Wakefields and Impedances
Part II

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CAS
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computer programs

time domain solver for short range wakes

in principle, resolution and numerical dispersion, indirect wake integration, extrapolation to wake function, incomplete overview of codes

eigenmode solver for long range interaction

in principle, problems without/with losses, perturbation methods, example, notation

modal part of wake and long range wake

splitting into resonant part and rest
two particle interaction per mode
azimuthal symmetry: monopole and dipole modes
longitudinal wakes and impedances
modal and total loss parameter
dipole wakes, damping and detuning
equivalent circuit model
losses

computer programs:

time-domain solver
for short range wakes

in principle

$$\frac{d\mathbf{E}}{dt} = \varepsilon^{-1}\,\mathrm{curl}\,\mu^{-1}\mathbf{B} - \varepsilon^{-1}\mathbf{J}$$

$$\frac{d\mathbf{B}}{dt} = -\mathrm{curl}\,\mathbf{E}$$

spatial discretization (resolution ~ Δ)+ recursion in time: $t^{(n)} = n\Delta t$

$$\begin{pmatrix}\mathbf{E}\\ \mathbf{B}\end{pmatrix}^{(1)}$$

initial fields

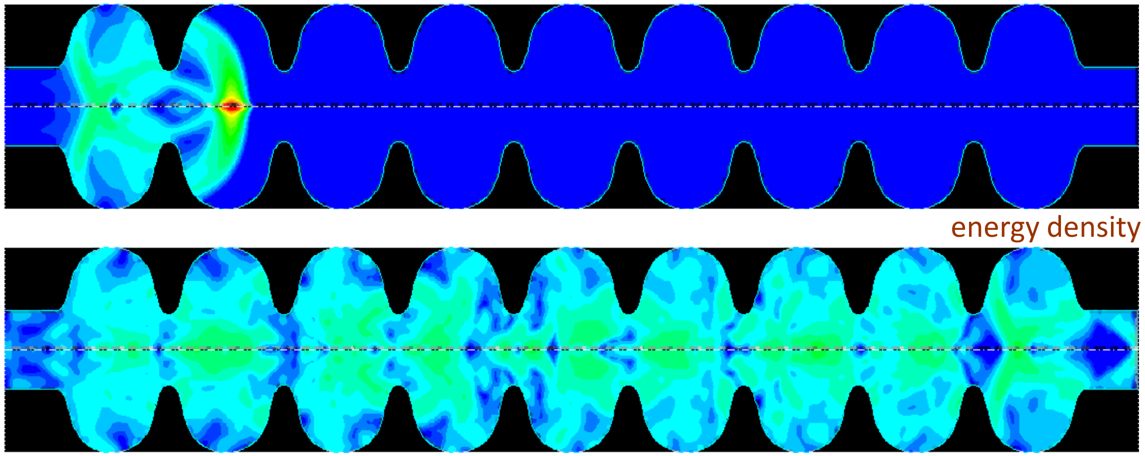
$$\begin{pmatrix}\mathbf{E}\\ \mathbf{B}\end{pmatrix}^{(n+1)} = \begin{pmatrix}\mathbf{E}\\ \mathbf{B}\end{pmatrix}^{(n)} + \Delta t \begin{pmatrix}\varepsilon^{-1}\,\mathrm{curl}\,\mu^{-1}\mathbf{B} - \varepsilon^{-1}\boxed{\mathbf{J}}\\ -\mathrm{curl}\,\mathbf{E}\end{pmatrix}^{(n+1/2)}$$

source term
(the beam)

or leap frog schemes

boundary conditions: perfect electric/magnetic conducting
"beam boundary"
waveguide
finite conductivity

fixed volume



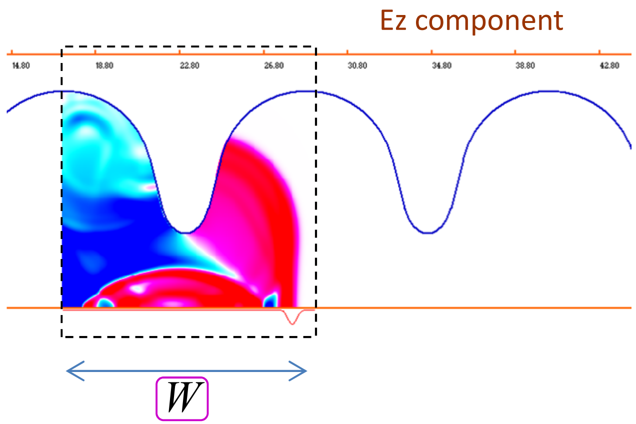
bunch enters and exits domain through boundary → "beam boundary"

excited EM fields propagate through boundary → "waveguide boundary"
"open boundary"

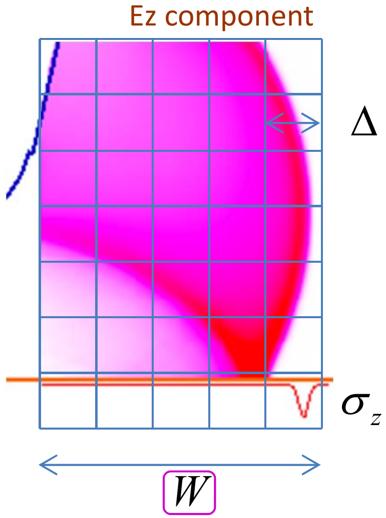
decay of fields can be observed
"middle" range wakes

window (moving mesh)

beam, waveguide & open boundaries
are not required
bunch starts with surrounding field
→short and ultra-short range wakes



resolution and numerical dispersion



spatial resolution: cavity geometry and bunch
mesh has to be fine compared to bunch
$\Delta << \sigma_z$

accuracy and stability: $c\Delta t \le \Delta$

propagation of waves: numerical dispersion $\rightarrow \lambda_{\text{numerical}} = c\frac{\omega}{2\pi} + \text{error}$

$$v_{\text{phase,numerical}} = c + \text{error}$$

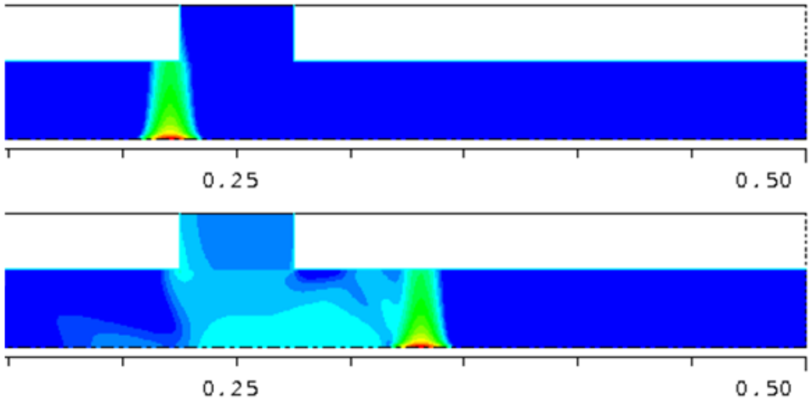
1st generation of wake-field codes: FDTD (finite-differences-time domain) methods; criterion: $\Delta^2 << \sigma_z^3 / L$
with $L >> W$, the length of interaction

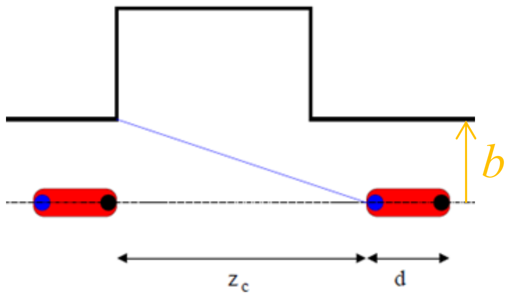
2nd generation: codes without dispersion error in z-direction; for instance DG (discrete Galerkin) methods; see table

wake integration:

$$\int_{-\infty}^{\infty} E(\cdots, z-s, z/c)dz \rightarrow \int_{-L/2}^{L/2} E(\cdots, z-s, z/c)dz$$

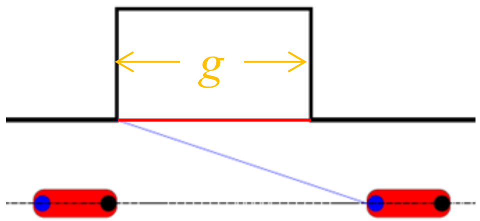
indirect wake integration





long catch-up distance $z_c = \frac{b^2}{2d} - \frac{d}{2}$ $\rightarrow$ short integration path

example: monopole wake of pillbox cavity



$$W_{\parallel}^{(0)}(0,0,0,0,s) = W_{\parallel}^{(0)}(0,0,b,0,s) = W_{\parallel}^{(0)}(s)$$

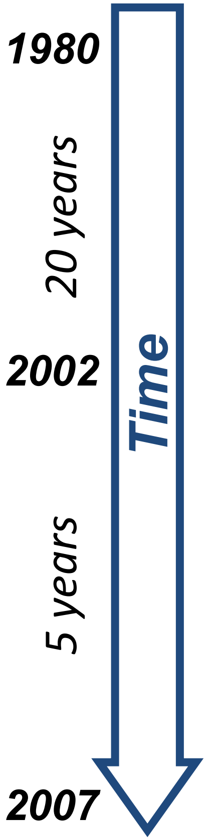
$$\rightarrow \int_{-\infty}^{\infty} E_z(b,0,z-a,z/c)dz = \int_{-g/2}^{g/2} E_z(b,0,z-a,z/c)dz$$

this simple method is not always applicable, but there are schemes that are!
f.i. use decomposition into waveguide modes in beam pipes

see:

I. Zagorodnov: Indirect Methods for Wake Potential Integration, Phys. Rev. ST Accel. Beams 9, (2006).
Henke, W. Bruns: Calculation of Wake Potentials in General 3D Structures, Proc. of EPAC’06, Edinburgh, UK (2006), WEPCH110.
E. Gjonaj, T. Lau, T. Weiland, R. Wanzenberg: Computation of Short Range Wake Fields with PBCI, BD-Newsletter No 45.

an (incomplete) survey of available codes



	Dimensions	*Nondispersive*	*Parallelized*	*Moving window*	
BCI / TBCI	2.5D	No	No	Yes	FDTD (IW)
NOVO	2.5D	Yes	No	No	
ABCI	2.5D	No	No	Yes	FDTD IW
MAFIA	2.5/3D	No	No	Yes	FDTD (IW) LC
GdfidL	3D	Yes	Yes	Yes	IW
Tau3P	3D	No	Yes	No	
ECHO	2.5/3D	Yes	No	Yes	IW LC
CST Particle Studio	3D	No	No	No	FDTD IW LC
PBCI	3D	Yes	Yes	Yes	DG IW
NEKCEM	3D	Quasi	Yes	No	DG

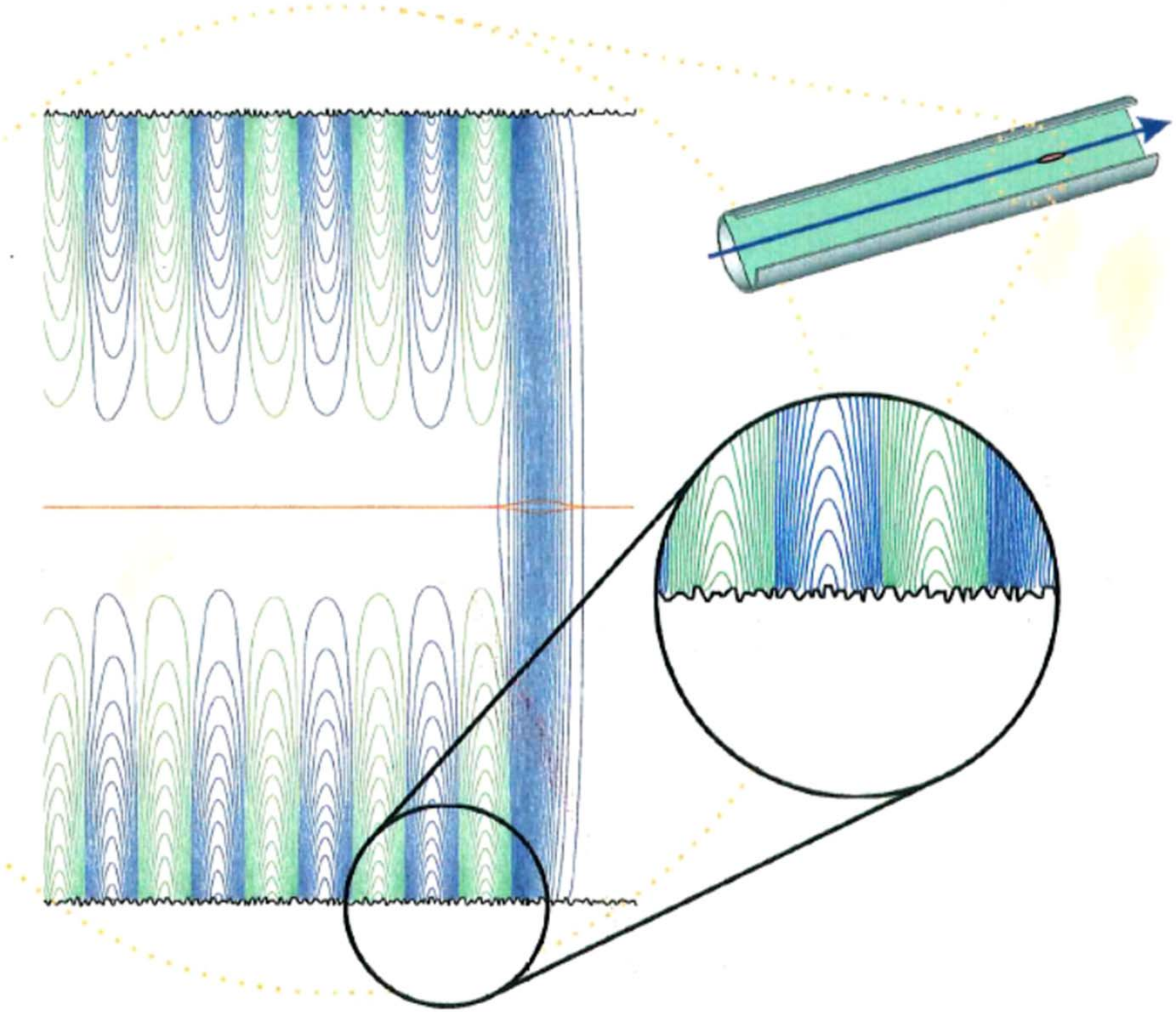
FDTD = finite differences time d.
DG = discontinuous Galerkin
IW = indirect wake integration
LC = large finite conductivity

table from: S. Schnepp, W. Ackermann, E. Arevalo, E. Gjonaj, T. Weiland: Large Scale 3D Wakefield Simulations with PBCI, ILC wakefield workshop at SLAC 2007

