Ions

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<u>Content</u>:

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Part I

Subjects Treated:

- **Why ions in the vacuum chamber? Ionisation process**
- **Ion motions due to the EM fields of the stored beam**
- **Impact of residual gases & ions on the stored beam**

Ultra-high vacuum/Residual gases

A stored beam should ideally not get any disturbance along its trajectory to reach the maximal ring performance → Need of Ultra-High Vacuum (UHV) inside the beam duct

Today, we can reach the vacuum level of $10^{-9} \sim 10^{-10}$ mbar, but the residual gases could still become significant sources of beam perturbations.

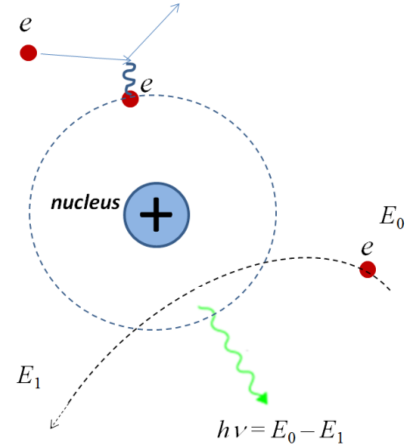
- Collisions, scattering (elastic and inelastic) → lifetime drops/beam losses
- Ionisation
- Two-beam interactions → emittance blow-ups/beam losses

With the general trend of using narrower beam ducts (e.g. in insertion devices), beam physics and technical issues related to vacuum (vacuum conductance, NEG coating, pressure bumps, outgassing due to heating, …), hence to ions, still remain as the 1st degree concerns for accelerators.

Collisions with residual gases

Several mechanisms of particle collision with residual gases:

- Møller scattering: Due to an atomic electron
- Rutherford scattering: Due to the EM field of a nucleus (elastic)
- Bremsstrahlung: Due to the EM field of a nucleus (inelastic)



The collision rate is given by

$$\boxed{\frac{1}{\tau_{Col}} = \sigma_{Total} \cdot d_m \cdot \beta c}$$

where

$$\begin{cases} \sigma_{Total} & : \text{Total collision cross section } [m^2] \\ d_m & : \text{Density of molecule of specie } m \ [m^{-3}] \\ \beta c & : \text{Velocity of the stored beam } [m/s] \end{cases}$$

$$\sigma_{Total} = Z_i \cdot \sigma_{Møller} + \sigma_{Rutherford}(Z_i) + \sigma_{Bremsstrahlung}(Z_i) \qquad (Z_i \text{ : Atomic number of the molecule})$$

d_m is related to the partial pressure P_m at 20°C via $d_m\,[m^{-3}] = 2.47\times10^{22} \cdot P_m\,[mbar]$

In case the molecule consists of several atoms and/or there are several species in the residual gases, we take a sum of all contributions:

$$\frac{1}{\tau_{Col}} = \sum_m \sum_k \frac{1}{(\tau_{Col})_{mk}}$$

(m: Molecule species, k: Different atoms in a molecule)

Ionisation of residual gases

In a similar way, we can obtain the ionisation rate $1/\tau_{ion,m}$ for a stored particle, or equivalently the ionisation time $\tau_{ion,m}$ by replacing σ_{Col}^{Total} by the **ionisation cross section** $\boldsymbol{\sigma_{ion,m}}$ in the previous formula:

$$\frac{1}{\tau_{ion,m}} = d_m \cdot \sigma_{ion,m} \cdot \beta c$$

In general, $\sigma_{ion,m}$ only depends on the species *m* and the velocity of the stored particle βc:

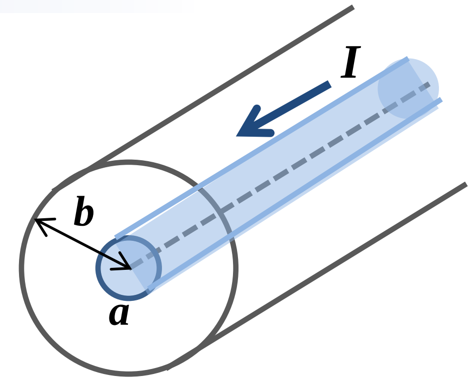
$$\sigma_{ion,m} = 4\pi \left(\frac{\hbar}{m_e c} \right)^2 \cdot (M^2 \cdot x_1 + C \cdot x_2)$$

where m_e : Electron mass, $4\pi \left(\frac{\hbar}{m_e c} \right)^2 = 1.874\times10^{-24}\ [\mathrm{m}^2]$, $x_1 = \beta^2 \cdot \ln(\frac{\beta^2}{1-\beta^2}) - 1$, $x_2 = \beta^2$

M and C are molecule dependent constants:

Molecule	M^2	C	Z	A
H_2	0.5	8.1	2	2
N_2	3.7	34.8	14	28
CO	3.7	35.1	14	28
O_2	4.2	38.8	16	32
H_2O	3.2	32.3	10	18
CO_2	5.75	55.9	22	44
C_6H_4	17.5	162.4	46	76

Beam-induced EM fields and their characteristics



Let us review the static Electro-Magnetic (EM) field created by a round coasting beam of radius a and current I in a circular chamber of radius b:

Using $\iint \vec{E}\cdot d\vec{a} = \int \frac{\rho}{\varepsilon_0} dV$ (ρ: Charge density) and $\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I$,

we get $E_r = \begin{cases} \dfrac{e\lambda}{2\pi\varepsilon_0}\dfrac{r}{a^2} & (0<r<a) \\ \dfrac{e\lambda}{2\pi\varepsilon_0}\dfrac{1}{r} & (a<r) \end{cases}$ and $B_\varphi = \begin{cases} \dfrac{\mu_0 I}{2\pi a^2}\cdot r & (0<r<a) \\ \dfrac{\mu_0 I}{2\pi r} & (a<r) \end{cases}$

where $\lambda = I/(e\beta c)$: Line density of electrons, βc: Speed of electrons

An ion (of $+1e$) having longitudinal speed of $\beta_i c$ gets a force from the EM field of

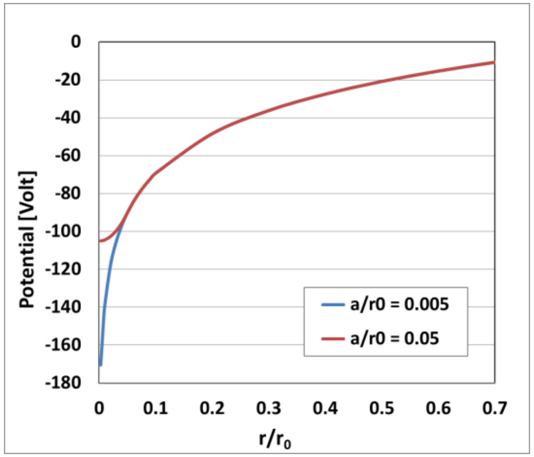
$$F_r^E = eE_r \quad \text{and} \quad F_r^B = e\beta_i c B_\varphi$$

For all values of r, $\dfrac{F_r^B}{F_r^E} = \beta_i \beta \simeq \beta_i \ll 1$ as ions move relatively slow. Therefore, the magnetic force due to the beam can usually be ignored.

Ion trapping: Coasting beam

Using the static electric field E_r created by the beam, the electric potential created by a coasting beam is given by

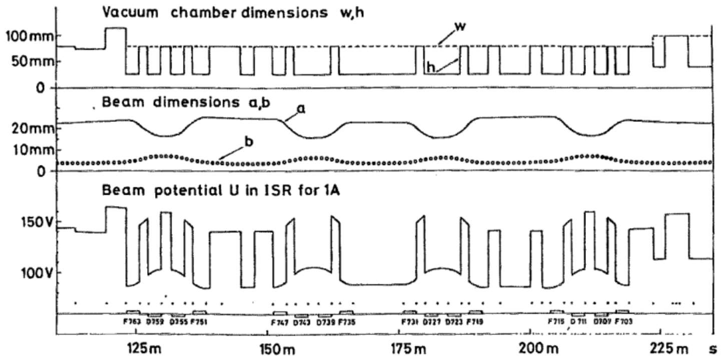
$$V(r) = -\int_{r_0}^{r} E_r \, dr = \begin{cases} \dfrac{e\lambda}{2\pi\varepsilon_0}\left(\dfrac{r^2}{2a^2} - \dfrac{1}{2} + \ln\dfrac{a}{r_0}\right) & (0 < r < a) \\ \dfrac{e\lambda}{2\pi\varepsilon_0} \cdot \ln\dfrac{r}{r_0} & (a < r) \end{cases}$$



- Evaluating the depth of the potential for realistic cases (see Fig.), one finds that the ions having only thermal energy in the order of $k_B T$ ($\sim 10^{-21}$ J) cannot escape from the potential having the order of some tens of volt (therefore the energy of $\sim 10^{-18}$ J).

