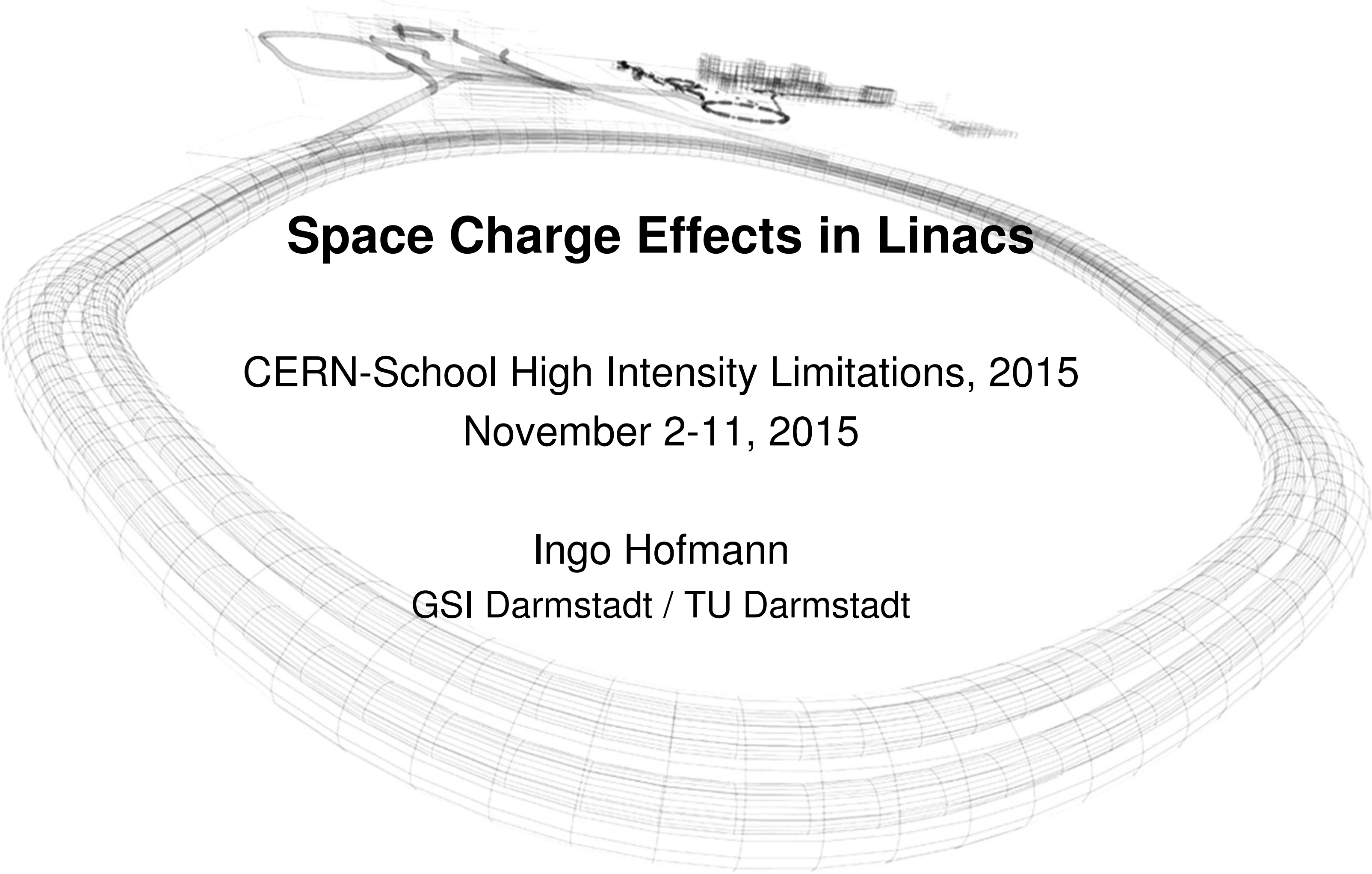


Space Charge Effects in Linacs

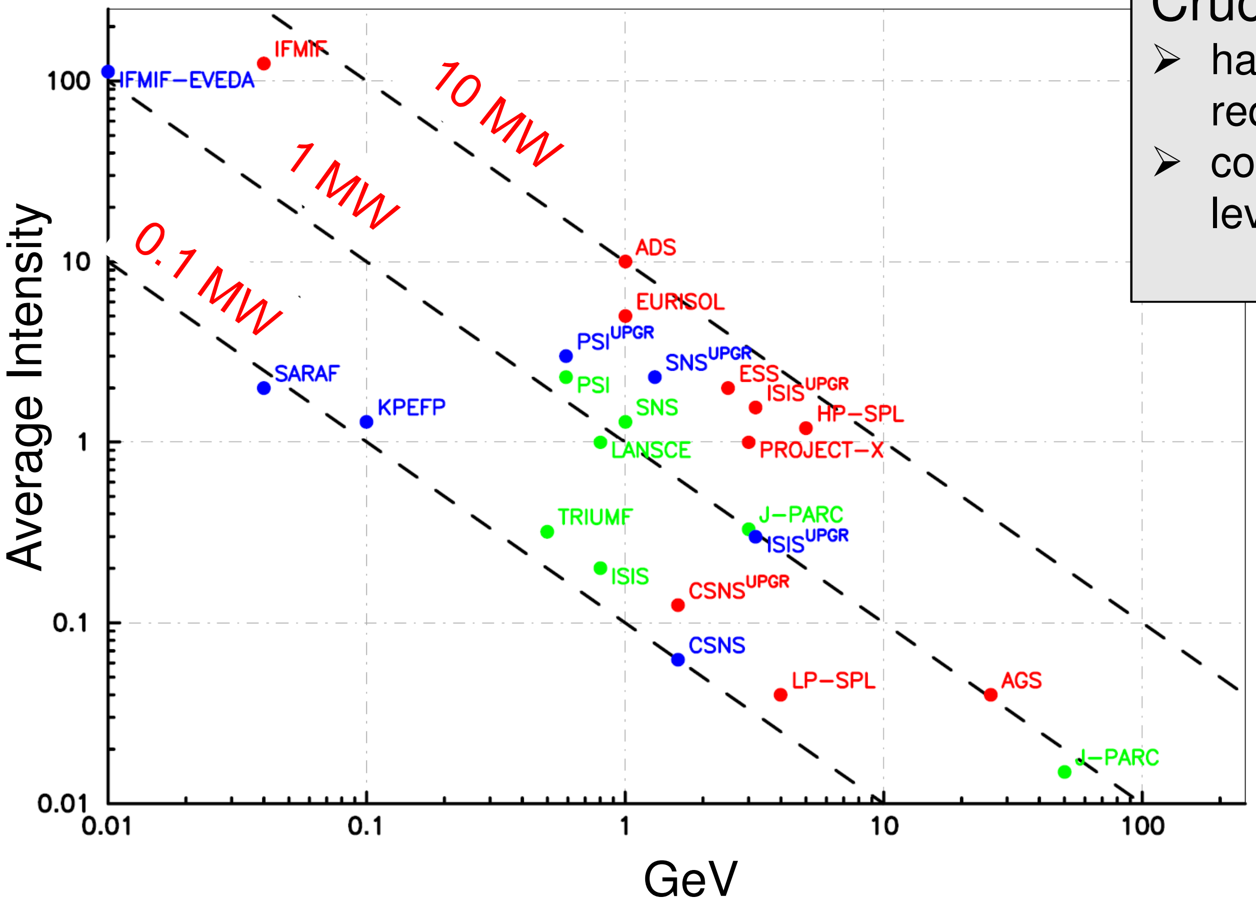
CERN-School High Intensity Limitations, 2015
November 2-11, 2015

Ingo Hofmann
GSI Darmstadt / TU Darmstadt

Overview

- ☐ This lecture focuses on direct space charge
  - p or heavy ion high intensity linacs at non- or weakly relativistic energies
  - electrostatic interaction – ignorable image charge effects
  - several mechanisms also relevant to circular accelerators!
  - limited relevance to space charge at injection of e^- linacs
- ☐ Introduction to envelopes and space charge
- ☐ Space charge resonances & instabilities
  - nearly all sources of emittance growth are of resonant nature (why?)
  - discuss **three main criteria** for linac design
- ☐ Mismatch, errors, halo → Beam loss
- ☐ Summary