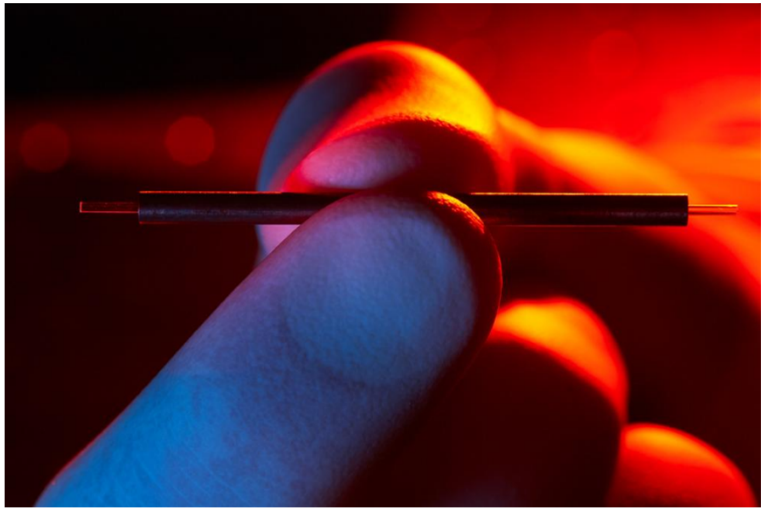
ABCI, Yong Ho Chin, KEK http://abci.kek.jp/abci.htm
Echo 2D, Igor Zagorodnov, DESY http://www.desy.de/~zagor/WakefieldCode_ECHOz/

NOVO: beam-pipe with random surface roughness

M. Timm, PhD Thesis (2000); S. Novokhatsky

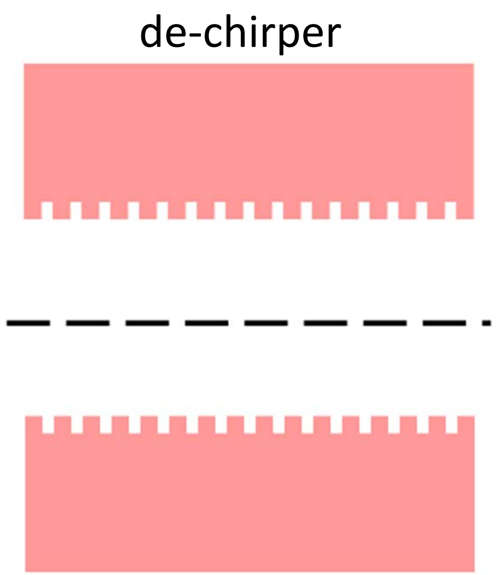


wakefield accelerator



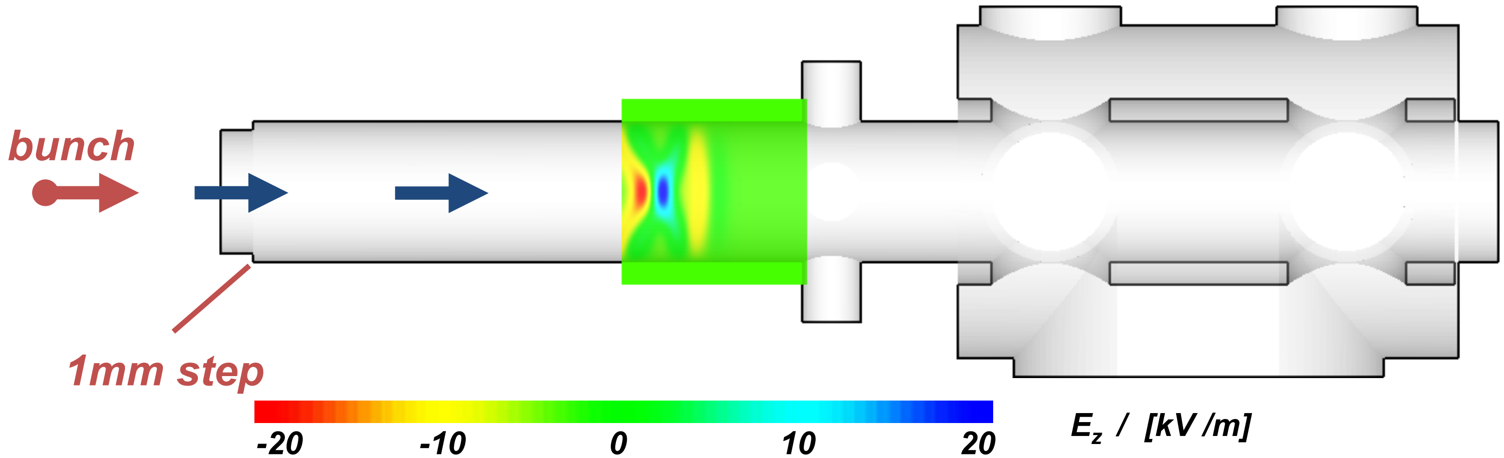
de-chirper

FELs: chirp (longitudinal energy distribution with slope) is needed for bunch compression; it can be reduced (afterwards) by a de-chirper

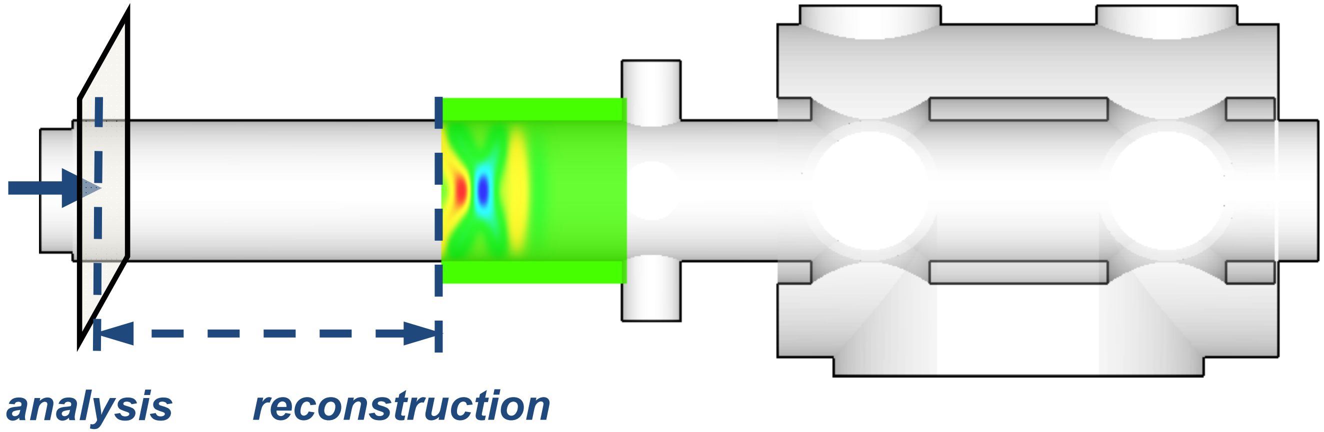


PBCI: diagnostic cross in PITZ (injector section)

a) full time domain calculation: effect of a small step before the cross



b) quasi-analytic field propagation in long intermediate pipes



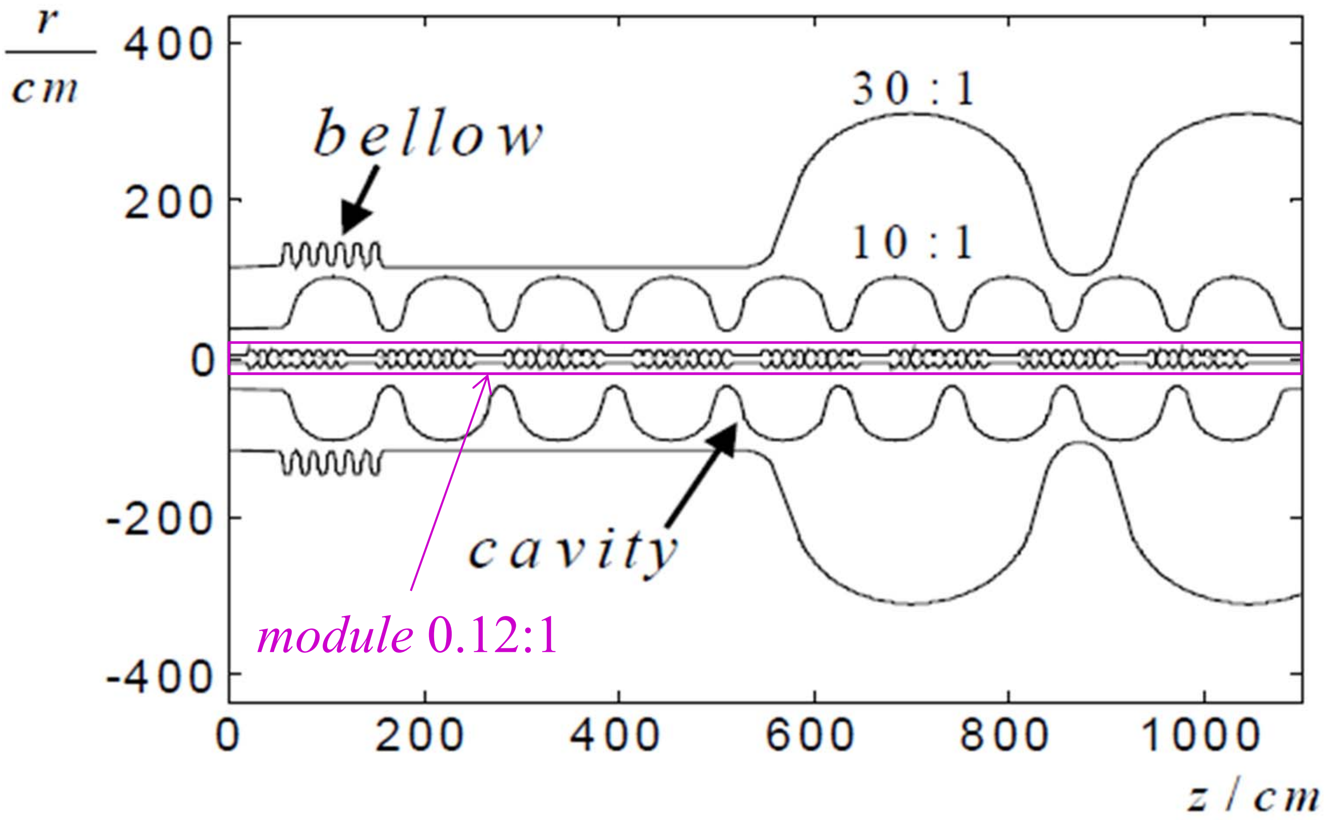
from: E. Gjonaj, et. al.: Large Scale Parallel Wake Field Computations with PBCI, ICAP 2006

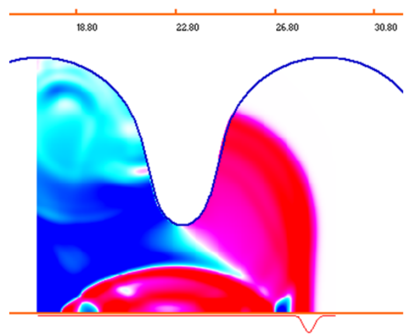
Echo: ultra short-range wake in module with TESLA-cavities

contribution of 3rd module (with 8 cavities)

$L \approx$ 0 .. 25 m for module 1 & 2, 25 .. **37 m** for module 3

geometry



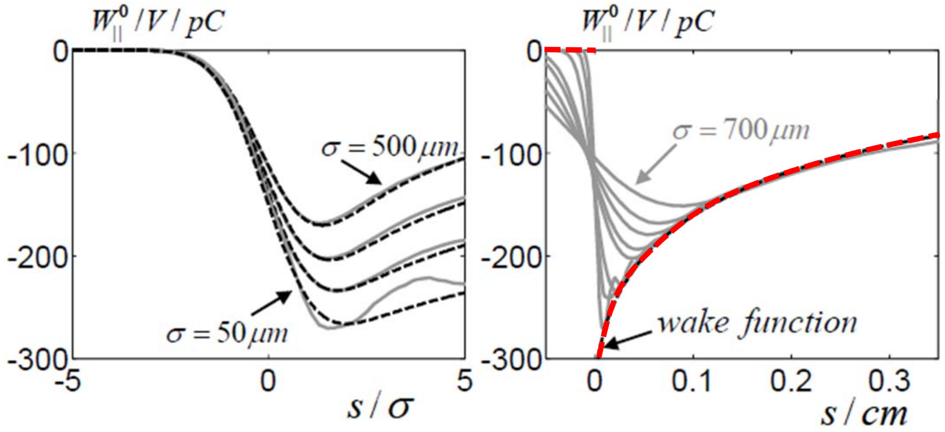


σ = 700 ... **50 μm**

extrapolation: bunch-length $\to 0$

contribution of 3rd module (with 8 cavities)

longitudinal wake (monopole)

