BEAM INSTABILITIES IN LINEAR MACHINES 2

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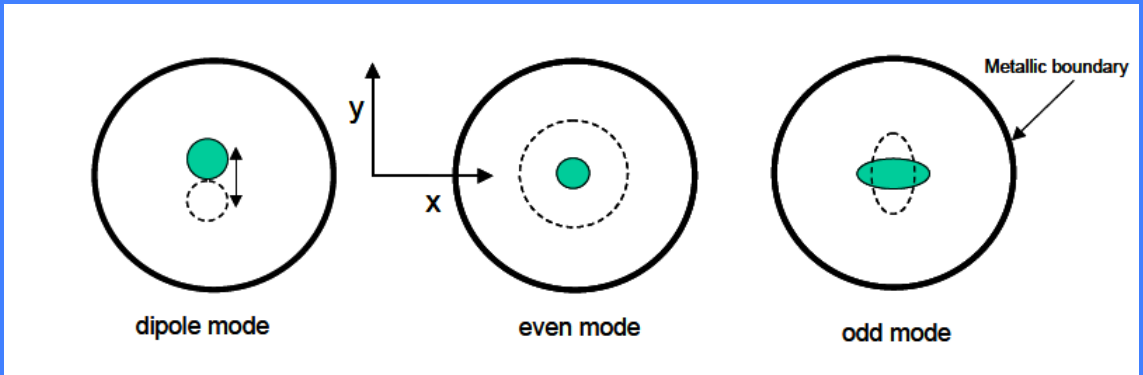

The CERN Accelerator School

CERN – 5 November 2015

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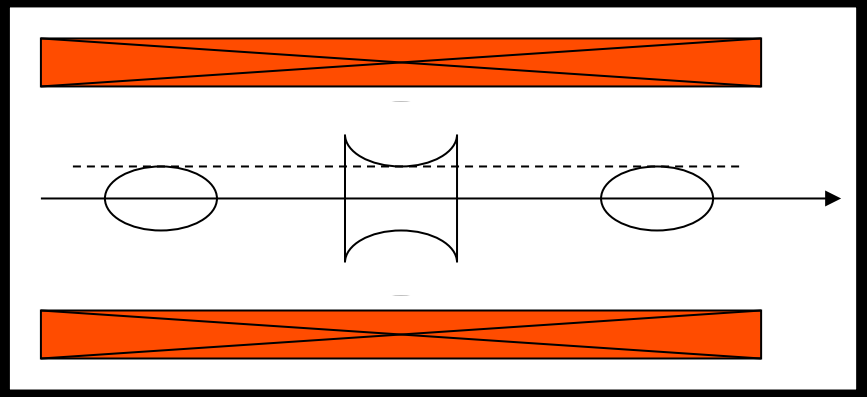
Single Component Relativistic Plasma

$$\sigma'' + k_s^2 \sigma = \frac{k_{sc}(s,\gamma)}{\sigma}$$

$$k_s = \frac{qB}{2mc\beta\gamma}$$

Equilibrium solution:

$$\sigma_{eq}(s,\gamma) = \frac{\sqrt{k_{sc}(s,\gamma)}}{k_s}$$



Small perturbation:

$$\sigma(\zeta) = \sigma_{eq}(s) + \delta\sigma(s)$$

$$\delta\sigma''(s) + 2k_s^2 \delta\sigma(s) = 0$$

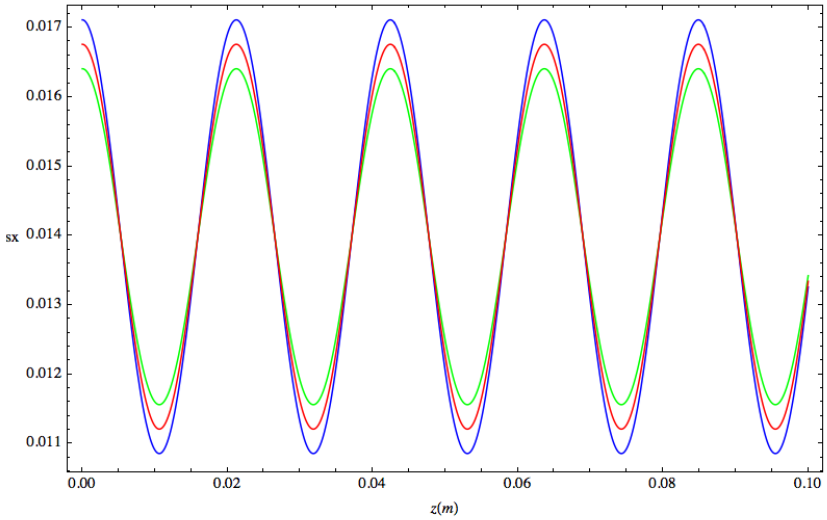
$$\delta\sigma(s) = \delta\sigma_o(s)\cos\left(\sqrt{2}k_s z\right)$$

Perturbed trajectories oscillate around the equilibrium with the same frequency but with different amplitudes:

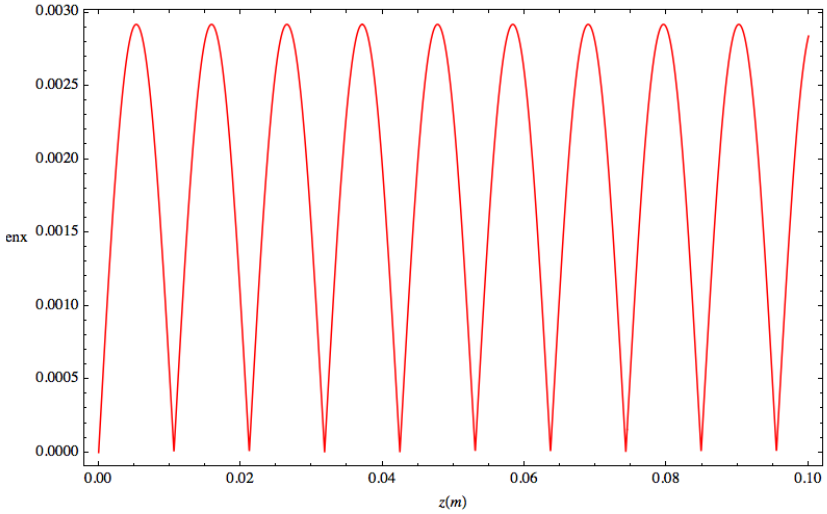
$$\sigma(s) = \sigma_{eq}(s) + \delta\sigma_o(s)\cos\left(\sqrt{2}k_s z\right)$$

Envelope oscillations drive Emittance oscillations

$\sigma(z)$



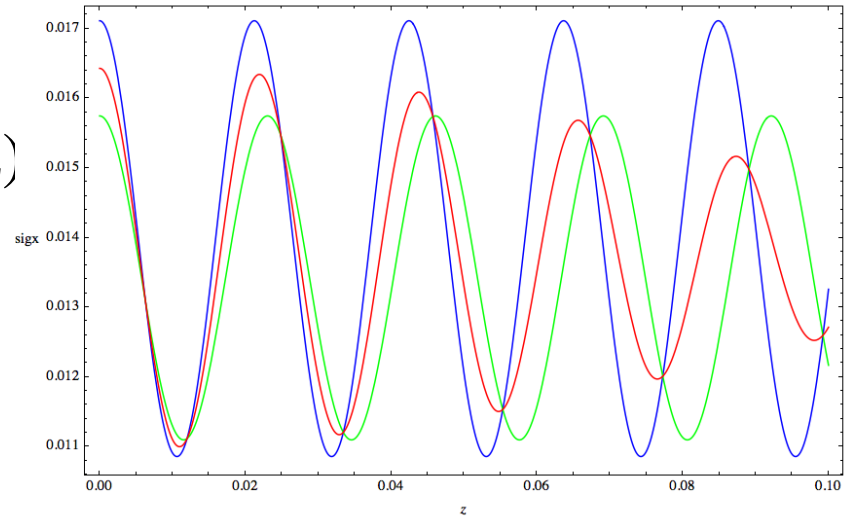
$\varepsilon(z)$



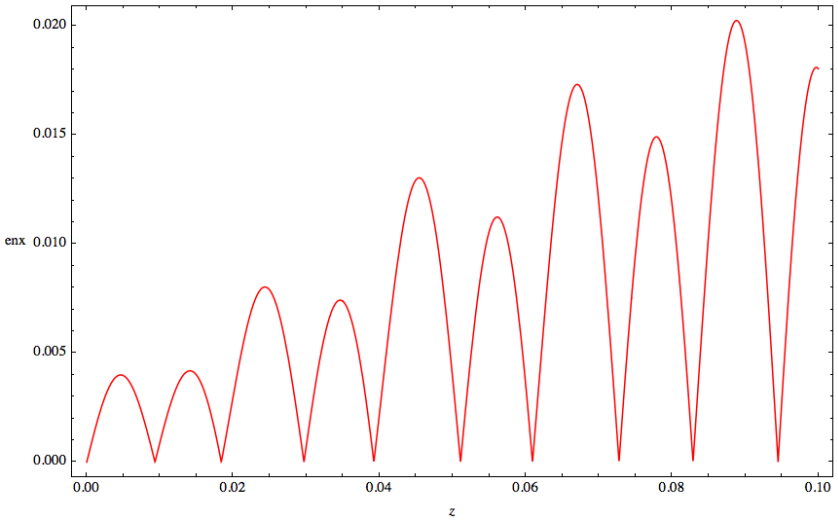
$$\varepsilon_{rms} = \sqrt{\sigma_x^2 \sigma_{x'}^2 - \sigma_{xx'}{}^2} = \sqrt{\left(\left\langle x^2 \right\rangle \left\langle x'^2 \right\rangle - \left\langle xx' \right\rangle^2\right)} \approx \left| sin\left(\sqrt{2} k_s z\right) \right|$$

energy spread induces decoherence

$\sigma(z)$



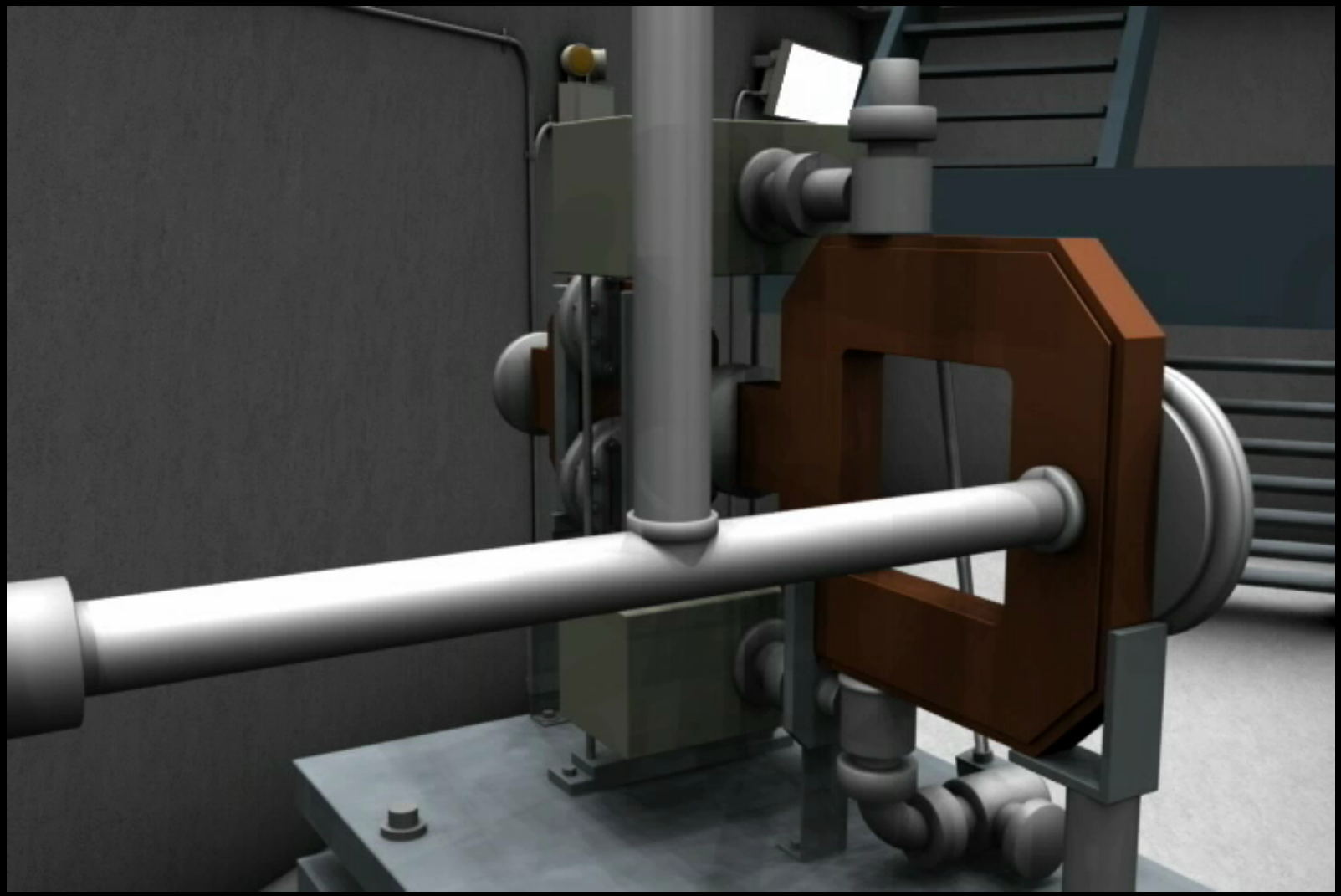
$\varepsilon(z)$



OUTLINE

- The rms emittance concept
- rms envelope equation
- Space charge forces
- Beam/Envelope emittance oscillations
- Matching conditions in a linac and emittance compensation