- Potential depth increases as the beam emittance decreases and the beam intensity increases

Neutralisation



Beam potential calculated for the ring ISR. Locations of clearing electrodes are indicated by dots (*Y. Baconnier, CERN 85-19 (1985), p.267*)

- As the trapping of ions progresses, the potential depth decreases due to neutralisation of opposing charges, which saturates the trapping process. The degree of neutralisation is defined by

$$\boxed{\eta = \frac{N_i}{N}}$$

N_i and N: Total number of ions and electrons in the ring

If all ions are +1e charged, then $0 \leq \eta \leq 1$.

- For a proton ring, on the contrary, the electrons created in ionisation could be trapped. Ions instead could be repelled by the proton potential and bombard the chamber surface, which in turn induce outgassing. This could lead to a cascading phenomenon called the "*pressure bump*".

Bassetti-Erskine formula

Analytical expressions for the transverse electric fields E_x and E_y created by an electron bunch having Gaussian distributions were derived by M. Bassetti and E.A. Erskine (*CERN-ISR-TH/80-06*).

$$\rho(x,y) = \frac{Q}{2\pi\sigma_x\sigma_y}\cdot\exp\left[-\left(\frac{x^2}{2\sigma_x^2}+\frac{y^2}{2\sigma_y^2}\right)\right]$$

(Q: Total charge over the transverse distribution)

Starting from the equation $\nabla^2\phi = \frac{\rho}{\varepsilon_0}$ and assuming $\sigma_x > \sigma_y$, the potential ϕ was solved analytically in an integral form, leading to

$$E_x - iE_y = -i\frac{Q}{2\varepsilon_0\sqrt{2\pi(\sigma_x^2-\sigma_y^2)}}\cdot\left\{w(a+ib) - e^{[-(a+ib)^2+(ar+ib/r)^2]}\cdot w(ar+ib/r)\right\}$$

$$\left(a = \frac{x}{\sqrt{2(\sigma_x^2-\sigma_y^2)}},\ b = \frac{y}{\sqrt{2(\sigma_x^2-\sigma_y^2)}},\ r = \frac{\sigma_y}{\sigma_x}\right)$$

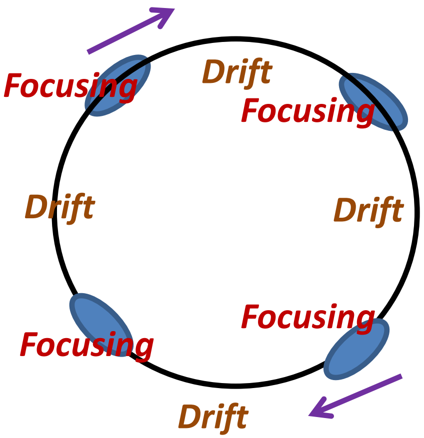
$w(z)$: Complex error function

$$\boxed{E_x = \frac{Q}{2\pi\varepsilon_0\sigma_x(\sigma_x+\sigma_y)}\cdot \boldsymbol{x} + \textit{higher-order terms},\quad E_y = \frac{Q}{2\pi\varepsilon_0\sigma_y(\sigma_x+\sigma_y)}\cdot \boldsymbol{y} + \textit{higher-order terms}}$$

Since electron beams are usually Gaussian and the transverse ion distributions are often approximated as those of the stored beam, the Bassetti-Erskine formula is frequently used in evaluating the electric forces felt by the two beams.

Ion trapping: Bunched beam

With bunched beams, ions are attracted during passage of a bunch and drift freely in between two bunches (in places where there are no magnets).



Transverse motions of an ion thus resemble those of a circulating electron. Their stability (i.e. trapping) can be argued using transfer matrices in the linear approximation.

Consider the vertical motion of an (+1e charged) ion in a symmetric beam filling.

- During passage of a bunch, Newton's equation of an ion reads

$$M_{ion}\ddot{y}_i = e(E_e)_y = -e\frac{Q}{2\pi\varepsilon_0\sigma_y(\sigma_x+\sigma_y)}\cdot y_i = -\frac{N}{n_b L_b}\frac{2r_p c^2 m_p}{\sigma_y(\sigma_x+\sigma_y)}\cdot y_i$$

where $M_{ion} = A\cdot m_p$, N: Total number of stored electrons, n_b: Number of bunches, r_p: Classical proton radius (= $e^2/4\pi\varepsilon_0 m_p c^2$), L_b: Total bunch length, m_p: Proton mass

Namely, $$\begin{pmatrix} y_i \\ \dot{y}_i \end{pmatrix}_{new} = \begin{pmatrix} 1 & 0 \\ -a & 1 \end{pmatrix}\begin{pmatrix} y_i \\ \dot{y}_i \end{pmatrix}_{old}$$ with $$a = \frac{N}{n_b}\frac{2r_p c}{\beta\sigma_y(\sigma_x+\sigma_y)}\frac{1}{A}$$ (βc: Speed of electrons)

- In between two bunches, $$\begin{pmatrix} y_i \\ \dot{y}_i \end{pmatrix}_{new} = \begin{pmatrix} 1 & \tau \\ 0 & 1 \end{pmatrix}\begin{pmatrix} y_i \\ \dot{y}_i \end{pmatrix}_{old}$$ with $\tau = 2\pi R/n_b\beta c$ (R: Ring radius)

Critical mass

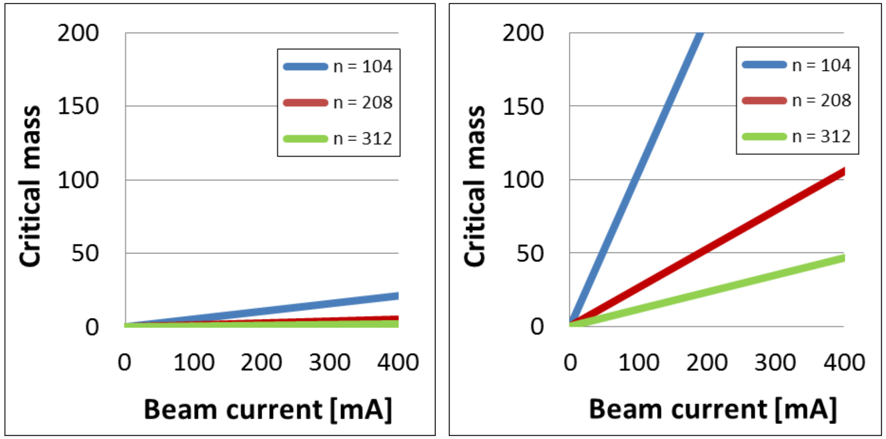
Transfer matrix for one period is therefore
$$M_{period} = \begin{pmatrix} 1 & \tau \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -a & 1 \end{pmatrix}$$

Condition for any linear motions to be bounded $-2 \le Tr(M_{period}) \le 2$ leads to

$$\boxed{A \ge A_c \equiv \frac{N}{n_b} \frac{r_p}{n_b} \frac{\pi R}{\beta^2 \sigma_y (\sigma_x + \sigma_y)}}$$

(A_c: Critical mass)

- Ions having $A < A_c$ cannot be trapped
- Since $A_c \propto 1/n_b^2$, ions do not tend to be trapped in a few-bunch mode
- Since $A_c \propto 1/(\sigma_x \cdot \sigma_y) \propto 1/\varepsilon_H^2$ (ε_H: Horizontal emittance), ions do not tend be trapped in a low-emittance ring
- Evidence of A_c observed in ADONE (1.5 GeV e+e-) in Frascati, Italy (*M.E. Biagini et al., 1980*)



Comparison at two different values of the horizontal emittance ε_H (Left: 4 nm, Right 0.2 nm)
Circumference = 354 m of SOLEIL.

Betatron tune shifts

A cloud of trapped ions generally gives a transversely focusing force to the stored beam, inducing betatron tune shifts $\Delta\nu_{x,y}$.

They can be evaluated by the well-known formula

$$\Delta\nu = \frac{1}{4\pi}\oint \beta(s)\cdot\Delta k(s)\, ds$$

where $\Delta k(s)$ represents the quadrupolar errors in a ring.