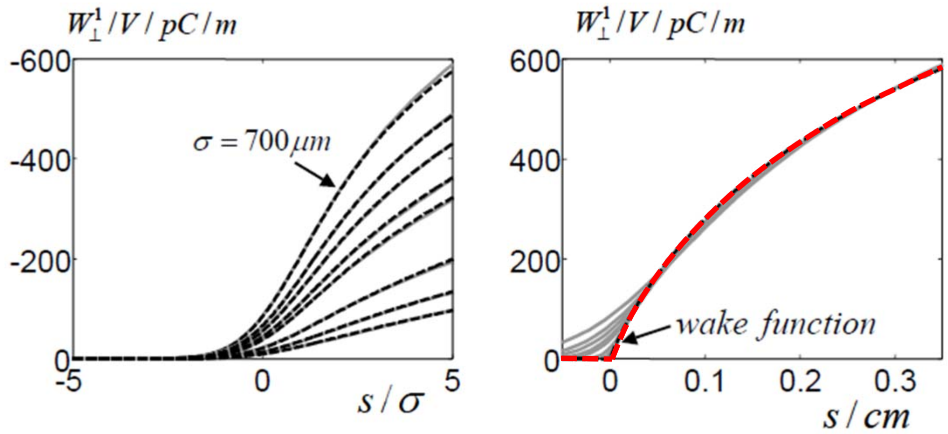


asymptotic model:

$$w_{\parallel}(s) = A \exp\left(-\sqrt{s/s_0}\right)$$

$$w_{\perp}(s) = B\left(1 - \left(1 + \sqrt{s/s_1}\right)\right)\exp\left(-\sqrt{s/s_1}\right)$$

transverse wake (dipole)



from: The Short-Range Transverse Wake Function for TESLA Accelerating Structure
T. Weiland, I. Zagorodnov, TESLA Report 2003-19

computer programs:

eigenmode solver
for long range interaction

eigenmode solver, in principle

$$\frac{d\mathbf{E}}{dt} = \varepsilon^{-1}\,\mathrm{curl}\,\mu^{-1}\mathbf{B} - \varepsilon^{-1}\mathbf{J}$$

$$\frac{d\mathbf{B}}{dt} = -\,\mathrm{curl}\,\mathbf{E}$$

case “no losses”

$$i\hat{\omega}\begin{pmatrix}\hat{\mathbf{E}}\\ \hat{\mathbf{B}}\end{pmatrix} = \begin{pmatrix}\varepsilon^{-1}\,\mathrm{curl}\,\mu^{-1}\hat{\mathbf{B}}\\ -\,\mathrm{curl}\,\hat{\mathbf{E}}\end{pmatrix}$$

curl-curl equation

$$\hat{\omega}^2\,\hat{\mathbf{E}} = \varepsilon^{-1}\,\mathrm{curl}\,\mu^{-1}\,\mathrm{curl}\,\hat{\mathbf{E}}$$

(similar for B)

eigenvalue eigenvector

problem: curl-curl operator has an (in)finite number of “static” eigenvalues
but we are interested in “dynamic” solutions

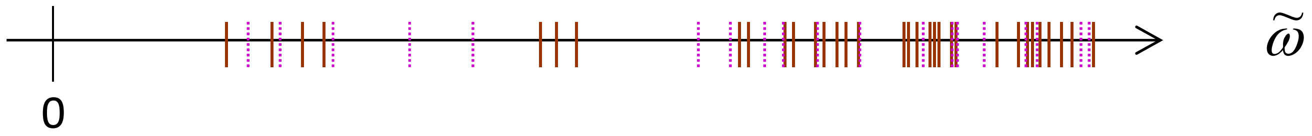
$\hat{\omega} = 0 \qquad \mathrm{curl}\,\hat{\mathbf{E}} = 0$

$\hat{\omega}^2 > 0 \qquad \mathrm{curl}\,\hat{\mathbf{E}} \neq 0$

trick: modify curl-curl equation so that "dynamic" solutions are not changed, but eigenvalues of "static" solutions are shifted from zero; this is done by adding the grad-div equation

$$\tilde{\omega}^2\,\hat{\mathbf{E}} = \varepsilon^{-1}\,\mathrm{curl}\,\mu^{-1}\,\mathrm{curl}\,\hat{\mathbf{E}} + f\cdot\mathrm{grad}\,\mathrm{div}\,\varepsilon\hat{\mathbf{E}}$$

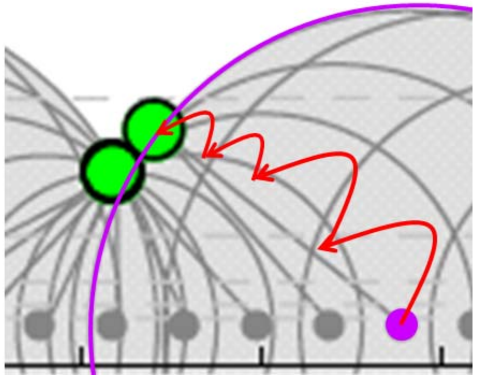
shifted eigenvalues: "static" and "dynamic"

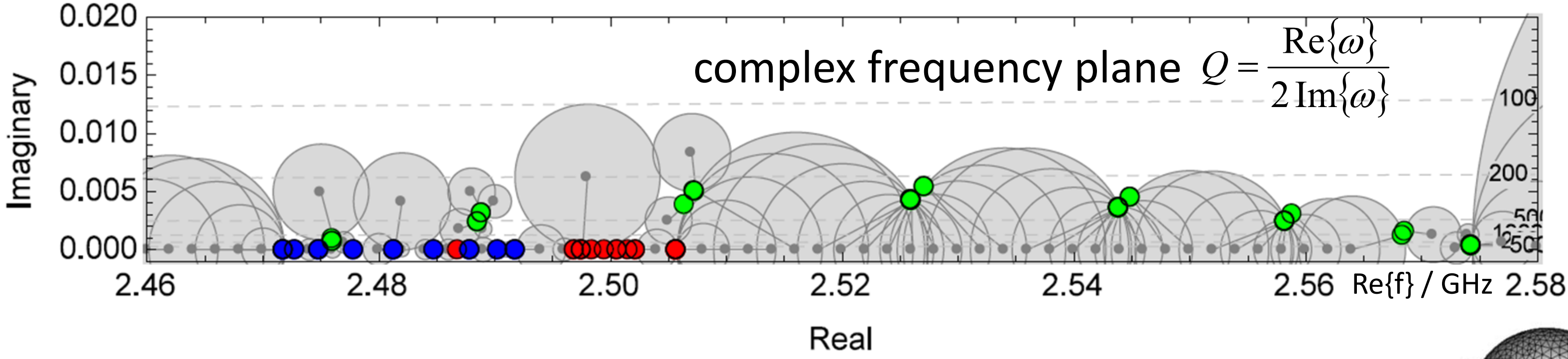


(density increases with frequency)

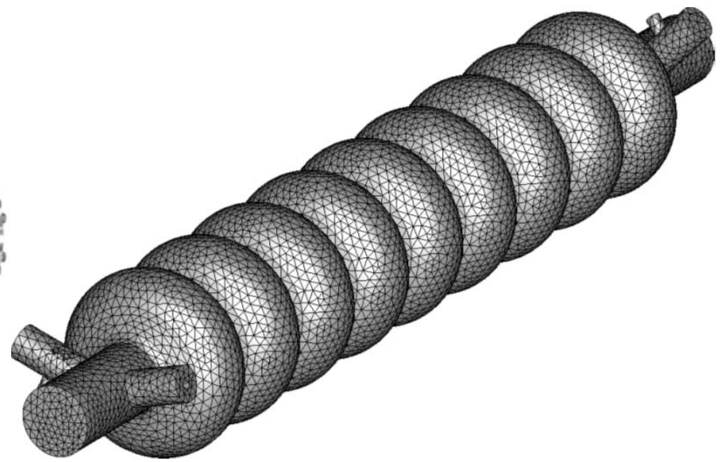
loss-free problems: there are very effective **simultaneous** eigenmode solvers; usually they find a specified number of lowest eigen-solutions and distinguish between static and dynamic modes

lossy eigenmode problems: f.i. Jacobi-Davidson eigenvalue solver in the complex plane; modes are iterative searched, **one-by-one** $\rightarrow$ large numerical effort





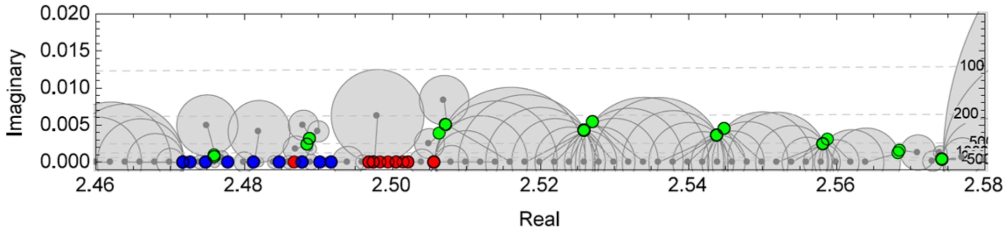
example: eigenmodes in TESLA cavity

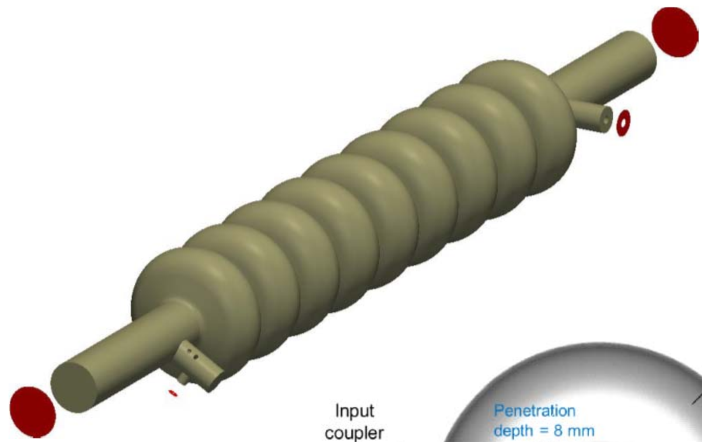


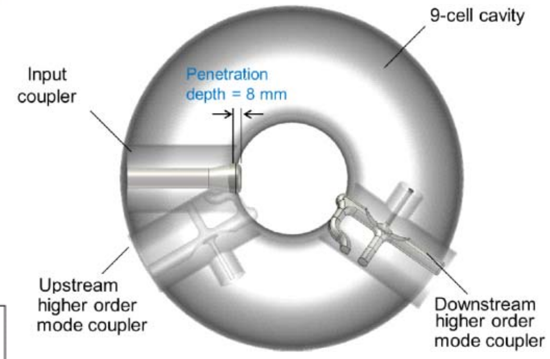
pictures from W. Ackermann and Cong Liu, TEMF-TU-Darmstadt

example: eigenmodes in TESLA cavity

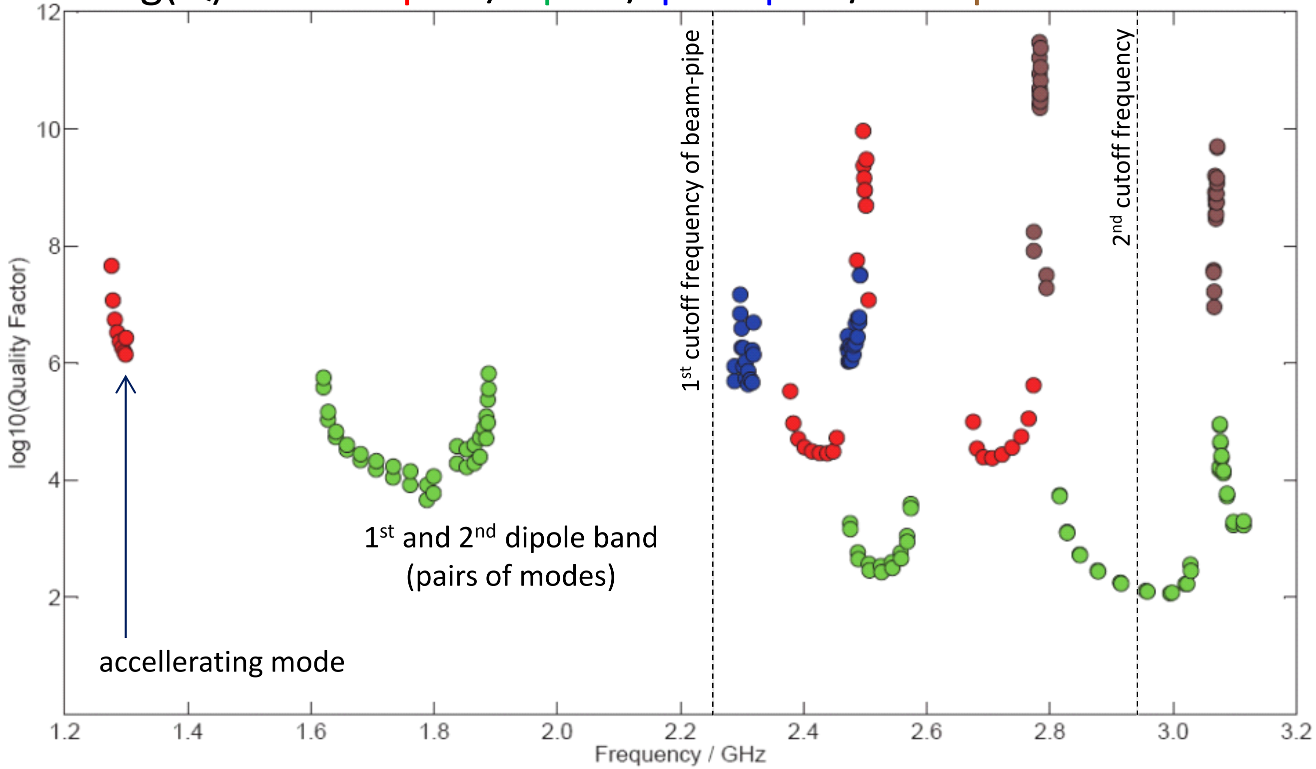
losses by waveguide modes above f_c







log(Q) of monopole/dipole/qudrupole/sextupole-modes



pictures from W. Ackermann and Cong Liu, TEMF-TU-Darmstadt

example: eigenmodes in TESLA cavity

accelerating mode

1. MONOPOLE PASSBAND

MODE 9

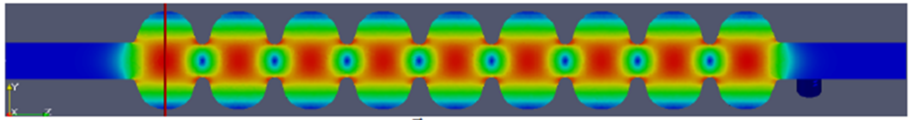


Figure 46: Electric field strength $|\vec{E}|$ of monopole mode 9 in the vertical plane.

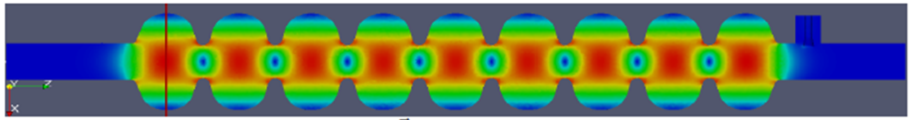


Figure 47: Electric field strength $|\vec{E}|$ of monopole mode 9 in the horizontal plane.

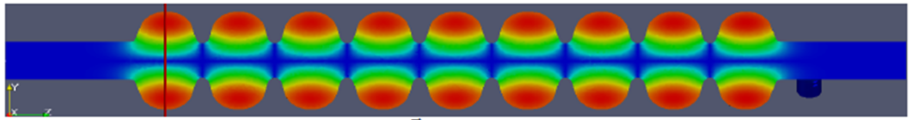


Figure 48: Magnetic flux density $|\vec{B}|$ of monopole mode 9 in the vertical plane.