Overview on high power linacs



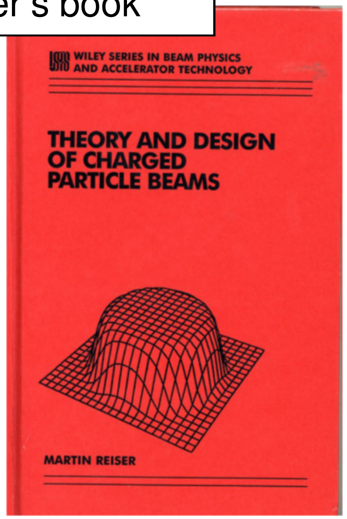
Crucial issue:

- hands-on maintenance requires beam loss < 1W/m
- control of beam power loss at level 10^{-6} for MW beam power

C. Prior, HB2010

Levels of description in linacs

Analytical basis:
Reiser's book


WILEY SERIES IN BEAM PHYSICS AND ACCELERATOR TECHNOLOGY

THEORY AND DESIGN OF CHARGED PARTICLE BEAMS

MARTIN REISER

Envelope dynamics with linear space charge in linear optics

design

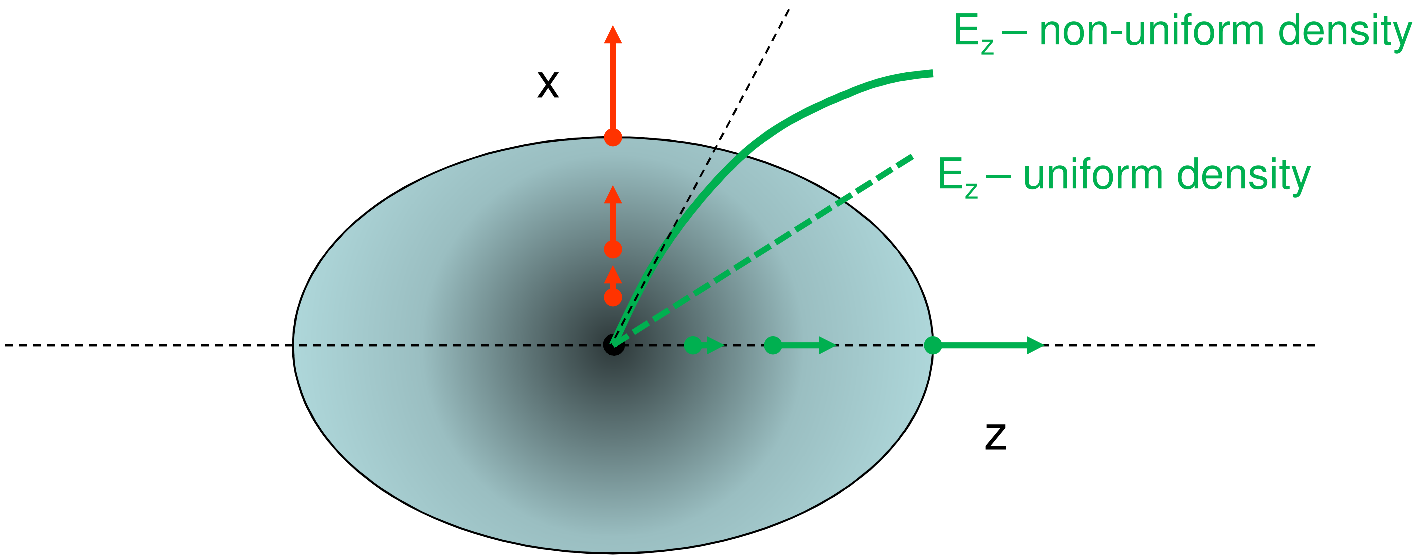
Multi-particle beam dynamics in idealized linear (nonlinear) optics with **nonlinear space charge**

verification of design

Multi-particle beam dynamics in optics with **random errors**

beam halo and loss prediction

Calculation of direct space charge force



- bunches usually close to spherical (within factor of 2)
- → image charges usually negligible (pipe far away)
- forces $E_{x,y,z}$ = linearly increasing with amplitudes in <u>uniform bunch</u>
- in non-uniform bunch non-linear $E_{x,y,z}$ not negligible →major source of ε growth

$$E_x = \frac{3qN(1-f)}{4\pi\varepsilon_0(r_x+r_y)r_z}\frac{x}{r_x}, \qquad E_z = \frac{3qNf}{4\pi\varepsilon_0 r_x r_y}\frac{z}{r_z}$$

for uniform ellipsoid with semi - axi $\mathrm{r}_{x,y,z}$

Sacherer's r.m.s envelope equations

and the equilibrium problem in 2d (infinitely long) beams

- Linear force (lattice + space charge) predicts rms emittances are constant!
  - with space charge exact self-consistent solution is 2D – KV
  - equivalent to envelope equation (transversely uniform density, infinitely long)

- for non-KV distribution the r.m.s envelope equations still hold – in good approximation! (Sacherer, ~1973)
  - non-uniform density leads → nonlinear space charge force
  - **surprisingly r.m.s. envelope equations still very good approximation - if emittances constant!**
  - **applies ~ also to 3D case of "bunched beam" !**

Rms envelope equations

- valid under assumption of constant emittances -

$$a_x'' + \kappa_x(s)a_x - \frac{\varepsilon_x^2}{a_x^3} - \frac{3K(1-f)}{(a_x+a_y)a_z} = 0$$

$$a_y'' + \kappa_y(s)a_y - \frac{\varepsilon_y^2}{a_y^3} - \frac{3K(1-f)}{(a_x+a_y)a_z} = 0$$

$$a_z'' + \kappa_z(s)a_z - \frac{\varepsilon_z^2}{a_z^3} - \frac{3Kf}{a_x a_y} = 0$$

rms beam sizes : $a_{x,y,z} = r_{x,y,z} / \sqrt{5}$

rms emittances : $\varepsilon_x^2 = \overline{x^2}\,\overline{x'^2} - \overline{xx'}^2$

space charge parameter :

$$K = \frac{qN}{20\sqrt{5}\pi\varepsilon_0\beta^2\gamma^3 mc^2}$$

When are the rms emittances constant?

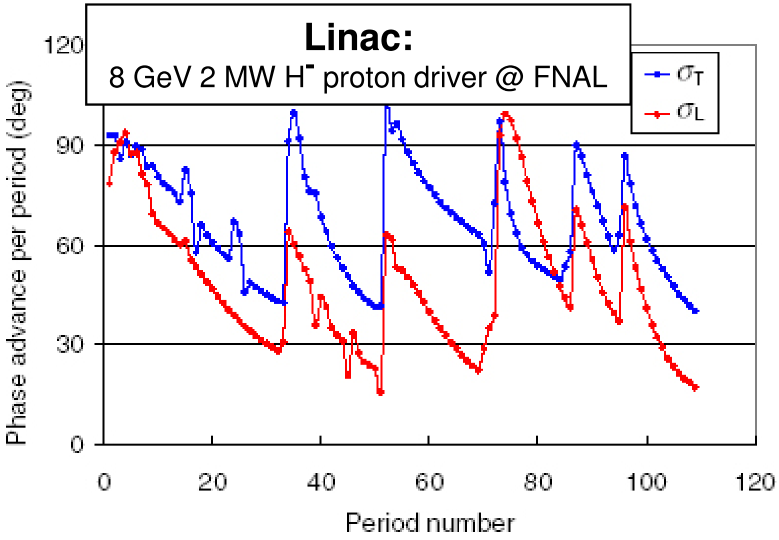
$\varepsilon_{99\%}$, $\varepsilon_{99.99\%}$ equally important!

numerous studies:

Struckmeier and Reiser, Part. Accel. 14 (1984) ....Li and Zhao, PRSTAB 17 (2014)

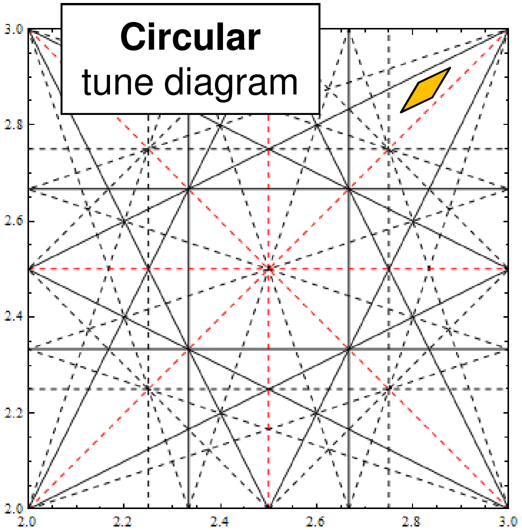
Linac beam dynamics is different!