High Brightness Photo-Injector



Envelope Equation with Longitudinal Acceleration

$$p_o = \gamma_o m_o \beta_o c$$
$$p_x << p_o$$
$$p = p_o + p'z$$
$$p' = (\beta\gamma)' m_o c$$

$$\frac{dp_x}{dt} = \frac{d}{dt}(px') = \beta c \frac{d}{dz}(px') = 0$$

$$x'' + \frac{p'}{p} x' = 0$$

$$x'' = -\frac{(\beta\gamma)'}{\beta\gamma} x'$$

$$\langle xx'' \rangle = -\frac{(\beta\gamma)'}{\beta\gamma}\langle xx' \rangle = -\frac{(\beta\gamma)'}{\beta\gamma}\sigma_{xx'}$$

$$\sigma_x'' = \frac{\varepsilon_{rms}^2}{\sigma_x^3} + \frac{\langle xx'' \rangle}{\sigma_x}$$

$$\frac{d\sigma_x}{dz} = \sigma_x' = \frac{\sigma_{xx'}}{\sigma_x}$$

Space Charge De-focusing Force

$$\sigma_x'' + \frac{(\beta\gamma)'}{\beta\gamma}\sigma_x' + k^2\sigma_x = \frac{\varepsilon_n^2}{(\beta\gamma)^2\sigma_x^3} + \frac{k_{sc}}{\sigma_x}$$

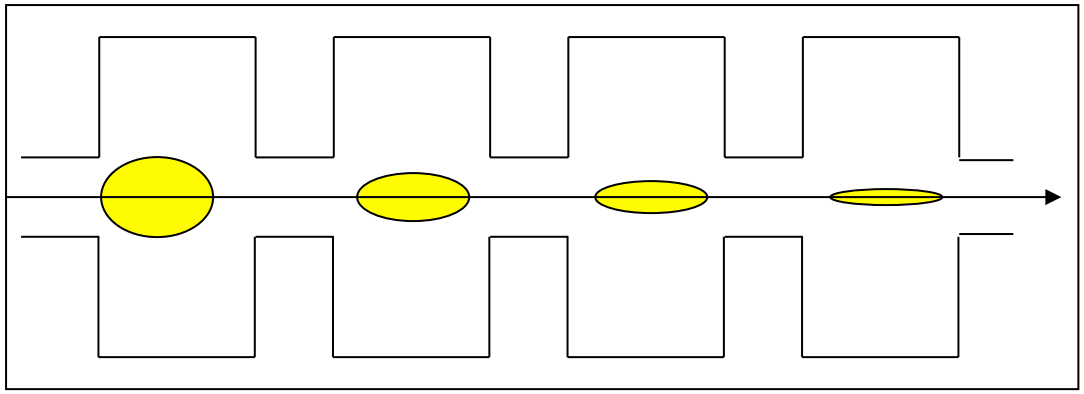
Adiabatic Damping

Emittance Pressure

Other External Focusing Forces

$$\varepsilon_n = \beta\gamma\varepsilon_{rms}$$

Beam subject to strong acceleration



$$\sigma_x'' + \frac{\gamma'}{\gamma}\sigma_x' + \frac{k_{RF}^2}{\gamma^2}\sigma_x = \frac{\varepsilon_n^2}{\gamma^2\sigma_x^3} + \frac{k_{sc}^o}{\gamma^3\sigma_x}$$

We must include also the RF focusing force: $k_{RF}^2 = \frac{\gamma'^2}{2}$

$$k_{sc}^o = \frac{2I}{I_A} g(s,\gamma)$$

$$\sigma_x'' + \frac{\gamma'}{\gamma}\sigma_x' + \frac{k_{RF}^2}{\gamma^2}\sigma_x = \frac{\varepsilon_n^2}{\gamma^2\sigma_x^3} + \frac{k_{sc}^o}{\gamma^3\sigma_x}$$

$\gamma = 1 + \alpha z$ ==> $\gamma'' = 0$

Looking for an "equilibrium" solution $\sigma_{inv} = \sigma_o \gamma^n$

==> all terms must have the same dependence on γ

$$\sigma_{inv}' = n\sigma_o\gamma^{n-1}\gamma'$$

$$\sigma_{inv}'' = n(n-1)\sigma_o\gamma^{n-2}\gamma'^2$$

$$n(n-1)\sigma_o\gamma^{n-2}\gamma'^2 + n\sigma_o\gamma^{n-2}\gamma'^2 + k_{RF}^2\sigma_o\gamma^{n-2} = \frac{k_{sc}^o}{\sigma_x}\gamma^{-3-n}$$

$$n-2 = -3-n \Rightarrow n = -\frac{1}{2}$$

$$\sigma_x'' + \frac{\gamma'}{\gamma}\sigma_x' + \frac{k_{RF}^2}{\gamma^2}\sigma_x = \frac{\varepsilon_n^2}{\gamma^2\sigma_x^3} + \frac{k_{sc}^o}{\gamma^3\sigma_x}$$

$\gamma = 1 + \alpha z$ ==> $\gamma'' = 0$

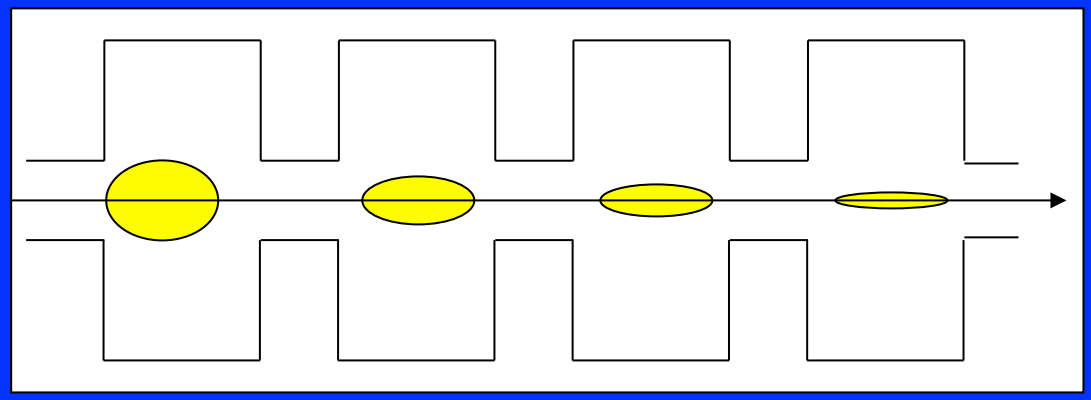
Looking for an "equilibrium" solution $\sigma_{inv} = \sigma_o \gamma^n$
==> all terms must have the same dependence on γ

Laminar beam $\rho >> 1 \Rightarrow n = -\frac{1}{2}$ $\sigma_q = \frac{\sigma_o}{\sqrt{\gamma}}$

Thermal beam $\rho << 1 \Rightarrow n = 0$ $\sigma_\varepsilon = \sigma_o$

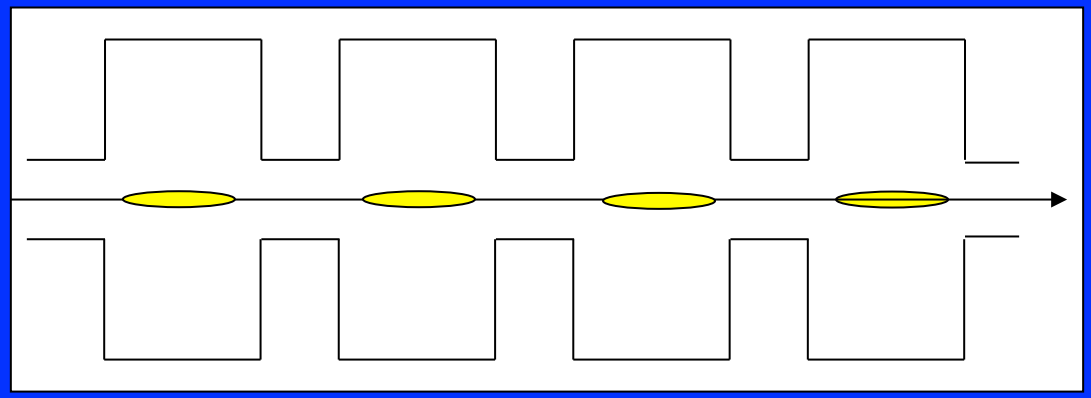
Space charge dominated beam (Laminar)

$$\sigma_q = \frac{l}{\gamma'}\sqrt{\frac{2I}{I_A\gamma}}$$

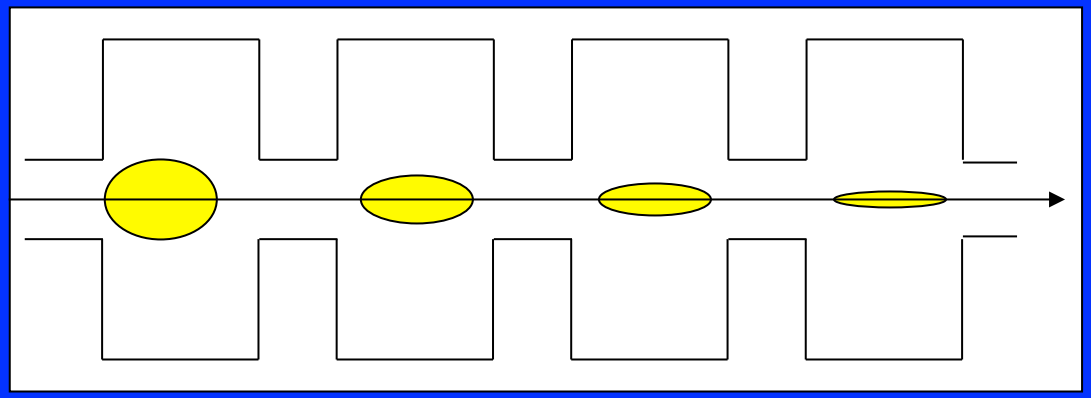


Emittance dominated beam (Thermal)

$$\sigma_\varepsilon = \sqrt{\frac{2\varepsilon_n}{\gamma'}}$$



$$\sigma_q = \frac{1}{\gamma'}\sqrt{\frac{2I}{I_A \gamma}}$$



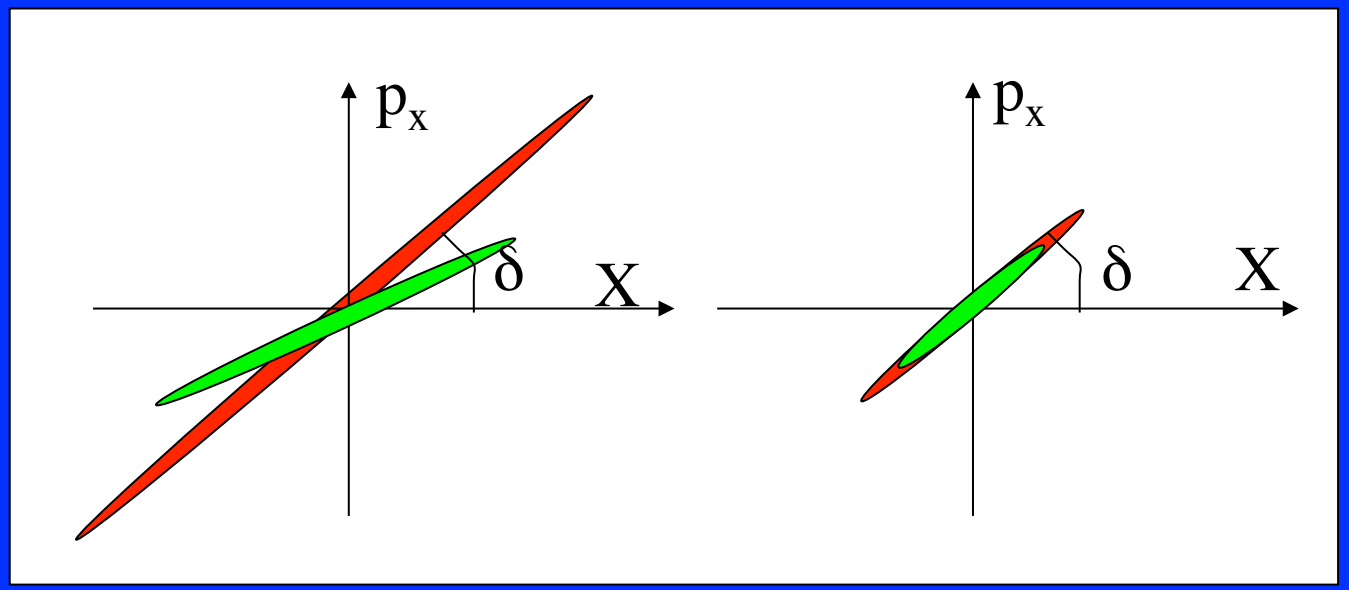
This solution represents a **beam equilibrium mode** that turns out to be the transport mode for achieving minimum emittance at the end of the **emittance correction process**

An important property of the laminar beam

$$\sigma_q = \frac{1}{\gamma'}\sqrt{\frac{2I}{I_A \gamma}}$$

$$\sigma_q^{'} = -\sqrt{\frac{2I}{I_A \gamma^3}}$$

Constant phase space angle: $\delta = \frac{\gamma \sigma_q^{'}}{\sigma_q} = -\frac{\gamma'}{2}$