Assuming that the ions have the same Gaussian distributions as the electrons and are charged to +1e, we can use the Bassetti-Erskine formula to get the focusing strength $\Delta k_i(s)$ due to ions,

$$(\Delta k_i)_{x,y}(s) = \frac{1}{E_0/e}\frac{\partial(E_i)_{x,y}}{\partial x, y} = \frac{1}{E_0/e}\frac{d_i}{\varepsilon_0}\cdot\frac{e}{1+\sigma_{x,y}/\sigma_{y,x}}$$

(d_i [m^{-3}] : ion density)

Similarly, the focusing strength $\Delta k_{SC}(s)$ due to an electron beam's own space-charge force is given by

$$(\Delta k_{SC})_{x,y}(s) = \frac{1}{\gamma^2}\frac{1}{E_0/e}\frac{d_e}{\varepsilon_0}\cdot\frac{e}{1+\sigma_{x,y}/\sigma_{y,x}}$$

(d_e [m^{-3}] : electron density)

However, since usually $\frac{d_e}{d_i}\cdot\frac{1}{\gamma^2} \ll 1$, we have $(\Delta\nu_{x,y}^{SC}) \ll (\Delta\nu_{x,y}^{ions})$

Ion motions

- When $A >> A_c$ and if there are no magnetic fields, the drift motions in between two bunches can be neglected. The resultant motions become approximately harmonic oscillations:

$$\ddot{u}_i \approx -\omega_{iu}^2 u_i \quad \text{with} \quad \omega_{iu}^2 = \frac{2\lambda r_p c^2}{A} \frac{1}{\sigma_u(\sigma_x + \sigma_y)} \quad (u = x, y)$$

$\lambda = I/(e\beta c)$: Line density of electrons

- Inside a bending magnet where $\boldsymbol{B} \neq 0$, the equations of motion are then

$$\begin{pmatrix} \ddot{s} \\ \ddot{x} \\ \ddot{y} \end{pmatrix} = \begin{pmatrix} 0 \\ -\omega_{ix}^2 \cdot x \\ -\omega_{iy}^2 \cdot y \end{pmatrix} + \omega_c \begin{pmatrix} -\dot{x} \\ \dot{s} \\ 0 \end{pmatrix} \quad \text{with} \quad \omega_c = \frac{eB}{M_{ion}}$$

Solutions are off-centred sinusoidal motions for *x* and *s* at the frequency $\omega = \sqrt{\omega_{ix}^2 + \omega_c^2}$

In particular, ions drift *longitudinally* at the average speed of

$$< \dot{s} > = \left(\frac{\omega_{ix}}{\omega_c}\right)^2 \left[\omega_c \cdot x(0) + \dot{s}(0)\right]$$

- Ions generally tend to move longitudinally towards a minimum of the potential *V(r)* of the stored beam. Since $V(0) = e\lambda/(2\pi\varepsilon_0)\cdot[\ln(a/r_0) - ½]$, they gather where a/r_0 is small (i.e. where the stored beam size is small and the chamber aperture is large), called *neutralisation spots*.

Ion distributions

Many studies assume that ions created by the collision with beam have the same transverse distributions as the beam (usually Gaussian).

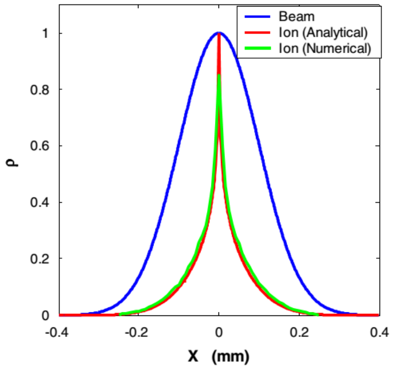
The above assumption is correct regarding the initial ion distribution when ions are created.
However, an equilibrium reached under the beam electric potential turns out to be significantly different from the original Gaussian distribution due to focusing
(*P.F. Tavares, CERN PS/92-55 LP 1992; L. Wang, Y. Cai, T. Raubenheimer, H. Fukuma, PRSTAB* ***14*** *084401, 2011*)

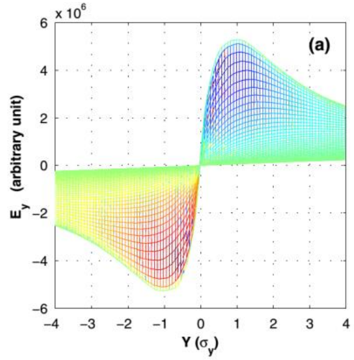
Analytically we find in the linear regime,

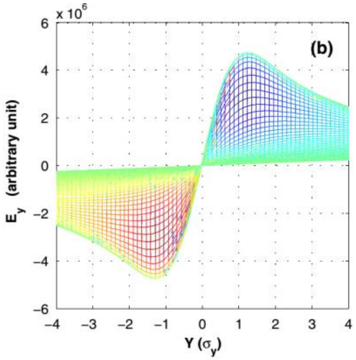
$$\rho(y) = \frac{1}{\sqrt{2\pi}\sigma_e} \cdot e^{-\frac{y^2}{2\sigma_e^2}} \Rightarrow \boxed{\frac{1}{\pi\sqrt{2\pi}\sigma_e} \cdot e^{-\frac{y^2}{4\sigma_e^2}} \cdot K_0\left(\frac{y^2}{4\sigma_e^2}\right)}$$

$K_0(z)$: Modified Bessel function of the 2nd kind

Despite this fact, the conventional treatment of assuming a Gaussian distribution for the ions and applying the Bassetti-Erskine formula with the relation $\sigma_i = \sigma_e / \sqrt{2}$ closely reproduces the electric field created by the ions.







Left: Electric field distribution of ions with their true 2-dimentional distributions. Right: Using bi-Gaussian distribution with $\sigma_i = \sigma_e / \sqrt{2}$

(*Taken from L. Wang, Y. Cai, T. Raubenheimer, H. Fukuma, PRSTAB* ***14*** *084401, 2011*)

Lifetime reduction and effective pressure rise due to trapped ions

When ions are trapped, they are populated on the stored beam trajectory whose distributions can be approximated as Gaussian as described previously with $\sigma_{ix} \sim \sigma_x / \sqrt{2}$ and $\sigma_{iy} \sim \sigma_y / \sqrt{2}$

If they are uniformly distributed in the ring, their density may be given by

$$d_i = \frac{2N_i}{\pi \sigma_x \sigma_y L}$$

(L: Ring circumference)

Using this localised density (i.e. around the beam trajectory) in the previous beam collision rate formula, we can estimate the lifetime reduction due to trapped ions,

$$\frac{1}{\tau_{ions}} = \sigma_{Total} \cdot d_i \cdot \beta c$$

Also, if we apply the relation $d_m \, [m^{-3}] = 2.47 \times 10^{22} \cdot P_m \, [mbar]$ introduced earlier, we can discuss the effective pressure rise due to trapped ions on the beam trajectory,

$$P_{ions} \, [mbar] = \frac{1}{2.47 \times 10^{22}} \cdot \frac{2\eta N}{\pi \sigma_x \sigma_y L}$$

(η: Neutralisation factor, N: Total number of stored particles)

Part II

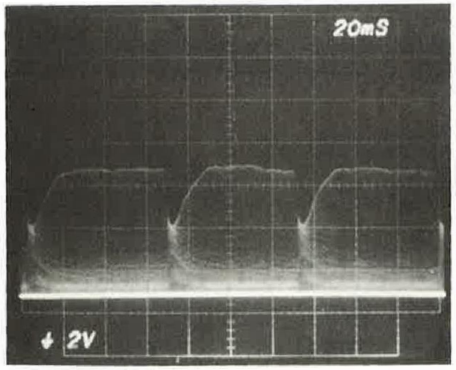
Subjects Treated:

- **Two-beam instabilities**
  - **Trapped ion case**
  - **Fast Beam-Ion Instability (FBII)**

Trapped ion case (1/3)

(*We closely follow S. Sakanaka, “Ion trapping in storage rings”, OHO 1986, KEK*)

With trapped ions, a resonant coupling between the two beams could arise that could lead to an instability → This type of instability was observed in many (2nd generation) light source rings



Vertical beam pulsation observed at Photon Factory, KEK (horizontal unit: 20 ms/div)

Description with a simplified model treating the centre of mass (CM) oscillations of the two beams:

(*E. Keil and B. Zotter, CERN-ISR-TH/71-58, 1971; D.G. Koshkarev, P.R. Zenkevich, Part. Accel.* **3** *1972, p1; etc.*)

- Consider only vertical oscillations, since the two beams interact strongly in this plane
- Electron centre of mass y_{cme} oscillating in the ring with the $Q_{\beta y}\omega_0$ ($Q_{\beta y}$: betatron tune) and under a linear force from the ion CM represented by the frequency ω_e
- Ion centre of mass y_{cmi} only feeling a linear force from the electron CM represented by the frequency ω_i
- As before, assume Gaussian distributions and $+1e$ charge for ions

Trapped ion case (2/3)

The coupled linear equations read

$$\ddot{y}_{cme} + Q_{\beta y}^2 \omega_0^2 \, y_{cme} = -\omega_e^2 \cdot (y_{cme} - y_{cmi})$$
$$\ddot{y}_{cmi} = -\omega_i^2 \cdot (y_{cmi} - y_{cme})$$

where

$$\omega_e^2 = \frac{2\lambda_i r_e c^2}{\gamma} \frac{1}{\sigma_y(\sigma_x + \sigma_y)} \qquad \left(r_e = \frac{e^2}{4\pi\varepsilon_0 m_e c^2}\text{: Classical electron radius}\right)$$

$$\omega_i^2 = \frac{2\lambda_e r_p c^2}{A} \frac{1}{\sigma_y(\sigma_x + \sigma_y)} \qquad (\lambda_e,\ \lambda_i\text{: Line densities } [m^{-1}] \text{ of electrons and ions})$$