- varying structures, focusing and tunes –


Linac:
8 GeV 2 MW H$^-$ proton driver @ FNAL

Linac:

- ✓ **single pass**
- ✓ optics ~ linear
- ✓ **space charge potential**
  - – nonlinear
  - – periodically varying
- ✓ → **resonances may exist**
- ✓ often transient and not separable
- ✓ avoid by design – if possible


Circular
tune diagram

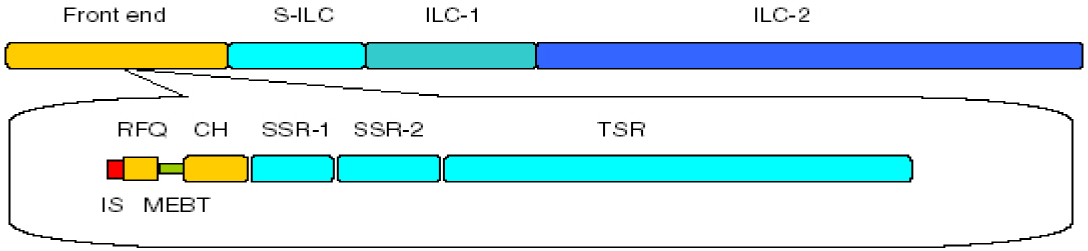
Circular:

- ✓ **many turns**
- ✓ optics nonlinear effects matter
- ✓ **space charge potential ~ a correction**
- ✓ → many resonances exist
  - – avoid or compensate

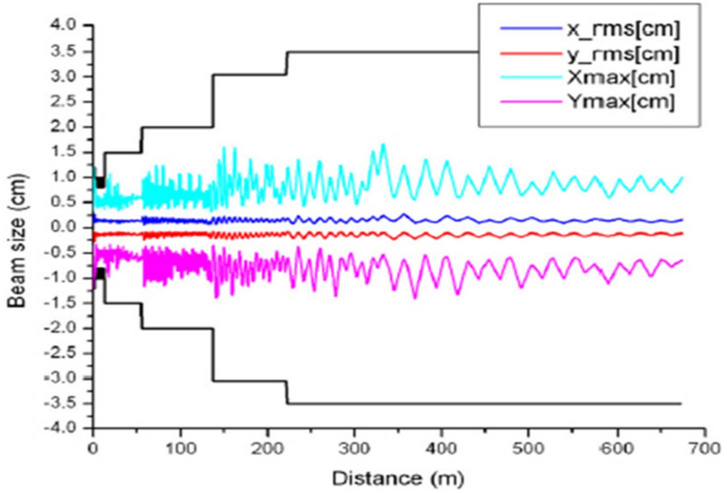
Example of linac structure effect on beam dynamics

- varying structures and focusing –

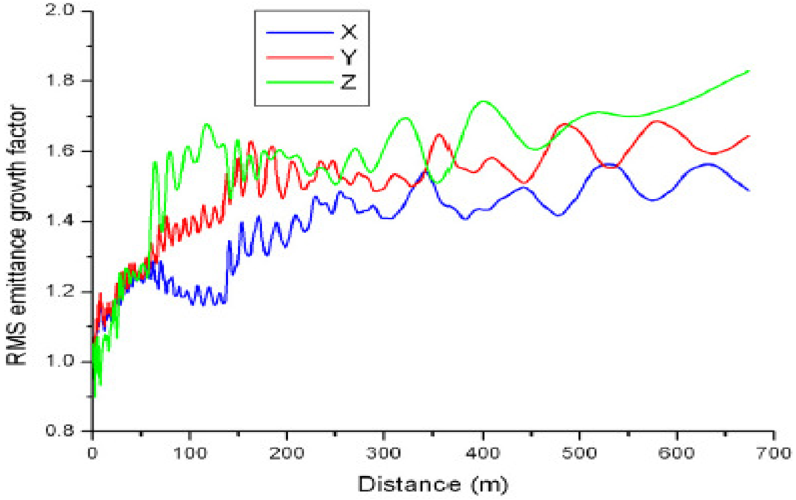
concern: emittance increase, halo, beam loss & activation



Proposal of a sc 8 GeV H$^-$ proton driver for Fermilab **(Project X)**
P. Ostroumov (ANL), 2006



Transverse envelopes of 32 mA beam along the linac. The black solid line shows the aperture.



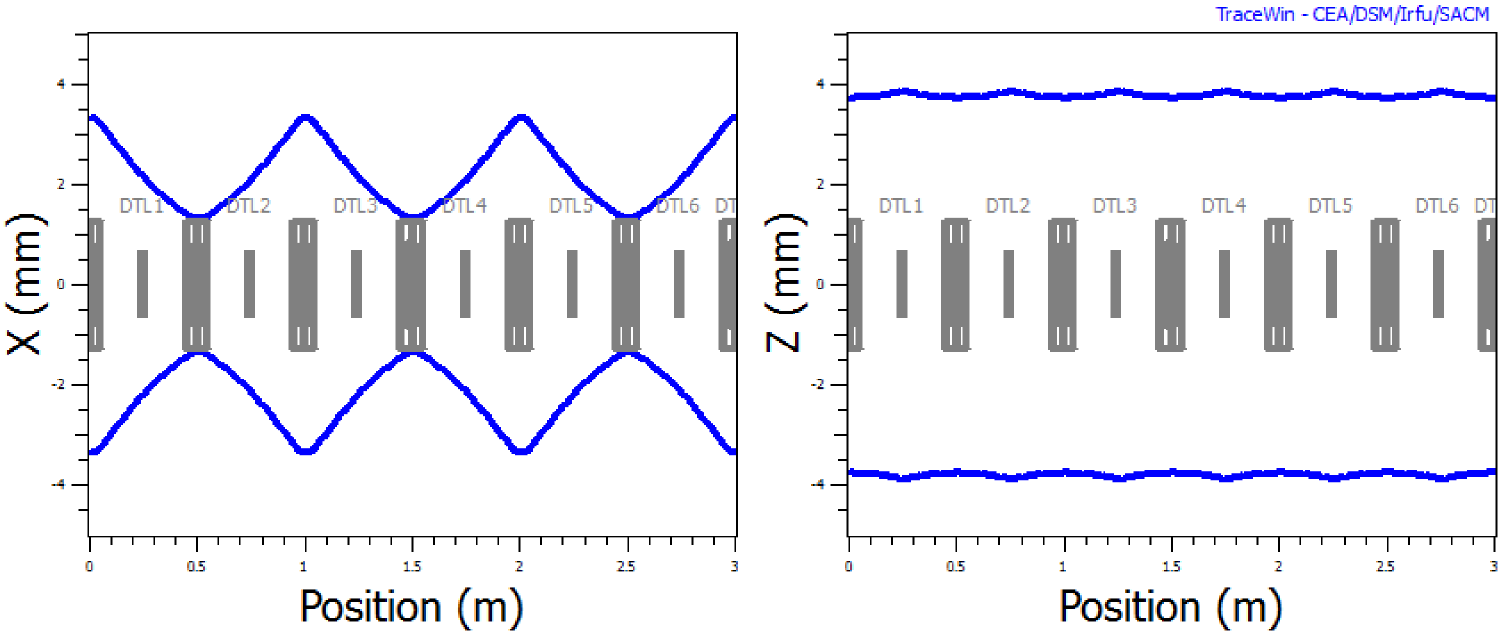
How to characterize space charge strength?

