin{pmatrix} a^\dagger \\ a \end{pmatrix} = F \begin{pmatrix} c^\dagger \\ c \end{pmatrix} = \begin{pmatrix} A^T & B^T \\ B^\dagger & A^\dagger \end{pmatrix} \begin{pmatrix} c^\dagger \\ c \end{pmatrix}$$

$$a_\alpha^+ = \Sigma_{i=1}^M (A_{i\alpha} c_i^+ + B_{i\alpha} c_i)$$

$$a_\alpha = \Sigma_{i=1}^M (B_{i\alpha}^* c_i^+ + A_{i\alpha}^* c_i)$$

Quasi-particle vacuum

$$|F\rangle = \prod_{\alpha=1}^{M'} a_\alpha |0\rangle \qquad \text{with} \quad \left\{ \begin{array}{lr} a_\alpha|0\rangle \neq 0 \quad \text{for } \alpha = 1, ..., M' \leq M \\ a_\alpha|0\rangle = 0 \qquad\qquad \text{else} \end{array} \right\}$$

$$\hat{\Theta}^s_{MK} \equiv \hat{P}(I; MK)\hat{Q}(N)\hat{Q}(Z)\hat{p}(\pi)$$

Variational procedures

complex Vampir approach

$$E^s[F_1^s] = \frac{\langle F_1^s | \hat{H} \hat{\Theta}_{00}^s | F_1^s \rangle}{\langle F_1^s | \hat{\Theta}_{00}^s | F_1^s \rangle}$$

$$|\psi(F_1^s); sM\rangle = \frac{\hat{\Theta}_{M0}^s |F_1^s\rangle}{\sqrt{\langle F_1^s | \hat{\Theta}_{00}^s | F_1^s \rangle}}$$

complex Excited Vampir approach

$$|\psi(F_2^s); sM\rangle = \hat{\Theta}^s_{M0}\left\{|F_1^s\rangle\alpha_1^2 + |F_2^s\rangle\alpha_2^2\right\}$$

$$|\psi(F_i^s); sM\rangle = \Sigma_{j=1}^{i}\,|\phi(F_j^s)\rangle\,\alpha_j^i \qquad \text{for} \quad i = 1, ..., n-1$$

$$|\phi(F_i^s); sM\rangle = \hat{\Theta}^s_{M0}|F_i^s\rangle$$

$$|\psi(F_n^s); sM\rangle = \Sigma_{j=1}^{n-1}\,|\phi(F_j^s)\rangle\,\alpha_j^n + |\phi(F_n^s)\rangle\,\alpha_n^n$$

$$\alpha_n^n = \langle\phi^n|[1 - \Sigma_{j,l=1}^{n-1}\,|\phi^j\rangle\,(A^{-1})_{jl}\langle\phi^l|]|\phi^n\rangle^{-1/2}$$

$$A_{jl} \equiv \langle\phi^j|\phi^l\rangle \quad i, l = 1, ..., n-1$$

$$\alpha_j^n = -\Sigma_{l=1}^{n-1}\,(A^{-1})_{jl}\langle\phi^l|\phi^n\rangle\alpha_n^n$$

$$\hat{S} \equiv \Sigma_{j,l=1}^{n-1} |\phi^j\rangle (A^{-1})_{jl} \langle \phi^l|$$

$$E_1^n \equiv \langle \psi^n | \hat{H} | \psi^n \rangle = \frac{\langle \phi^n | (1-\hat{S}) \hat{H} (1-\hat{S}) | \phi^n \rangle}{\langle \psi^n | (1-\hat{S}) | \phi^n \rangle}$$

$$(H - E^{(n)} N) f^n = 0$$

$$(f^{(n)})^+ N f^{(n)} = 1$$

$$|\Psi_\alpha^{(n)}; sM > = \sum_{i=1}^{n} |\psi_i; sM > f_{i\alpha}^{(n)}, \qquad \alpha = 1, ..., n$$

A$=70-90$ mass region

^{40}Ca - core

model space (π, ν):

$1p_{1/2}\ 1p_{3/2}\ 0f_{5/2}\ 0f_{7/2}\ 1d_{5/2}\ 0g_{9/2}$

(charge-symmetric basis + Coulomb contributions to the π-spe from the core)

renormalized G–matrix (OBEP, Bonn A) (Bonn CD)

- short range Gaussians in the *nn, pp, np channels*
- monopole shifts:

$$\langle 0g_{9/2}0f;T=0|\hat{G}|0g_{9/2}0f;T=0\rangle$$

$$\langle 1p1d_{5/2};T=0|\hat{G}|1p1d_{5/2};T=0\rangle$$

Superallowed Fermi β decay

Superallowed Fermi β decay between 0^+ T=1 analog states

test of the CVC hypothesis

test of the unitarity of the CKM matrix

$$ft(1+\delta_R)(1-\delta_c) = \frac{K}{2G_v^2(1+\Delta_R^v)}$$

δc – isospin-symmetry-breaking-correction

Charge-symmetric effective Hamiltonian:

- same single particle energies for π and υ

∀ • Bonn A potential

Isospin-symmetry-breaking contributions:

⁎ electromagnetic interaction

- Coulomb contribution to the single particle energies resulting from the Ca core
- Coulomb two-body matrix elements

⁎ charge-dependent strong interaction

- Bonn CD potential

Isospin-symmetry-breaking effective Hamiltonians:

⁎ **Bonn A + Coulomb**

⁎ **Bonn CD + Coulomb**

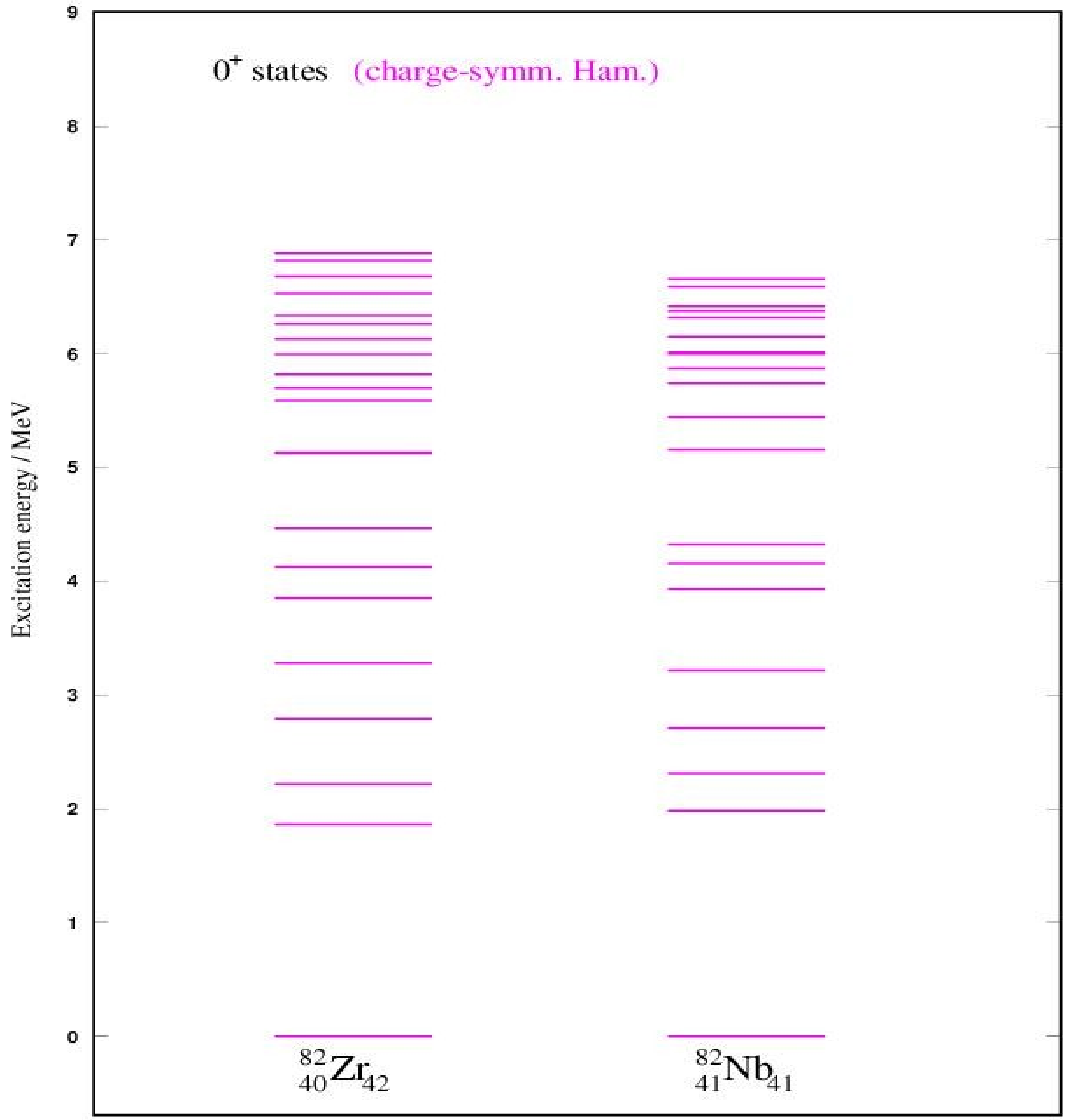
Radial mismatch problem - *avoided*

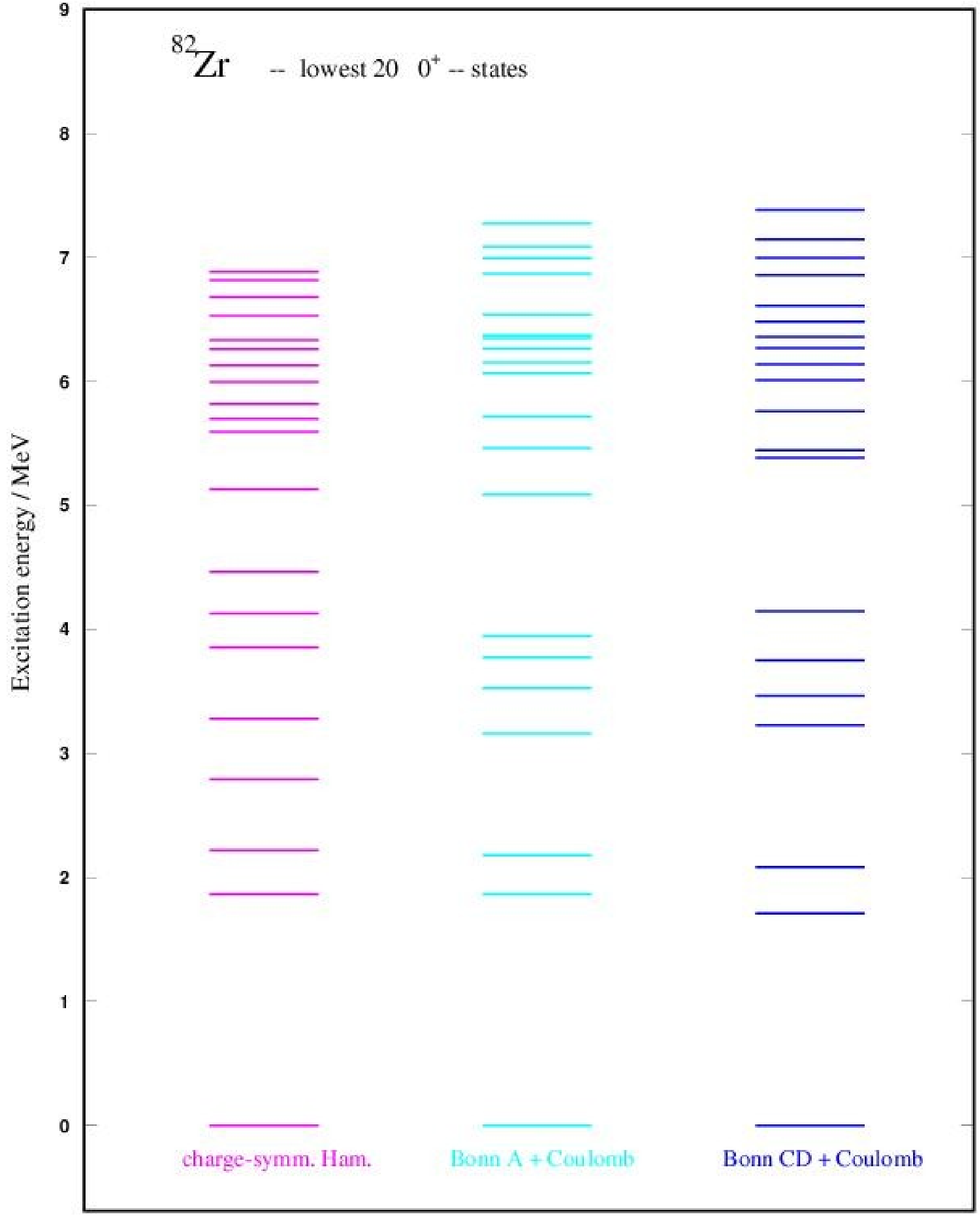
$$\tau_+ = \Sigma_\alpha a_a^+ b_\alpha$$

$$M_F = \langle f | \tau_+ | i \rangle$$

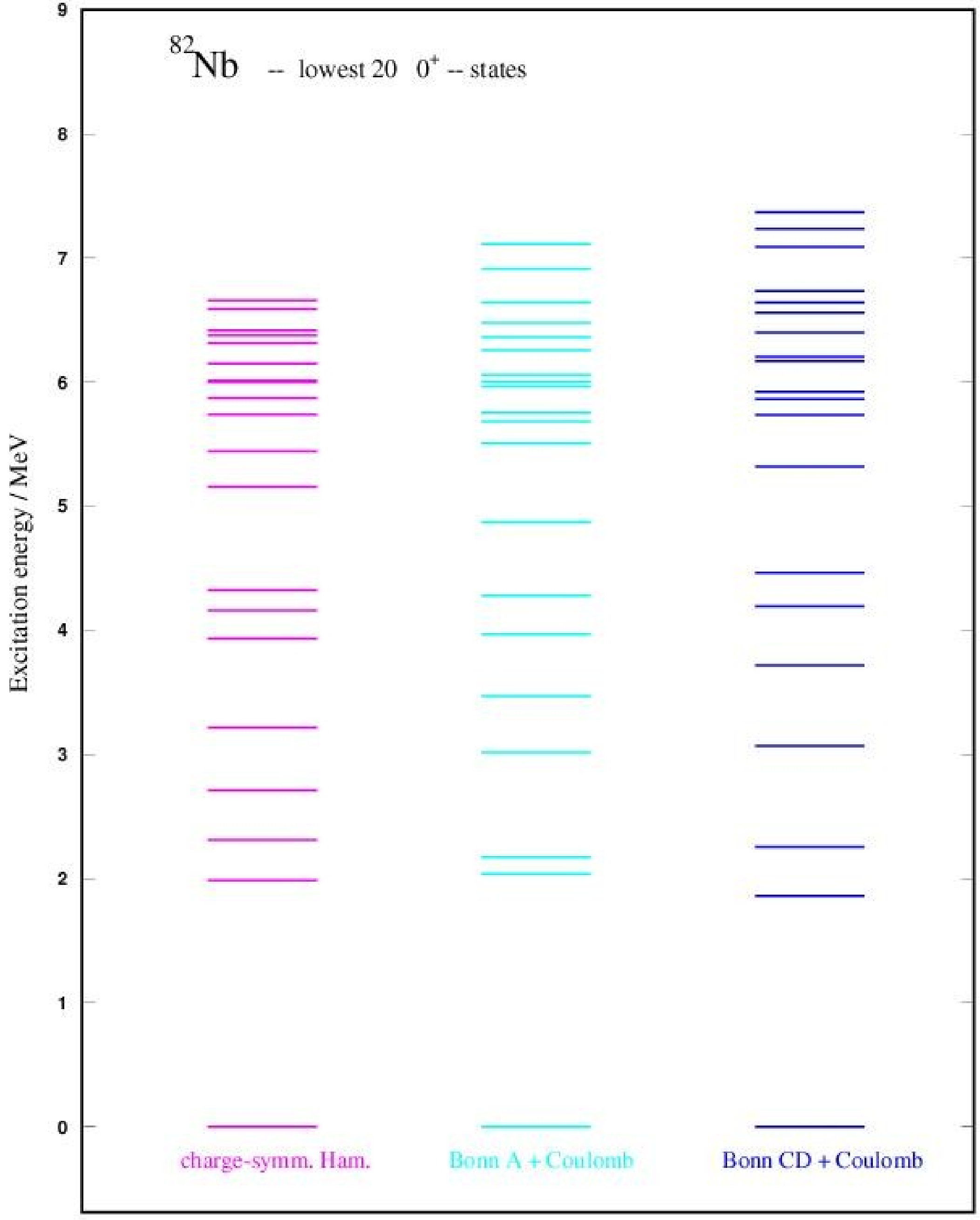
A=82 $_{41}Nb_{41} \rightarrow {}_{40}Zr_{42}$ $0^+ \rightarrow 0^+$ *GANIL, J. Garces Narro, PRC63(2001)044307*

$T_{1/2}$= 52(6)ms





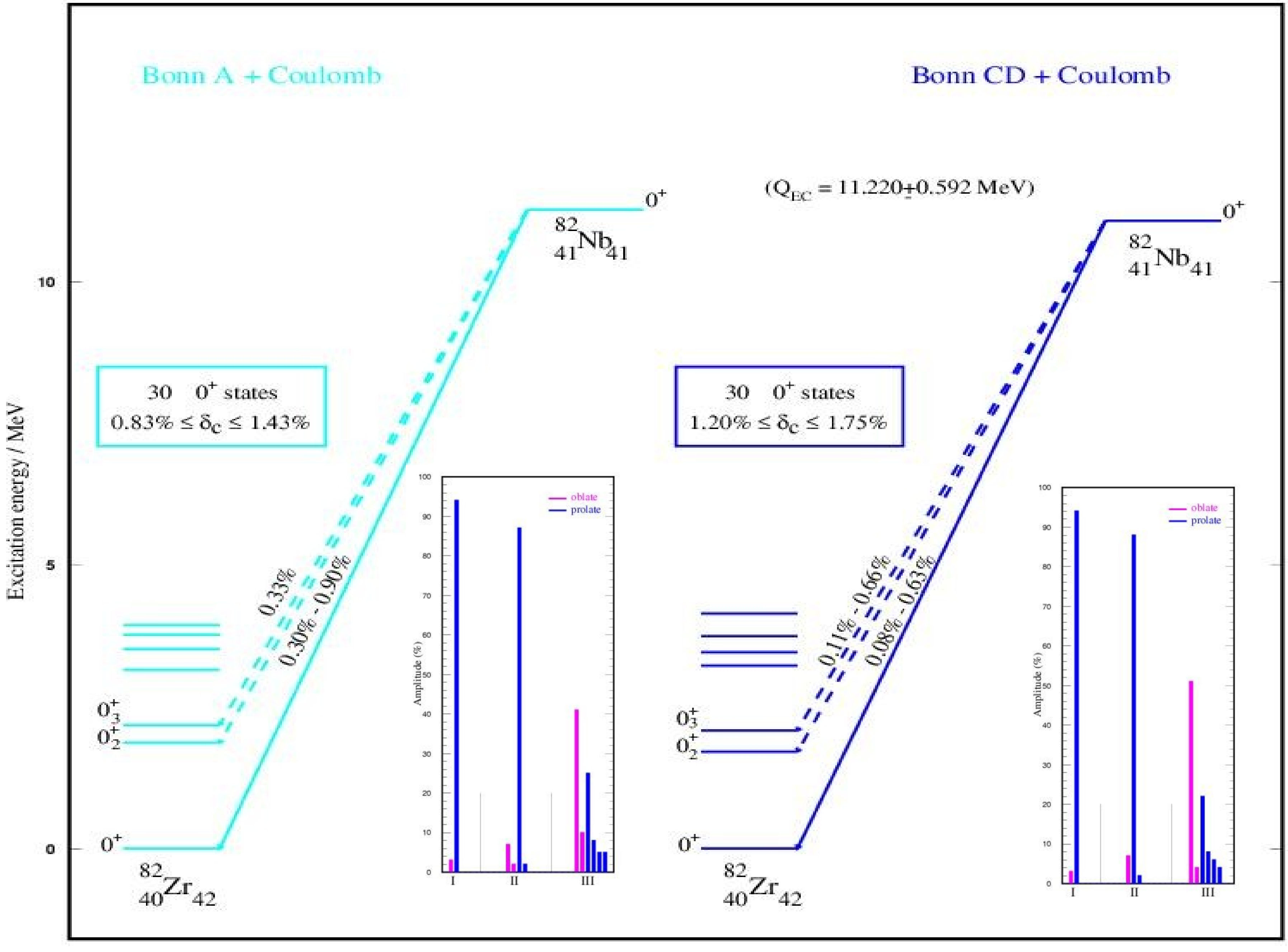
82Zr -- lowest 20 0+ -- states
Excitation energy / MeV
0
1
2
3
4
5
6
7
8
9
charge-symm. Ham.
Bonn A + Coulomb
Bonn CD + Coulomb