Find a solution in the form $y_{cme} = A_e \cdot \exp[i(n\omega_0 - \omega)t + i\theta_0]$ and $y_{cmi} = A_i \cdot \exp(-i\omega t)$ we get

$$(x^2 - Q_i^2) \cdot \left[(x-n)^2 - Q_y^2 - Q_e^2\right] = Q_e^2 \cdot Q_i^2$$

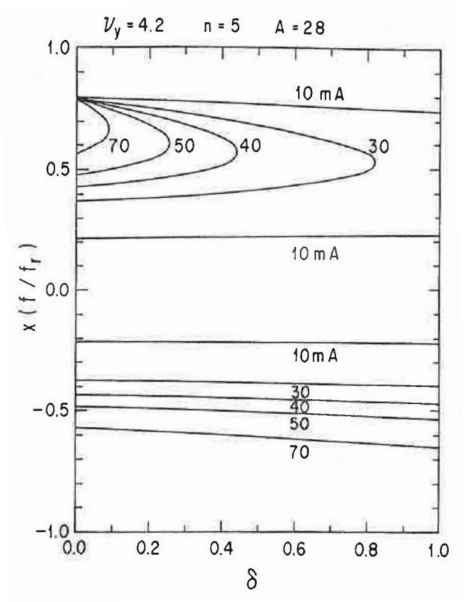
where $x = \omega/\omega_0$, $Q_e = \omega_e/\omega_0$ and $Q_i = \omega_i/\omega_0$.

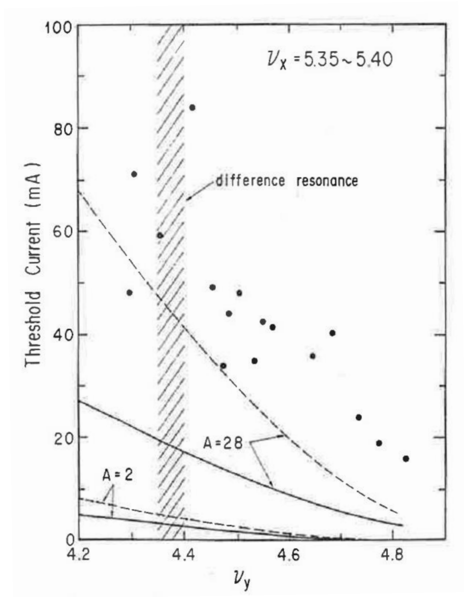
If the solution consists of complex numbers, it always appears in the form $a \pm ib$ (a, b: real), signifying that the two-beam motion is unstable.

Numerical studies indicate that instability is likely to appear for an n just above the value of $Q_{\beta y}$.

Trapped ion case (3/3)

Studies made for the Photon Factory KEK (*S. Sakanaka et al.)*:





Energy E [GeV]	2.5
Circumference L [m]	187.07
Revolution frequency f_0 [MHz]	1.6026
Momentum compaction α	0.04
Horizontal emittance ε_x [m·rad]	$4.1\pi\times10^{-7}$
Vertical emittance ε_y [m·rad]	$1.2\pi\times10^{-8}$
Horizontal tune $Q_{\beta x}$	5.2 - 5.5
Vertical tune $Q_{\beta y}$	4.1 - 4.2
Horizontal damping time $\tau_{\beta x}$ [ms]	9.1
Vertical damping time $\tau_{\beta y}$ [ms]	7.8

Main Photon Factory machine parameters

(Left): Real solutions at different beam currents as a function of the neutralisation factor $\delta = N_i/N$.

(Right): Instability threshold versus vertical tune. Solid lines: $\delta = 1.0$. Dashed lines: $\delta = 0.1$. Black circles: Measured values. $A = 28$: CO^+. $A = 2$: H_2^+.

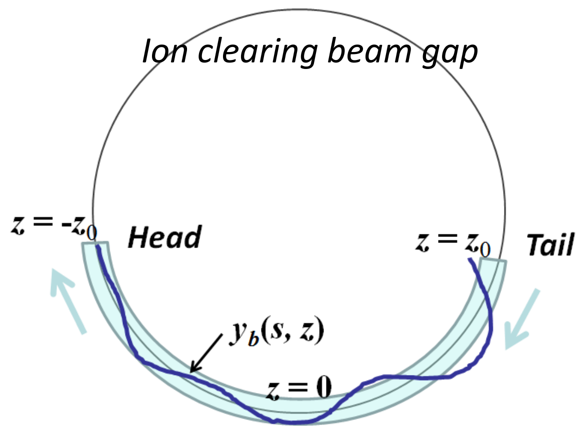
Comparisons with experiments indicate that the employed two centres of mass model describes the essential features of the dynamics

Fast Beam-Ion Instability (1/9)

Linear theory and simulations developed by Raubenheimer and Zimmermann (*Phys. Rev. E**52**, 5487, 1995*)

We saw that the linear forces between the two beams represented by ω_e^2 and ω_i^2 depend linearly on the beam intensity and inverse linearly on the product of the transverse beam sizes $\sigma_y(\sigma_x+\sigma_y)$

$\Rightarrow$ For modern and future accelerators producing a high intensity and low emittance beam, the "single pass" interaction between the two beams may become strong enough to jeopardise the performance.



z : relative to beam
s : ring position/time

This type of two-beam interaction resembles "beam breakup in linacs" and does not involve ion trapping, and an ion clearing beam gap may not be helpful.

$$\frac{d^2 y_b(s,z)}{ds^2} + \omega_\beta^2 \cdot y_b(s,z) = K\cdot[y_i(s,t)-y_b(s,z)]\cdot\int_{-\infty}^{z}\rho(z')dz'$$

Magnitude of interaction depends on the intensity of upstream particles in the bunch train

K : Corresp to ω_e^2 in the previous Eq.
$\rho(\mathrm{z})$: *bunch train distribution*

$$\frac{d^2 \tilde{y}_i(s,t)}{dt^2} + \omega_i^2 \cdot \tilde{y}_i(s,t) = \omega_i^2 \cdot y_b(s,z)$$

$y_b(s,z)$: beam centroid, $y_i(s,t)$: ion centroid,
$\tilde{y}_i(s,t)$: transverse ion slice

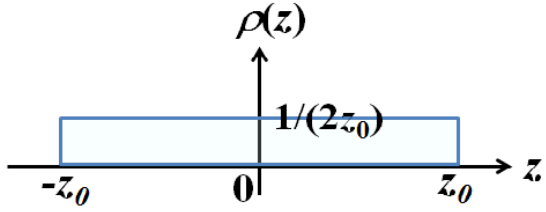
Fast Beam-Ion Instability (2/9)

Due to $M_{ion}/m_e \gg 1$, the wave length of ions $2\pi/\omega_i$ generally extends over multiple beam bunches
$\Rightarrow$ FBII leads to a coupled-bunch instability, while an electron cloud to a single bunch instability

Initial conditions for a transverse ion slice: $\tilde{y}_i(s,\ t'|t') = y_b(s,\ z')$ and $d\tilde{y}_i(s,\ t'|t')/dt = 0$

Ion centroid $y_i(s, t)$ is obtained by averaging $\tilde{y}_i(s,\ t|s+z')$ over all possible creation times:

$$y_i(s,t) = \frac{\int_{-\infty}^{z} dz'\rho(z')\cdot\tilde{y}_i(s,\ t|s+z')}{\int_{-\infty}^{z}\rho(z')dz'}$$



Solution of coupled linear equations:

Simplified constant beam distribution → ω_i assumed constant along the bunch train

Solution via perturbation series in K/ω_β by setting $y_b(s,t) = \sum_{n=0}^{\infty} y_b^{(n)}(s,z)$

Search for an asymptotic solution $\eta \equiv \dfrac{K\cdot\omega_i\cdot(z+z_0)^2 s}{16\omega_\beta z_0} \gg 1$

$$y_b(s,z) \approx \frac{e^{2\sqrt{\eta}}}{\eta^{1/4}}\sin(\omega_i z - \omega_\beta s + \theta - \phi)$$

Fast Beam-Ion Instability (3/9)

Asymptotic growth rate at the tail of a bunch train $z = z_0$: $y_b(s,\ z_0)\ \sim\ e^{\sqrt{t/\tau_{asmp}}}$ with

$$\boxed{\tau_{aymp}^{-1}(s^{-1}) \quad \approx \quad \frac{N_e^{3/2} n_b^2}{\gamma} \times \left[5 p_{gas}(\mathrm{Torr})\ \frac{r_e r_p^{1/2} L_{sep}^{1/2} c}{\sigma_y^{3/2} (\sigma_x + \sigma_y)^{3/2} A^{1/2} \omega_\beta} \right]}$$

N_e: Number of particles per bunch
n_b: Number of bunches
L_{sep}: Bunch spacing

Growth rate $1/\tau_{asymp}$ is not an e-folding time $exp[\ (t/\tau_{asymp})^{1/2}\ ]$

It depends strongly on

- Number of bunches $\propto\ n_b^2$
- Number of particles per bunch $\propto\ N_e^{3/2}$
- Beam size $\propto\ \sigma_y^{-3/2} \cdot (\sigma_x + \sigma_y)^{-3/2}$

Assumed linear model is supposed to break down when the amplitude of the oscillation $y_b(s, z)$ exceeds the vertical beam size σ_y where the coupling force between the two beam falls off.