- lattice: k_{0x}, $k_{0y,}$ k_{0z} describe lattice
- reduced by space charge to k_x, $k_{y,}$ k_z (k^2 ~ force)
- “tune depression” k_x/k_{0x} or k_z/k_{0z} relative importance of space charge;
- “convention” in p linacs: $k_x/k_{0x} < 0.7$ ~ “space charge dominated”: effective force ~ reduced to half by space charge
- $k_x/k_{0x} \rightarrow 0$ strict space charge limit
- =0 is “cold” beam with zero emittance

Idealized "demo lattice" - for simplicity

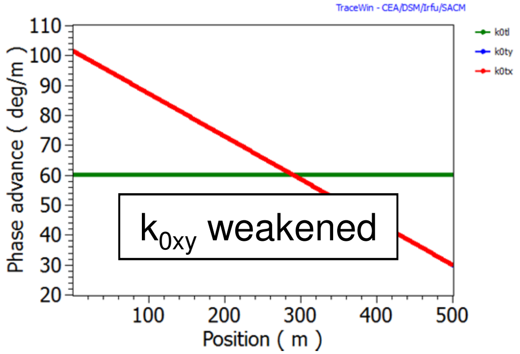
periodic cells / RF gaps + well-separated resonant effects

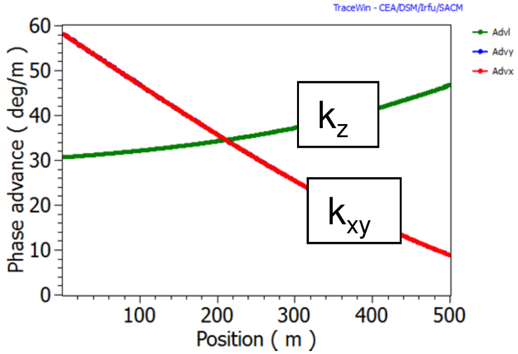
F/2 – O – D – O – F/2 with symmetric RF gaps

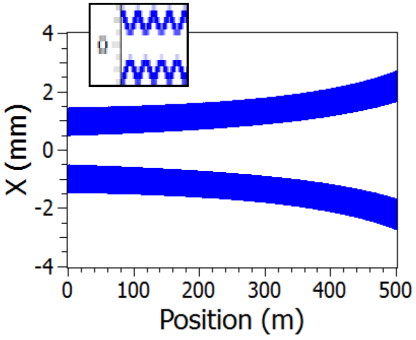


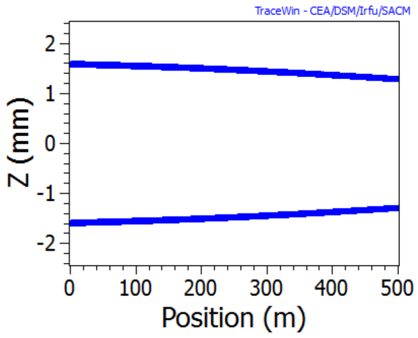
How to get more space charge dominated?

downwards k_{0xy}-ramp – envelope model “demo lattice”

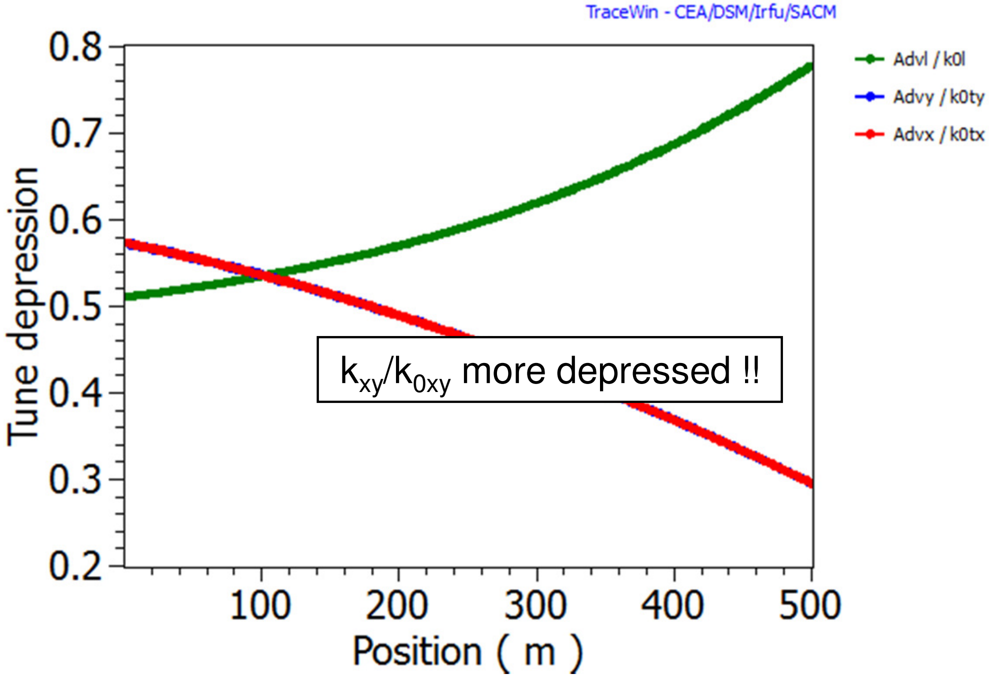








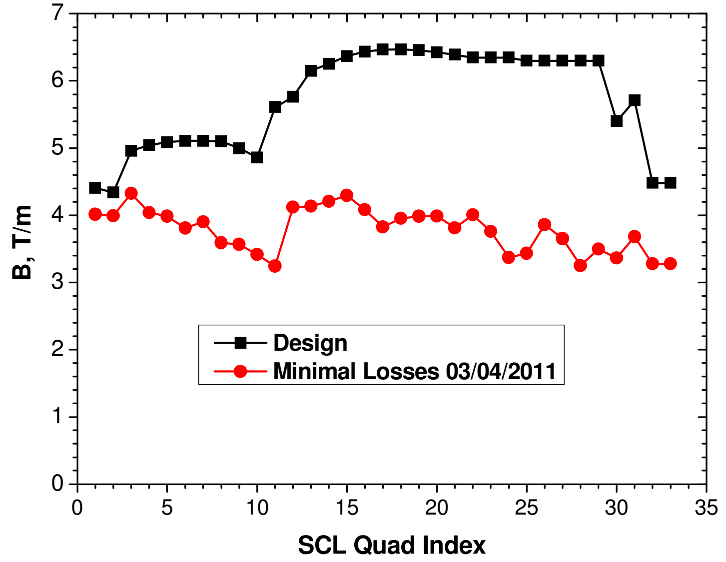
- ✓ k_{0xy} → 0 weaker focusing
- ✓ → **beam size grows**
- ✓ → **more space charge dominated**
- ✓ although absolute space charge force weaker!