^{82}Nb -- lowest 20 0^+ -- states
Excitation energy / MeV
9
8
7
6
5
4
3
2
1
0
charge-symm. Ham.
Bonn A + Coulomb
Bonn CD + Coulomb

The total (S_T) and analog (S_{g-g}) Fermi β decay strengths for the charge-symmetric , Bonn A + Coulomb, and Bonn CD + Coulomb effective Hamiltonian

charge-symmetric Ham.		Bonn A + Coulomb		Bonn CD + Coulomb	
S_T	S_{g-g}	S_T	S_{g-g}	S_T	S_{g-g}
1.9715	1.9626	1.9761	1.9357	1.9752	1.9293



Bonn A + Coulomb
Bonn CD + Coulomb
(Q_EC = 11.220±0.592 MeV)
Excitation energy / MeV
30 0+ states
0.83% ≤ δc ≤ 1.43%
30 0+ states
1.20% ≤ δc ≤ 1.75%
0.33%
0.30% - 0.90%
0.11% - 0.66%
0.08% - 0.63%
82 41 Nb 41
82 40 Zr 42
oblate
prolate
Amplitude (%)
I
II
III

Gamow-Teller β decay of ^{72}Kr

CERN/ISOLDE   I. Piqueras, Eur. Phys. J. A16(2003)313

$^{72}\mathrm{Kr} \rightarrow {}^{72}\mathrm{Br}$

$Q_{EC} = 5.040 \pm 0.375$ MeV

$0^+_{ground-state} \rightarrow 1^+$

$0^+_{first-excited} \rightarrow 1^+$

$E_{0_2^+} = 0.671$ MeV

$2^+_{yrast} \rightarrow 1^+$

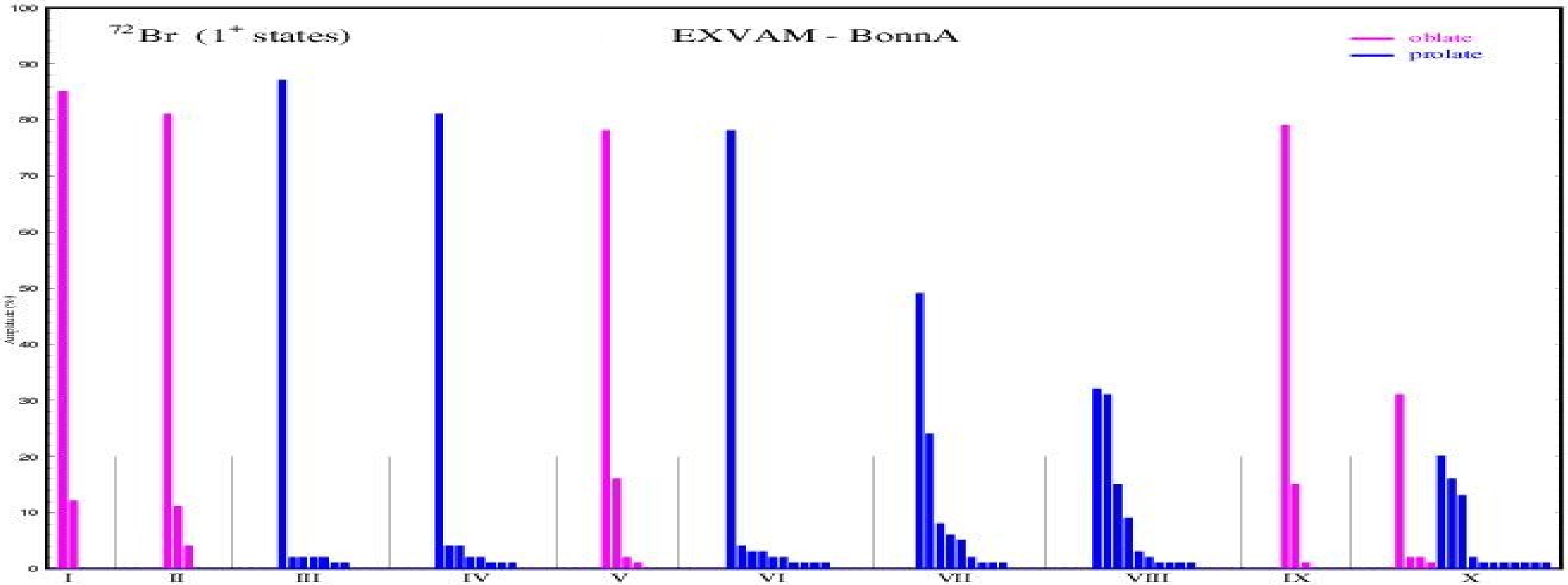
$E_{2_1^+} = 0.710$ MeV

$\rightarrow 2^+$

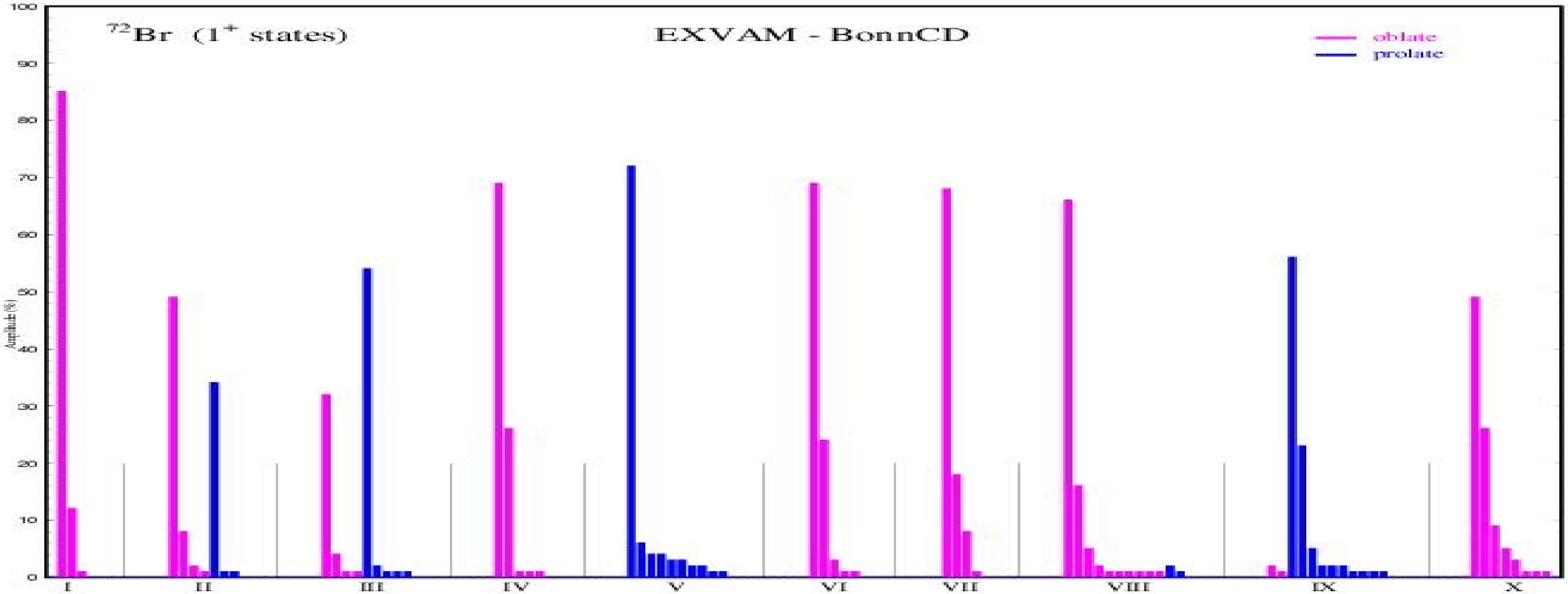
$\rightarrow 3^+$

The amount of mixing for the considered states of the ^{72}Kr nucleus (ms3).

$I[\hbar]$	Bonn A o-mixing	p-mixing	Bonn CD o-mixing	p-mixing
0_1^+	64(2)%	29(2)(1)(1)%	50(3)%	38(5)(3)%
0_2^+	35(2)%	57(3)(1)(1)%	49(2)%	46(3)%
2_1^+	92(1)%	6%	76(1)%	20(3)%



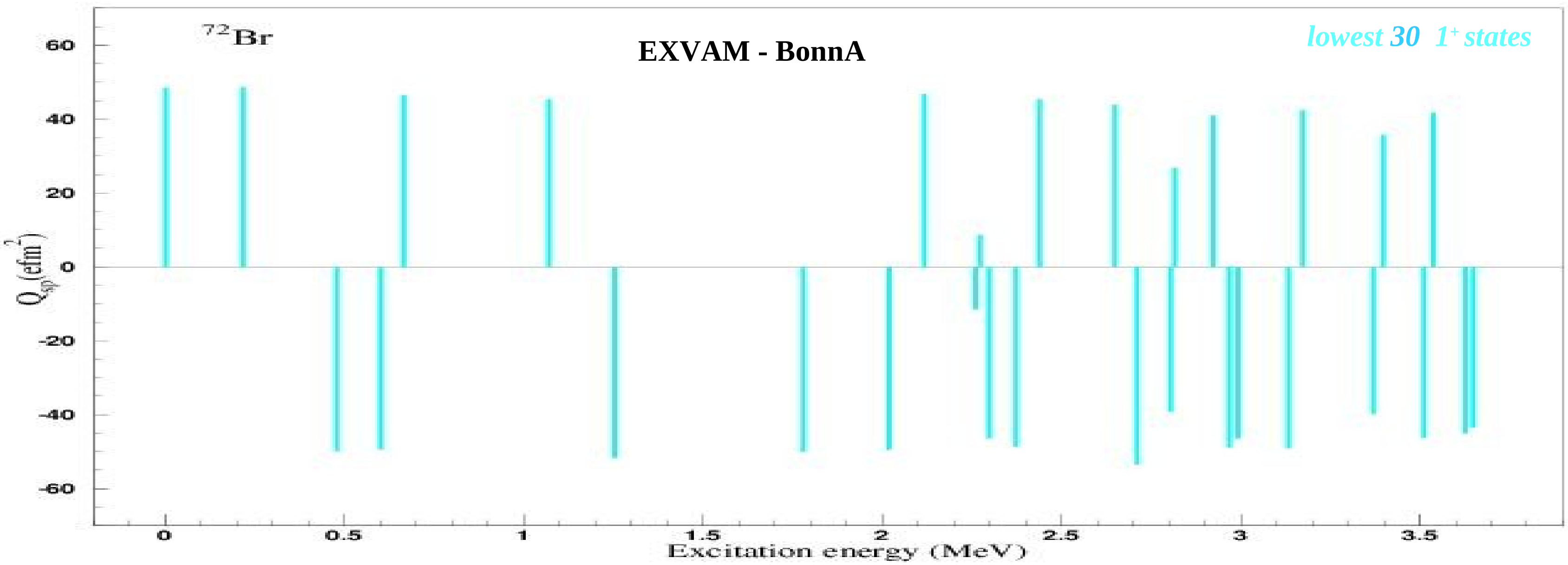
^{72}Br (1$^+$ states)
EXVAM - BonnA
oblate
prolate
Amplitude (%)
I
II
III
IV
V
VI
VII
VIII
IX
X