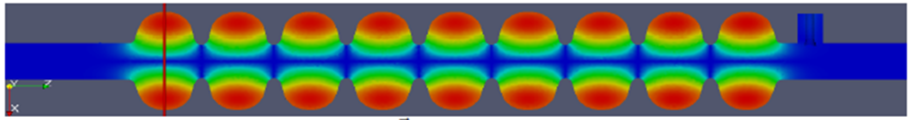
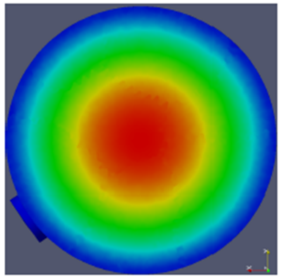
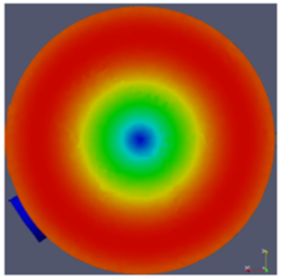


Figure 49: Magnetic flux density $|\vec{B}|$ of monopole mode 9 in the horizontal plane.



(a) Electric field strength $|\vec{E}|$



(b) Magnetic flux density $|\vec{B}|$

1. SEXTUPOLE PASSBAND

MODE 5

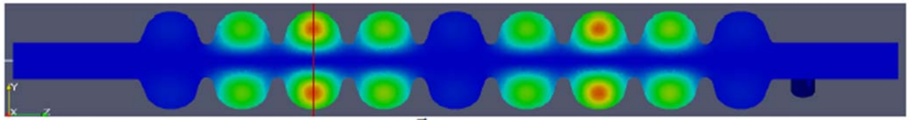


Figure 816: Electric field strength $|\vec{E}|$ of sextupole mode 5 in the vertical plane.

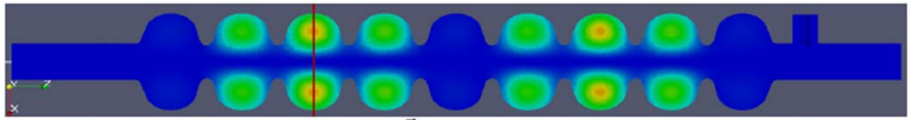


Figure 817: Electric field strength $|\vec{E}|$ of sextupole mode 5 in the horizontal plane.

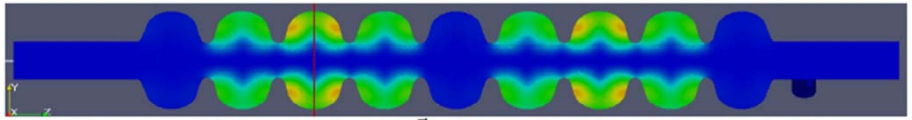


Figure 818: Magnetic flux density $|\vec{B}|$ of sextupole mode 5 in the vertical plane.

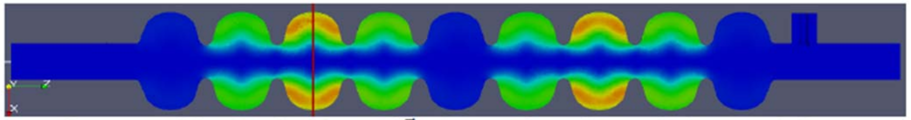
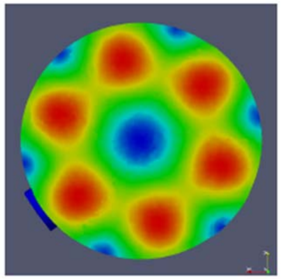
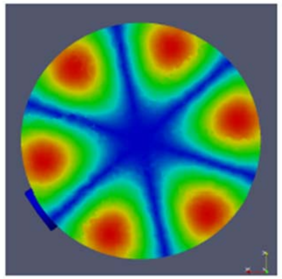


Figure 819: Magnetic flux density $|\vec{B}|$ of sextupole mode 5 in the horizontal plane.



(a) Electric field strength $|\vec{E}|$



(b) Magnetic flux density $|\vec{B}|$

pictures from W. Ackermann and Cong Liu, TEMF-TU-Darmstadt

example: eigenmodes in TESLA cavity

a pair of “dipole” modes

2. DIPOLE PASSBAND

MODE 5

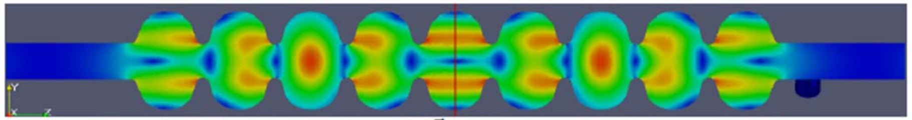


Figure 296: Electric field strength $|\vec{E}|$ of dipole mode 5 in the vertical plane.

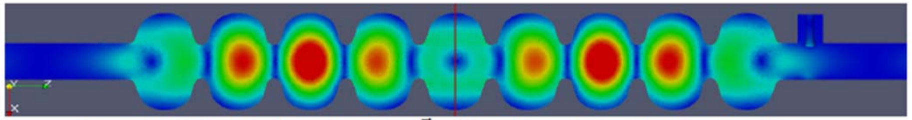


Figure 297: Electric field strength $|\vec{E}|$ of dipole mode 5 in the horizontal plane.

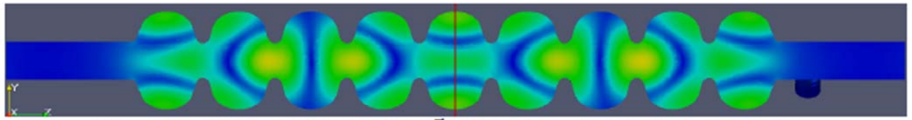


Figure 298: Magnetic flux density $|\vec{B}|$ of dipole mode 5 in the vertical plane.

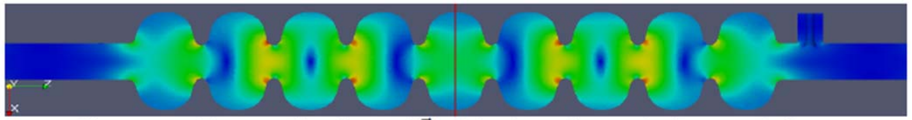
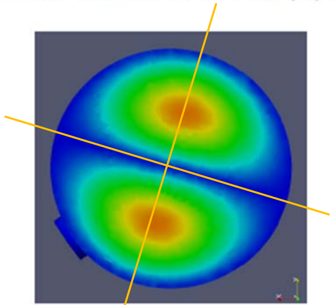
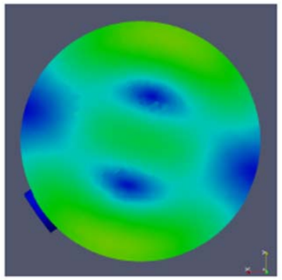


Figure 299: Magnetic flux density $|\vec{B}|$ of dipole mode 5 in the horizontal plane.



(a) Electric field strength $|\vec{E}|$



(b) Magnetic flux density $|\vec{B}|$

2. DIPOLE PASSBAND

MODE 6

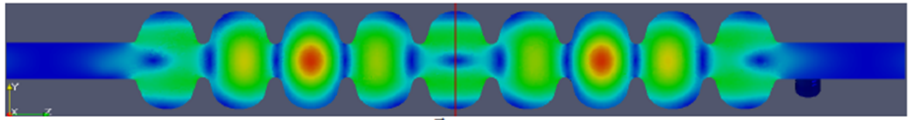


Figure 301: Electric field strength $|\vec{E}|$ of dipole mode 6 in the vertical plane.

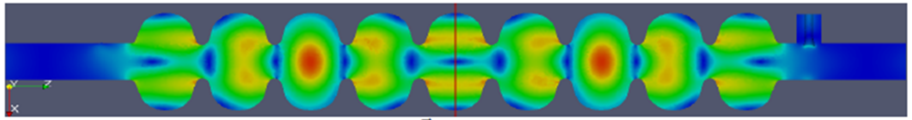


Figure 302: Electric field strength $|\vec{E}|$ of dipole mode 6 in the horizontal plane.

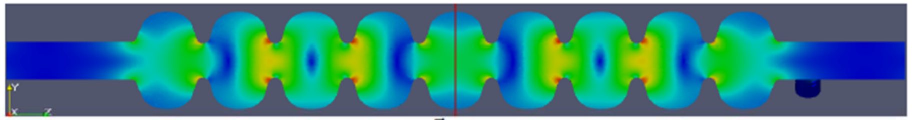


Figure 303: Magnetic flux density $|\vec{B}|$ of dipole mode 6 in the vertical plane.

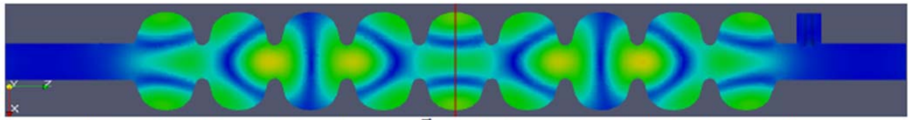
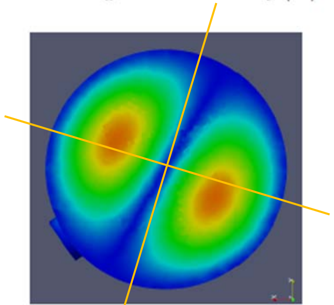
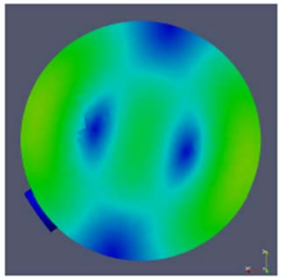


Figure 304: Magnetic flux density $|\vec{B}|$ of dipole mode 6 in the horizontal plane.



(a) Electric field strength $|\vec{E}|$



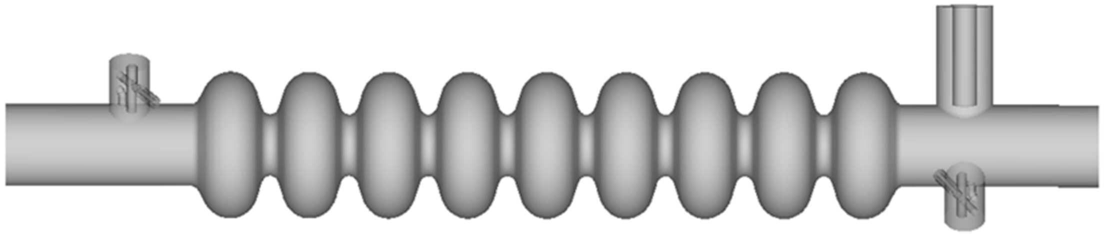
(b) Magnetic flux density $|\vec{B}|$

not quite perpendicular!