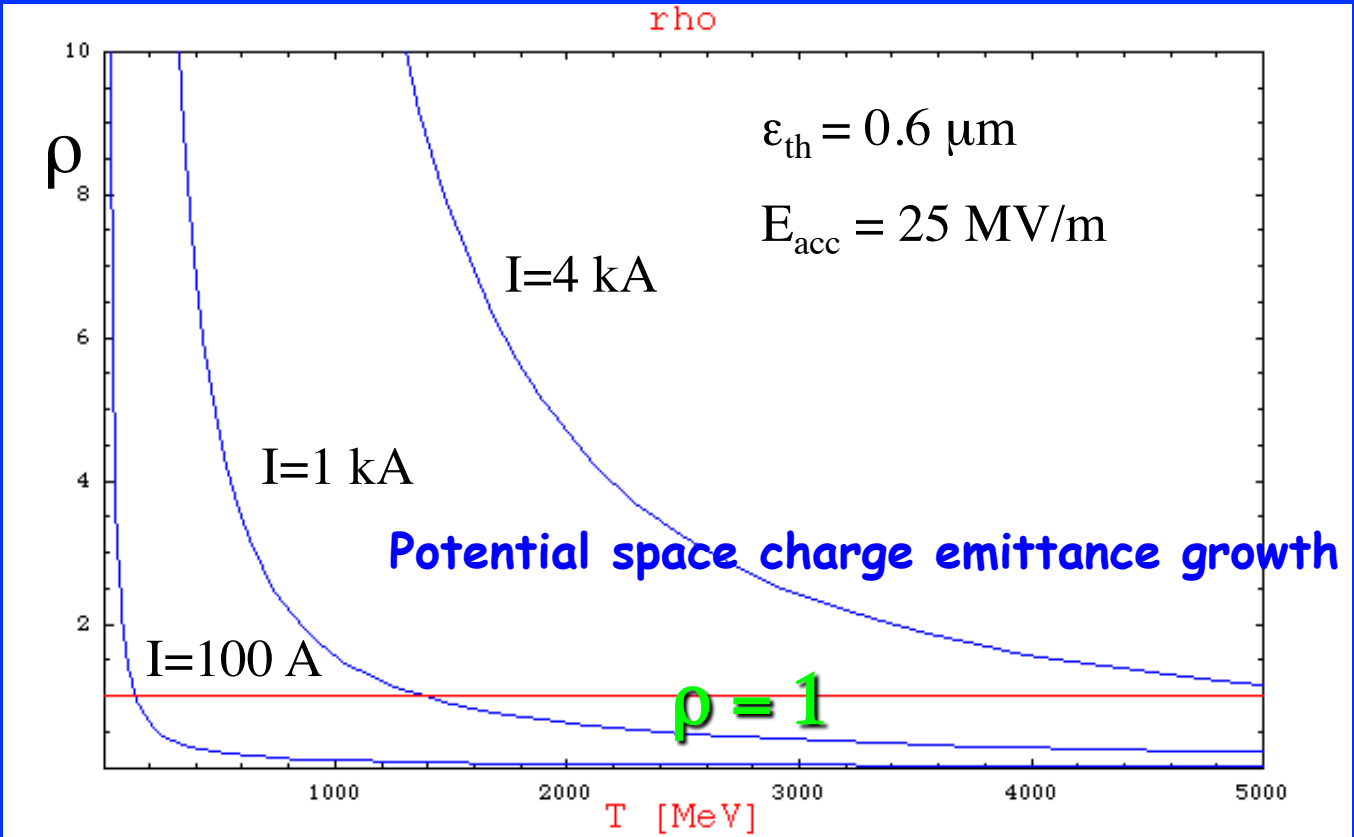


Laminarity parameter

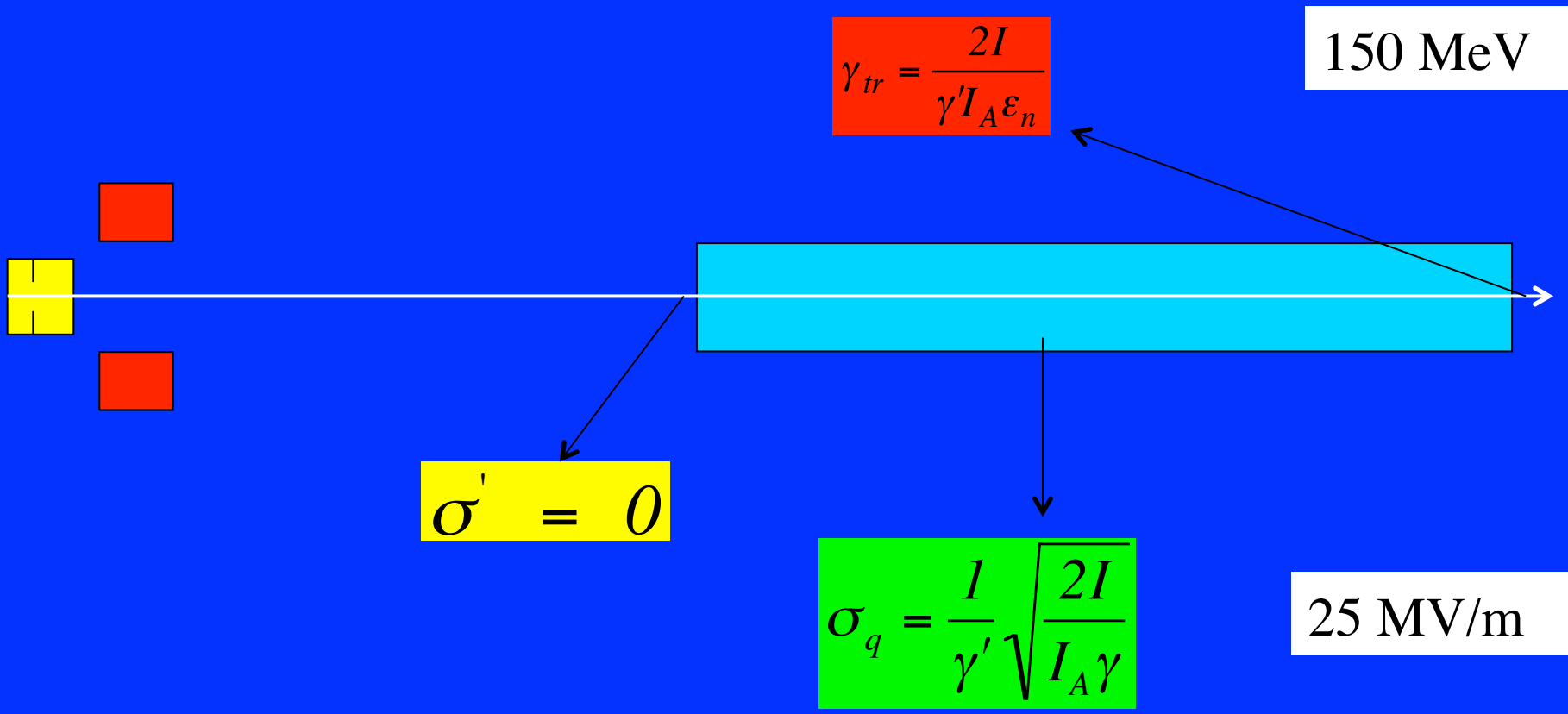
$$\rho = \frac{2I\sigma^2}{\gamma I_A \varepsilon_n^2} \equiv \frac{2I\sigma_q^2}{\gamma I_A \varepsilon_n^2} = \frac{4I^2}{\gamma'^2 I_A^2 \varepsilon_n^2 \gamma^2}$$

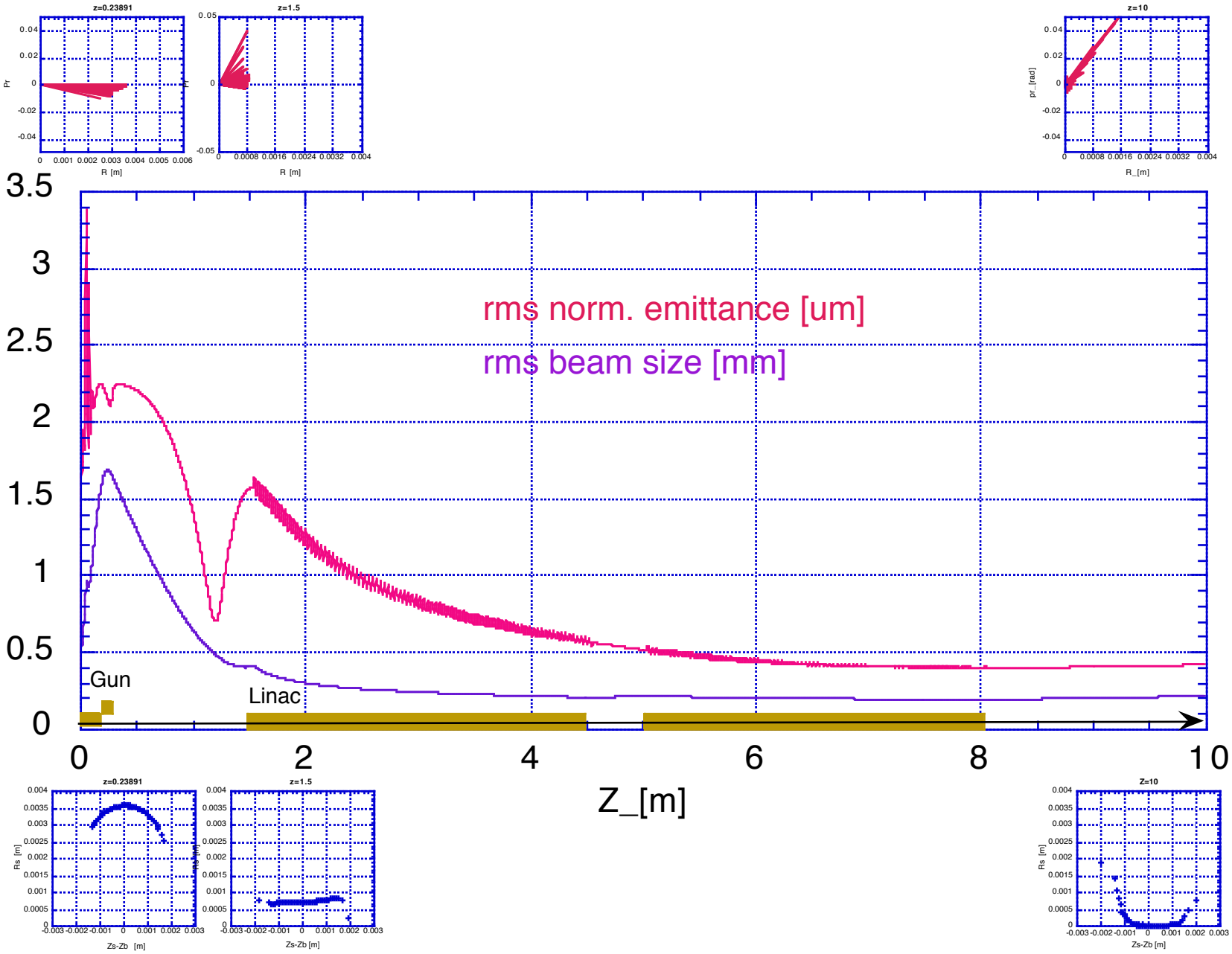
Transition Energy (ρ=1)

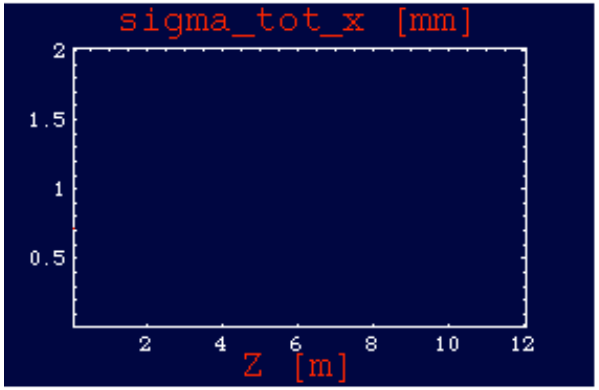
$$\gamma_{tr} = \frac{2I}{\gamma' I_A \varepsilon_n}$$

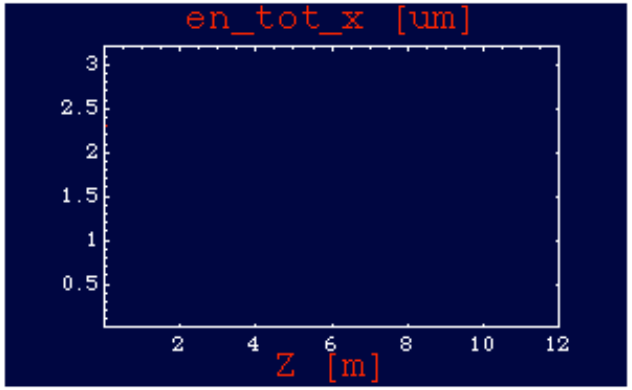


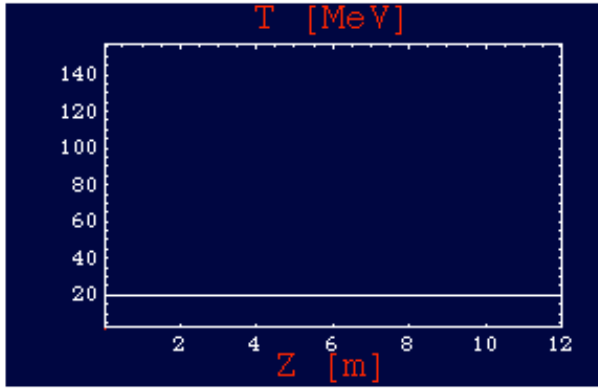
Matching Conditions with a TW Linac

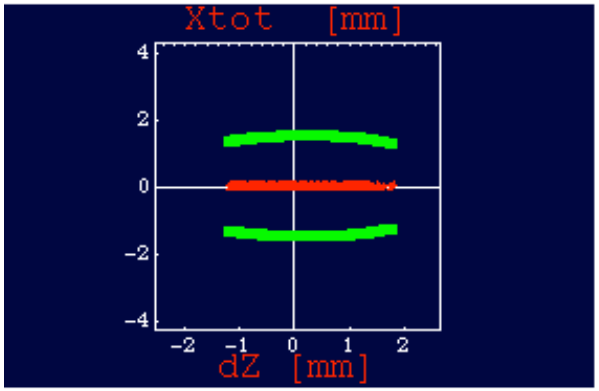


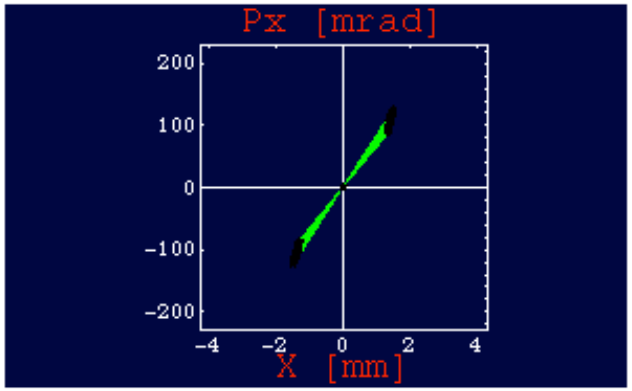

z=0.23891
z=1.5
z=10
Pr
R [m]
pr_ [rad]
R_[m]
rms norm. emittance [um]
rms beam size [mm]
Gun
Linac
Z_[m]
Rs [m]
Zs-Zb [m]

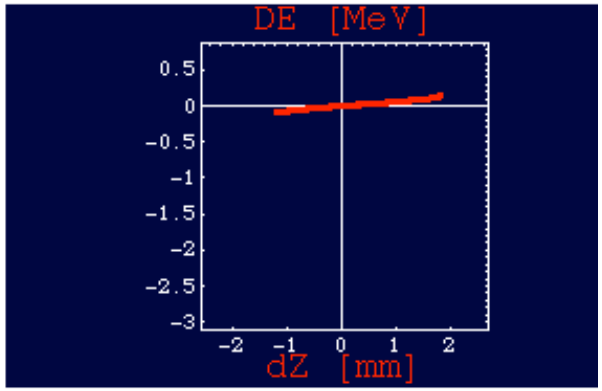

sigma_tot_x [mm]
Z [m]