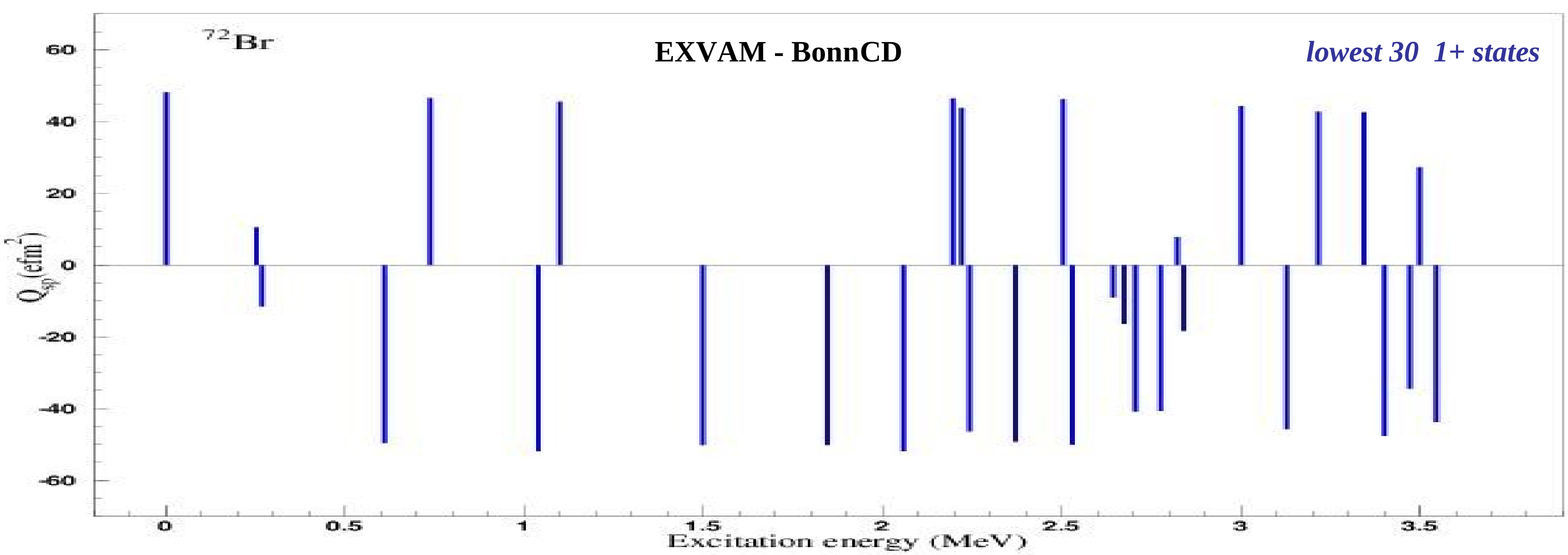
^{72}Br (1$^+$ states)
EXVAM - BonnCD
oblate
prolate
Amplitude (%)
I
II
III
IV
V
VI
VII
VIII
IX
X

The amount of mixing for the lowest calculated 1^+ states of ^{72}Br with significant B(GT) (Bonn A/Bonn CD).

o-mixing /p-mixing
85(12)%
81(11)(4)%
87(2)(2)(2)(2)(1)(1)%
81(4)(4)(2)(2)(1)(1)(1)%
78(16)(2)(1)%
78(4)(3)(3)(2)(2)(1)(1)(1)(1)%
49(24)(8)(6)(5)(2)(1)(1)(1)%
32(31)(15)(9)(3)(2)(1)(1)(1)(1)%
79(15)(1)%
31(2)(2)(1)%20(16)(13)(2)(1)(1)(1)(1)(1)(1)(1)%
85(12)(1)%
49(8)(2)(1)% 34(1)(1)%
32(4)(1)(1)%54(2)(1)(1)(1)%
69(26)(1)(1)(1)%
72(6)(4)(4)(3)(3)(2)(2)(1)(1)%
69(24)(3)(1)(1)%
68(18)(8)(1)%
66(16)(5)(2)(1)(1)(1)(1)(1)(1)% 2(1)%
2(1)% 56(23)(5)(2)(2)(2)(1)(1)(1)(1)%
49(26)(9)(5)(3)(1)(1)(1)%



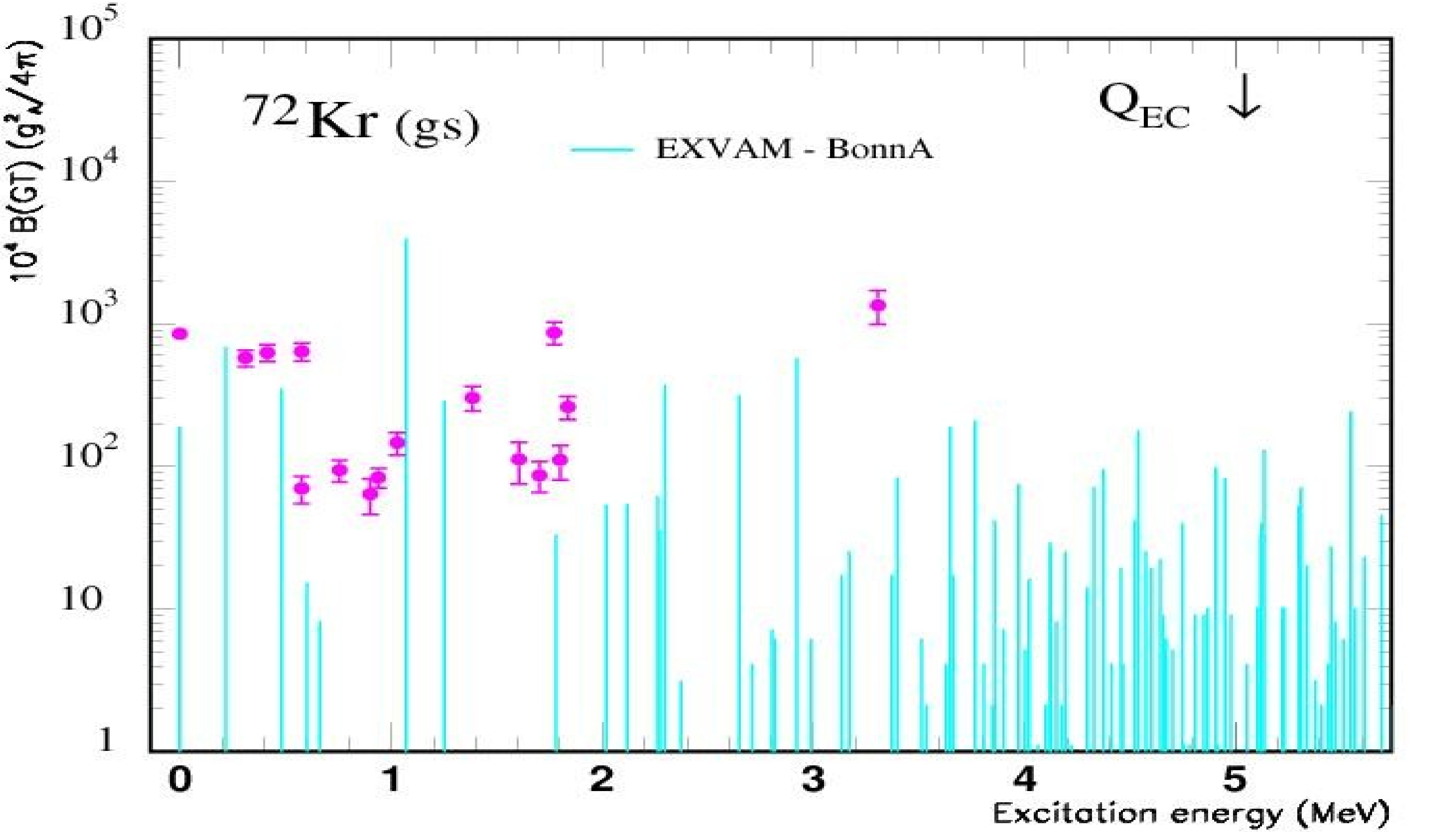
72Br
EXVAM - BonnA
lowest 30 1+ states
Q_sp (efm^2)
60
40
20
0
-20
-40
-60
0
0.5
1
1.5
2
2.5
3
3.5
Excitation energy (MeV)