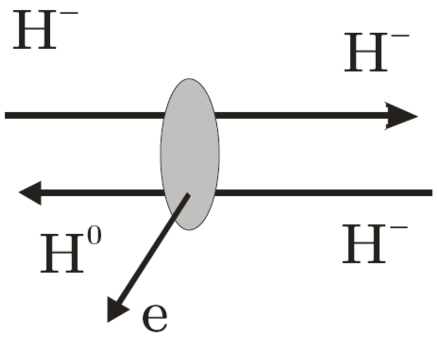


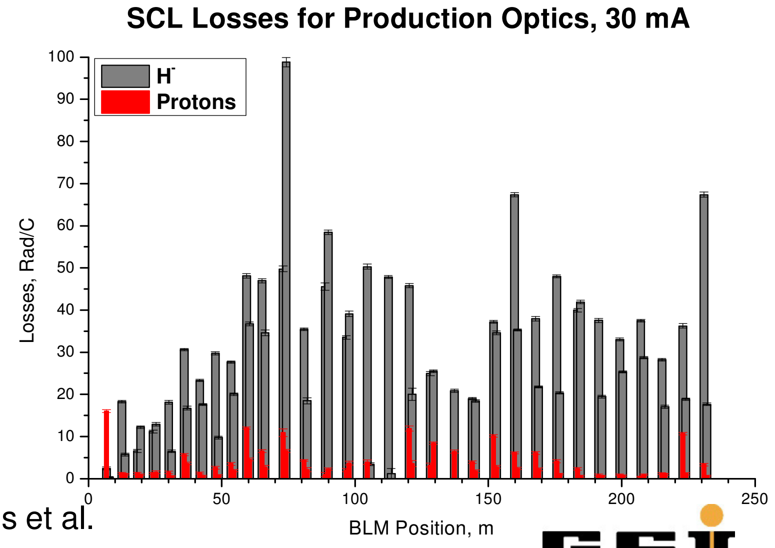
Application to intrabeam stripping

serious issue in H^- high power linacs → cure: expand beam!

- 2010: SCL losses can be caused by Intra Beam Stripping of H^- (Valeri Lebedev, FNAL)
- By lowering SCL quads' field gradients the losses were reduced to an acceptable level.
- Weaker focusing – more space charge dominated



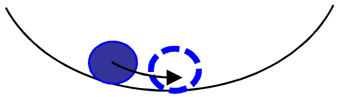




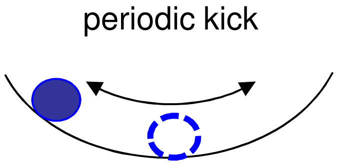
source: J. Galambos et al.

Equilibrium - Resonance – Instability

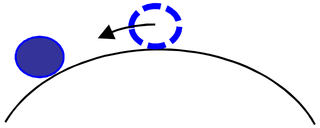
- sources of emittance growth – any accelerator -



deviation from stable equilibrium = „mismatch“
- small deviations → response bounded by initial value
- return to initial position if „damping“ exists – here particles
- → energy into „damping particles“



resonant excitation
- increasing amplitude
- limited by de-tuning or loss



instability

small deviations → runaway
- no return to initial position
- → instability (also resonant)

Beam: potential from magnets/RF **and** self-consistent electric field

all 3 involve **resonant** mechanisms – also in linac!

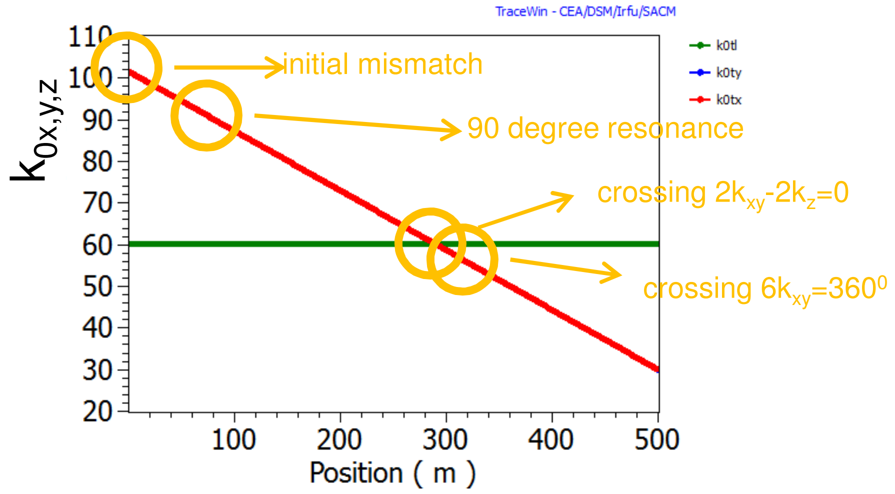
Full particle-in-cell simulation

TRACEWIN code for linac design and verification

- **TRACEWIN: design and verification**
  - **http://irfu.cea.fr/Sacm/logiciels/.**
- Grid-based Poisson solver “inside” bunch
- analytical continuation outside
  - model halo particles accurately far away from core
- free boundary:
  - ignore image charges – direct space charge dominant
- # simulation particles ~ 10^7
  - worry about loss at level 10^{-6}
- “error studies”: statistics with ~ 10^3 error seeded linacs
  - → effect on beam loss
- limited spatial resolution
  - → noise needs to be checked

Full particle-in-cell (PIC) in “demo-lattice”

100^0 downwards k_{0xy}-ramp – demonstration of main resonant effects

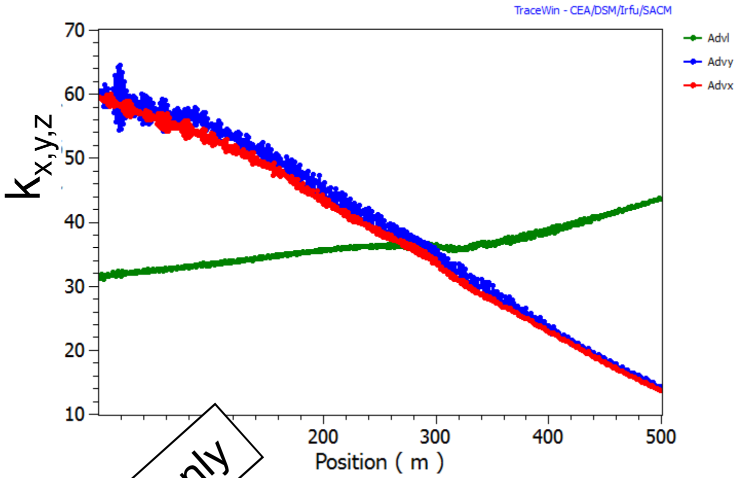


initial mismatch

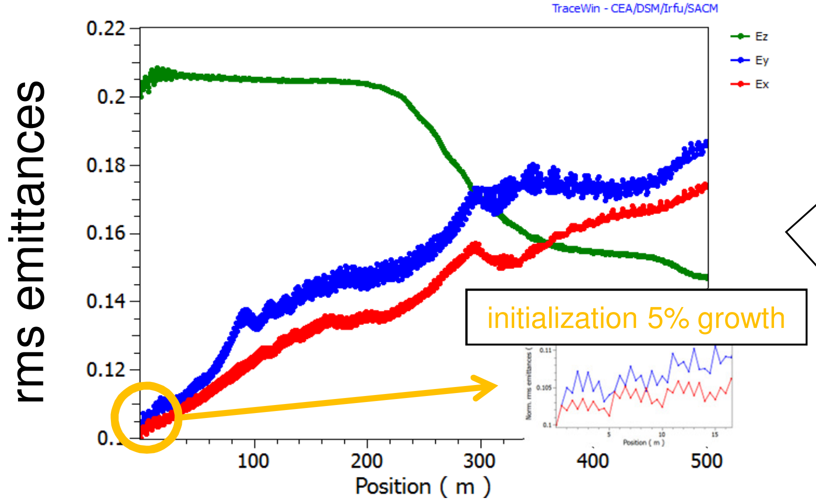
90 degree resonance

crossing $2k_{xy}-2k_z=0$

crossing $6k_{xy}=360^0$



not a linac- lattice demo only



initialization 5% growth

Sources of emittance growth in linacs

in principle also relevant to circular accelerators

Non-resonant

Initial density profile mismatch

- if starting with non-selfconsistent initial distribution
- evolves very fast: ~¼ plasma period (typically < 1 betatron period)

Resonant instability by periodic structure

“90 degree” stopband envelope instability”

- exponential growth from initial noise
- involves a **resonance** condition
- requires time (distance) to develop

Distinction instability – resonance sometimes confused

“Classical” resonances

1. Structure resonances
   - driven by periodically modulated space charge force → resonance condition
2. Anisotropy
   - driven by energy (emittance or "temperature") difference between degrees of freedom
   - is a difference resonance - only exchange of emittances (rings: “Montague resonance”)