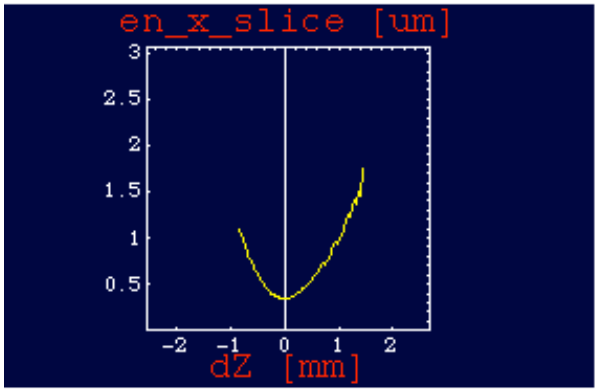

en_tot_x [um]
Z [m]

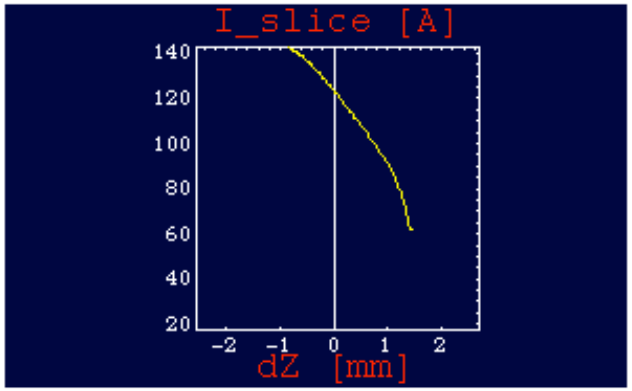

T [MeV]
Z [m]

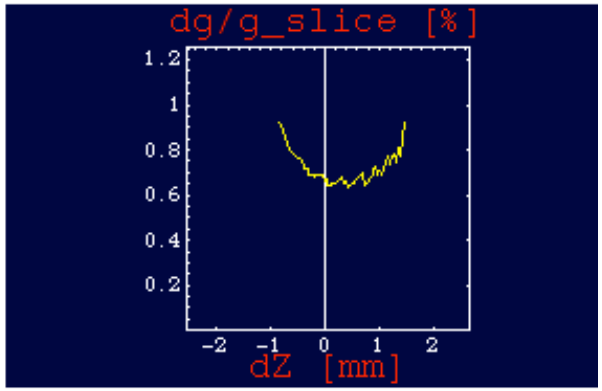

Xtot [mm]
dZ [mm]


Px [mrad]
X [mm]


DE [MeV]
dZ [mm]


en_x_slice [um]
dZ [mm]


I_slice [A]
dZ [mm]


dg/g_slice [%]
dZ [mm]

Emittance Compensation for a SC dominated beam: Controlled Damping of Plasma Oscillations

- ε_n oscillations are driven by Space Charge

- propagation close to the laminar solution allows control of ε_n oscillation "phase"

- ε_n sensitive to SC up to the transition energy

References:

[1] T. Shintake, Proc. of the 22nd Particle Accelerator Conference, June 25-29, 2007, Albuquerque, NM (IEEE, New York, 2007), p. 89.

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[3] M. Reiser, "Theory and Design of Charged Particle Beams" , Wiley, New York, 1994

[4] J. B. Rosenzweig, "Fundamentals of beam physics", Oxford University Press, New York, 2003

[5] T. Wangler, "Principles of RF linear accelerators", Wiley, New York, 1998

[6] S. Humphries, "Charged particle beams", Wiley, New York, 2002

[7] F. J. Sacherer, F. J., IEEE Trans. Nucl. Sci. NS-18, 1105 (1971).

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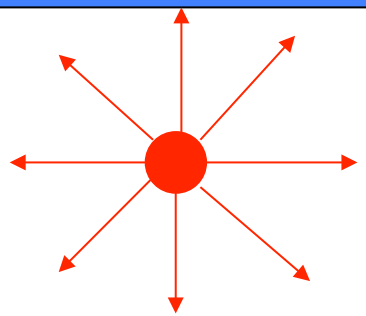
[9] J. Buon, "Beam phase space and emittance", in CERN 94-01

SELF FIELDS AND WAKE FIELDS

The realm of collective effects



Direct self fields

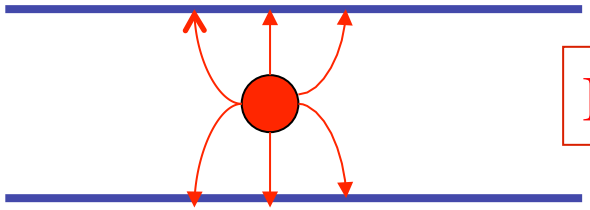
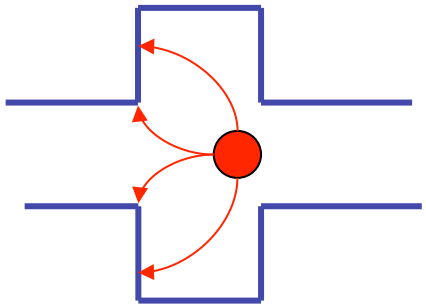


Image self fields

Space Charge



Wake fields

OUTLINE

- Introduction and Heuristic model
- Basic Concepts
- Beam Break Up in Linear Accelerators
- BNS damping

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

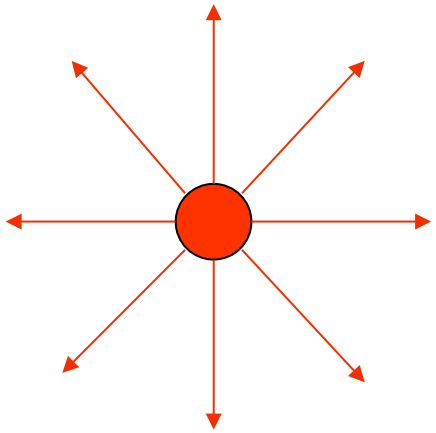
$$\vec{E} = \frac{q}{4\pi\varepsilon_o}\frac{\left(1-\beta^2\right)}{\left(1-\beta^2\,sin^2\,\theta\right)^{3/2}}\frac{\vec{r}}{r^3}$$

$$\beta = 0 \Rightarrow \vec{E} = \frac{q}{4\pi\varepsilon_o}\frac{\vec{r}}{r^3}$$

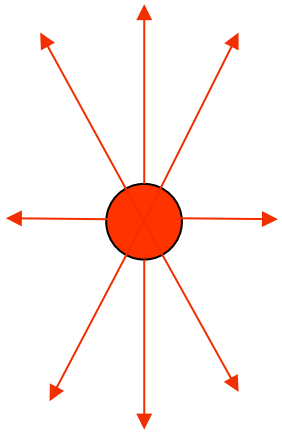
$$\theta = 0 \Rightarrow E_{//} = \frac{1}{\gamma^2}\frac{q}{4\pi\varepsilon_o}\frac{\vec{r}}{r^3} \xrightarrow{\gamma\to\infty} 0$$

$$\theta = \frac{\pi}{2} \Rightarrow E_{\perp} = \gamma\frac{q}{4\pi\varepsilon_o}\frac{\vec{r}}{r^3} \xrightarrow{\gamma\to\infty} \infty$$

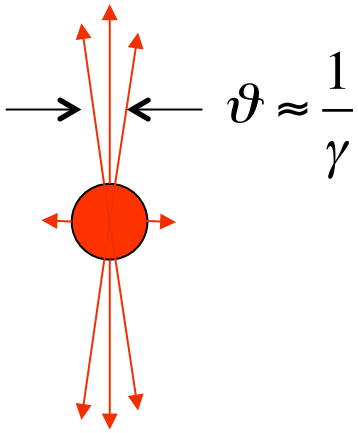
γ=1



γ≠0



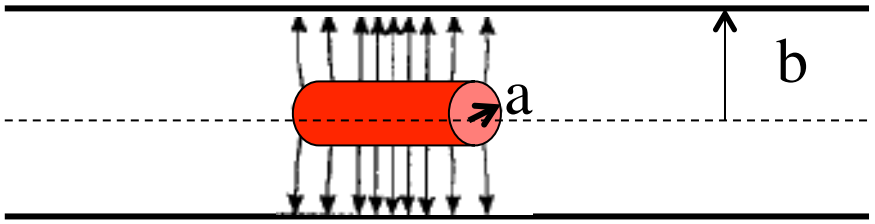
γ >>1



Bunched beam - Circular Perfectly Conducting Pipe

- Beam at Centre- Static Approximation $\gamma \to \infty$

$$\rho = \frac{I}{\pi a^2 v}$$



$$\varphi(b) = 0$$

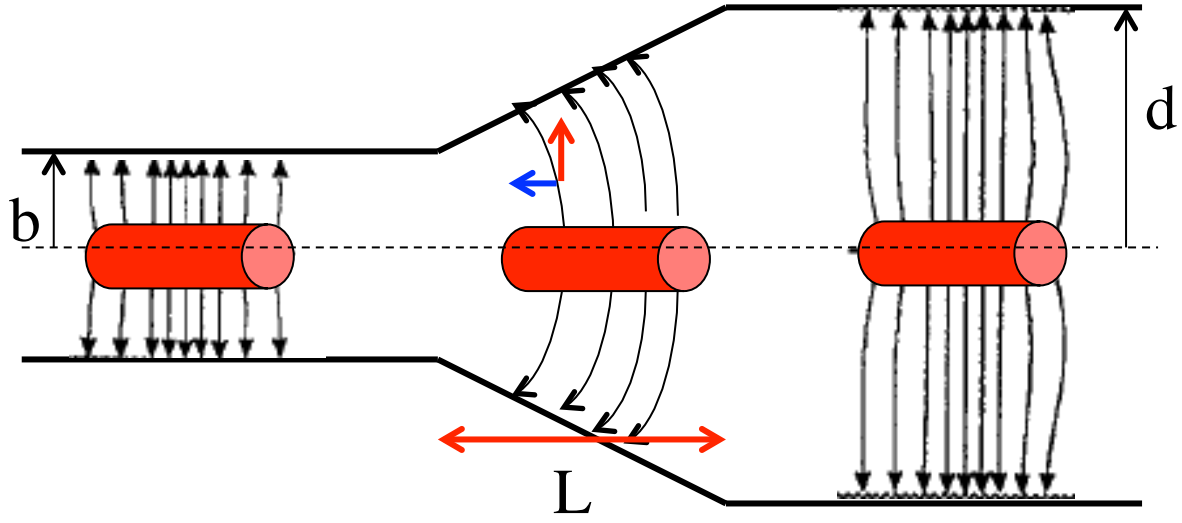
$$E_r = \frac{I}{2\pi\varepsilon_o a^2 v} r \quad \textit{for} \quad r \le a$$

$$E_r = \frac{I}{2\pi\varepsilon_o v}\frac{1}{r} \quad \textit{for} \quad r > a$$

$$B_\vartheta = \frac{\beta}{c} E_r$$

$$\varphi(r) = \int_r^b E_r(r')dr' \rightarrow \begin{cases} = \dfrac{I}{4\pi\varepsilon_o v}\left(1 + 2\ln\dfrac{b}{a} - \dfrac{r^2}{a^2}\right) & \textit{for} \quad r \le a \\ = \dfrac{I}{2\pi\varepsilon_o v}\ln\dfrac{b}{r} & \textit{for} \quad a \le r \le b \end{cases}$$

Circular Perfectly Conducting Pipe with Transition



There is a longitudinal $E_z(r,z)$ field in the transition and a test particle experience a voltage given by:

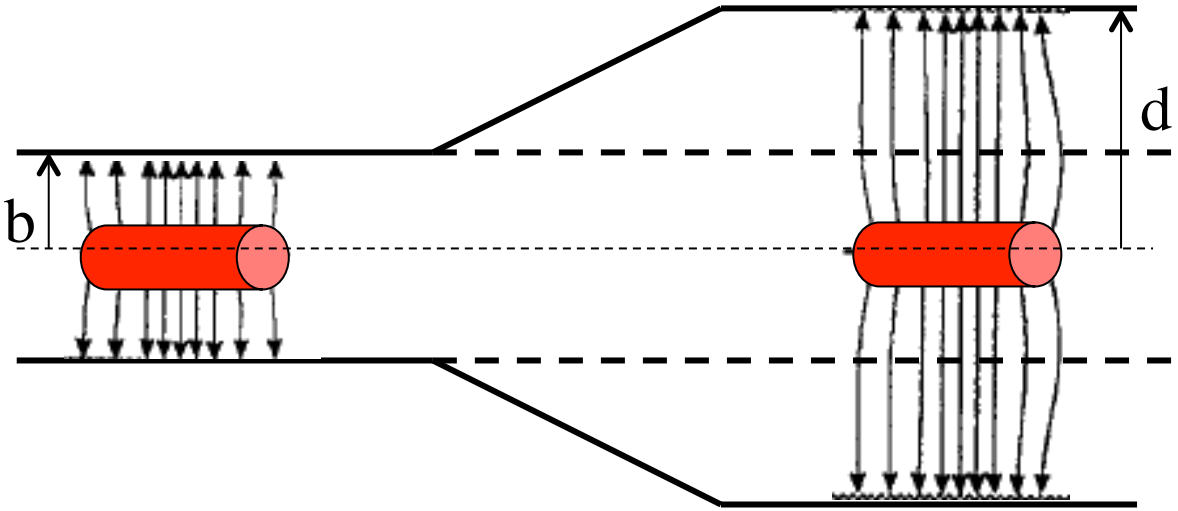
$$V = -\int_0^L E_z(r,z)dz = -\left(\varphi(r,L) - \varphi(r,0)\right) = -\frac{I}{2\pi\varepsilon_o v} ln\frac{d}{b}$$

decelerating if d > b

$$P_b = VI = \frac{I^2}{2\pi\varepsilon_o v} ln\frac{d}{b}$$

Power lost by the beam

For d > b the power is deposited to the energy of the fields: moving from left to right the beam induces the fields in the additional space available