pictures from W. Ackermann and Cong Liu, TEMF-TU-Darmstadt

example: accelerating mode in 3.9 GHz superconducting cavity

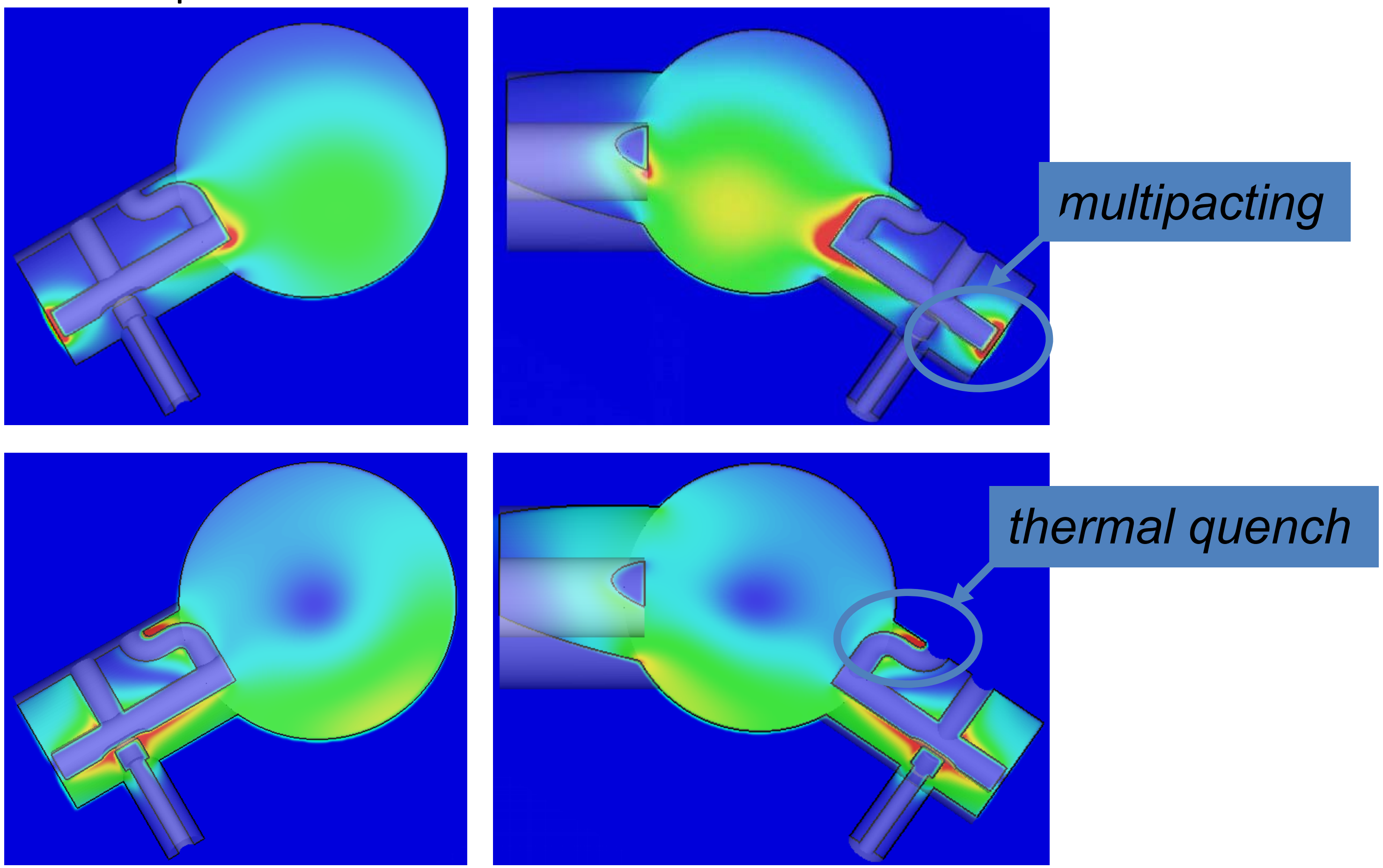
EM fields in HOM couplers



upstream | downstream

electric field strength

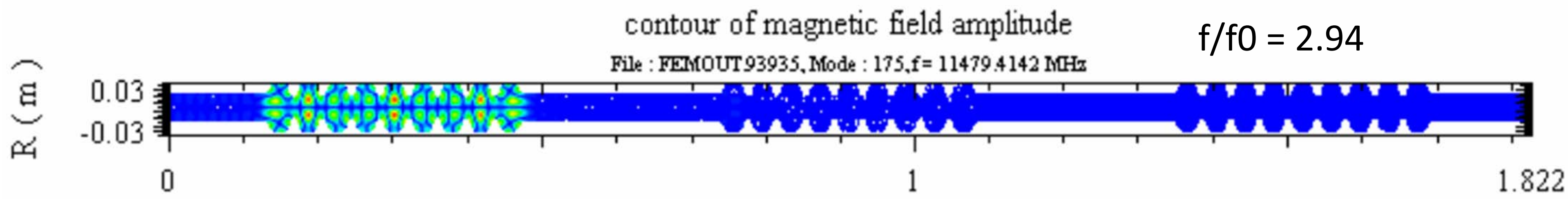
magnetic field strength

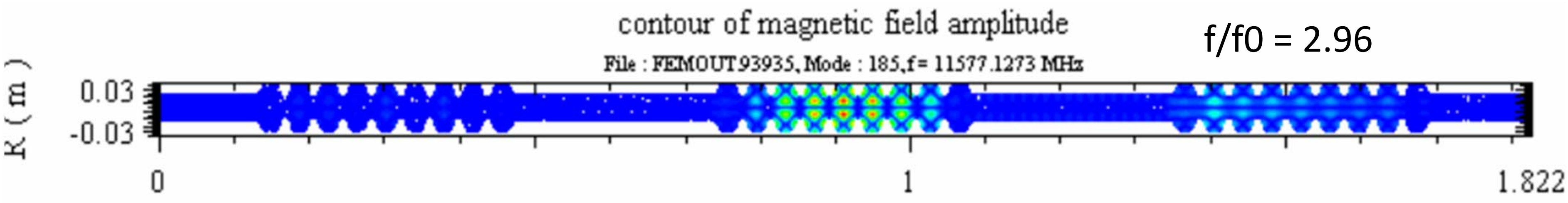


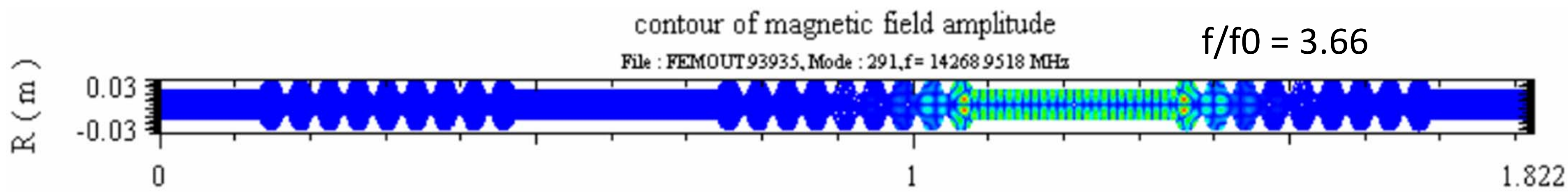
pictures from E. Gjonaj, TEMF-TU-Darmstadt

example: trapped modes in 3.9 GHz superconducting cavity

azimuthal geometry (without dampers), monopole modes
some (of many) trapped modes







pictures from J. Sekutovicz, TEMF-TU-Darmstadt

perturbation methods

surface losses

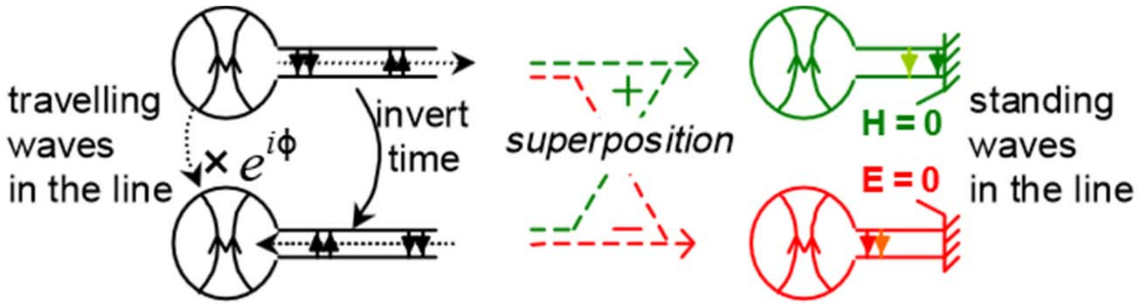
power-loss method: $\hat{Q} = \frac{\hat{\omega}\hat{W}}{\hat{P}}$ with $\hat{P} = \frac{1}{2}\mathrm{Re}\left\{\int \hat{\mathbf{E}} \times \hat{\mathbf{H}}^* \cdot d\mathbf{A}\right\} \approx \frac{1}{2}\int \mathrm{Re}\{Z_{\mathrm{surface}}\}\hat{H}^2 \cdot d\mathbf{A}$

$\mathrm{Re}\{Z_{\mathrm{surface}}\} = \sqrt{\frac{\omega\mu}{2\kappa}}$ conductivity κ

waveguide ports

Kroll-Yu method: calculate $\hat{\omega}(L)$ for resonator with perfect reflection in wave port, for different reference planes (length L to the port) $\rightarrow$ resonance frequency and quality; (it is more than a pert. method and works even for low Q)

Balleyguier method: calculate eigenmodes for two different boundary conditions (E=0, H=0); superimpose these modes to get the travelling wave in the waveguide; $\rightarrow$ power flow through waveguide



[1] N. Kroll, D. Yu: Computer determination of the external Q and resonant frequency of waveguide loaded cavities, SLAC-PUB-5171, Jan 1990

[2] P. Balleyguier: A straightforward method for cavity external Q computation, Particle Accelerators, Vol. 57, p113-127, 1997

notation

there are many eigenmodes "ν" with eigenvalues $\hat{\alpha}_\nu$, spatial fields $\hat{\mathbf{E}}_\nu(\mathbf{r})$, $\hat{\mathbf{B}}_\nu(\mathbf{r})$ and mode amplitudes $\hat{a}_\nu$

$$\begin{pmatrix} \mathbf{E}(\mathbf{r},t) \\ \mathbf{B}(\mathbf{r},t) \end{pmatrix} = \mathrm{Re}\left\{ \sum_\nu \hat{a}_\nu \begin{pmatrix} \hat{\mathbf{E}}_\nu(\mathbf{r}) \\ \hat{\mathbf{B}}_\nu(\mathbf{r}) \end{pmatrix} e^{i\hat{\alpha}_\nu t} \right\}$$

for simplicity we skip the index, but write every quantity with "hat"

complex eigenvalues: $\hat{\alpha} = -\frac{1}{\hat{\tau}_\nu} + i\hat{\omega}$ with

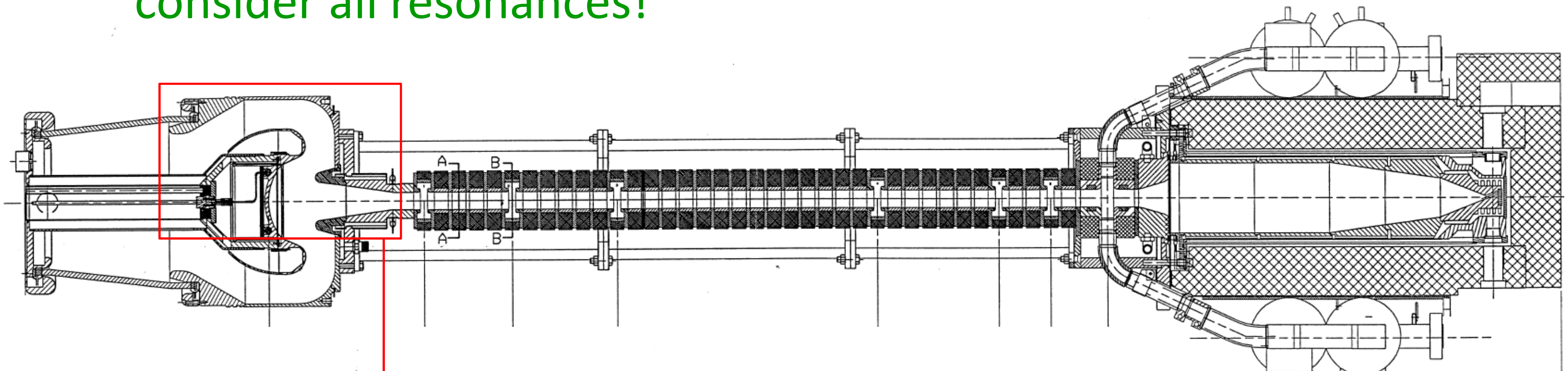
- $\hat{\tau}$ decay time
- $\hat{\omega}$ resonance frequency
- $\rightarrow \hat{Q} = \frac{\hat{\omega}\hat{\tau}}{2}$ quality factor

EM field energy, without losses

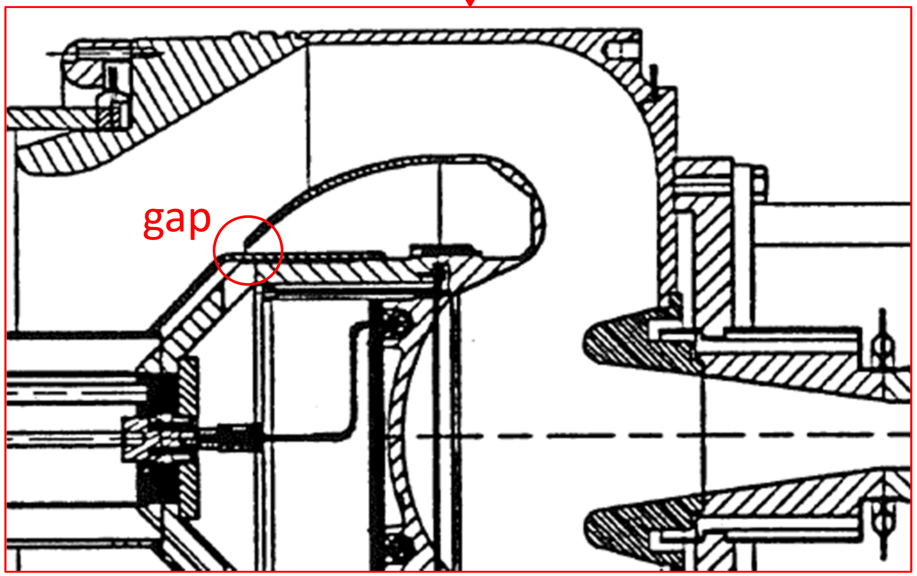
$$W_{EM} = \frac{1}{2}\int\left(\varepsilon\left\|\mathbf{E}(r,t)\right\|^2 + \mu^{-1}\left\|\mathbf{B}(r,t)\right\|^2\right)dV = \sum\left|\hat{a}\right|^2\hat{W}$$

$$\hat{W} = \frac{1}{2}\int\varepsilon\hat{E}^2 dV = \frac{1}{2}\int\mu^{-1}\hat{B}^2 dV$$

consider all resonances!



unexpected resonance in a klystron → instability



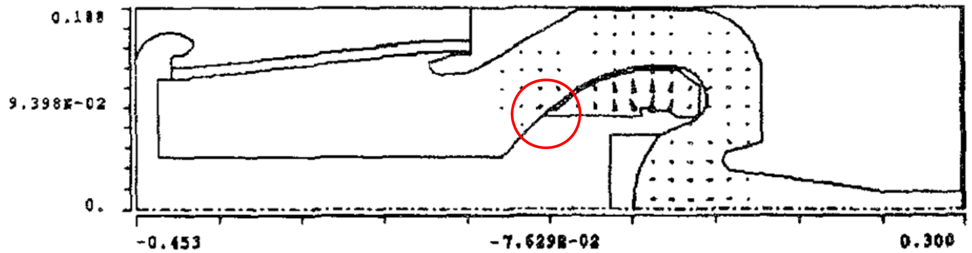
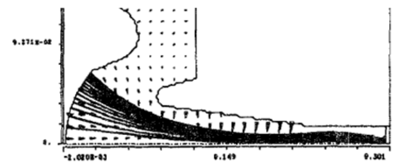


Figure 1. 1.365 GHz mode in the SLAC/DESY gun diode.



from: B. Krietenstein, K. Ko, T. Lee, U. Becker, T. Weiland, M. Dohlus: Spurious Oscillations in high Power Klystrons, SLAC-PUB-9957

modal part of wake
and long range wake

splitting into resonant part and rest

$$\mathbf{w}(x_1, y_1, x_2, y_2, s) = \sum \hat{\mathbf{w}}(x_1, y_1, x_2, y_2, s) + \mathbf{w}_{\text{rest}}(x_1, y_1, x_2, y_2, s)$$

this splitting is unique if the modes are fully excited (the source particle “1” is not longer in interaction with the field of the mode) and the mode is just ringing

$$\hat{\mathbf{w}}(x_1, y_1, x_2, y_2, \boxed{s > L}) = \operatorname{Re}\left\{\hat{\mathbf{f}}(x_1, y_1, x_2, y_2)\, e^{-i\hat{\omega}s/c}\right\}$$

the modal part contributes essentially to long range interactions (from bunch to bunch); usually the bunch distance is larger than L, the length of the field of the mode