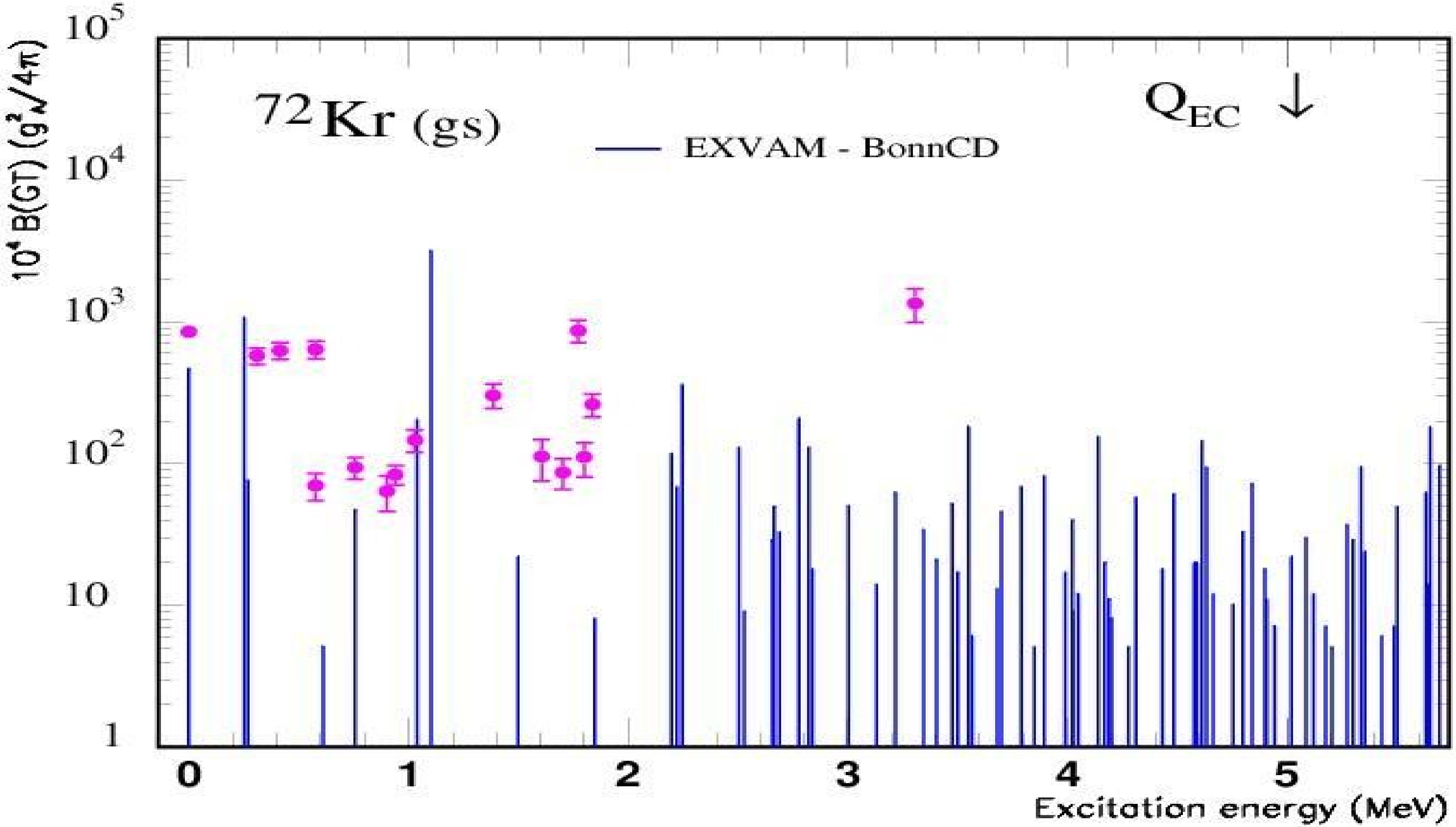
72Br
EXVAM - BonnCD
lowest 30 1+ states
Q_sp (efm^2)
60
40
20
0
-20
-40
-60
0
0.5
1
1.5
2
2.5
3
3.5
Excitation energy (MeV)

The spectroscopic quadrupole moments Q_2^{sp} (in efm^2) for the lowest 1^+ states of ^{72}Br (Bonn A/BonnCD).

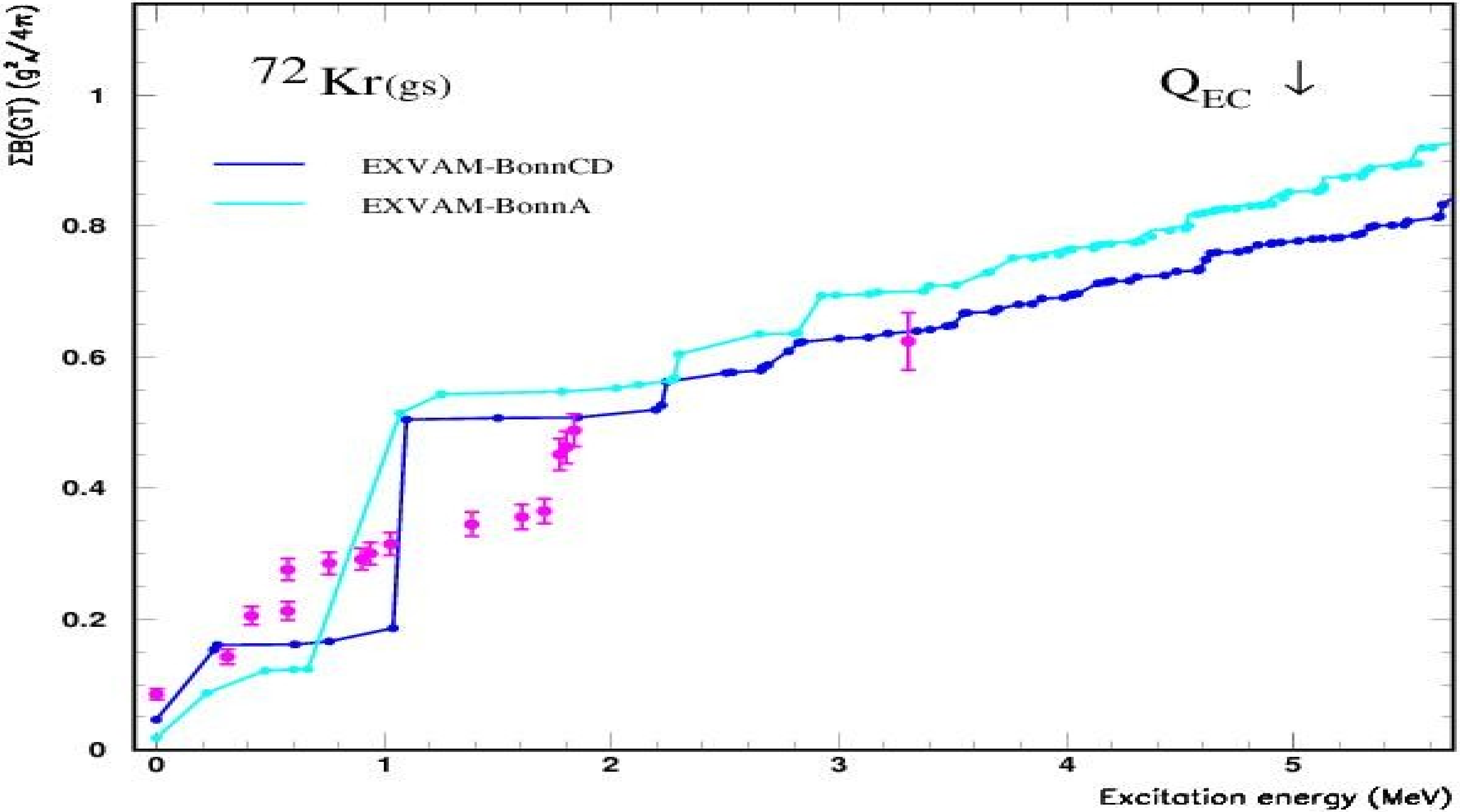
48.5	48.7	-49.9	-49.4	46.5	45.5	-51.6	-50.1	-49.5	46.8
-11.5	8.7	-46.5	-48.7	45.4	44.0	-53.5	-39.1	27.0	41.0
-48.9	-46.5	-49.2	42.5	-39.8	35.8	-46.3	41.8	-45.0	-43.5
48.2	10.5	-11.5	-49.6	46.6	-51.8	45.6	-50.2	-50.2	-51.8
46.5	43.8	-46.4	-49.2	46.3	-50.1	-9.0	-16.4	-40.8	-40.6
7.8	-18.4	44.3	-45.8	42.8	42.6	-47.7	-34.4	27.3	-43.8



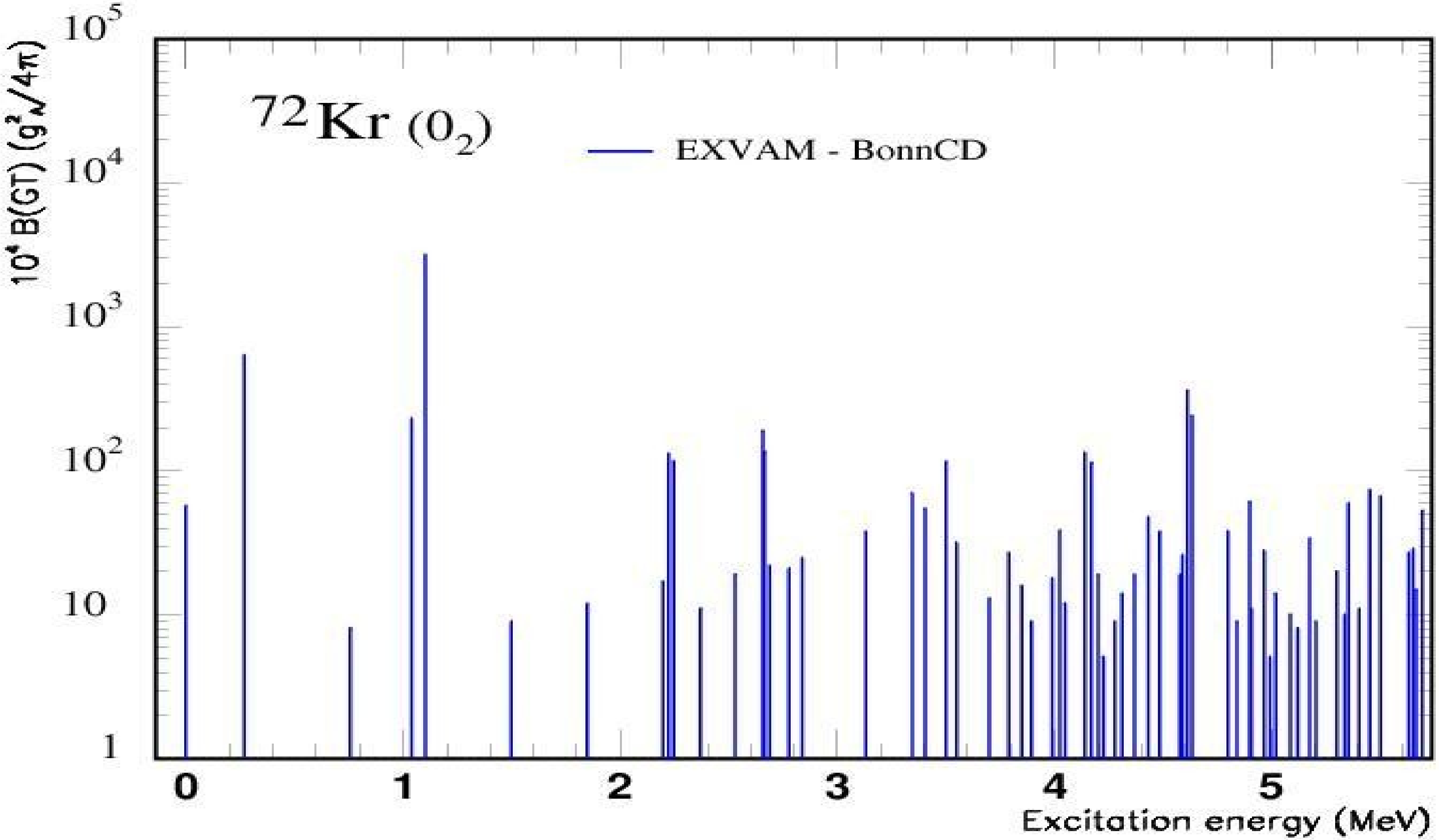
^{72}Kr (gs)
Q_{EC}
EXVAM - BonnA
10^4 B(GT) ($g_A^2/4\pi$)
Excitation energy (MeV)