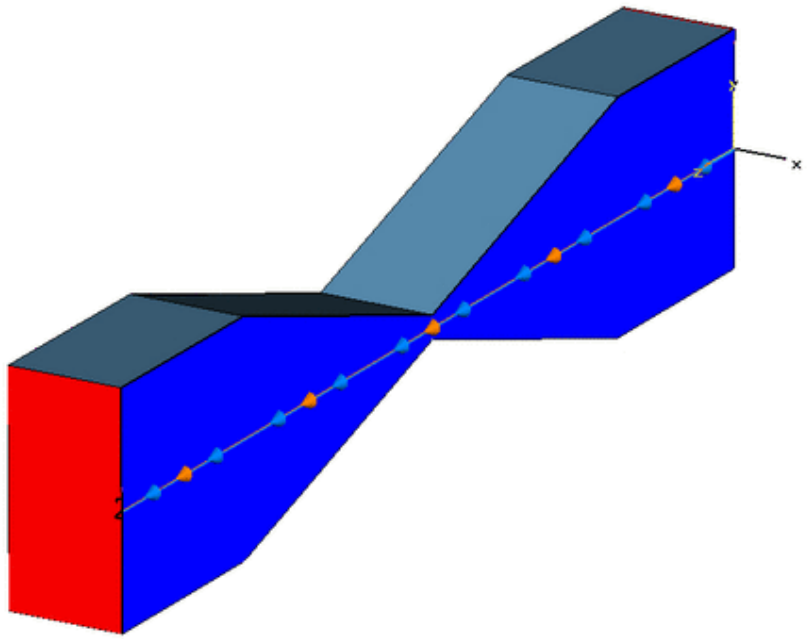


The additional power passing through the right part of the beam pipe is obtained by integrating the Poynting vector throught the sourface $\Delta S = \pi\left(d^2 - b^2\right)$

$$P_{em} = \int_{\Delta S}\left(\frac{1}{\mu}\vec{E}\times\vec{B}\right)\cdot d\vec{S} = \int_{b}^{d}\frac{E_r B_\vartheta}{\mu}2\pi r\,dr = \frac{I^2}{2\pi\varepsilon_o v}\ln\frac{d}{b}$$

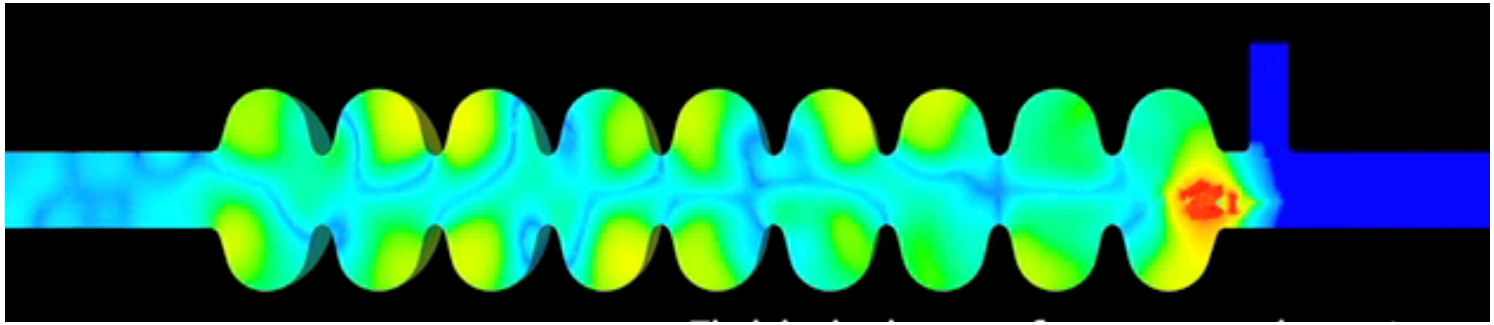
Notice that if d<b the beam gains energy. If d-->∞ the power goes to infinity, such an unphysical result is nevertheless consistent with the original assumption of an infinite energy beam (γ->∞).

Reflected and Diffracted fiels

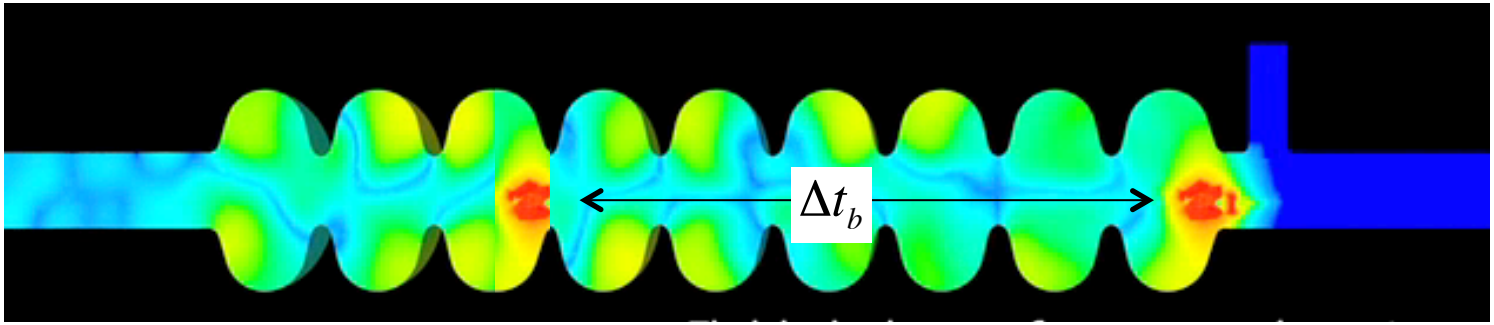


CST MICROWAVE STUDIO®

Short Range Wake Fields Effects ➔ head tail effects



Long Range Wake Fields Effects ➔ multibunch instabilities



$$\Delta t_b \approx \tau = \frac{2Q}{\omega} \begin{cases} \approx \mu\text{s} \Rightarrow \text{Normal Conducting Cavities} \\ \approx \text{ms} \Rightarrow \text{Superconducting Cavities} \end{cases}$$

Causality and the Catch-Up distance

The induced charges travel with the same particle velocity v.
Since both the particles and the image charges move on parallel paths, in the limit **v = c** they do not interact with each other, no matter how close to the wall the particles are.

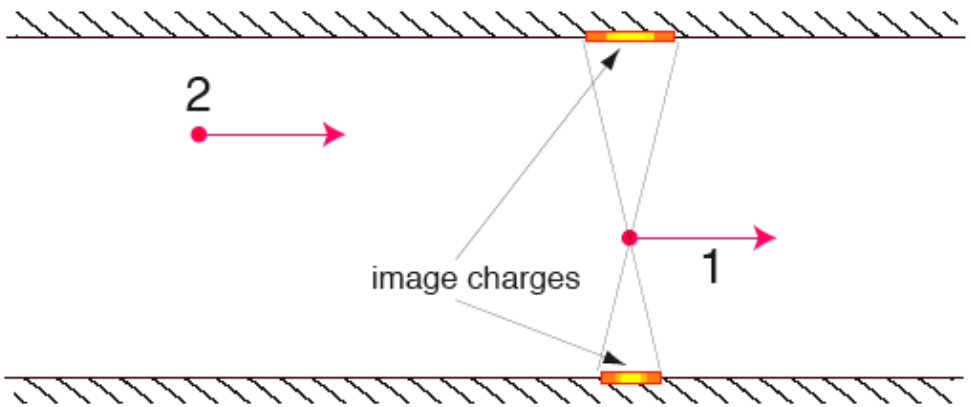


FIGURE 2. Particles traveling inside a perfectly conducting pipe of arbitrary cross section. Shown are the image charges on the wall generated by the leading charge.

If a particle moves along a straight line with the speed of light, the electromagnetic field of this particle scattered off the boundary discontinuities will not overtake it and, furthermore, will not affect the charges that travel ahead of it.

The field can interact only with the trailing charges in the beam that move behind it. This constitutes the principle of causality in the theory of wake fields

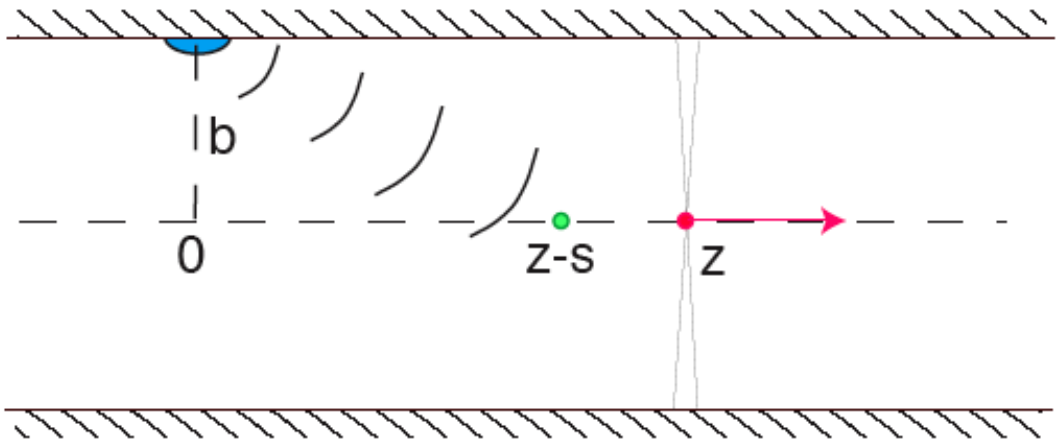
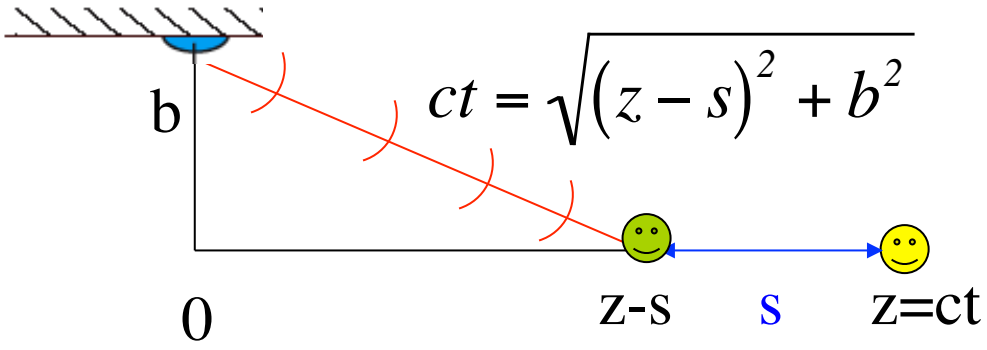


FIGURE 3. A wall discontinuity located at $z = 0$ scatters the electromagnetic field of an ultrarelativistic particle. When the particle moves to location z, the scattered field arrives to point $z - s$.

We can estimate the distance at which the electromagnetic field produced by a leading charge reaches a trailing particles traveling at a distance s behind.



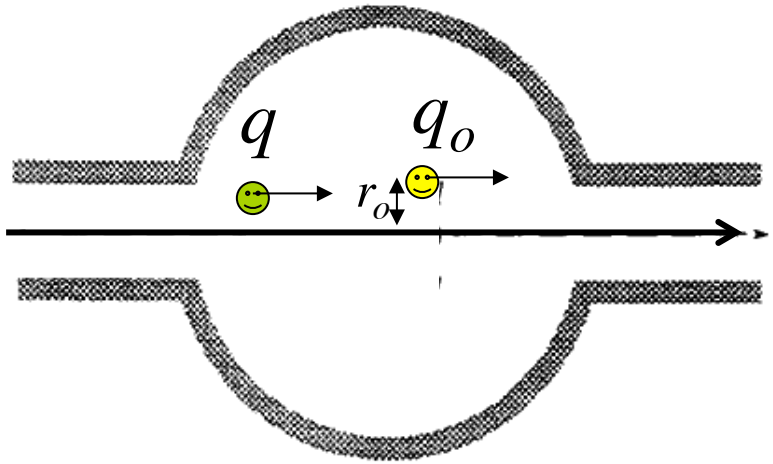
$$z^2 = (z-s)^2 + b^2 \quad \Longrightarrow \quad z_{catch-up} \approx \frac{b^2}{2s} \quad \textit{for } s \ll b$$

Only after the leading charge has traveled $z_{catch\text{-}up}$ away from the discontinuity, can a particle at point **s behind it** feel the field generated by the discontinuity.

OUTLINE

- Heuristic model
- Basic Concepts
- Beam Break Up in Linear Accelerators
- BNS damping

Wake Potentials



$$\boldsymbol{F} = q\left[E_z\hat{z} + \left(E_x - vB_y\right)\hat{x} + \left(E_y + vB_x\right)\hat{y}\right] \equiv \boldsymbol{F}_{//} + \boldsymbol{F}_{\perp}$$

there can be two effects on the **test charge** :

1) a longitudinal force which changes its energy,

2) a transverse force which deflects its trajectory.

If we consider a device of length L:

the Energy Gain is:

$$U = \int_0^L F_z ds$$

the Transverse Deflecting Kick is:

$$\boldsymbol{M} = \int_0^L \mathbf{F}_\perp ds$$

These quantities, normalised to the charges, are called ***wake-potentials*** and are both function of the distance z.

Note that the integration is performed over a given path of the trajectory.

Longitudinal wake potential

[V/C]

$$w_{//} = -\frac{U}{q_o q}$$

Energy Loss

Transverse wake potential

[V/Cm]

$$\boldsymbol{w}_{\perp} = \frac{1}{r_o}\frac{\boldsymbol{M}}{q_o q}$$

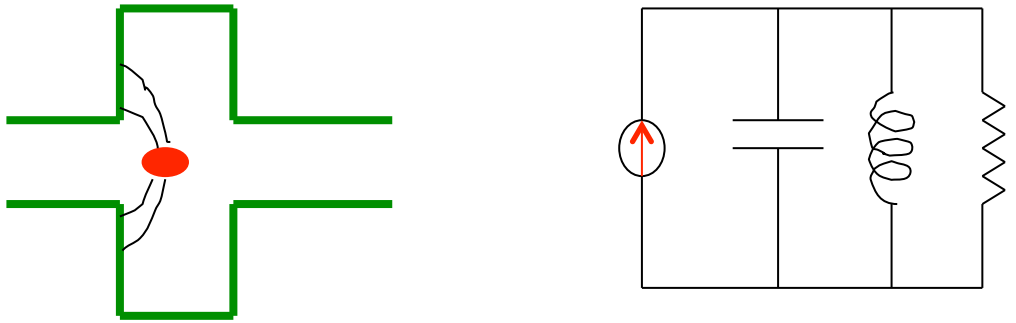
Transverse Kick

The sign minus in the longitudinal wake-potential means that the test charge loses energy when the wake is positive.