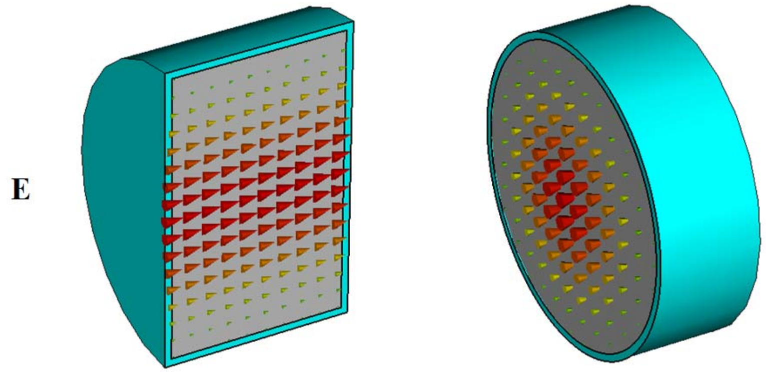
for ultra-relativistic bunches the wake function is causal $\mathbf{w}(x_1, y_1, x_2, y_2, s < 0) = \mathbf{0}$ and it is useful to **define**:

$$\hat{\mathbf{w}}(x_1, y_1, x_2, y_2, s < 0) = \mathbf{0}$$

$$\rightarrow \mathbf{w}_{\text{rest}}(x_1, y_1, x_2, y_2, s < 0) = \mathbf{0}$$

closed cavities without losses

no beam pipes, perfect conductivity



it can be shown* that the expansion into an infinite set of modes is complete, and:

$$\mathbf{w}_{\text{rest}}(x_1, y_1, x_2, y_2, s) \equiv 0$$

$$\hat{\mathbf{w}}(x_1, y_1, x_2, y_2, s) = h(s)\,\mathrm{Re}\left\{\hat{\mathbf{f}}(x_1, y_1, x_2, y_2)\, e^{i\hat{\omega}s/c}\right\}$$

with $h(s) = \begin{cases} 0 & \text{for } s < 0 \\ 1 & \text{for } s = 0 \\ 2 & \text{otherwise} \end{cases}$

therefore $\mathbf{w}(x_1, y_1, x_2, y_2, s) = h(s)\,\mathrm{Re}\left\{\sum \hat{\mathbf{f}}(x_1, y_1, x_2, y_2) e^{i\hat{\omega}s/c}\right\}$

* see: T. Weiland, R. Wanzenberg: Wake Fields and Impedances, DESY M91-06

two particle interaction per mode

for simplicity we choose zero offset of both particles and skip the offset coordinates

particle 1 travels alone through the cavity and excites the mode

energy loss of the particle $\Delta W_1 = q_1^2 \hat{k} = q_1^2 \hat{w}_{\|}(0)$

mode rings with complex amplitude $A_1 \sim q_1 \sqrt{\hat{k}}$

with $\hat{k}$ an unknown constant

particle 2 travels alone through the cavity, but shifted in time $\Delta t = s/c$

energy loss of the particle $\Delta W_2 = q_2^2 \hat{k} = q_2^2 \hat{w}_{\|}(0)$

mode rings with different phase $A_2 \sim q_2 \sqrt{\hat{k}} e^{i\hat{\omega}s/c}$

both particles together, in any distance

energy loss $\Delta W_1 + \Delta W_2 = q_1^2 \hat{w}_{||}(0) + q_1 q_2 \hat{w}_{||}(s) + q_2 q_1 \hat{w}_{||}(-s) + q_2^2 \hat{w}_{||}(0)$

mode $A = A_1 + A_2 \sim \left(q_1 + q_2 e^{i\hat{\omega}s/c}\right)\sqrt{\hat{k}}$

$$W_{\text{mode}} \sim |A|^2$$

$$W_{\text{mode}} = q_1^2 \hat{k} + q_1 q_2 2\hat{k}\cos(\hat{\omega}s/c) + q_2^2 \hat{k}$$

with energy conservation $\Delta W_1 + \Delta W_2 + W_{\text{mode}} = 0$ follows

$$\hat{w}_{||}(0) = -\hat{k}$$

$$\hat{w}_{||}(s) + \hat{w}_{||}(-s) = -2\hat{k}\cos(\hat{\omega}s/c)$$

with (defined) causality for ultra-relativistic bunches

$$\hat{w}_{||}(s) = -\hat{k}\cos(\hat{\omega}s/c)\,h(s)$$

$$h(s) = \begin{cases} 0 & \text{for } s < 0 \\ 1 & \text{for } s = 0 \\ 2 & \text{otherwise} \end{cases}$$

modal loss parameter

particle 1 travels through the cavity and excites the mode

$$W_{\text{mode}} = q_1^2 \hat{k}$$

particle 2 is a test charge (q2→0)

$$\Delta W_1 + \Delta W_2 \to q_1^2 \hat{w}_{\|}(0) + \underbrace{q_2 q_1 \hat{w}_{\|}(s)}$$

$$- q_1 2\hat{k} \cos(\hat{\omega} s/c)$$

voltage observed by test particle $V_{\text{mode}} = q_1 2\hat{k}$

→ modal loss parameter $\boxed{\hat{k} = \dfrac{V_{\text{mode}}^2}{4W_{\text{mode}}}}$

with $\hat{W}_{\text{mode}} = \frac{1}{2}\int \varepsilon \hat{E}^2 dV = \frac{1}{2}\int \mu^{-1} \hat{B}^2 dV$ and $\hat{V}_{\text{mode}} = \left| \int E_z(0,0,z) e^{i\tilde{\omega} z/c} dz \right|$

two particle interaction per mode, more general

arbitrary offset of source particle (index=1) and test particle (index=2)

$$\hat{w}_{\|}(x_1, y_1, x_2, y_2, s) = -h(s)\,\mathrm{Re}\left\{\hat{v}_{\|}^{*}(x_1, y_1)\,\hat{v}_{\|}(x_2, y_2)\,e^{i\hat{\omega}s/c}\right\}$$

with $\hat{v}_{\|}(x, y) = \dfrac{\int \hat{E}_z(x, y, z)\,e^{i\tilde{\omega}z/c}\,dz}{2\sqrt{\hat{W}}}$

modal loss-parameters and voltages are post-processing results from eigenmode solvers; transverse wake functions are

$$\hat{w}_{x}(x_1, y_1, x_2, y_2, s) = -h(s)\,\mathrm{Re}\left\{\hat{v}_{\|}^{*}(x_1, y_1)\,\hat{v}_{x}(x_2, y_2)\,e^{i\hat{\omega}s/c}\right\}$$

$$\hat{w}_{y}(x_1, y_1, x_2, y_2, s) = -h(s)\,\mathrm{Re}\left\{\hat{v}_{\|}^{*}(x_1, y_1)\,\hat{v}_{y}(x_2, y_2)\,e^{i\hat{\omega}s/c}\right\}$$

the transverse voltages v_x, v_y are either calculated directly (as $v_{\|}$), or with help of the Panofsky-Wenzel theorem:

$$\hat{v}_x(x, y) = i\frac{c}{\hat{\omega}}\frac{\partial}{\partial x}\hat{v}_{\|}(x, y) \qquad \hat{v}_y(x, y) = i\frac{c}{\hat{\omega}}\frac{\partial}{\partial y}\hat{v}_{\|}(x, y)$$

azimuthal symmetry

longitudinal field (of TM modes):

$$\hat{E}_z = \hat{E}_z^{(m)}(r,z)\begin{Bmatrix}\cos(m\varphi)\\ \sin(m\varphi)\end{Bmatrix}$$

with $m = 0 \rightarrow$ monopole modes
1 dipole modes
2 sextupole modes
…

for $m > 0$ there are always pairs of modes (with $\cos(m\varphi)$ and $\sin(m\varphi)$)

$$\hat{v}_{\|}(x,y) = \frac{\int \hat{E}_z^{(m)}(r,z) e^{i\tilde{\omega}z/c}\,dz}{2\sqrt{\hat{W}}}\begin{Bmatrix}\cos(m\varphi)\\ \sin(m\varphi)\end{Bmatrix}$$

ultra-relativistic case:

$$\nabla_{\perp}^2 \hat{v}_{\|} = 0 \rightarrow \hat{v}_{\|}(x,y) = \hat{v}^{(m)} \cdot r^m \begin{Bmatrix}\cos(m\varphi)\\ \sin(m\varphi)\end{Bmatrix}$$

$\hat{v}^{(m)}$ can be chosen real

normalized loss-parameter: $\hat{k}^{(m)} := \left(\hat{v}^{(m)}\right)^2$

monopole modes: $\hat{v}_{\parallel}(x,y) = \hat{v}^{(0)} = const$

$$\hat{w}_{\parallel}^{(0)}(\cdots,s) = -h(s)\hat{k}^{(0)}\cos(\hat{\omega}s/c)$$

$$\hat{w}_{\perp}^{(0)}(\cdots,s) = 0$$

dipole modes: $\hat{v}_{\parallel}(x,y) = \hat{v}^{(1)} r \begin{Bmatrix} \cos\varphi \\ \sin\varphi \end{Bmatrix} = \hat{v}^{(1)} \begin{Bmatrix} x \\ y \end{Bmatrix}$

the pair of modes is combined to

$$\hat{w}_{\parallel}^{(1)}(x_1,y_1,x_2,y_2,s) = -h(s)\hat{k}^{(1)}(x_1x_2 + y_1y_2)\cos(\hat{\omega}s/c)$$

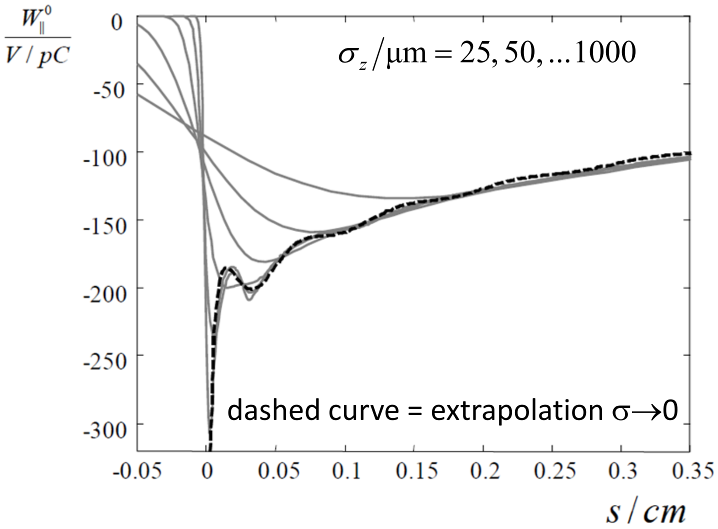
$$\hat{w}_{x}^{(1)}(x_1,y_1,x_2,y_2,s) = h(s)\frac{c}{\hat{\omega}}\hat{k}^{(1)}x_1\sin(\hat{\omega}s/c)$$

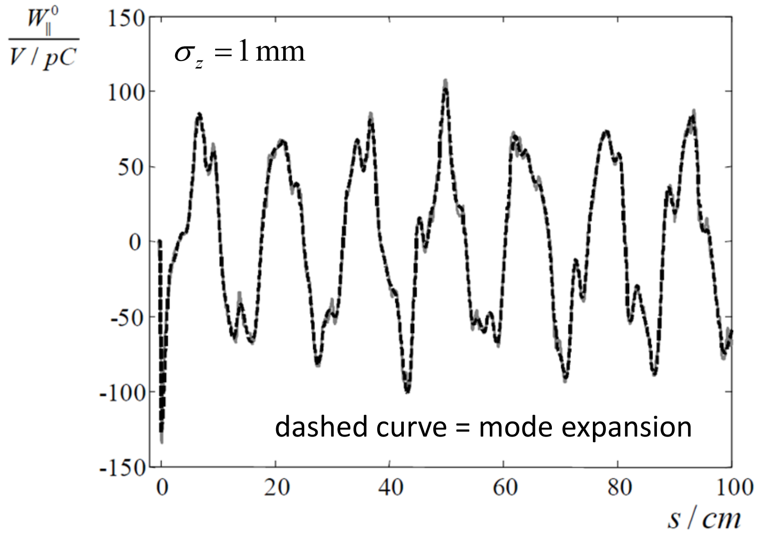
$$\hat{w}_{y}^{(1)}(x_1,y_1,x_2,y_2,s) = h(s)\frac{c}{\hat{\omega}}\hat{k}^{(1)}y_1\sin(\hat{\omega}s/c)$$

longitudinal kick is second order;
transverse kick depends linear on the offset of the source particle,
it does **not** depend on the offset of the test particle

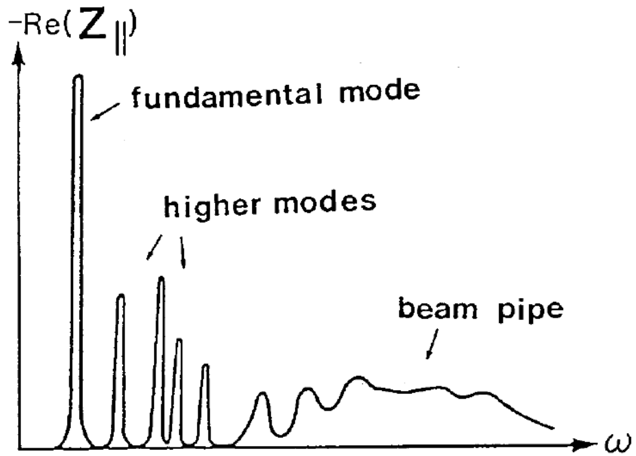
longitudinal wakes and impedances

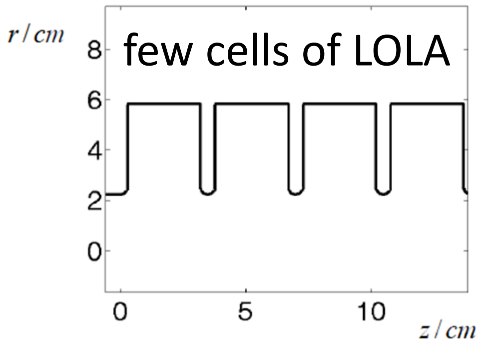
longitudinal wake for a multicell structure (transverse deflecting cavity LOLA)





real part of impedance, in principle





pictures from: Zagorodnov, T. Weiland, M.Dohlus: Wake Fields Generated by the LOLA-IV Structure and the 3rd Harmonic Section in TTF-II, TESLA Report 2004-01 and T. Weiland, R. Wanzenberg: Wake Fields and Impedances, DESY M91-06

modal and total loss parameters for finite bunch length

splitted longitudinal wake function: $w_{\|}(\cdots,s)=\sum \hat{w}_{\|}(\cdots,s)+w_{\|,\mathrm{rest}}(\cdots,s)$

calculation by time-domain wake code gives wake potential:

$$W_{\|}(\cdots,s)=\int w_{\|}(\cdots,u)\lambda(s-u)du$$

with normalized line charge density $\lambda(s)$, in particular $\lambda_{\sigma}(s)=(2\pi\sigma)^{-1}\exp\left(-s^2/\left(2\sigma^2\right)\right)$