Resonant halo formation

driven by rms mismatch → periodic force from space charge

- pushes particles into a halo
- also caused by random errors in magnet optics

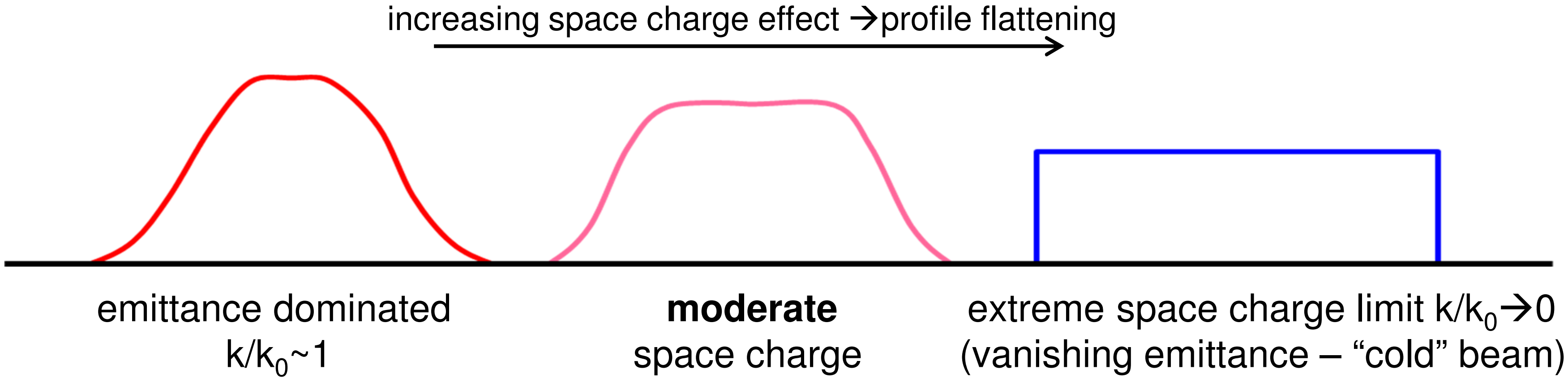
Not all equally serious

Initial density profile mismatch – rms matched!

“un-matched” nonlinear field energy → emittance growth

- discovered in 1980's under **"nonlinear field energy"**
  - **1D:** Wangler et al., IEEE Trans. Nucl. Sci. NS·32, 2196 (1985)
  - **3D:** Hofmann and Struckmeier, Part. Accel. 81, 69 (1987)
- always present at injection of a space charge dominated beam
- reason: space charge repulsion wants to **flatten the beam** the more the closer to space charge limit ($k/k_0 \rightarrow 0$) (self-consistent solution including **non-parabolic space charge potential**)
- “Plasma effect” known as “Debye shielding” – **a non-resonant effect** (only one here!)

matched density profiles (schematic – Gaussian distribution):



Initial density effect cont'd

- Uniform density bunch has minimum electrostatic Coulomb energy - comparing bunches with *same* charge and *same* rms size
- → if non-uniform density is injected at high space charge **and ignoring profile flattening** the **extra electrostatic energy** ΔW transforms into additional **rms emittance**

$$\Delta W \equiv \left[\frac{\varepsilon_0}{2}\iiint E^2\,dxdydz\right]_{\text{initial}} - \left[\frac{\varepsilon_0}{2}\iiint E^2\,dxdydz\right]_{\text{profile-matched}} \qquad \rightarrow \Delta\varepsilon_{x,y,z}$$

$$\frac{\varepsilon_{final}}{\varepsilon_{initial}} \approx \left[1-\frac{1}{3}\left(\frac{k_0^2}{k^2}-1\right)\left(U_{final}-U_{initial}\right)\right]^{1/2}$$

see: Hofmann and Struckmeier, 1987

Initial density effect cont'd

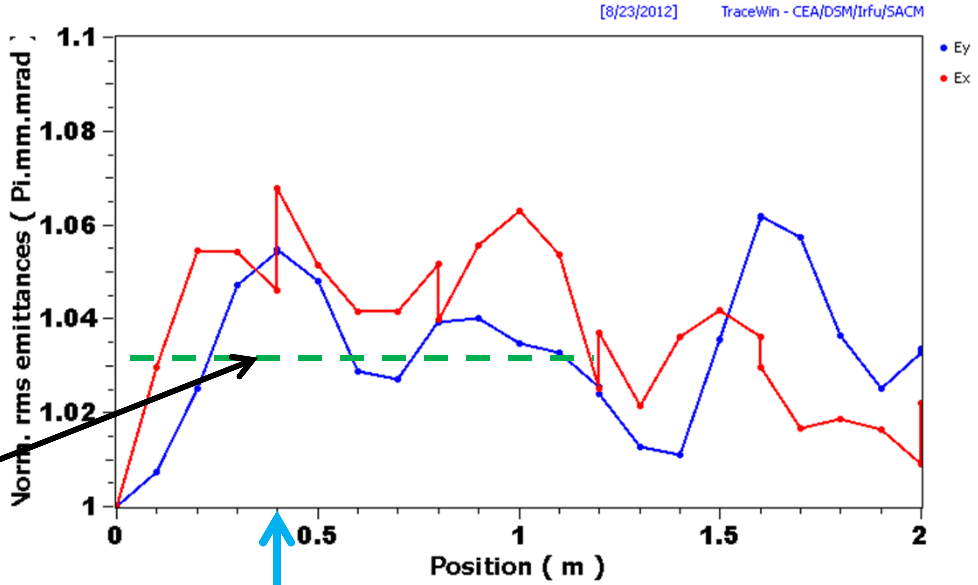
case 1

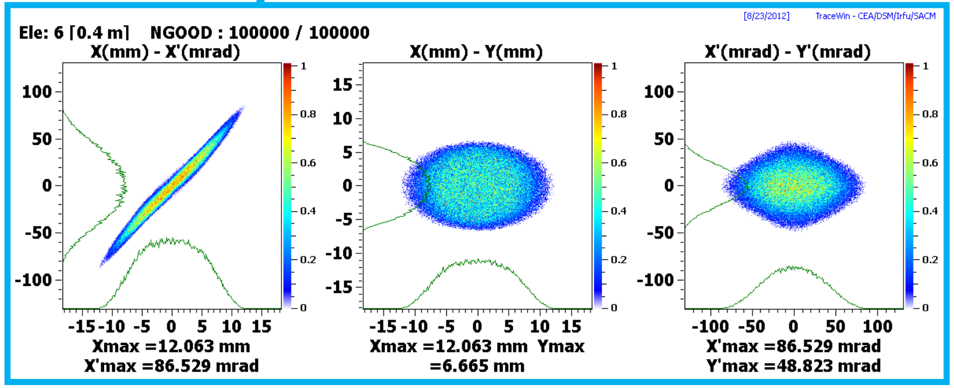
- Simulation example $k_{0x,y}=85^0$ $k_{x,y}=43^0$ $(k_{0z}=85^0)$
- Initial WB & after perfect matching with r.m.s. envelope equations

Analytical estimate in spherical approximation and assuming $U_{final}=0$:

k/k_0	$U_{initial}$	$\Delta\varepsilon/\varepsilon_{initial}$
0.5	**0.06 (WB)**	**3 %**
0.25	"	13 %
0.5	0.26 (Gauss)	12 %
0.25	"	51 %

~ good agreement!
phase space plot suggests a space charge octupole as driving force!





