Tutorial exercises on "Beam-beam Effects in Hadron Colliders":

1 Exercise I

1.1 Problem:

Assume the beam-beam force for a round beam in the simplified form:

$$f(x) \;=\; \frac{1}{x}(1 - e^{-\frac{x^2}{2}})$$

We know from the lecture that the beam-beam tune shift for very large amplitude particles goes to zero.

a) Explain with your own words, using the above formula as input

b) Show it analytically, using the above formula as input

2 Exercise II

2.1 Problem:

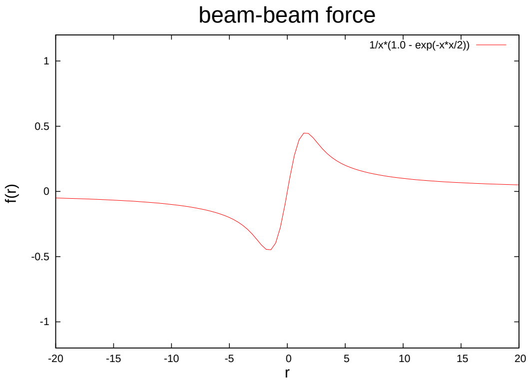
Do beam-beam effects (e.g. head-on and long range interactions) contribute to chromaticity ?
Explain your answer.
Try to make an estimate for the case of long range interactions.

2.2 Solution I:

The beam-beam force in its simple form:

$$f(x) \;=\; \frac{1}{x}(1 - e^{-\frac{x^2}{2}})$$

as a plot: The tune shift of a particle with the amplitude a is the "average" slope

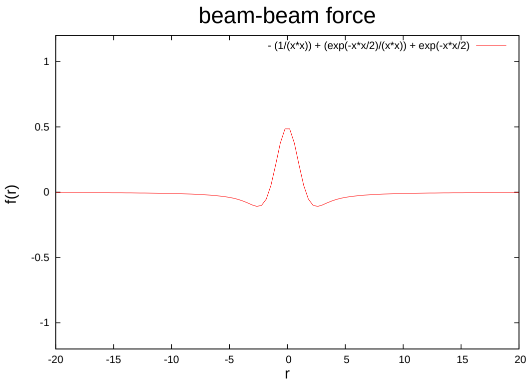


over a full oscillation.

The derivative of the force f(x) is F(x)

$$F(x) = -\frac{1}{x^2} + e^{-\frac{x^2}{2}} + \frac{e^{-\frac{x^2}{2}}}{x^2}$$

and is shown below: The tuneshift for the particle with amplitude a is therefore:



$$\Delta Q \;=\; \frac{1}{2a}\int_{-a}^{+a} F(x)dx$$

For a particle with very large amplitude, i.e.

$$\Delta Q \;=\; \int_{-\infty}^{+\infty} F(x)dx \;=\; 0$$

2.3 Solution II:

Both head-on as well as long range interactions contribute to chromaticity.
Head-on:

The chromaticity Q'_{bb} is due to the modulation of thev β-functions:

$$Q'_{x,y} \;=\; \Delta Q_{x,y} \cdot \; \frac{1}{\beta_{x,y}} \frac{\partial \beta_{x,y}}{\partial \delta}$$

Long range:

The chromaticity Q'_{bb} is due to a finite dispersion at the parasitic encounters, thus changing the separation $d_{eff} \;=\; d_0 + D \cdot \delta$.

$$\Delta Q_0 \;\propto\; \frac{1}{d_0^2} \;\;\rightarrow\;\; \frac{1}{(\; d_0 + D \cdot \delta)^2}$$

We are interested in:

$$\frac{\partial}{\partial \delta} \Delta Q$$

We get:

$$\frac{\partial}{\partial \delta} \Delta Q \;=\; \frac{-2D}{(d_0 + D \cdot \delta)^3}$$

and also:

$$\frac{\partial}{\partial d} \Delta Q \;=\; \frac{-2}{(d_0 + D \cdot \delta)^3}$$

we can therefore re-write:

$$\frac{\partial}{\partial \delta} \Delta Q \;=\; \frac{-2D}{d_0} \cdot \Delta Q$$

The ratio $\frac{-2D}{d_0}$ can be quite large so for large ΔQ it can be very significant, e.g. for a pretzl scheme with arc dispersion.