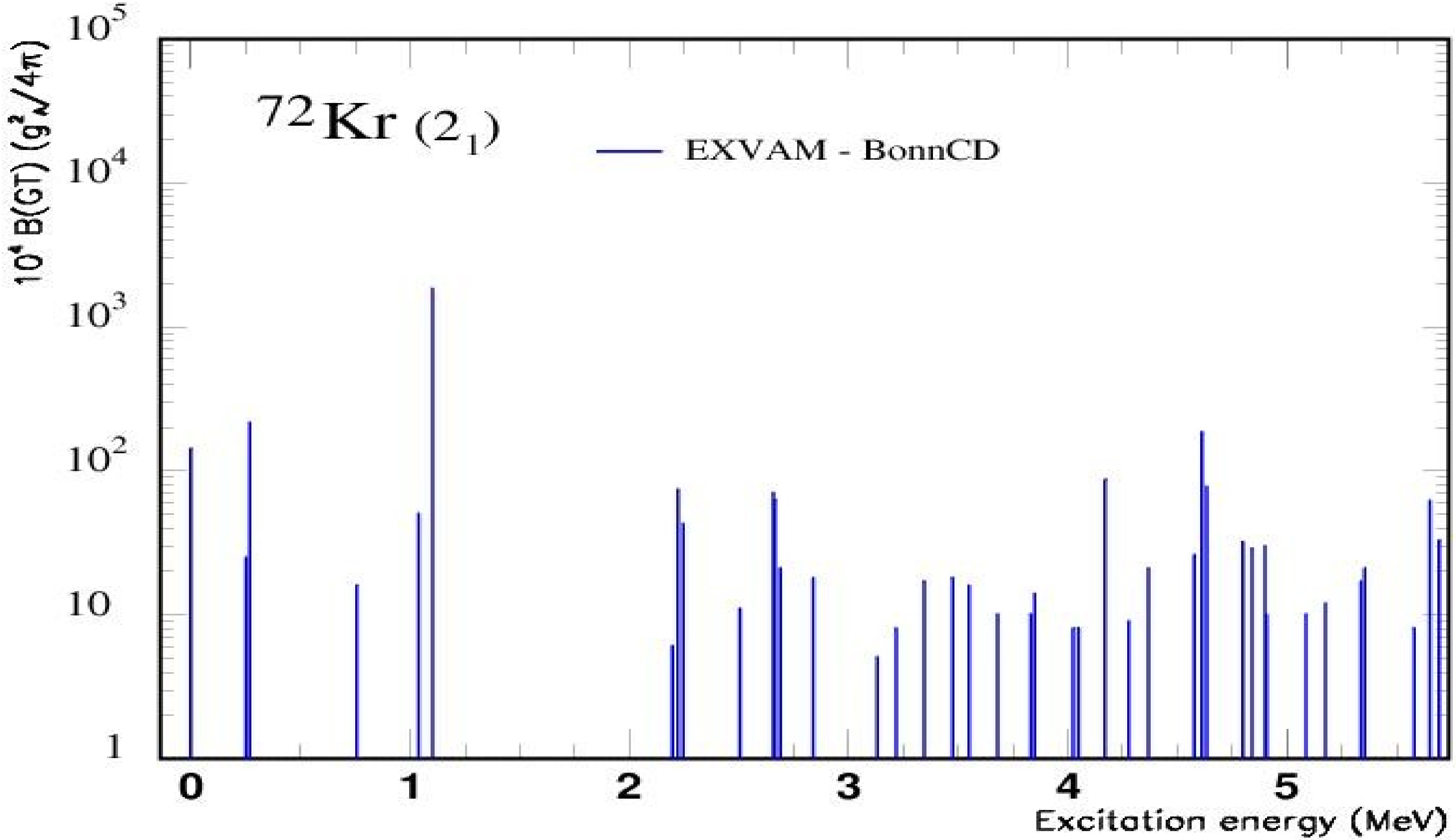
10^5
10^4
10^3
10^2
10
1
10^4 B(GT) $(g_A^2/4\pi)$
^{72}Kr (gs)
Q_{EC}
EXVAM - BonnCD
0
1
2
3
4
5
Excitation energy (MeV)



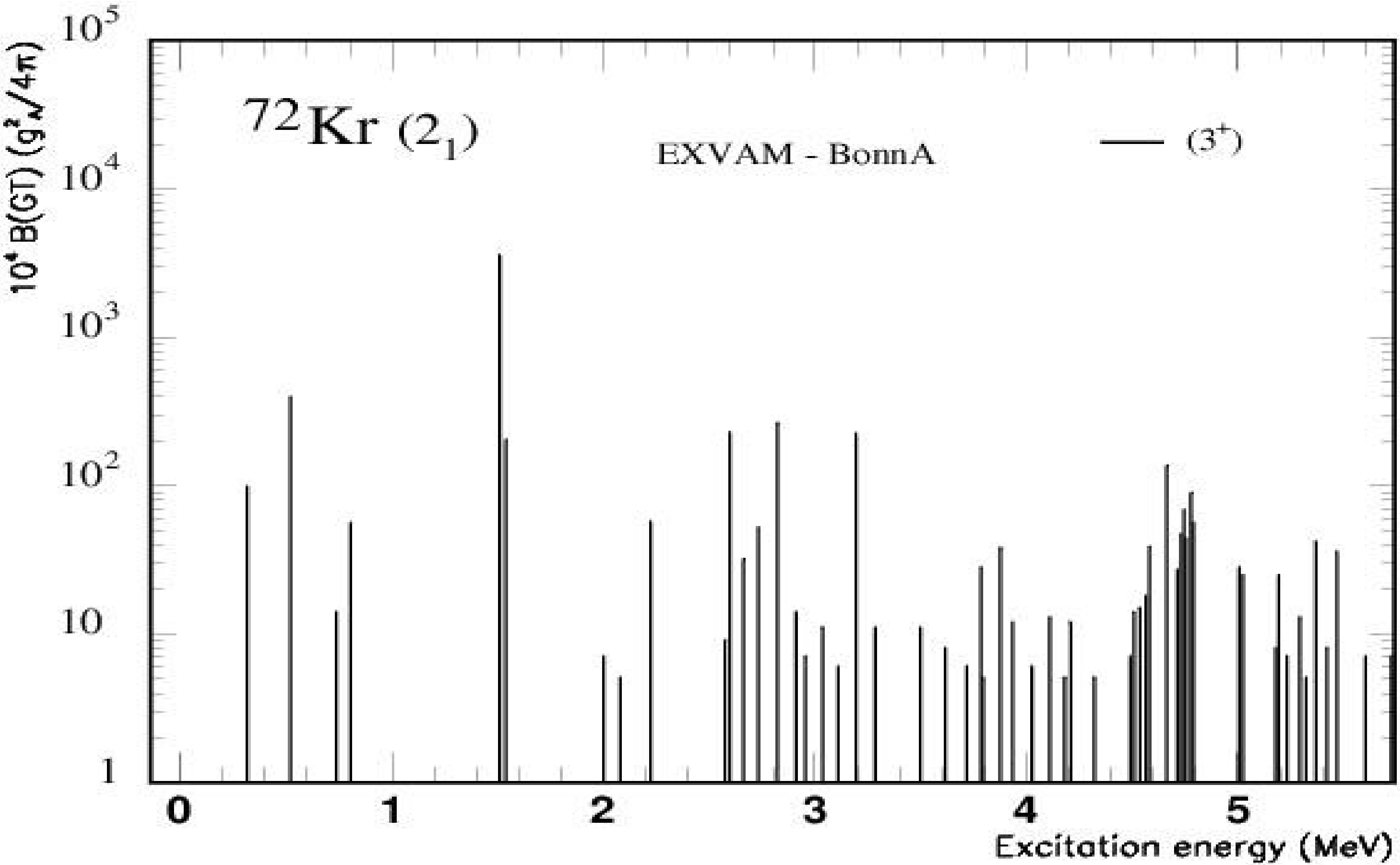
^{72}Kr(gs)
Q_{EC} ↓
EXVAM-BonnCD
EXVAM-BonnA
ΣB(GT) ($g_A^2/4\pi$)
Excitation energy (MeV)
0
0.2
0.4
0.6
0.8
1
0
1
2
3
4
5



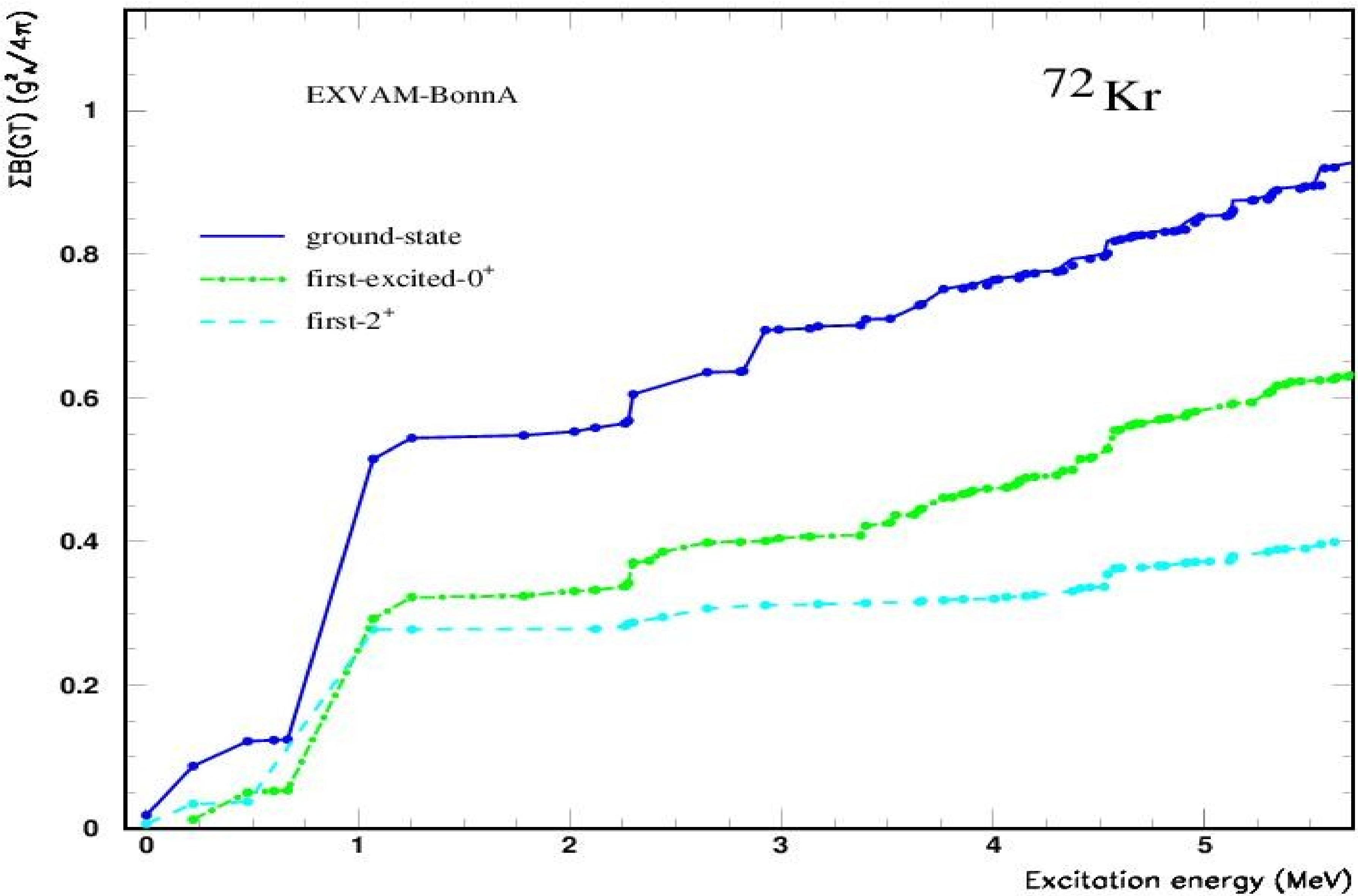
^{72}Kr (0_2)
EXVAM - BonnCD
10^4 B(GT) $(g^2_A/4\pi)$
Excitation energy (MeV)
10^5
10^4
10^3
10^2
10
1
0
1
2
3
4
5