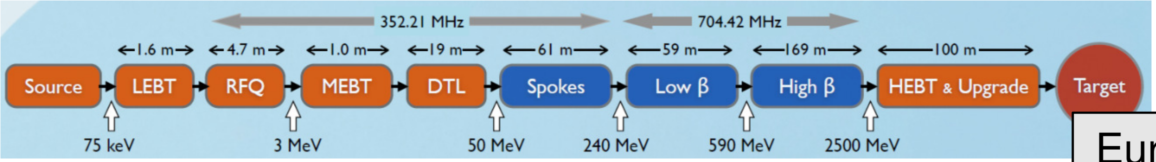
→1st criterion for high-current linac design:
"smooth real estate phase advance"

Unmatched density profile:

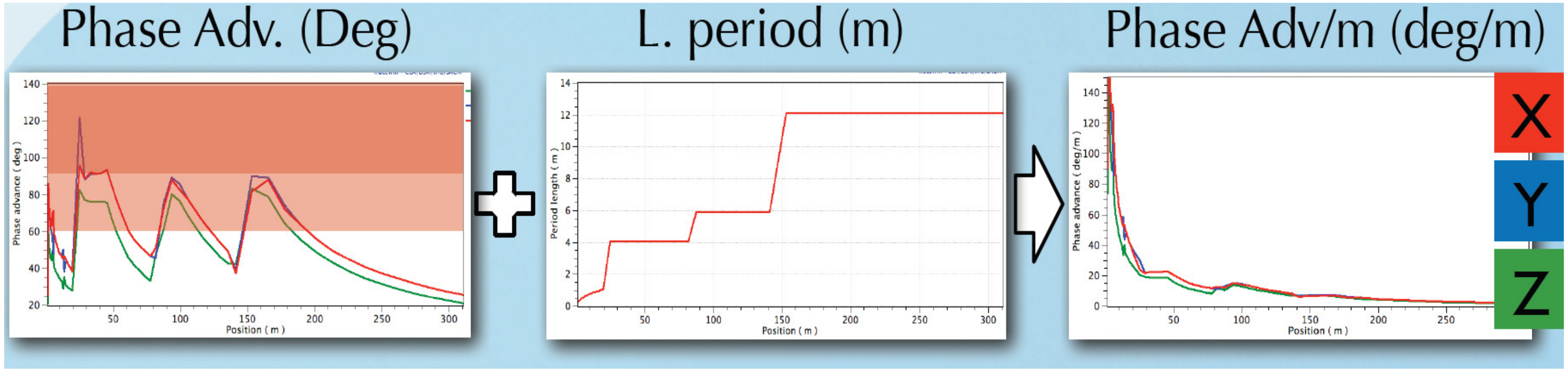
- ✓ inevitable at injection: cannot match injection density profile to profile of self-potential → self-matching with ε-growth
- ✓ occurs again, if sudden jump in focusing strength (phase advance per meter!) often required by different RF structures
- ✓ **avoid:** need to **design linac lattice smoothly** by inserting gradual transitions to allow adiabatic density adjustment ("smooth real estate phase advance")

Example ESS: “smooth” design

smooth real estate phase advance (deg/m)

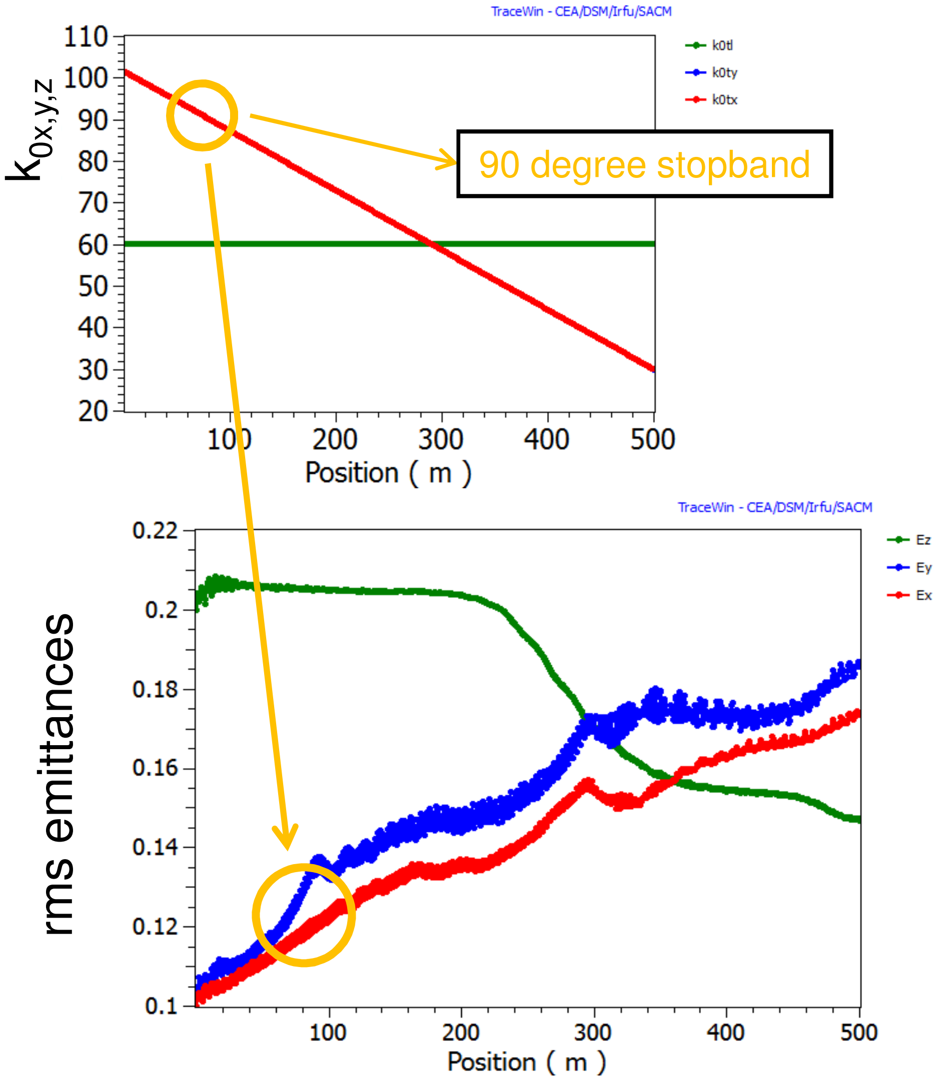


European Spallation Source
2.5 GeV 5 MW p linac



M. Eshraqi, HB2010

Second candidate (in “demo” lattice):



symbolic nomenclature:

	Linac	Circular machine
Envelope instability	$2k_{xy}\sim180^0$	$2Q_{xy}\sim½$
4th order resonance*)	$4k_{xy}\sim360^0$	$4Q_{xy}\sim1$

*) driven by space charge pseudo-octupole

Do we expect
2nd order envelope instability
or 4th order resonance?
Let experiment decide!

Structure resonance / instability in periodic focusing

Mathieu equation: parametric resonance

Avoid Mathieu instability at $k_0 = 180^0$

$$x'' = (a - 2\,q\cos 2\phi)x = 0$$

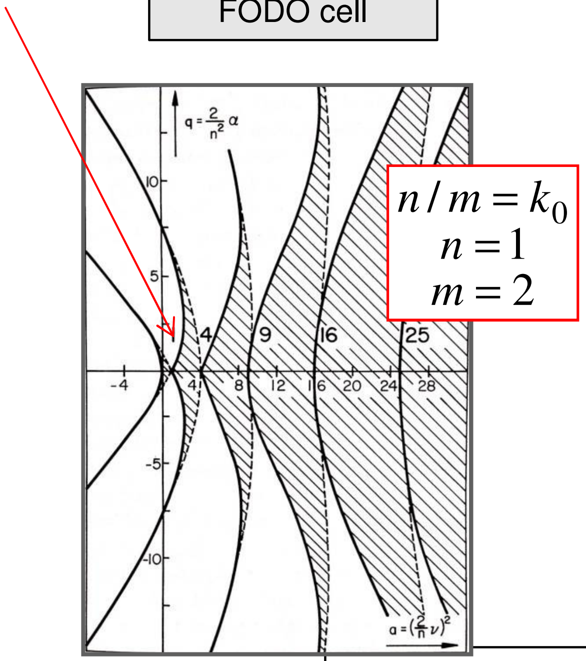
2:1 structure resonance :

- particle motion is unstable due to structure of fundamental focusing cell

Resonance or instability?