Fast Beam-Ion Instability (4/9)

Evaluation of growth rate $1/\tau_{asymp}$ for some existing rings:

TABLE II. Parameters and oscillation growth rates for some existing accelerators.

Accelerator	SLC arc	SLC e^+ DR	ALS	HERA e^-	CESR	ESRF
ϵ_x^N (m)	5×10^{-5}	3×10^{-5}	1.2×10^{-5}	2×10^{-3}	2.7×10^{-3}	7.5×10^{-5}
ϵ_y^N (m)	5×10^{-6}	3×10^{-6}	2×10^{-7}	1.1×10^{-4}	1.2×10^{-4}	7.5×10^{-6}
n_b	1	1	328	210	7	330
N_b	3.5×10^{10}	4×10^{10}	7×10^{9}	3.7×10^{10}	4.6×10^{11}	5×10^{9}
$\beta_{x,y}$ (m)	4	1,3	2.5,4	25	14,13	8,8
$\overline{\beta_y}$ (m)	4	3	4	25	13	8
σ_x (μm)	50	114	101	1000	2000	224
σ_y (μm)	15	62	17	230	400	70
z_0 (σ_z)	1 mm	5.9 mm	100 m	3024 m	335 m	140 m
E (GeV)	46	1.2	1.5	26	5	6
p (Torr)	10^{-5}	10^{-8}	10^{-9}	10^{-9}	5×10^{-9}	2×10^{-9}
particle species	e^+	e^+	e^-	e^-	e^-	e^-
$\omega_{\rm ion}/2\pi$ (MHz)	4×10^{5}	5×10^{4}	25	0.8	0.6	8.3
Single or multibunch	single	single	multibunch	multibunch	multibunch	multibunch
$\tau_{\rm asym}$ ($z\approx z_0$)	1.1 μs	490 μs	2.4 μs	211 μs	3.9 ms	50 μs

(*Table taken from Raubenheimer and Zimmermann, Phys. Rev. E* ***52****, 5487 1995*)

Significantly short growth times found for ALS and ESRF, i.e. the light source rings. However, no clear evidence of FBII observed for these machines.

<u>Possible explanations</u>:

- Model assumes constant ω_i whereas in light source have strongly varying β functions due to DBA and TBA lattices, → ω_i could effectively be varying significantly around the ring
- Presence of Landau damping sources such as strong sextupoles and non-zero chromaticity
- Other important nonlinear effects not considered in the linear model

Fast Beam-Ion Instability (5/9)

Simulation of FBII (continue to follow the work of Raubenheimer-Zimmermann):

Simulation of beam-ion interactions using macroparticles for both beams

- Ionisation processes via beam - residual gas collisions
- Application of space-charge force of each beam to macroparticles of the opponent beam
- Discarding all ions at the end of each beam passage

<u>Advantages of numerical simulations</u>:

- Integration of nonlinear effects such as due to finite beam sizes
- Capacity to follow self-consistently and dynamically the evolution of (bunch) distributions of the two beams

Main features of the simulation:

- Motions of macroparticles described with (x, x', y, y', $\delta E/E$)
- Beam bunches are initially Gaussian in longitudinal and transverse
- Collision with gas takes place at some specified points in a ring (or a linac)
- A beam bunch is divided into ~5 slices longitudinally
- Each macroparticle is free to move in x and y according to the E-fields, but fixed in z
- Two-dimensional grids (e.g. 25×25) w.r.t. the CM of each slice
- At each grid,
  - Ions are created according to the specified pressure and collisional cross section
  - Ions have zero initial velocity
  - E-field of the beam evaluated with Bassetti-Erskine formula and applied to ions
  - Ion density and ion-induced E-field calculated and applied to beam macroparticles

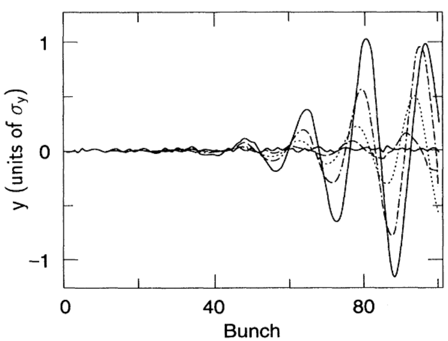
Fast Beam-Ion Instability (6/9)

<u>Major findings from the simulations</u>:

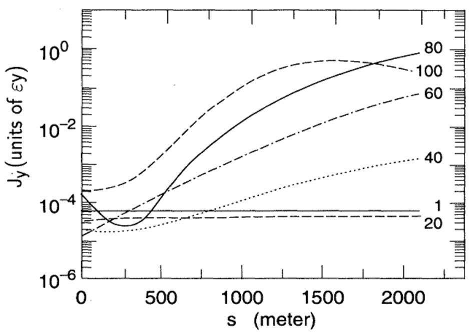
- Simulated growth rates $(1/\tau)_{sim}$ depend sensitively on initial conditions
- Apart from this uncertainty, $(1/\tau)_{sim}$ agree well with the theory

If the bunch distribution of the beam can be assumed not to change, the beam bunch can be treated as a rigid object (i.e. one macroparticle) → Great simplification and reduction in CPU time (cf. Code developed by K. Ohmi at KEK)

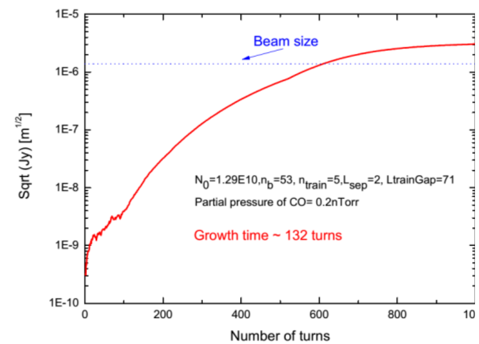
Code *mbtrack* (developed at SOLEIL) integrates both features in addition to treating impedances (geometric and resistive-wall) and FIR filter-based bunch-by-by feedback

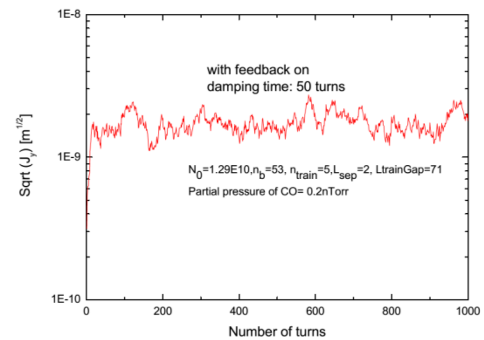


y_b vs bunch number for several different instants



Action J_y vs time (s) for several bunches in the train





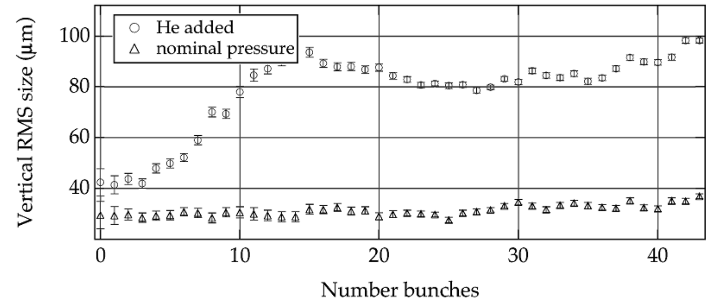
Amplitude growth versus time without (left) and with feedback (right) using Ohmi's code

(*From Raubenheimer & Zimmermann*)

(*G. Xia, K. Ohmi, E. Elsen, NIM **A593**, 2008, p.183*)

Fast Beam-Ion Instability (7/9)

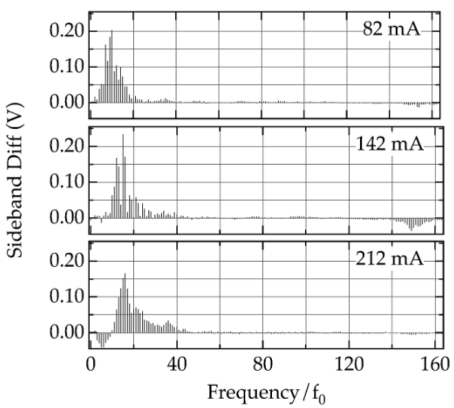
The phenomenon of FBII was experimentally demonstrated in ALS, PLS and KEK by artificially increasing the vacuum pressure.