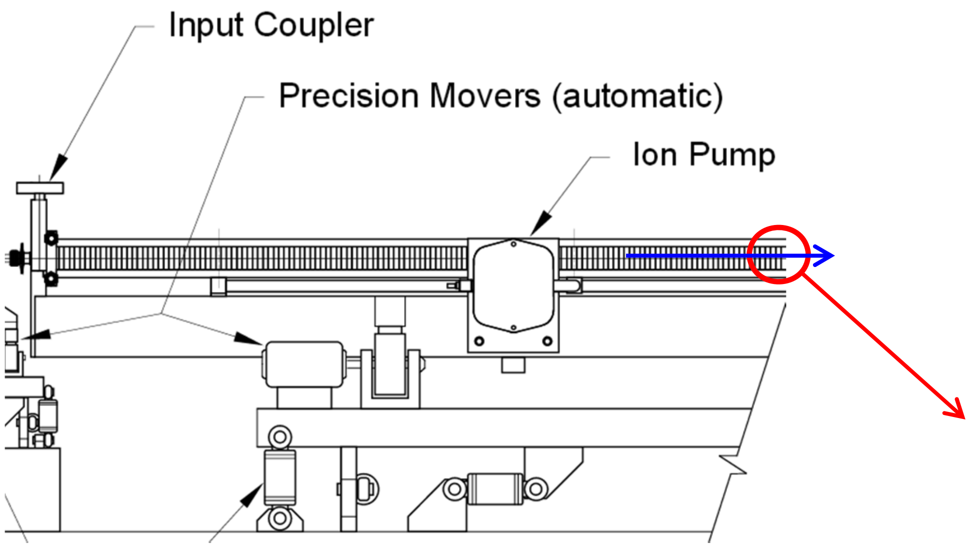
total loss parameter: $\boxed{k_{\mathrm{tot},\sigma}=-\int W_{\|}(\cdots,s)\lambda_{\sigma}(s)ds}$ and $k_{\mathrm{tot}}=-w_{\|}(\cdots,0)$

modal loss parameters with: $\hat{w}_{\|}(\cdots,s)=-h(s)\hat{k}(\cdots)\cos(\hat{\omega}s/c)$ and $h(s)=\begin{cases}0 & \text{for } s<0\\ 1 & \text{for } s=0\\ 2 & \text{otherwise}\end{cases}$

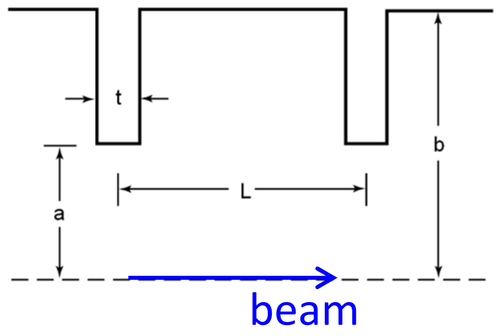
$$\hat{k}_{\sigma}=2\hat{k}\int_{-\infty}^{\infty}ds\times\lambda_{\sigma}(s)\int_{0}^{\infty}du\times\cos(\hat{\omega}u/c)\lambda_{\sigma}(s-u)$$

$$\boxed{\hat{k}_{\sigma}=\hat{k}\exp\left(-\hat{\omega}^2/\left(c^2\sigma^2\right)\right)}$$

travelling wave accelerator structure



one periode of a
„disc loaded“ structure:

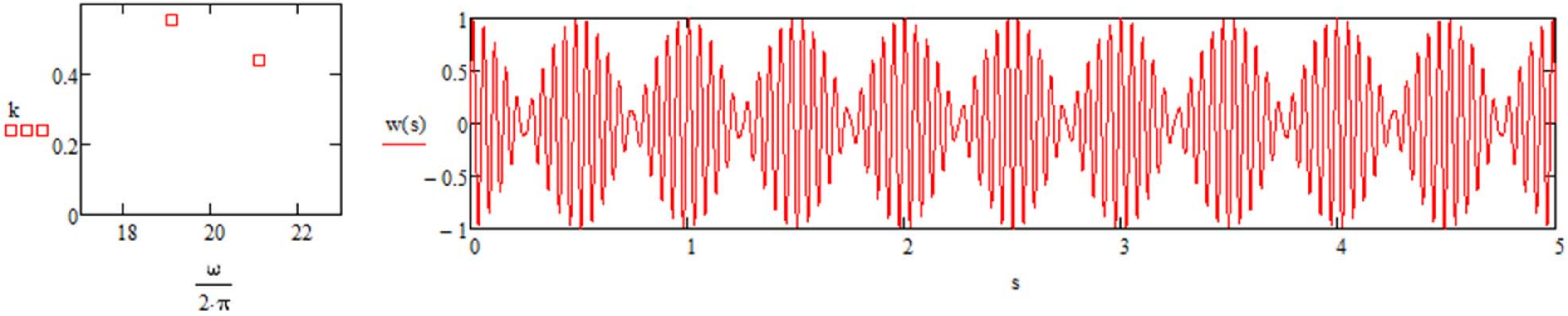


these structures are tapered, individual parameters for each cell: $a,\ b,\ t,\ \ldots$

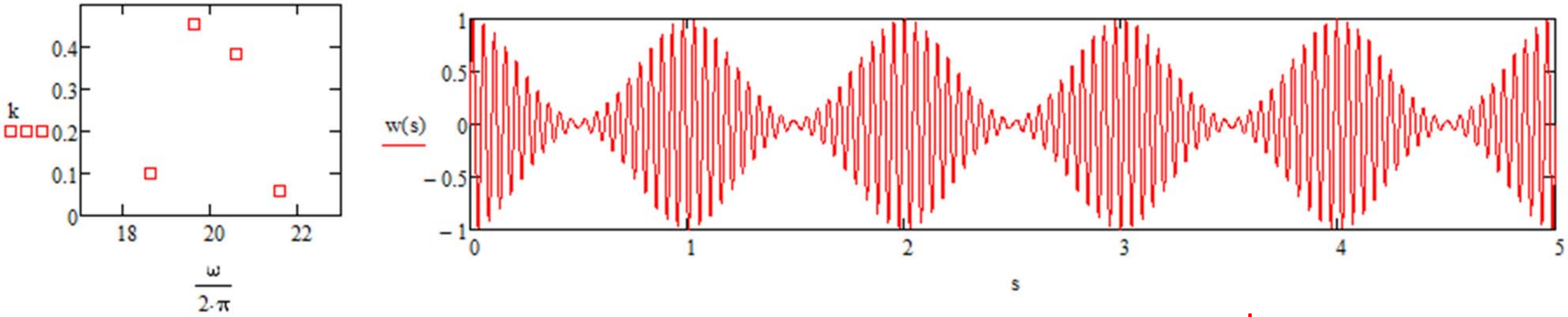
the parameters per period can be **tuned** to fulfill several conditions simultaneously, as resonance frequency and phase advance of the fundamental mode and frequency and loss-parameter of the 1st and 2nd dipole band

(de)tuning of dipole modes: adjust k and ω

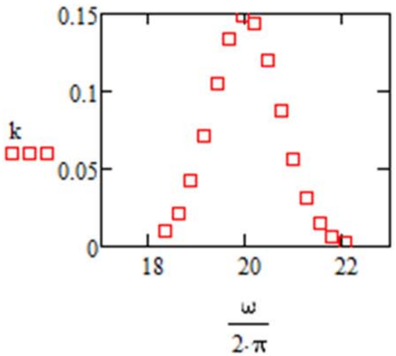
2 modes, with nearly the same strength ($k^{(1)}$)

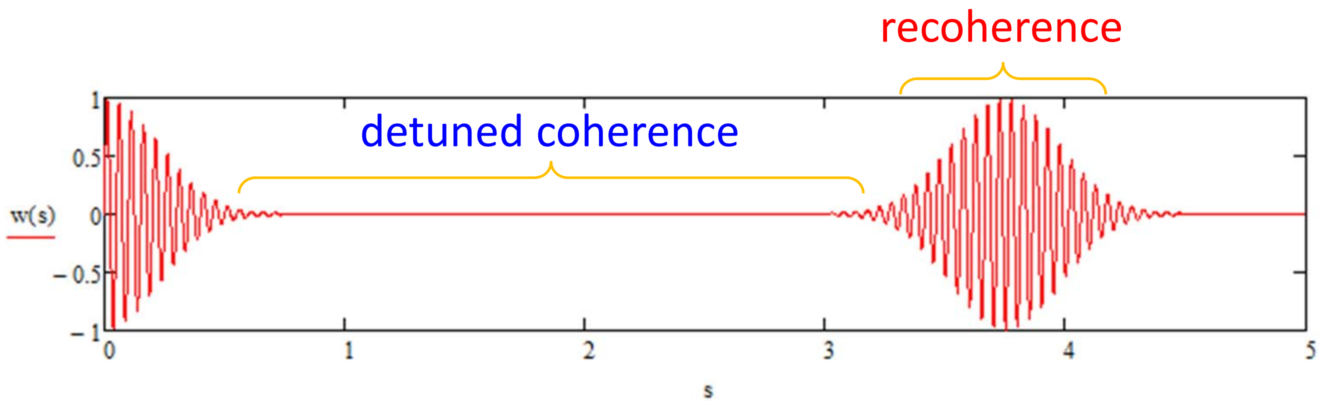


4 modes, „detuned“ k,ω distribution

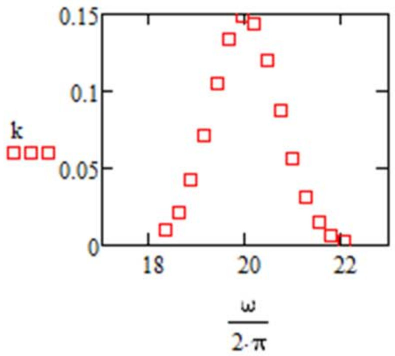


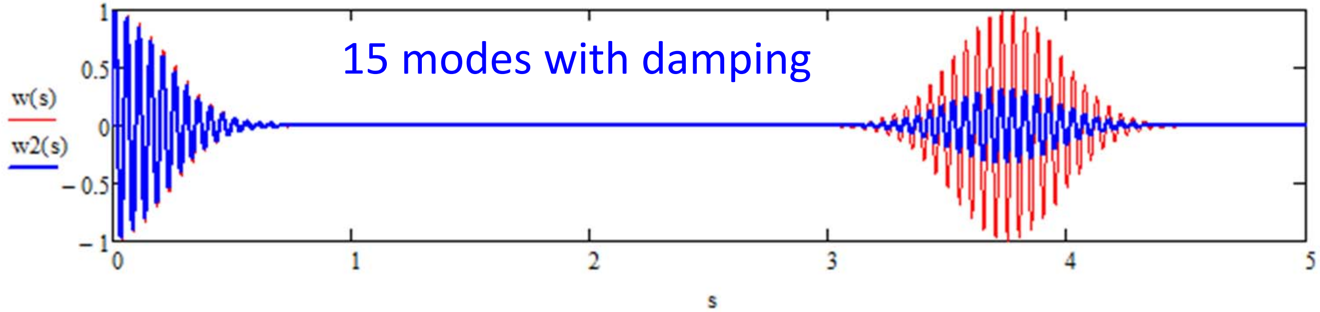
15 modes



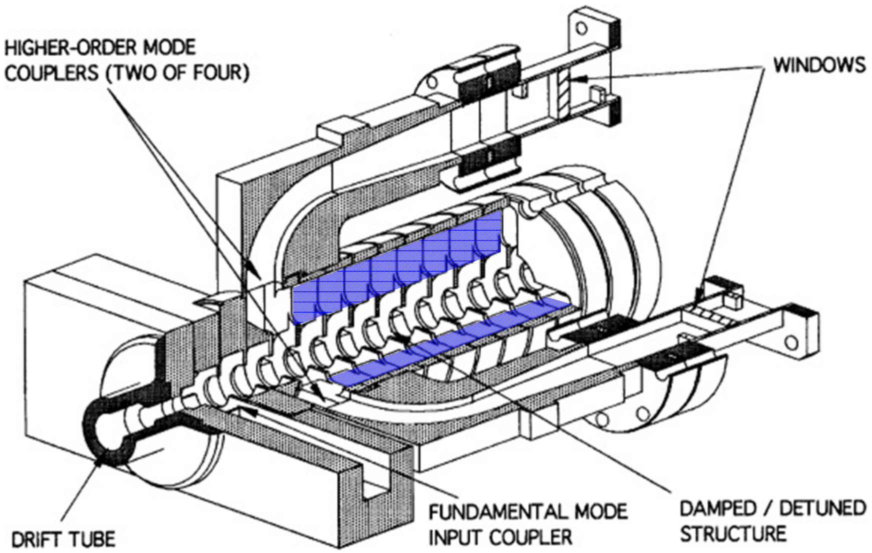


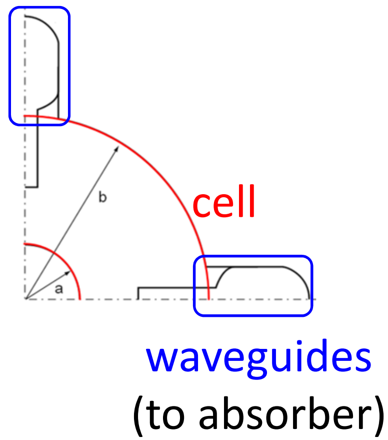
damping and detuning



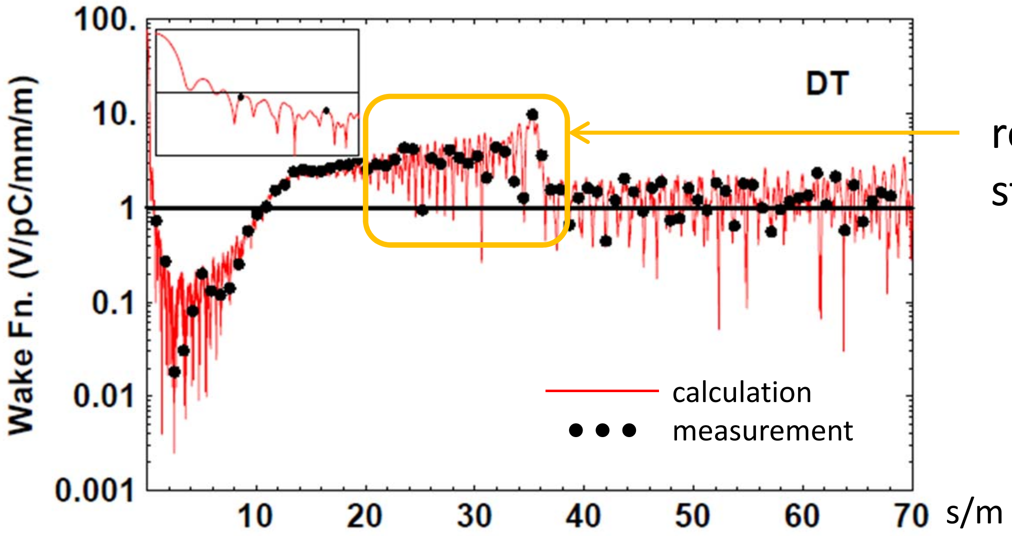


a multi-cell structure with damping and detuning (=DDS)