Positive transverse wake means that the transverse force is defocusing.

Longitudinal wake potential of a resonant HOM

When a charge crosses a resonant structure, it excites the fundamental mode and high order modes (HOM). Each mode can be treated as an electric RLC circuit loaded by an impulsive current.



Just after the charge passage, the capacitor is charged with a voltage $V_o=q/C$ and the electric field is $E_{so}= V_o/l_o$.

The time evolution of the electric field is governed by the same differential equation of the voltage

$$\ddot{V} + \frac{1}{RC}\dot{V} + \frac{1}{LC}V = \frac{1}{C}\dot{I}$$

The passage of the impulsive current charges only the capacitor, which changes its potential by an amount $V_c(0)$.

This potential will oscillate and decay producing a current flow in the resistor and inductance.

For t > 0 the potential satisfy the following equation and initial conditions:

$$\ddot{V} + \frac{1}{RC}\dot{V} + \frac{1}{LC}V = 0$$

$$V(t = 0^+) = \frac{q}{C} \equiv V_0$$

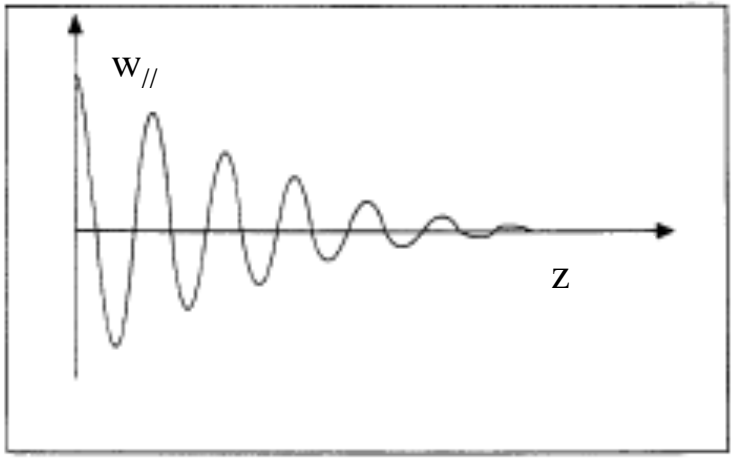
$$\dot{V}(t = 0^+) = \frac{\dot{q}}{C} = \frac{I(0^+)}{C} = \frac{V_0}{RC}$$

$$V(t) = V_0 e^{-\gamma t}\left[\cos(\bar{\omega}t) - \frac{\gamma}{\bar{\omega}}\sin(\bar{\omega}t)\right]$$

$$\bar{\omega}^2 = \omega_r^2 - \gamma^2 \quad 2\gamma = 1/RC \quad \omega_r^2 = 1/LC$$

putting z = -ct (z is negative behind the charge):

$$w_{//}(z) = \frac{V(z)}{q} = w_o e^{\gamma\, z/c}\left[\cos(\bar{\omega}z/c) + \frac{\gamma}{\bar{\omega}}\sin(\bar{\omega}z/c)\right]$$



…but what about the source charge?

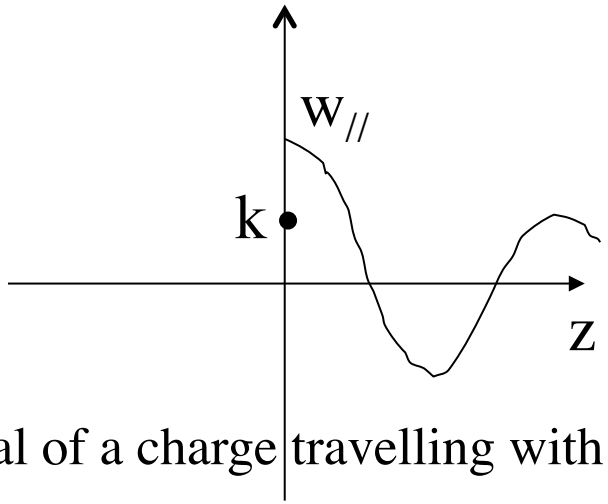
It is also useful to define the ***loss factor*** as the normalised energy lost by the **source charge q**

$$k = -\frac{U(z=0)}{q^2}$$

Although in general the loss factor is given by the longitudinal wake at z=0, for charges travelling with the light velocity the longitudinal wake potential is discontinuous at z=0

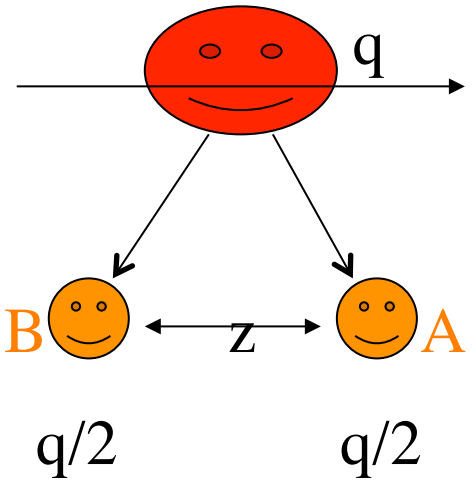
The exact relationship between *k* and *w(z=0)* is given by the ***beam loading theorem*:**

$$k = \frac{w_{//}(z \to 0)}{2}$$



Causality requires that the longitudinal wake potential of a charge travelling with the velocity of light is discontinuous at the origin.

$$U_o = -q^2 k$$



$$U_A = -q_A^2 k = -\frac{q^2}{4} k$$

$$U_B = -q_B^2 k - q_A q_B w_{//}(z)$$

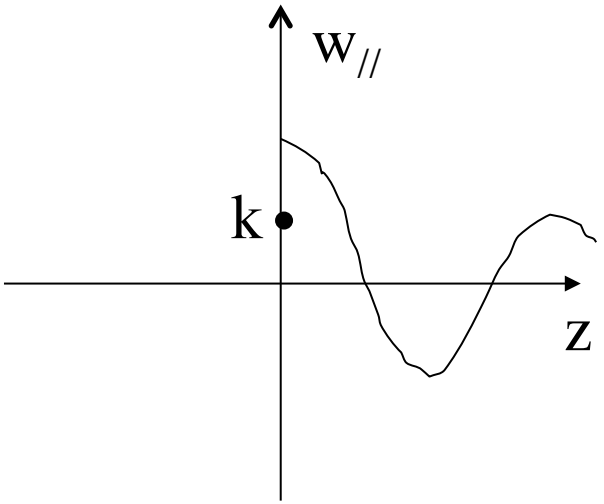
$$= -\frac{q^2}{4} k - \frac{q^2}{4} w_{//}(z)$$

$$U_A + U_B = -\frac{q^2}{2} k - \frac{q^2}{4} w_{//}(z)$$

$$z \to 0 \qquad U_o = U_A + U_B$$

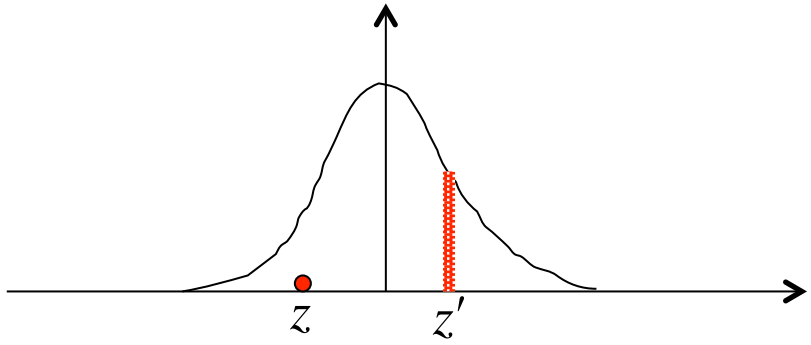
$$q^2 k = \frac{q^2}{2} k + \frac{q^2}{4} w_{//}(0)$$

$$k = \frac{w_{//}(0)}{2}$$



Wake potentials and energy loss of a bunched distribution

When we have a bunch with density λ(z), we may wander what is the amount of energy lost or gained by a single charge e in the beam



To this end we calculate the effect on the charge from the whole bunch by means of the convolution integral:

$$U(z) = -e\int_{-\infty}^{\infty} w_{//}(z'-z)\lambda(z')dz'$$

Which allows to define the ***wake potential of a distribution***

$$W_{//}(z) = -\frac{U(z)}{qe} = \frac{1}{q}\int_{-\infty}^{\infty} w_{//}(z'-z)\lambda(z')dz'$$

The total energy lost by the bunch is computed summing up the loss of all particles:

$$U_{bunch} = \frac{1}{e}\int_{-\infty}^{\infty} U(z)\lambda(z)dz = -q\int_{-\infty}^{\infty} W_{//}(z)\lambda(z)dz$$

Example: Energy spread and loss for a finite uniform beam due to a HOM

$$w_{//}(z) \approx w_0 \cos\left(\frac{\omega_r}{c} z\right) H(-z)$$

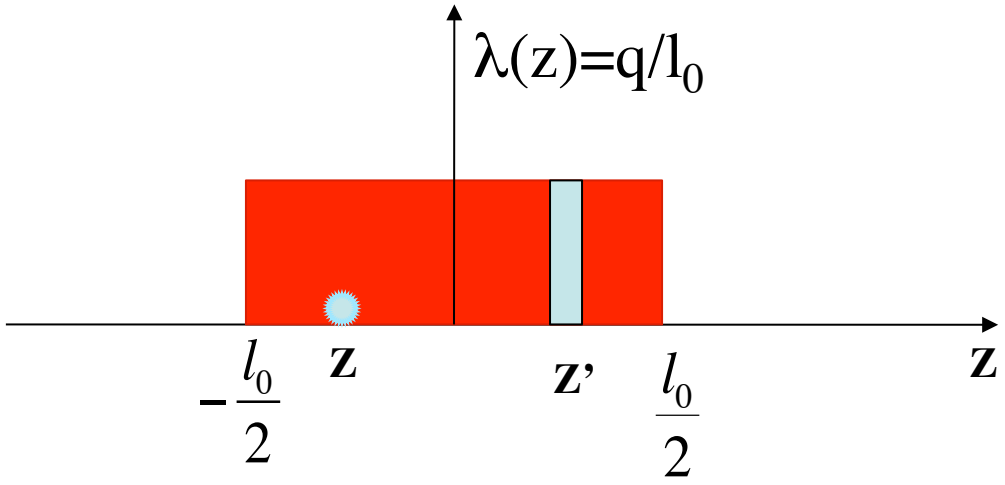
$$U(z) = -e \int_{-\infty}^{+\infty} w_{//}(z - z')\lambda(z')dz'$$

$$U(z) = -\frac{eqw_0}{l_0} \int_z^{\infty} \cos\left[\frac{\omega_r}{c}(z - z')\right] dz'$$

$$z - z' = x$$

$$U(z) = \frac{eqw_0}{l_0} \int_0^{(z-l_0/2)} \cos\left(\frac{\omega_r}{c} x\right) dx = \frac{eqw_0}{l_0} \left[\frac{\sin\left(\frac{\omega_r}{c} x\right)}{\left(\frac{\omega_r}{c}\right)}\right]_0^{(z-l_0/2)}$$

$$U(z) = -\frac{eqw_0}{2} \left[\frac{\sin\left[\frac{\omega_r}{c}\left(\frac{l_0}{2} - z\right)\right]}{\left(\frac{\omega_r}{c}\frac{l_0}{2}\right)}\right]$$



Wake potential?

Energy spread (U_{max}-U_{min})?

Energy loss

$$U_{bunch} = \frac{1}{e}\int_{-\infty}^{+\infty} U(z)\lambda(z)dz \approx \frac{-q^2 w_0}{2l_0\left(\frac{\omega_r}{c}\frac{l_0}{2}\right)} \int_{-\frac{l_0}{2}}^{\frac{l_0}{2}} \sin\left[\frac{\omega_r}{c}\left(\frac{l_0}{2}-z\right)\right] dz$$

$$U_{bunch} = \frac{-q^2 w_0 c}{\omega_r l_0^2}\left|\frac{-\cos\left[\frac{\omega_r}{c}\left(\frac{l_0}{2}-z\right)\right]}{-\frac{\omega_r}{c}}\right|_{-\frac{l_0}{2}}^{\frac{l_0}{2}}$$

$$U_{bunch} = -\frac{q^2 w_0 c^2}{\omega_r^2 l_0^2}\left[1-\cos\left(\frac{\omega_r l_0}{c}\right)\right] = -\frac{2q^2 w_0 c^2}{\omega_r^2 l_0^2}\sin^2\left(\frac{\omega_r l_0}{2c}\right)$$

$$U_{bunch} = -\frac{q^2 w_0}{2}\frac{\sin^2\left(\frac{\omega_r l_0}{2c}\right)}{\left(\frac{\omega_r l_0}{2c}\right)^2}$$