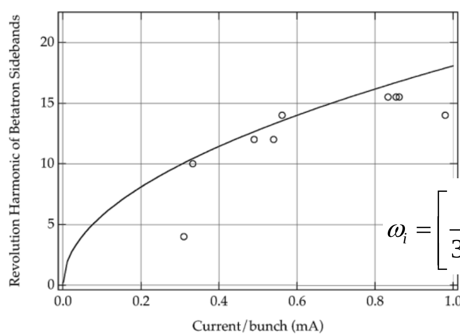
Measured vertical beam size as a function of the length of bunch train

<u>ALS experiment as first of such attempts</u>: (*J. Byrd et al., PRL **79**, 79, 1997*)

- *He* gas was injected in the ring to attain 80×10^{-9} Torr
- Length of a bunch train was increased in steps leaving a large beam gap (> 80 buckets)
- Transverse bunch-by-bunch feedback was turned on to eliminate usual instabilities
- Evolution of coherent signals versus beam intensity was followed
- Vertical scraper was inserted to probe the beam oscillation amplitude along the bunch train

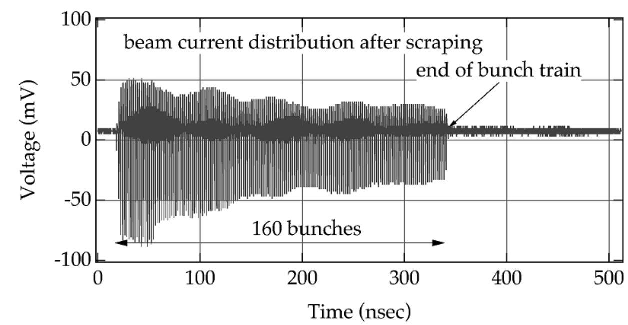


Spectra of beam vertical coherent oscillations



$$\omega_i = \left[\frac{4 N_e r_p}{3 L_{sep} \sigma_y (\sigma_x + \sigma_y) A} \right]^{1/2} \cdot c$$

Comparison of measured coherent frequencies with theory



Measured vertical amplitude along the bunch train after insertion of a vertical scraper

Fast Beam-Ion Instability (8/9)

Some further theoretical development related to FBII (1):

Effect of ion decoherence in the growth rate evaluations

(*Stupakov, Raubenheimer, Zimmermann, Phys. Rev. E**52**, 5499, 1995*)

Among several sources of ion frequency spreads, those due to

- Variation of electron density in the beam with horizontal displacement
- Amplitude-dependent frequency arising from nonlinearity of the electron potential

were considered

An extension to the linear theory was made by introducing a *Decoherence function* $D(z\text{-}z')$ defined such that

$$\frac{\partial^2 y_b(s,z)}{\partial s^2} + \frac{\omega_\beta^2}{c^2}\cdot y_b(s,z) = -\frac{K}{2z_0}\int_0^z z'\,\frac{\partial y_b(s,z')}{\partial z'}\,D(z-z')\,dz'$$

$D(z-z') = \int d\omega_i \cdot \cos[\frac{\omega_i}{c}(z-z')]\cdot f(\omega_i)$ with a distribution function $f(\omega_i)$ satisfying $\int f(\omega_i)d\omega_i = 1$

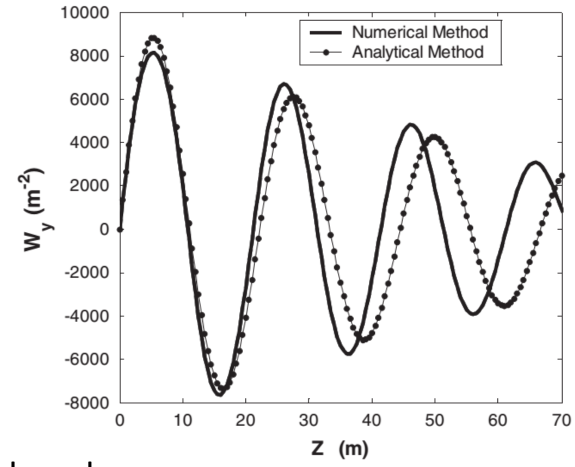
If no tune spread $f(\omega_i) = \delta(\omega_i - \omega_{i0})$ $\Rightarrow$ Equation reduces to the previous linear theory
Appropriate forms of $f(\omega_i)$ studied under the assumption that $L_{sep} << 2\pi/\omega_\beta,\ 2\pi/\omega_i$

$\Rightarrow$ In the treated cases, the growth rate was reduced by roughly a factor of 2 with ion frequency spread.
Good agreement with macroparticle simulations.

Fast Beam-Ion Instability (9/9)

Some further theoretical development related to FBII (2):

<u>Introduction of **a transverse wake function induced by an ion cloud**</u>



Following a successful attempt with an electron cloud, the notion of a wake function of ions was introduced:
(*L. Wang, Y. Cai, T.O. Raubenheimer, H. Fukuma, PRSTAB* ***14****, 084401, 2011; E.S. Kim, K. Ohmi, Jpn. J. Appl. Phys.* ***48****, 086501, 2009*)

Following the 1st group above, the wake function is defined by

$$W_y(s) \equiv \frac{\gamma}{N_e r_e} \cdot \frac{\Delta y'_e(s)}{\Delta y_e} \qquad \begin{cases} \Delta y_e & : \text{ Transverse displacement of the leading bunch} \\ \Delta y'_e(s) & : \text{ Transverse kick given to a trailing bunch at distance } s \text{ downstream} \end{cases}$$

Nonlinearity of space charge force between the two beams induces ion frequency spread, rendering the wake to decay exponentially in time. The wake can thus be parametrised as

$$W_y(s) = \hat{W}_y \cdot \exp\{-\omega_i s/(2Qc)\} \cdot \sin\left(\frac{\omega_i s}{c}\right) \quad \text{with} \quad \hat{W}_y = N_i \left(\frac{r_p L_{sep}}{A N_e}\right)^{1/2} \cdot \left[\frac{4}{3} \cdot \frac{1}{\sigma_y(\sigma_x + \sigma_y)}\right]^{3/2}$$

The corresponding impedance therefore takes the form
$$Z_{ion}(\omega) \approx \frac{\hat{W}_y}{\omega} \cdot \frac{Q}{1 + iQ\left(\frac{\omega_i}{\omega} - \frac{\omega}{\omega_i}\right)}$$

Beam instability can thus be studied in the conventional way using an ion wake and impedance.

Part III

Subjects Treated:

- **Mitigation methods**
- **Observations of ion effects**

Mitigation methods (1/2)

◊ Partial fillings/Multi-bunch trains/Bunch gaps

- Classical recipe of avoiding ion trapping
- FBII can still survive in a filling with a beam gap
- Generally effective to introduce multiple (shorter) bunch trains with (longer) beam gaps to fight against FBII

◊ Ion clearing electrodes

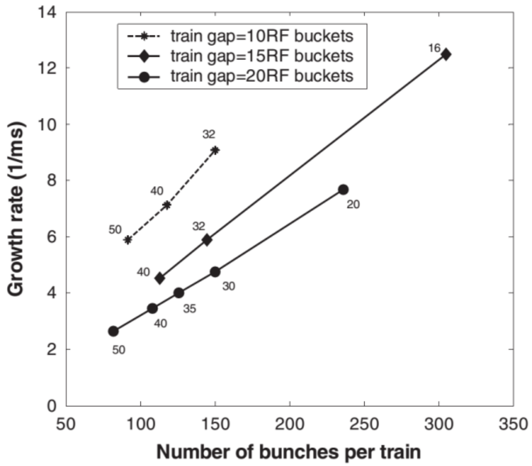
- Classical method successfully applied in a number of storage rings (Aladdin, SRS, ISR, …)
- Especially effective when installed where the beam potential is low
- However has a disadvantage of creating an impedance issue

◊ Positron beam storage

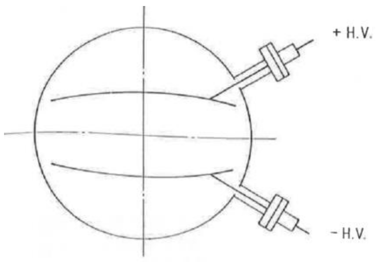
- Positive results obtained in a number of lepton storage rings (DCI, ACO, Photon Factory, APS, PETRA-III, …)
- Smallness of electron mass avoids electrons from being trapped
- However, may be exposed to electron cloud issues