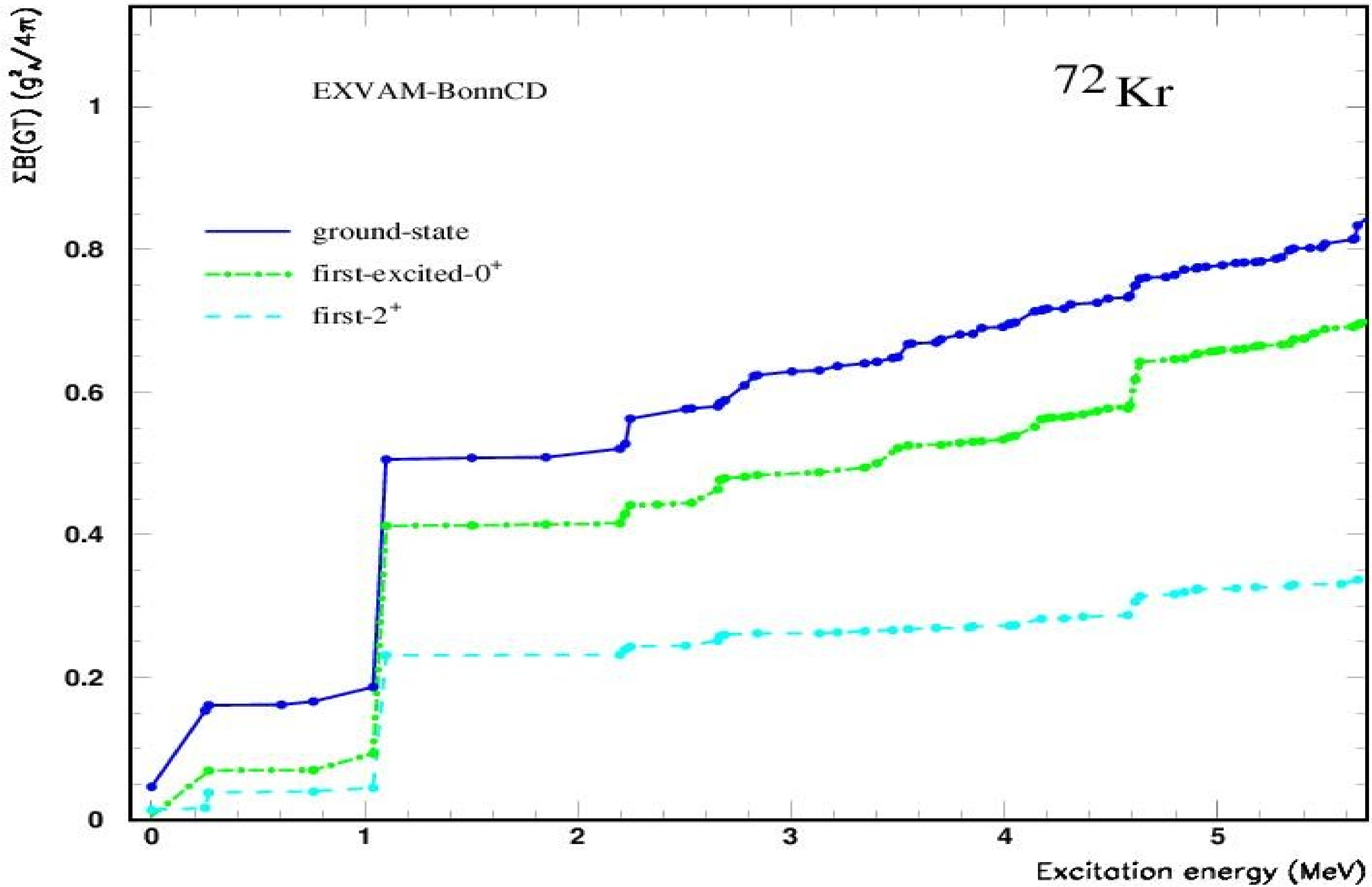
^{72}Kr (2_1)
EXVAM - BonnCD
10^4 B(GT) $(g_A^2/4\pi)$
10^5
10^4
10^3
10^2
10
1
0
1
2
3
4
5
Excitation energy (MeV)

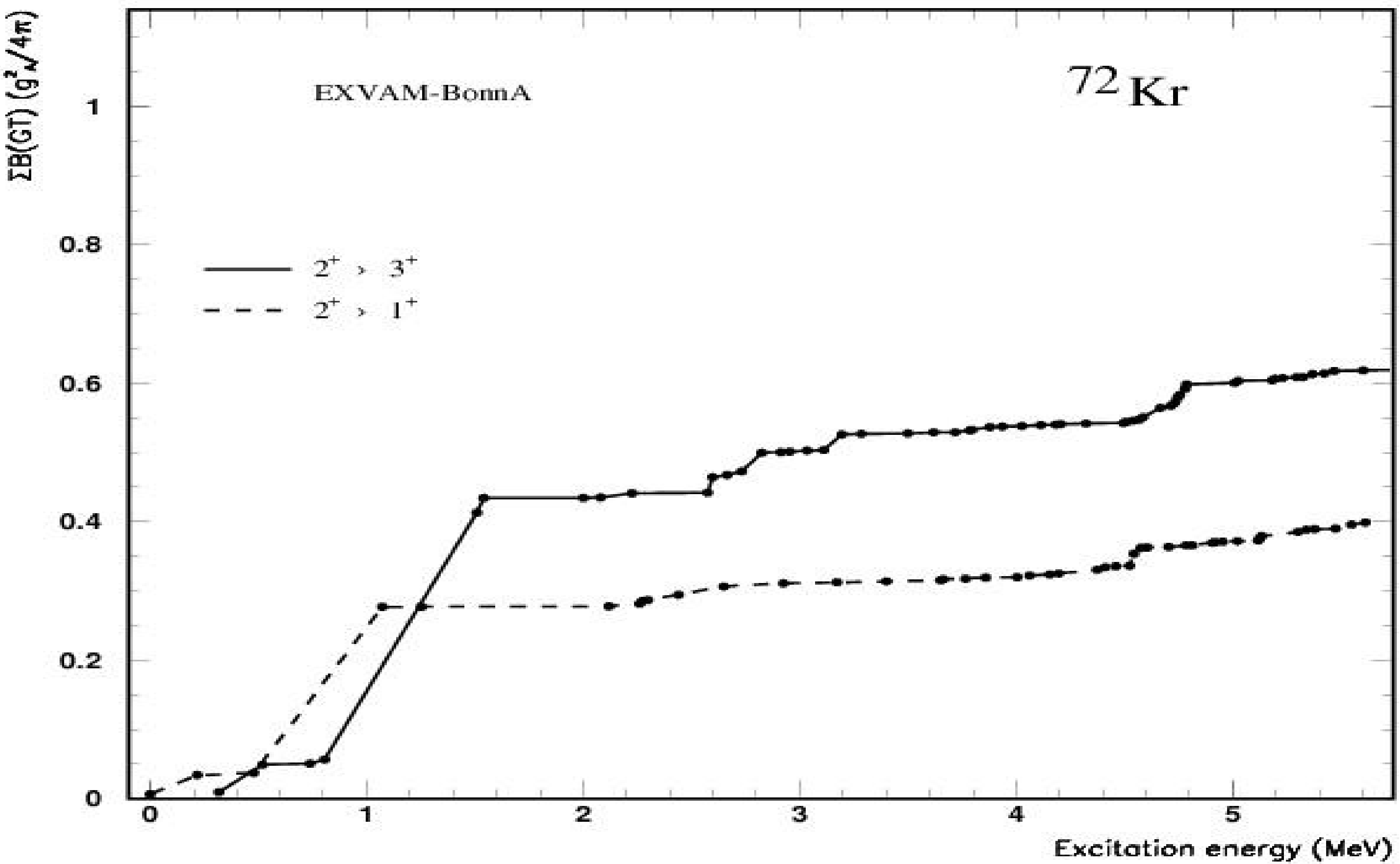


^{72}Kr (2_1)
EXVAM - BonnA
(3^+)
10^4 B(GT) $(g_A^2/4\pi)$
Excitation energy (MeV)



$\Sigma B(GT)\ (g_A^2/4\pi)$
EXVAM-BonnA
^{72}Kr
ground-state
first-excited-0$^+$
first-2$^+$
0
0.2
0.4
0.6
0.8
1
0
1
2
3
4
5
Excitation energy (MeV)


EXVAM-BonnCD
^{72}Kr
ground-state
first-excited-0^+
first-2^+
$\Sigma B(GT)$ $(g_A^2/4\pi)$
Excitation energy (MeV)
0
0.2
0.4
0.6
0.8
1
0
1
2
3
4
5



ΣB(GT) (g²A/4π)
EXVAM-BonnA
72 Kr
2+ > 3+
2+ > 1+
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
Excitation energy (MeV)

$$\frac{1}{T_{1/2}} = \frac{g_A^2}{D} \sum_i f(Z, E_i) | \left\langle 1_i^+ || \beta^+ || 0^+ \right\rangle |^2$$