$$\lim_{l_0\to 0}\left(U_{bunch}\right) = -\frac{q^2 w_0}{2}$$

Energy spread compensation

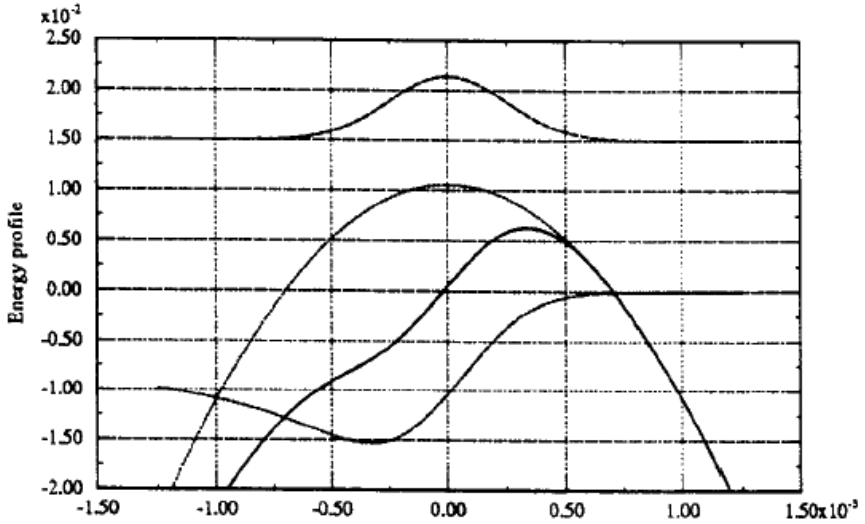


Fig. 5 Energy profile within the bunch sitting on the crest of the rf wave

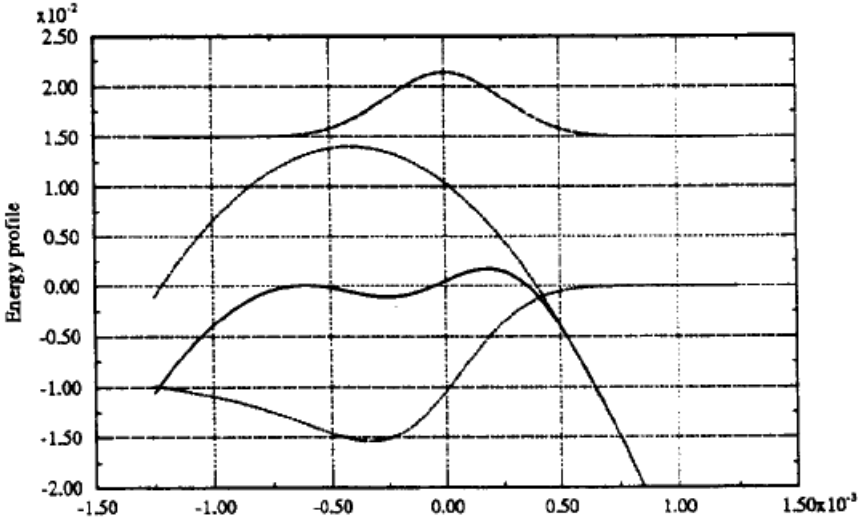
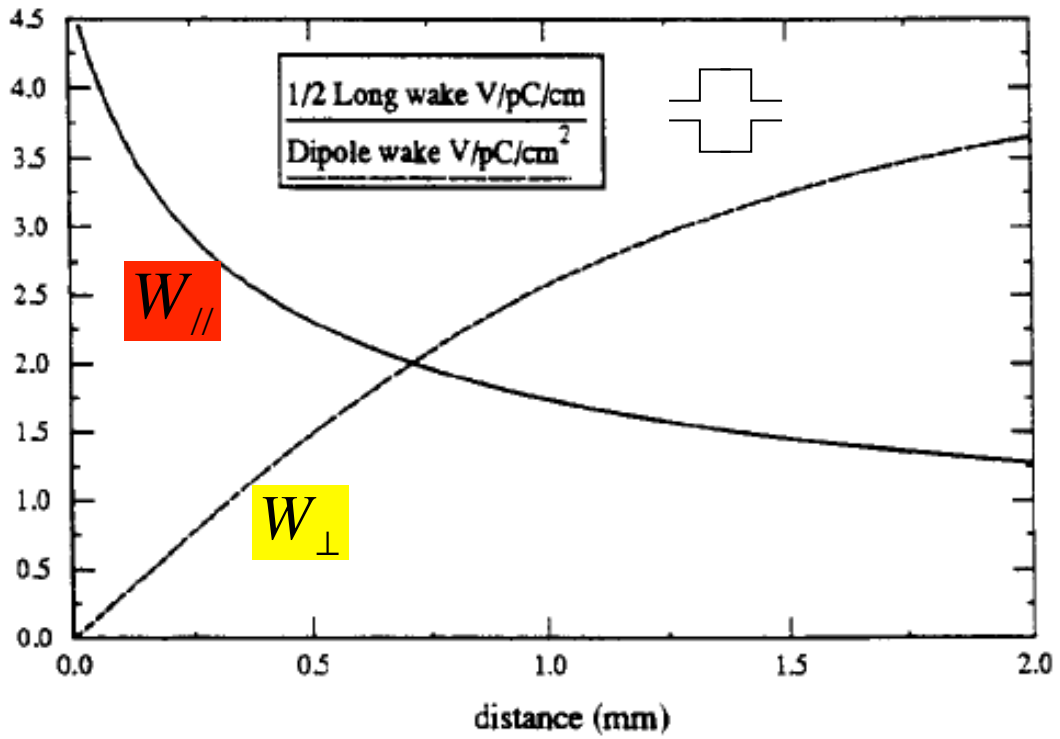


Fig. 6 Energy profile within the bunch after optimization of the rf phase

Relationship between transverse and longitudinal forces :
"**Panofsky-Wenzel theorem**".

$$\nabla_{\perp} F_{//} = \frac{\partial}{\partial z} \boldsymbol{F}_{\perp}$$

$$\nabla_{\perp} w_{//} = \frac{\partial}{\partial z} \boldsymbol{w}_{\perp}$$



Coupling Impedance

The wake potentials are used for to study the beam dynamics in the time domain ($s=vt$). If we take the equation of motion in the frequency domain, we need the Fourier transform of the wake potentials. Since these quantities have Ohms units are called ***coupling impedances***:

Longitudinal impedance (Ω)

$$Z_{//}(\omega) = \frac{1}{v}\int_{-\infty}^{\infty} w_{//}(z) e^{-i\frac{\omega z}{v}}\, dz$$

Transverse impedance (Ω/m)

$$\boldsymbol{Z}_{\perp}(\omega) = \frac{i}{v}\int_{-\infty}^{\infty} \boldsymbol{w}_{\perp}(z) e^{-i\frac{\omega z}{v}}\, dz$$

Z_R is responsible for the energy losses

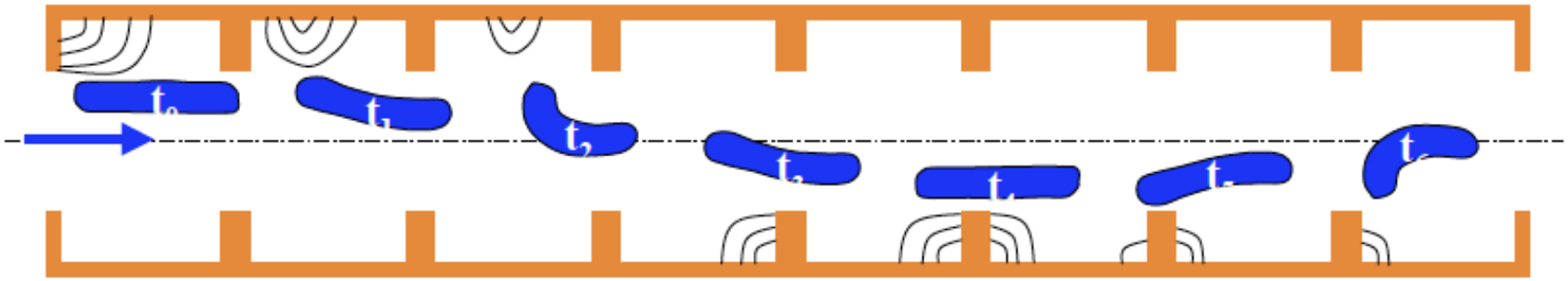
Z_j defines the phase between the beam response & exciting wake potential

OUTLINE

- Heuristic model
- Basic Concepts
- Beam Break Up in Linear Accelerators
- BNS damping

Beam Break Up

A beam injected off-center in a LINAC, because of the focusing quadrupoles, execute betatron oscillations. The displacement produces a transverse wake field in all the devices crossed during the flight, which deflects the trailing charges.



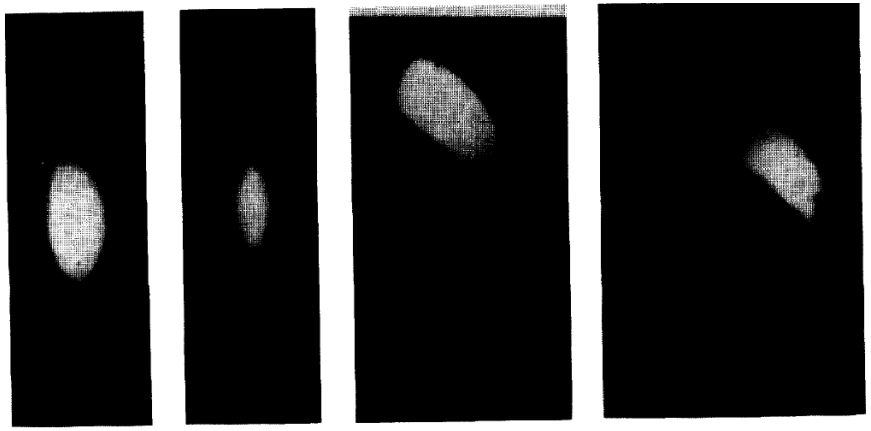
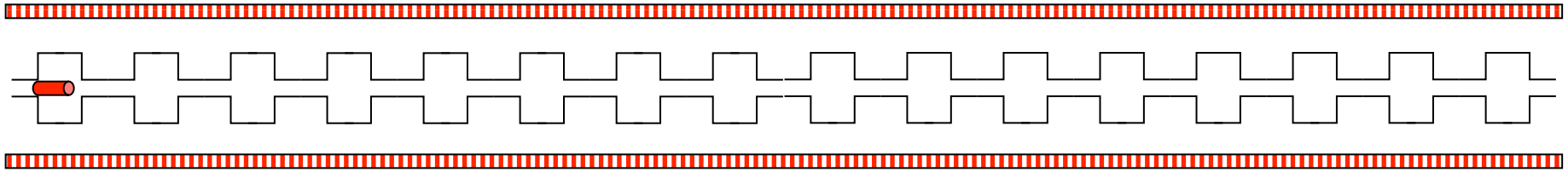
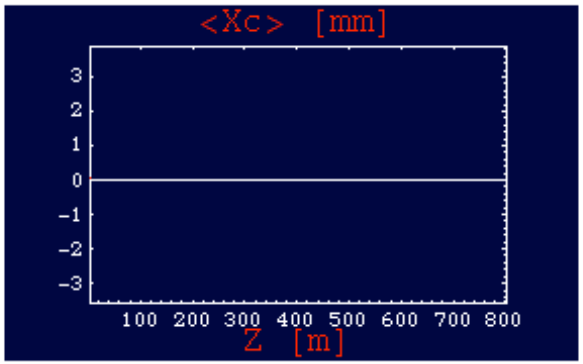
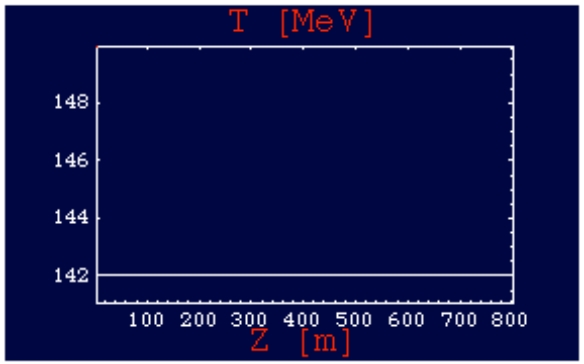
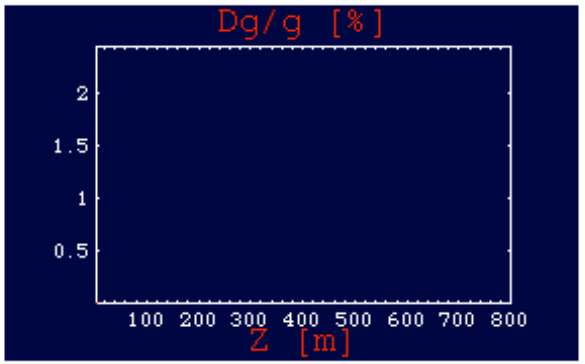


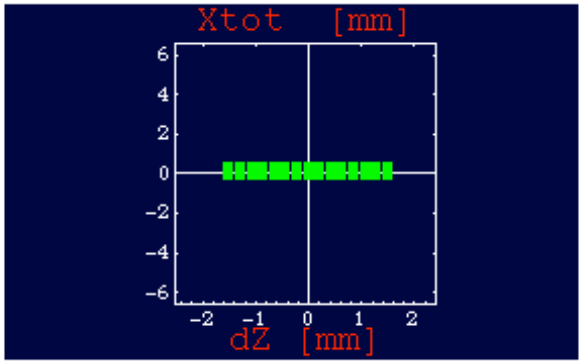
Figure 3.4. Four transverse beam profiles observed at the end of the SLAC linac are shown when the beam was carefully injected and injected with 0.2, 0.5, and 1 mm offsets. The beam sizes σ_x and σ_y are about 120 μm. (Courtesy John Seeman, 1991.)