◊ Use of octupoles/Chromaticity shifting

- Increasing the betatron tune spread of the beam could suppress beam-ion instability via Landau damping
- However, may not be compatible with dynamic acceptance constraints



Calculated growth rate of FBII for different beam fillings (*taken from L. Wang et al., PRSTAB* ***14****, 084401, 2011*)



Ion clearing electrode (*image taken from S. Sakanaka, OHO 1986, KEK*)

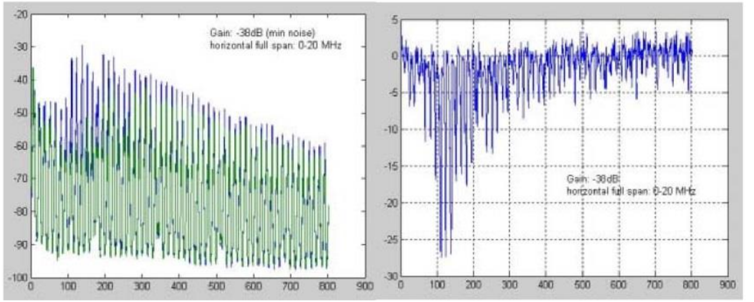
Mitigation methods (2/2)

◊ RF knockout

- Forcing the beam to oscillate at an externally given RF could stabilise the beam against two-beam instability, and successfully applied in some light source rings (Photon Factory, UVSOR, …)

◊ Transverse bunch-by-bunch feedback

- Considered as one of the most efficient methods against ion instabilities (whether ion trapping or FBII)
- Should work as long as a bunch exhibits dipolar CM motions (and not decohered).
- For future accelerators, challengingly short feedback damping times may be required to fight against FBII



Vertical beam spectra with (*green*) *and without* (*blue*) *feedback in the uniform filling measured at the ESRF. Horizontal axis spans 0-20 MHz* (*E. Plouviez et al., EPAC 2008*)

◊ Reduced vertical beam size (by more than a factor 2)

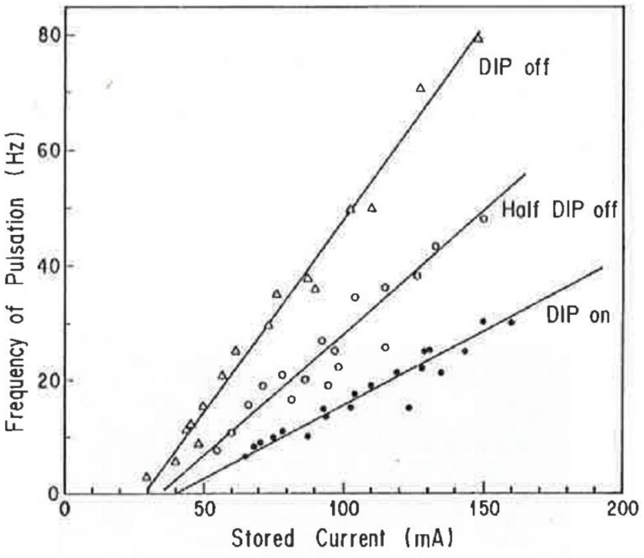
- Anticipating that when the beam size exceeds $\sim\sigma_y$ the inter-beam force saturates, one may reduce the vertical beam size by a factor of two w.r.t. the desired value (*K. Oide, KEK*)
- Should certify the absence of residual beam blow ups in the saturation regime

◊ Enhancing vertical beta function variation

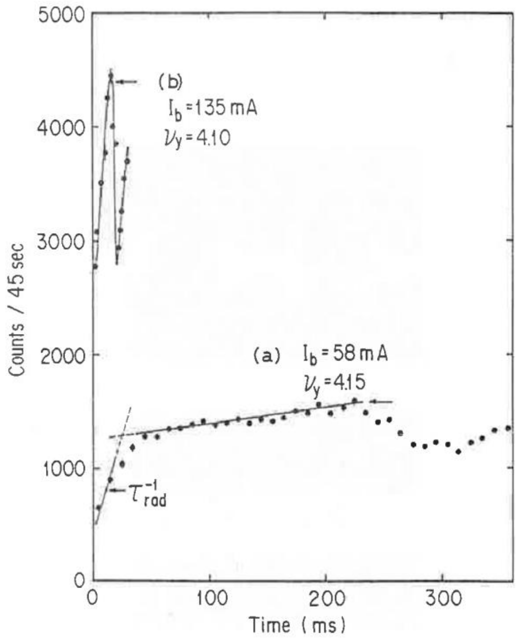
- Should increase the ion frequency spread along the machine and help Landau damp coherent ion motions
- Suspected to be an important source of stabilisation in light source rings that have large β_x and β_y variations

Observations of ion effects (1/4)

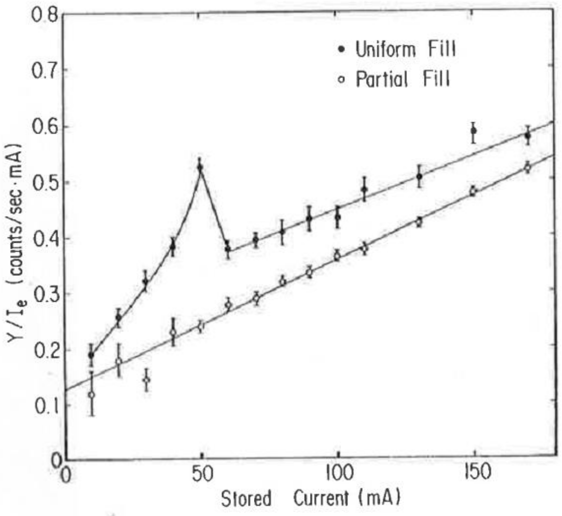
Experimental characterisations of ion trapping in Photon Factory: *S. Sakanaka et al., from OHO KEK 1986*



Dependence of vertical pulsation of the beam on beam intensity and vacuum



Time evolution of the Bremsstrahlung count rate at two different beam intensities



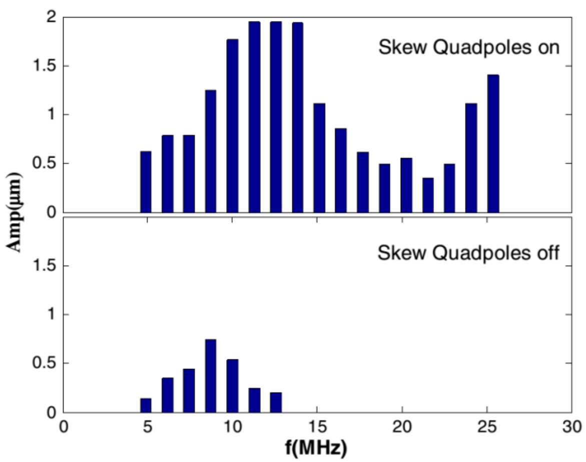
Bremsstrahlung count rate versus beam intensity for two different beam fillings

Observations of ion effects (2/4)

<u>Experimental characterisations of FBII in SPEAR3</u>: *L. Wang et al., PRSTAB* ***16*** *104402 (2013)*

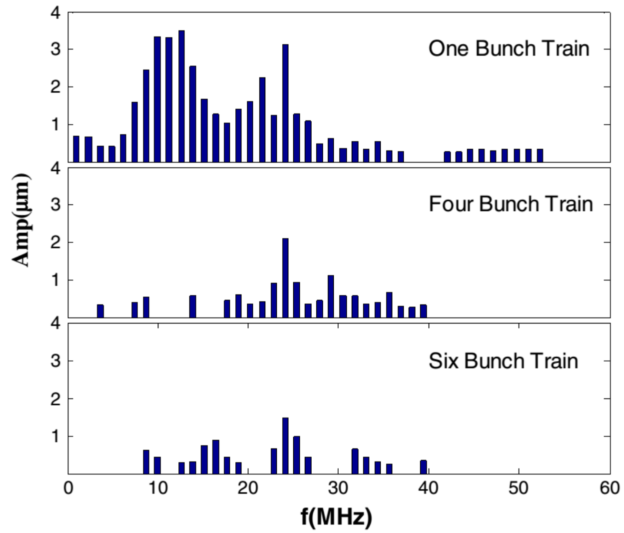
Follow the dependence of vertical betatron amplitude (lower sidebands) on;