D = 6146 s $\qquad$ g_A = 1.26

$T_{1/2}^{exp}$ = 17.1(2) s

$T_{1/2}$ (gs) = 20.8 s (Bonn A) $\qquad$ 18.9 s (Bonn CD)

$T_{1/2}$ (first-excited 0^+) = 17.3 s (Bonn A) $\qquad$ 12.9 s (Bonn CD)

$T_{1/2}$ (yrast $2^+ \rightarrow 1^+$) = 18.7 s (Bonn A) $\qquad$ 21.6 s (Bonn CD)

$T_{1/2}$ (yrast 2+ $\rightarrow$ 3+) = 19.5 s (Bonn A)

$$\lambda = \ln 2/K \, \Sigma_i [(2J_i+1)\, e^{-E_i/(kT)}] / G(Z,A,T) \, \Sigma_j B_{ij} \Phi_{ij}$$

i – parent states *j – daughter states*

$$G(Z,A,T) = \Sigma_i e^{-E_i/(\kappa T)}$$ *(partition function of the parent nucleus)*

$Bij = Bij\ (GT)$

Φ_{ij} *– phase space integral*

X-ray bursts $T < 2$ *GK*

In the astrophysical environment of the X-ray bursts the effect of the decay of the lowest excited states of ^{72}Kr is within the uncertainty of the ground-state half-life

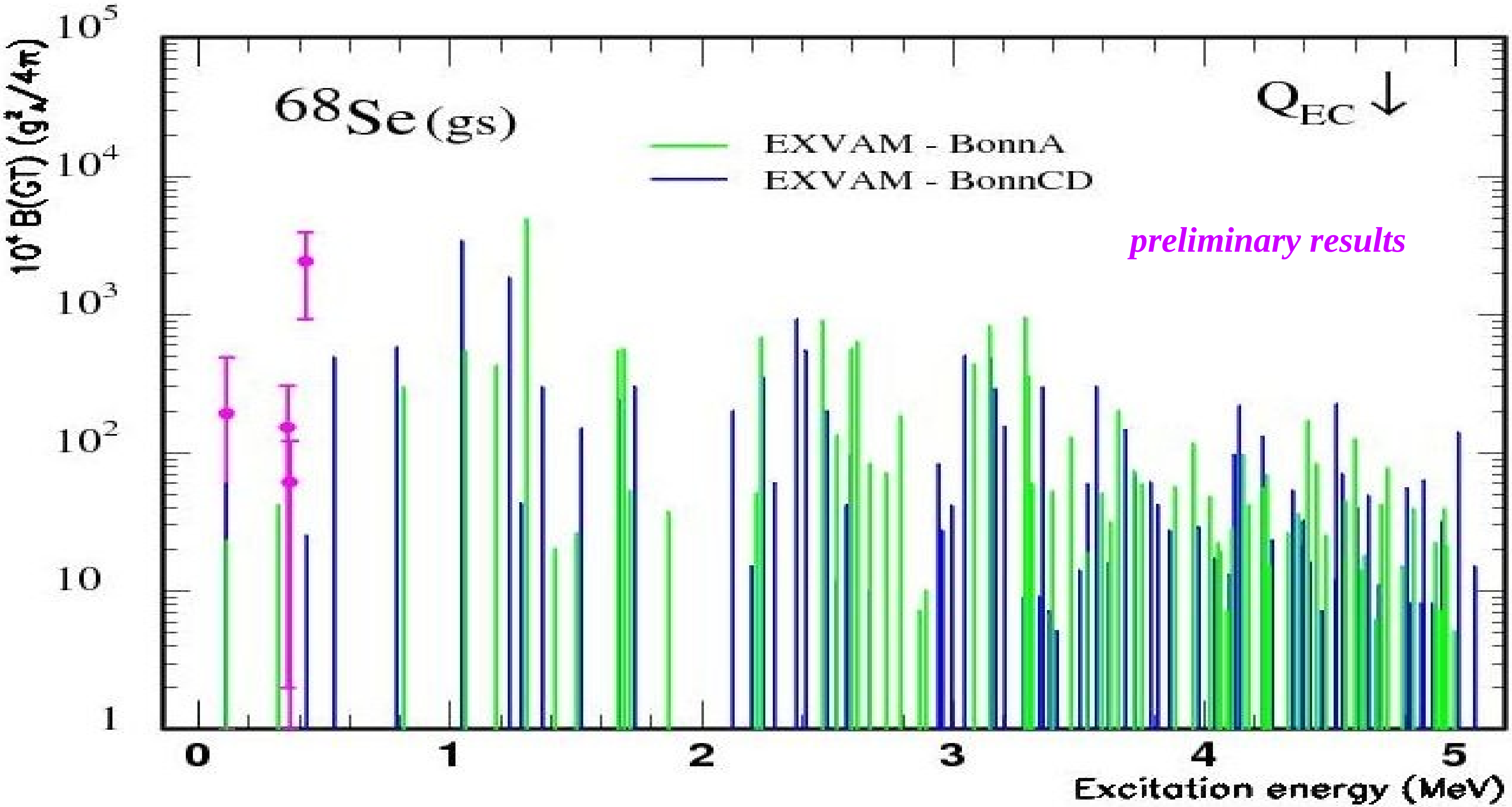
→ ***first theoretical results predicting no influence from the lowest excited states on the effective half-life***

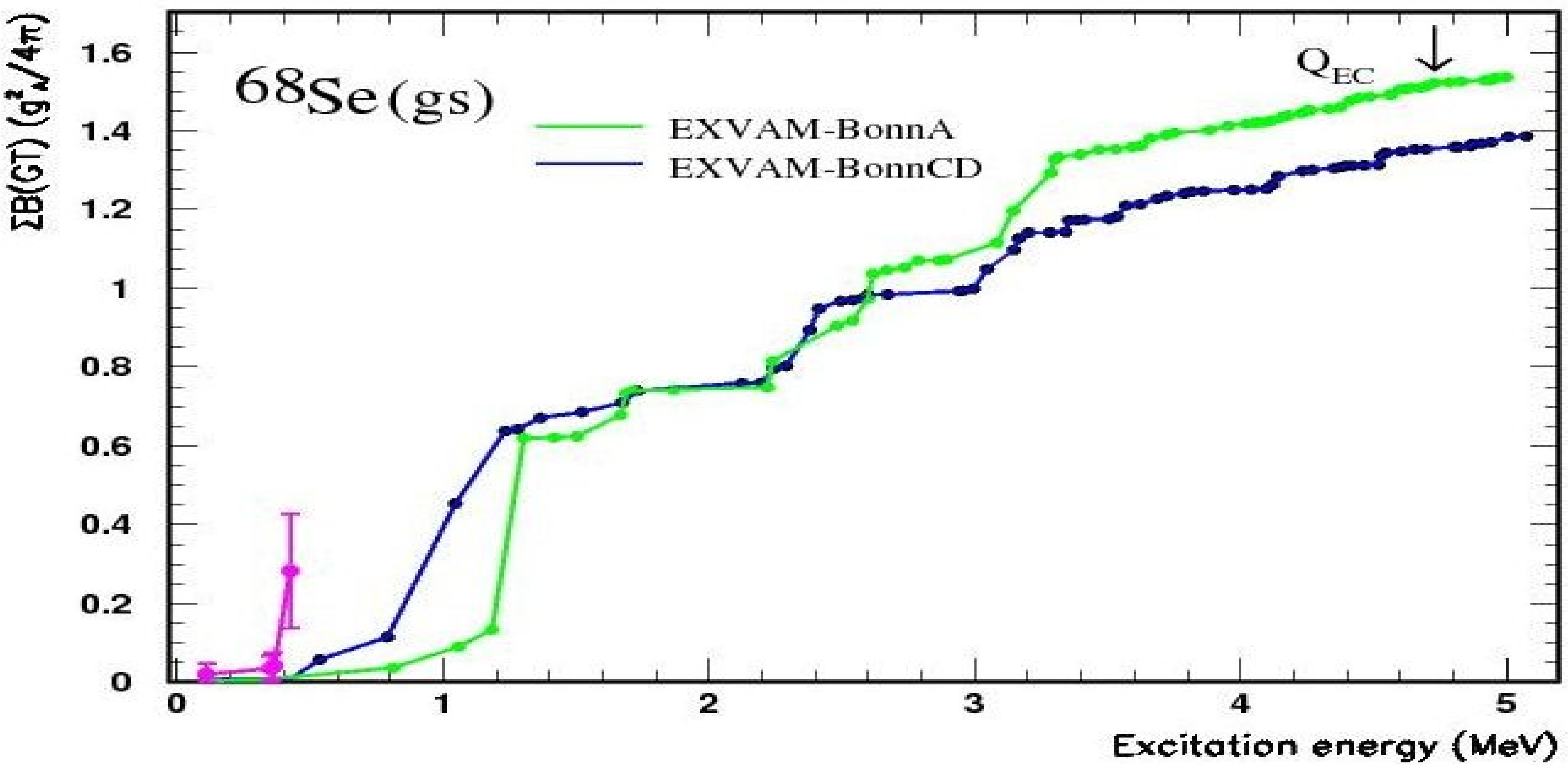
A. Petrovici et al., Phys. Rev. C78 (2008) 044315

Gamow-Teller β decay of ^{68}Se

CERN/ISOLDE P. Baumann et al., Phys. Rev. C50 (1994) 1180

$^{68}Se \rightarrow {}^{68}As$ $0^{+} \rightarrow 1^{+}$ $Q_{EC} = 4.730 \pm 0.310$ MeV





$T_{1/2}^{exp} = 35.5(7)$ s $\qquad$ $T_{1/2}^{BonnCD} = 33.9$ s

Summary and outlook

• improved formalism for calculating the isospin-symmetry-breaking effect on the superallowed Fermi β decay was applied for the first time to the ^{82}Nb $\rightarrow$ ^{82}Zr decay within the *complex* Excited Vampir model describing self-consistently both the analog and non-analog branches

• self-consistent approach to the Gamow-Teller β decay of the ground state, first-excited 0+ and yrast 2^+ of ^{72}Kr to the 1^+ (and for yrast 2^+, also to the 2^+ and 3^+) states in the β window in ^{72}Br gives good agreement with the available data

• at the temperatures of the X-ray bursts the decay of the lowest excited states will not influence the effective half-life of ^{72}Kr

• preliminary results on the Gamow-Teller β decay of the ground state of ^{68}Se to ^{68}As (dominated by shape coexistence) indicate agreement with the measured half-life

• the uncertainties in the effective interaction require systematic investigations

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