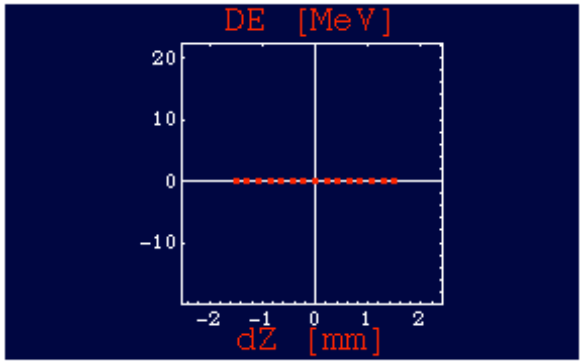


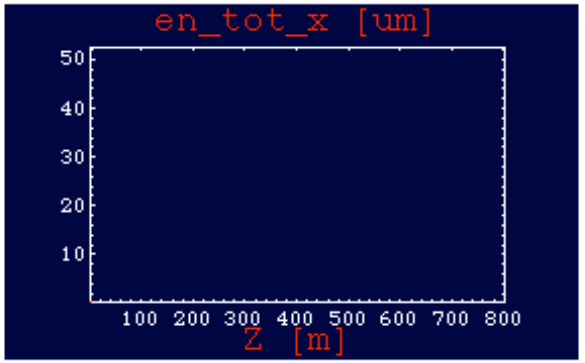

<Xc> [mm]
3
2
1
0
-1
-2
-3
100 200 300 400 500 600 700 800
Z [m]

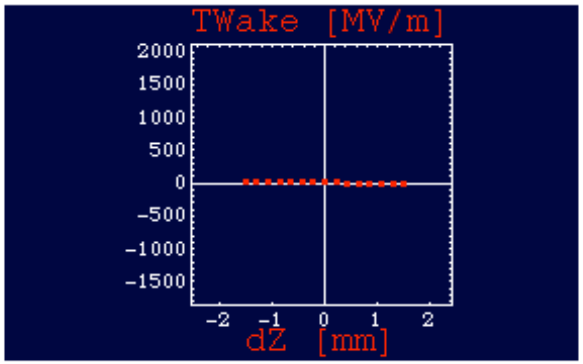

T [MeV]
148
146
144
142
100 200 300 400 500 600 700 800
Z [m]

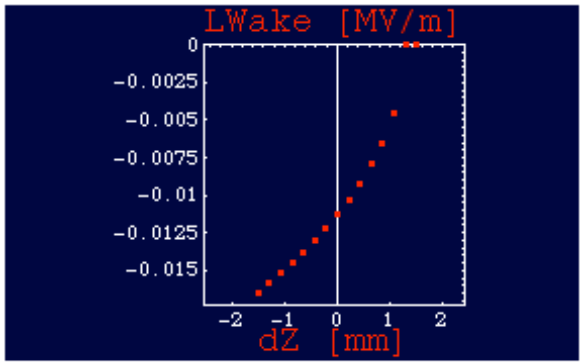

Dg/g [%]
2
1.5
1
0.5
100 200 300 400 500 600 700 800
Z [m]

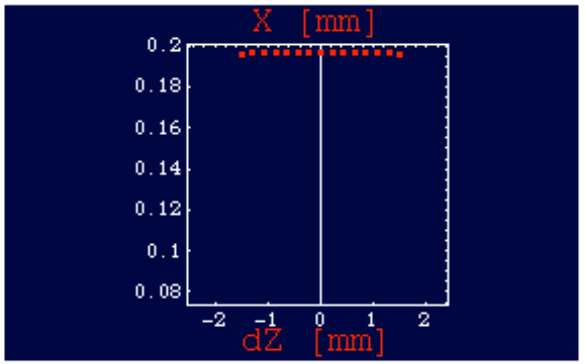

Xtot [mm]
6
4
2
0
-2
-4
-6
-2 -1 0 1 2
dZ [mm]


DE [MeV]
20
10
0
-10
-2 -1 0 1 2
dZ [mm]

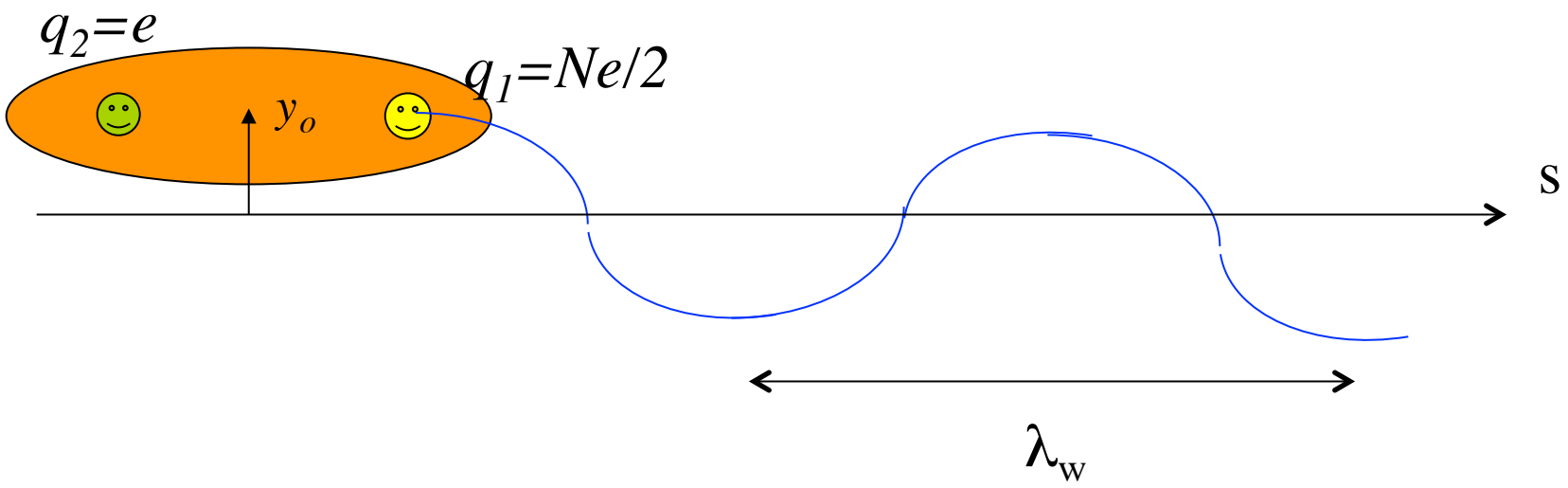

en_tot_x [um]
50
40
30
20
10
100 200 300 400 500 600 700 800
Z [m]


TWake [MV/m]
2000
1500
1000
500
0
-500
-1000
-1500
-2 -1 0 1 2
dZ [mm]


LWake [MV/m]
0
-0.0025
-0.005
-0.0075
-0.01
-0.0125
-0.015
-2 -1 0 1 2
dZ [mm]


X [mm]
0.2
0.18
0.16
0.14
0.12
0.1
0.08
-2 -1 0 1 2
dZ [mm]

In order to understand the effect, we consider a simple model with only two charges $q_1=Ne/2$ (leading = half bunch) and $q_2=e$ (trailing = single charge). Without acceleration.



the leading charge executes free betatron oscillations:

$$y_1(s) = \hat{y}_1 \cos\left(\frac{\omega_y}{c} s\right); \quad \frac{\omega_y}{c} = \frac{2\pi}{\lambda_w}$$

the test charge, at a distance z behind, over a length L_w experiences a deflecting force proportional to the displacement y_1, and dependent on the distance z:

$$w_\perp(z) = \frac{1}{r_0}\frac{M(r_0,z)}{q^2}$$

$$M(r_0,z) = \int_0^{L_w} F_\perp ds = \langle F_\perp(r_0,z)\rangle L_w \quad \Longrightarrow \quad \langle F_\perp(z,y_1)\rangle = \frac{Ne^2}{2L_w} w_\perp(z) y_1(s)$$

e Ne/2

This force drives the motion of the test charge:

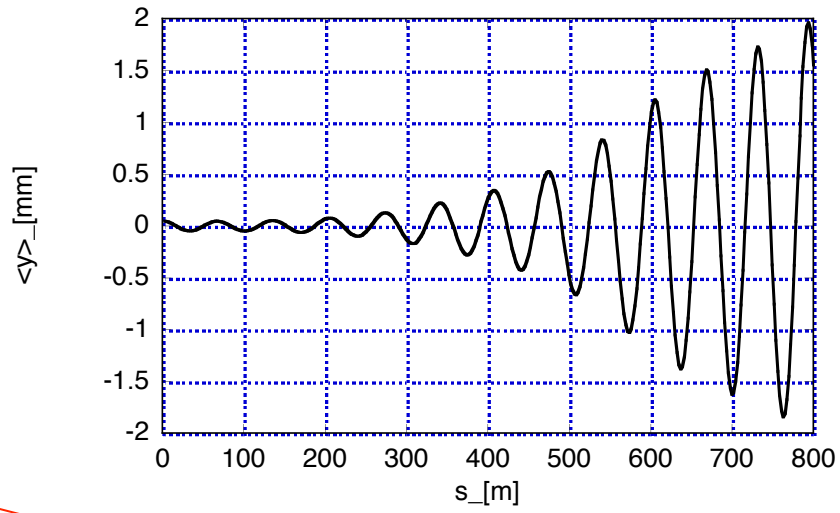
betatron equation of motion with coherent force

$$y_2'' + \left(\frac{\omega_y}{c}\right)^2 y_2 = \frac{1}{\beta^2 E_o}\langle F_\perp(z,y_1)\rangle = \frac{Ne^2 w_\perp(z)}{2\beta^2 E_o L_w}\hat{y}_1 \cos\left(\frac{\omega_y}{c}s\right)$$

This is the typical equation of an harmonic oscillator driven at the resonant frequency. The solution is given by the superposition of the "free" oscillation and a "driven" oscillation which, being driven at the resonant frequency, grows linearly with s.

$$y_2(s) = \hat{y}_2 \cos\left(\frac{\omega_y}{c} s\right) + y_2^{driven}$$

$$y_2^{driven} = \frac{cNe^2 w_\perp(z)}{4\omega_y E_o L_w} s\hat{y}_1 \sin\left(\frac{\omega_y}{c} s\right)$$



continuos growth

At the end of the LINAC of length L_L, the oscillation amplitude is grown by :

$$\left(\frac{\Delta\hat{y}_2}{\hat{y}_2}\right)_{\max} = \frac{cNe^2 w_\perp(z) L_L}{4\omega_y E_o L_w}$$

OUTLINE

- Heuristic model
- Basic Concepts
- Beam Break Up in Linear Accelerators
- BNS damping

Balakin-Novokhatsky-Smirnov Damping

The BBU instability is quite harmful and hard to take under control even at high energy with a strong focusing, and after a careful injection and steering.

A simple method to cure it has been proposed observing that the strong oscillation amplitude of the bunch tail is mainly due to the **"resonant" driving**.

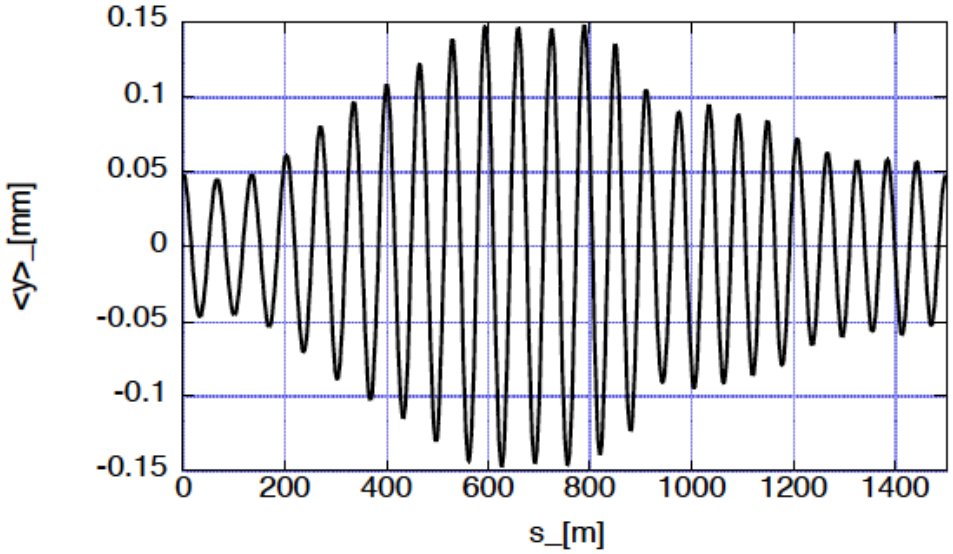
If the tail and the head move with a different frequency, this effect can be significantly removed.

Let us assume that the tail oscillates with a frequency $\omega_y+\Delta\omega_y$, the equation of motion reads:

$$y_2'' + \left(\frac{\omega_y + \Delta\omega_y}{c}\right)^2 y_2 = \frac{Ne^2 w_\perp(z)}{2\beta^2 E_o L_w}\hat{y}_1 \cos\left(\frac{\omega_y}{c}s\right)$$

the solution of which is:

$$y_2(s) = \hat{y}_2 \cos\left(\frac{\omega_y + \Delta\omega_y}{c} s\right) + \frac{c^2 N e^2 w_\perp(z)}{4\omega_y \Delta\omega_y E_o L_w} \hat{y}_1 \left[\cos\left(\frac{\omega_y}{c} s\right) - \cos\left(\frac{\omega_y + \Delta\omega_y}{c} s\right)\right]$$



$$y_2(s) = \hat{y}_2 \cos\left(\frac{\omega_y + \Delta\omega_y}{c} s\right) + \frac{c^2 N e^2 w_\perp(z)}{4\omega_y \Delta\omega_y E_o L_w} \hat{y}_1 \left[\cos\left(\frac{\omega_y}{c} s\right) - \cos\left(\frac{\omega_y + \Delta\omega_y}{c} s\right)\right]$$

by a suitable choice of $\Delta\omega_y$, it is possible to fully depress the oscillations of the tail.

$$\hat{y}_2 = \hat{y}_1 \qquad \frac{c^2 N e^2 w_\perp(z)}{4\omega_y \Delta\omega_y E_o L_w} = 1 \Rightarrow \qquad y_2(s) = \hat{y}_1 \cos\left(\frac{\omega_y}{c} s\right) = y_1(s)$$

$$\Delta\omega_y = \frac{c^2 N e^2 w_\perp(z)}{4\omega_y E_o L_w}$$

The extra focusing at the tail can be obtained by:

- Using an RFQ, where head and tail see a different focusing strength.
- Creating a correlated energy distribution along the bunch which, because of the chromaticity, induces a spread in the betatron frequencies. An energy spread correlated with the longitudinal position is attainable with the external accelerating voltage, or with the wake fields.

$$\frac{\Delta\omega_y}{\omega_y} = -\frac{\Delta E}{E}$$

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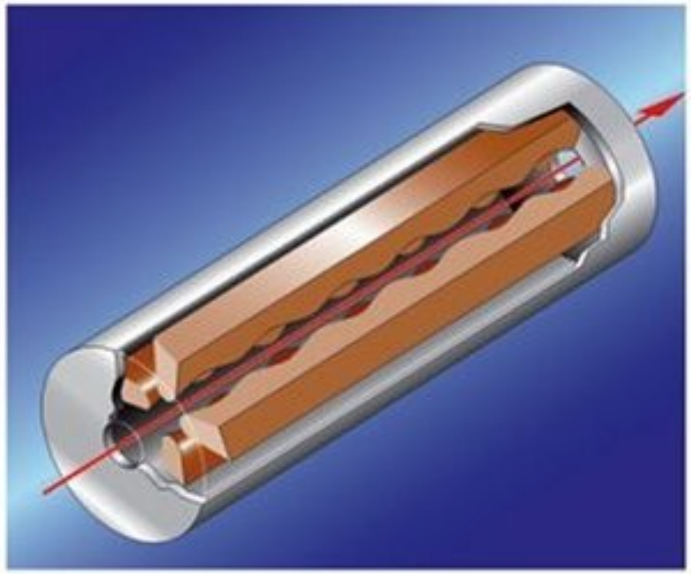


THE END