Vertical beam size



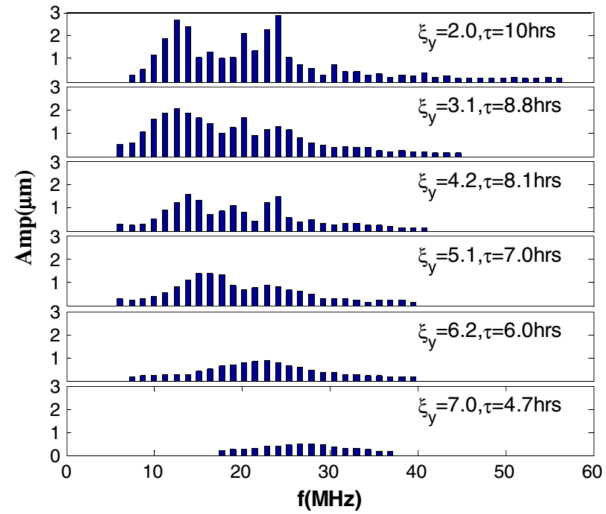
Single bunch-train (280 bunches) at 192 mA. When skew quads are off, the vertical beam size is ~2.3 times larger

Beam filling



In all cases, there are 280 bunches and the total current is 500 mA. The bunch train gap is 15 buckets (32 ns)

Vertical chromaticity

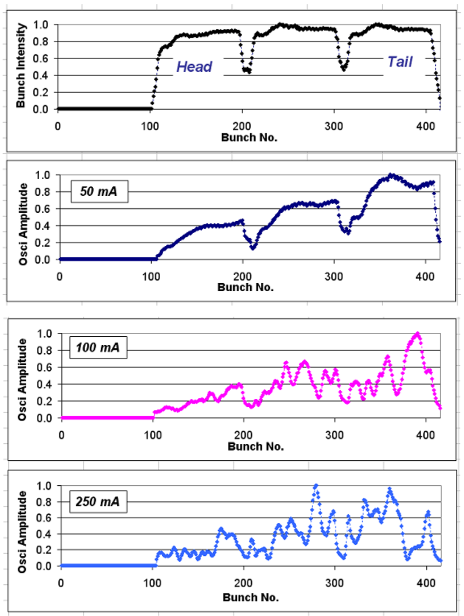


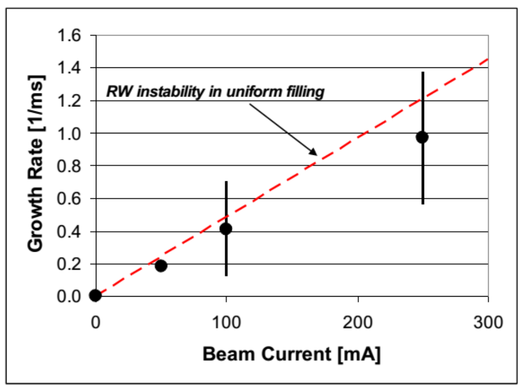
Single bunch-train (280 bunches) at 500 mA.
Horizontal chromaticity is kept at 2.

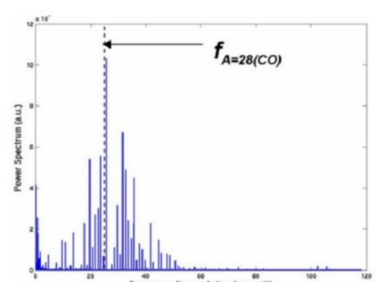
Observations of ion effects (3/4)

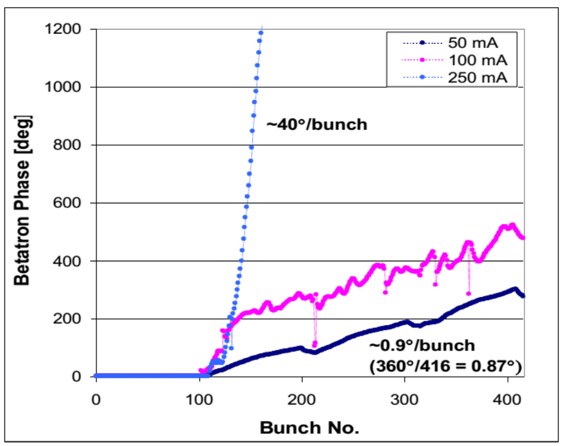
Beam losses due to a combined effect of FBII driven by beam-induced outgassing, resistive-wall instability and transverse feedback at SOLEIL: *R. Nagaoka et al., TWIICE workshop, SOLEIL 2014*

- At SOLEIL, transverse bunch-by-bunch feedback is routinely used to suppress resistive-wall (RW) instability.
- However, depending upon the beam filling and intensity, beam-induced heating could trigger FBII via outgassing and leads to ***total beam losses***.





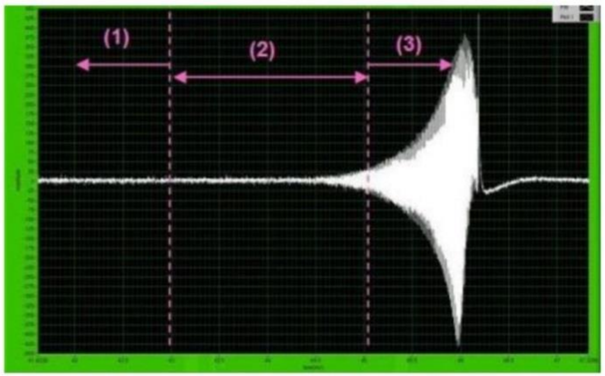


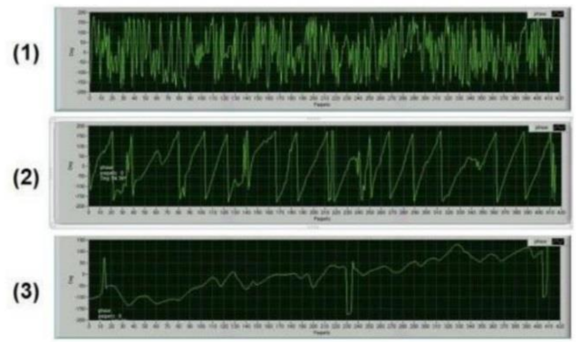


Signatures of the co-existence of FBII and resistive-wall instability

Observations of ion effects (4/4)

- Usually the beam is lost some 10 minutes ***after*** reaching the final current (500 mA)
- The above interval of time as well as the ***total beam loss*** due to FBII remained as a big puzzle

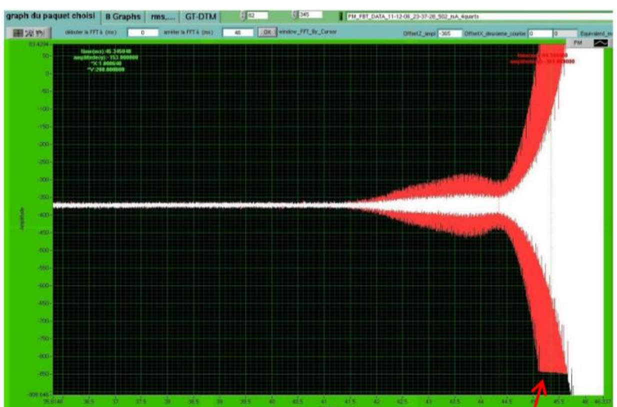




No phase correlation

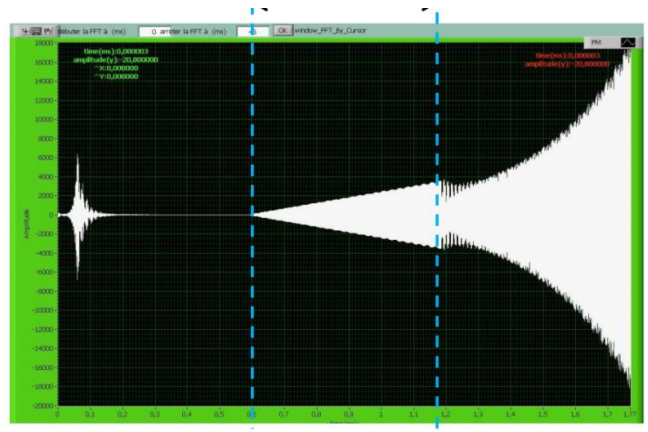
Ion regime

RW regime



Measured beam loss at 500 mA
White: Beam, Red: TFB kick

feedback saturation



Simulation: RW with temporal "shaker" excitation at F_{ion}

Experimental and numerical analyses lead us to conclude that over the time interval, the ***local pressure keeps rising*** up to the point when feedback hits its limit and becomes ***destructive***

Conclusion

◊ Due to difficulties of measurement and frequent non-reproducibility of vacuum conditions, beam instabilities due to ions are generally non straightforward to understand.

◊ However, theoretical, numerical and experimental studies made so far allow fairly good explanations and predictions of the beam dynamics with ions qualitatively and quantitatively.

◊ Due presumably to lower beam emittances in modern storage rings, ion trapping does not seem to be a big issue anymore. However, FBII could jeopardise the performance of future low emittance and high beam intensity accelerators as its growth rate tends to increase further.

◊ Continuation of beam-ion studies is of great importance in raising the performance of future accelerators.