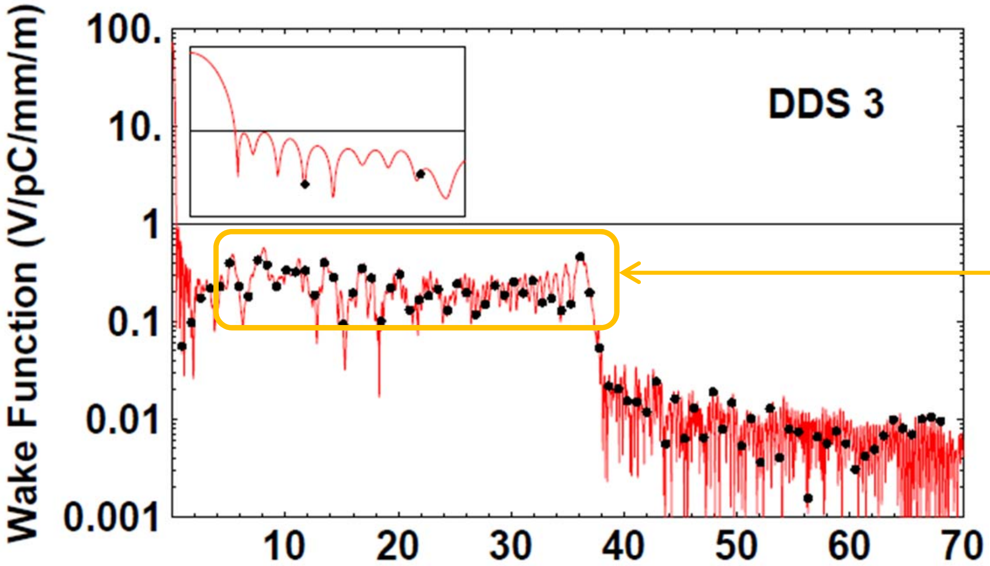




from: R. Jones, N. Kroll, R. Miller, R. Ruth, W. Wang: Advanced damped detuned structure development at SLAC, PAC 1997 a multi-cell structure with damping and detuning (=DDS)



recoherence in detuned structure with natural losses

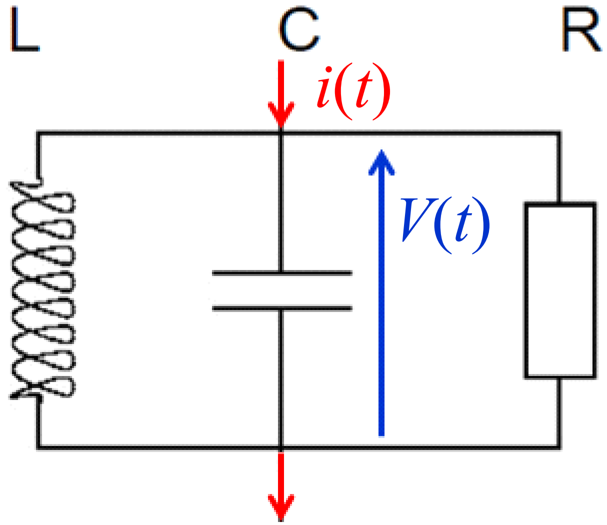


natural + external damping

transverse long-range wake is reduced by two orders of magnitude, in comparison to the short range wake

from: R. Jones, N. Kroll, R. Miller, R. Ruth, W. Wang: Advanced damped detuned structure development at SLAC, PAC 1997

equivalent circuit model for longitudinal wake



with $C = \left(2\hat{k}\right)^{-1}$

$LC = \hat{\omega}^{-2}$

no losses: $R \to \infty$

bunch current: $i(t) = q\delta(t)$

$\to$ voltage: $V(t>0) = -2\hat{k}\cos(\hat{\omega}t)$

the voltage $V(t)$ induced by a series of particles $i(t) = q_1\delta(t-t_1) + q_2\delta(t-t_2) + \ldots$
corresponds to voltage observed by a test particle at time t
(t, t1, t2, … corresponds to time in a certain reference plane)

with losses: $R = \dfrac{\hat{Q}}{\hat{\omega}C} = \hat{Q}\hat{\omega}L = \dfrac{2\hat{k}\hat{Q}}{\hat{\omega}}$ shunt impedance

longitudinal impedance (per mode):

$$\hat{Z}(\omega) = \frac{V(\omega)}{I(\omega)} = -\left(\frac{1}{i\omega L} + i\omega C + \frac{1}{R}\right)^{-1} = -\frac{2\hat{k}\hat{Q}}{\hat{\omega}}\left(1 + i\hat{Q}\left(\frac{\omega}{\hat{\omega}} - \frac{\hat{\omega}}{\omega}\right)\right)^{-1}$$

this is the Fourier-transform of $\hat{w}(s)$, as defined in part 1

some important cavity parameters

(angular) resonance frequency $\hat{\omega}$

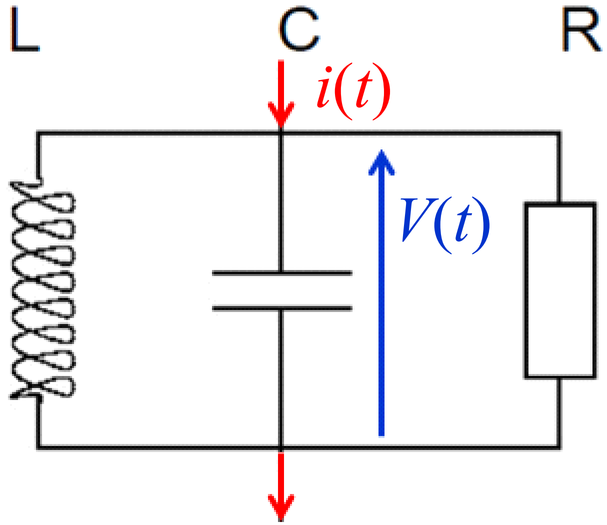
loss parameter $\hat{k} = \hat{W} \Big/ \left(4\hat{V}^2\right)$

quality $\hat{Q}$

shunt impedance $\hat{R} = 2\hat{k}\hat{Q} \big/ \hat{\omega}$

„R/Q“ $\hat{R} \big/ \hat{Q} = 2\hat{k} \big/ \hat{\omega}$

losses (absorption by R)



$$i(t) = \underbrace{I + 2I}_{\tilde{I}} \sum \cos\left(m \frac{2\pi}{T} t \right)$$

$$m_r \frac{2\pi}{T} = \omega_r \rightarrow \quad u(t) \approx \underbrace{2IR}_{\tilde{V}} \cos(\hat{\omega} t)$$

„single“ bunch losses

decay time τ is large compared to bunch distance T: $\hat{W}_1 = q^2 \hat{k} \rightarrow P_1 = \frac{q^2 \hat{k}}{T}$

fully resonant losses

for $\hat{\omega} T = n 2\pi$ and $\tau >> T$: $P_r = \frac{1}{2} \tilde{I}^2 R = I^2 \boxed{2R} \quad = P_1 \frac{2\hat{\tau}}{T}$

in general

$$P = \frac{\hat{k} q^2}{T} \frac{1 - e^{-2T/\hat{\tau}}}{\left| 1 - e^{\hat{\alpha} T} \right|^2} \quad \text{with } \hat{\alpha} = -\frac{1}{\hat{\tau}_v} + i \hat{\omega} \text{ and } \hat{Q} = \frac{\hat{\omega} \hat{\tau}}{2}$$

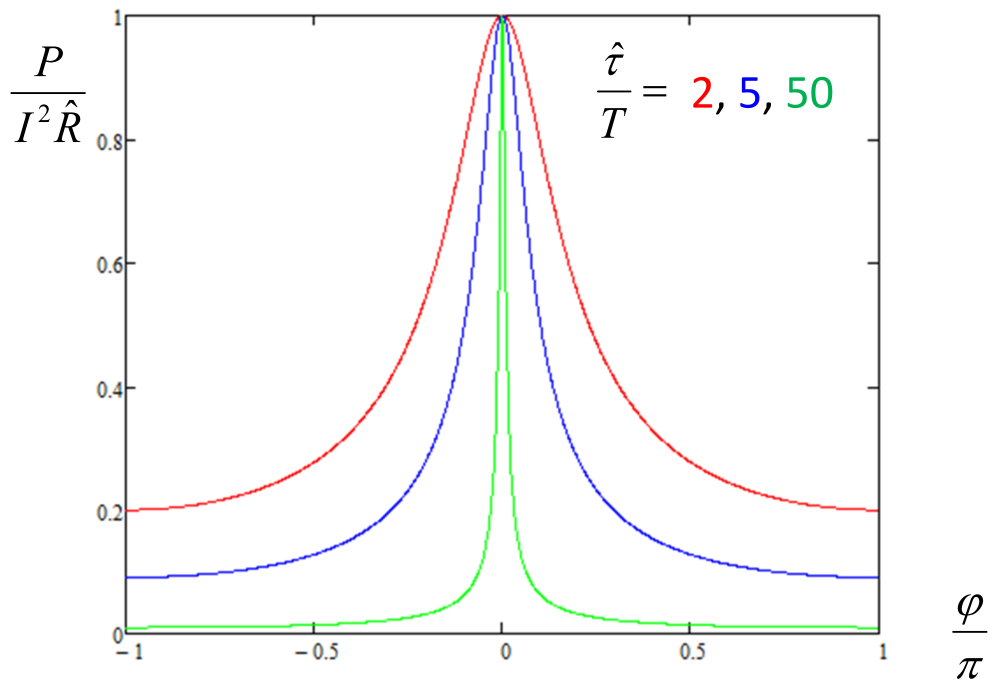
weak decay

$$P = \frac{\hat{k}q^2}{T}\frac{1-e^{-2T/\hat{\tau}}}{\left|1-e^{\hat{\alpha}T}\right|^2} \rightarrow I^2\hat{R}\frac{1}{\left|1+\frac{\hat{\tau}}{T}(1-e^{-i\varphi})\right|^2}$$

resonant losses | phase dependent term

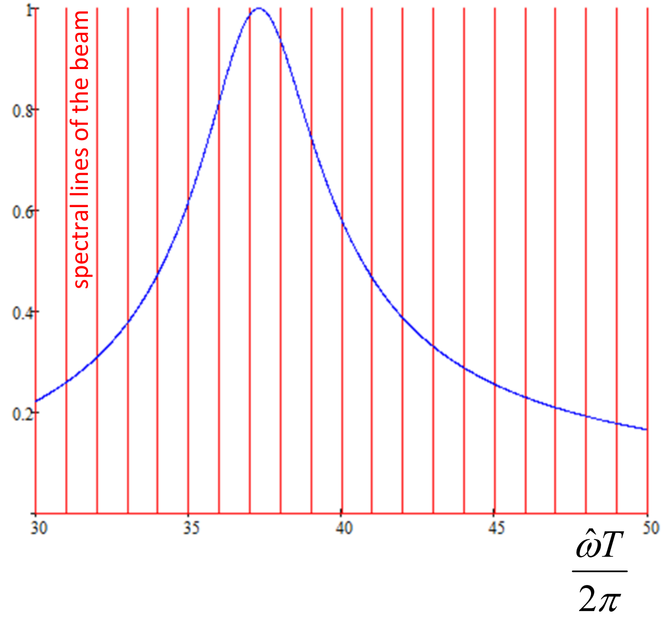
with $\varphi = \mathrm{mod}(\hat{\omega}T, 2\pi)$

the phase dependent term

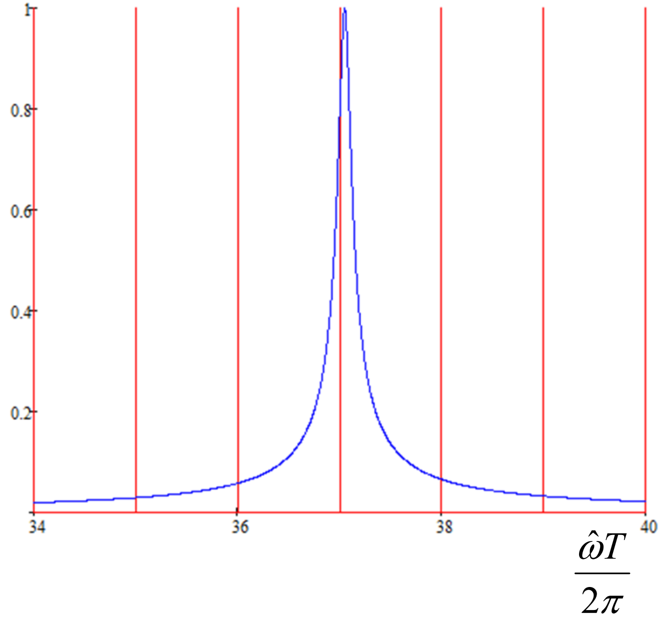


it is not very likely to excite a sharp resonance, and it is very difficult to determine the phase φ, but …

$T >> \tau \rightarrow$ single bunch losses P_1



$T < \tau \rightarrow$ losses may be resonant



the exact frequency of parasitic, unwanted resonances is usually not known and may depend on geometric parameters that are not exactly determined (f.i. length of bellows)

if the probability for φ is equally distributed or $\frac{\Delta\hat{\omega}T}{2\pi} > 1$

the worst case losses are resonant; in best case the losses are much smaller than P_1, but typically:

$$\langle P \rangle = I^2 R \left\langle \left| 1 + \frac{\hat{\tau}}{T}(1 - e^{-i\varphi}) \right|^{-2} \right\rangle_{\varphi} \rightarrow P_1 \qquad \text{with} \qquad \frac{1}{2\pi}\int_0^{2\pi} \frac{d\varphi}{\left| 1 + \frac{\hat{\tau}}{T}(1 - e^{-i\varphi}) \right|^2} \rightarrow \frac{T}{2\hat{\tau}}$$

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