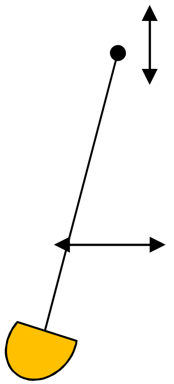
- instability of central orbit (zero amplitude - perturbed)
- perturbing force ~ initial amplitude perturbation → instability with exponential growth
- resonance: finite driving force already present by structure

structure = basic FODO cell

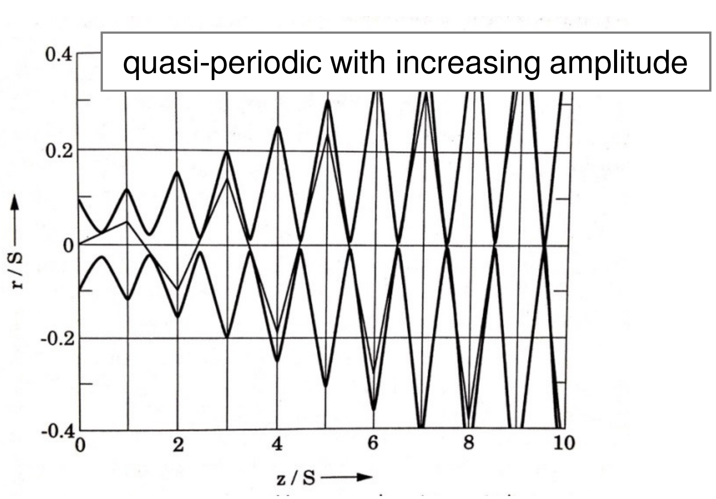


$$n/m = k_0$$
$$n = 1$$
$$m = 2$$

source: Reiser book



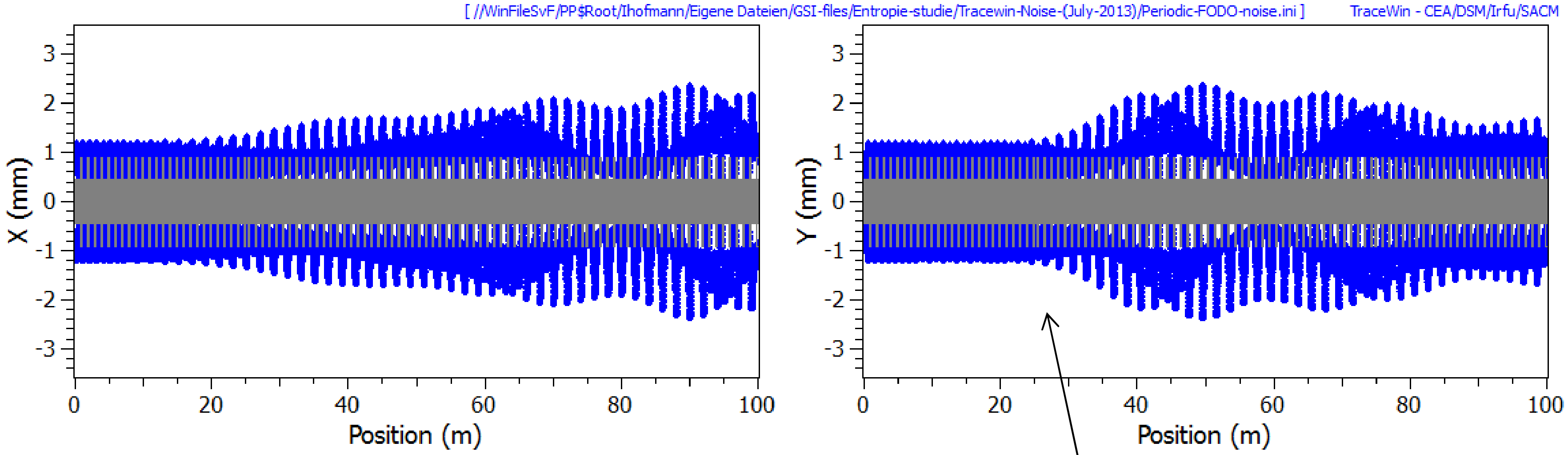
change length at double freuquency



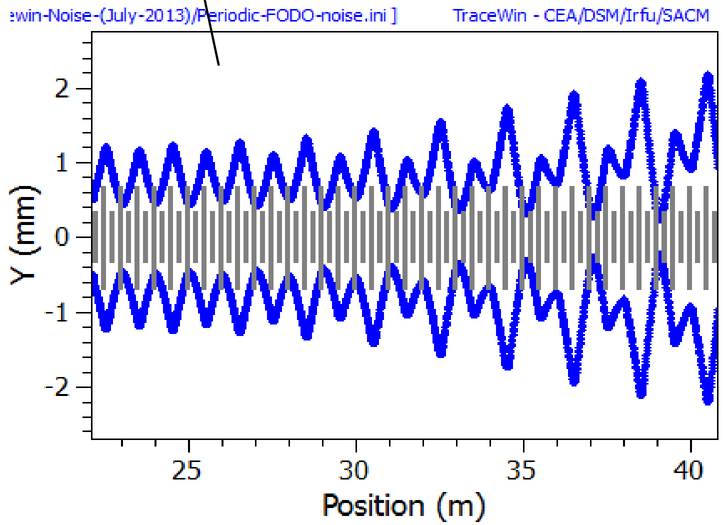
quasi-periodic with increasing amplitude

Adding space charge – additional mechanism:

„Envelope instability“ – a 2nd order structure instability



single particle k → 90^0 per focusing period
→ perturbed envelope “k” ~ 180^0 per period
→ also 2:1 relationship
→ particles driven exponentially unstable by envelope perturbation

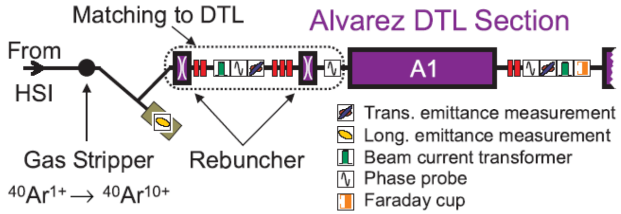


Experiment on $k_x \sim 90^0$ stopband in 2008 at GSI-UNILAC

first measurement of a **space charge structure resonance** in a linac!

L. Groening et al., PRL, 2009 (in context of HIPPI campaign)

16 cells!

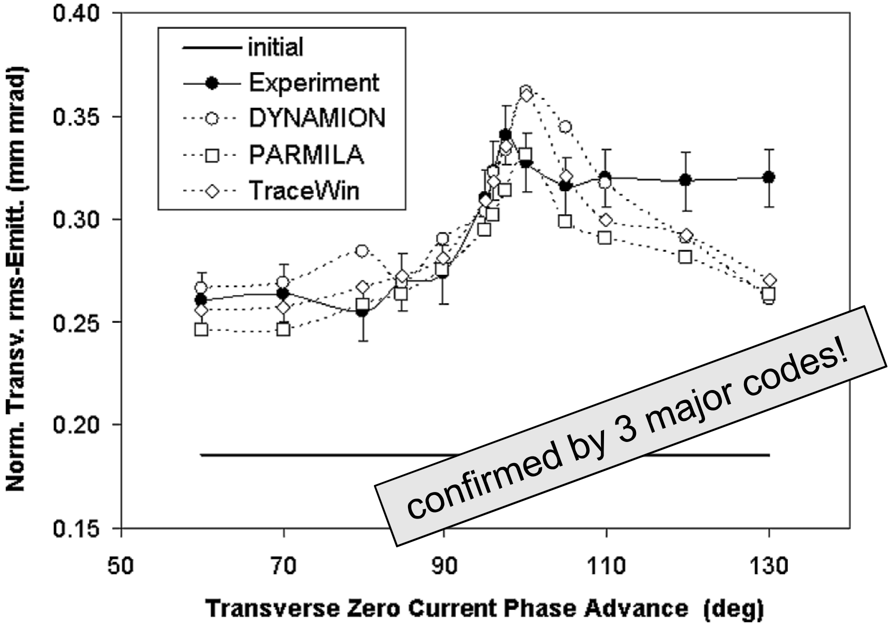


Main question:

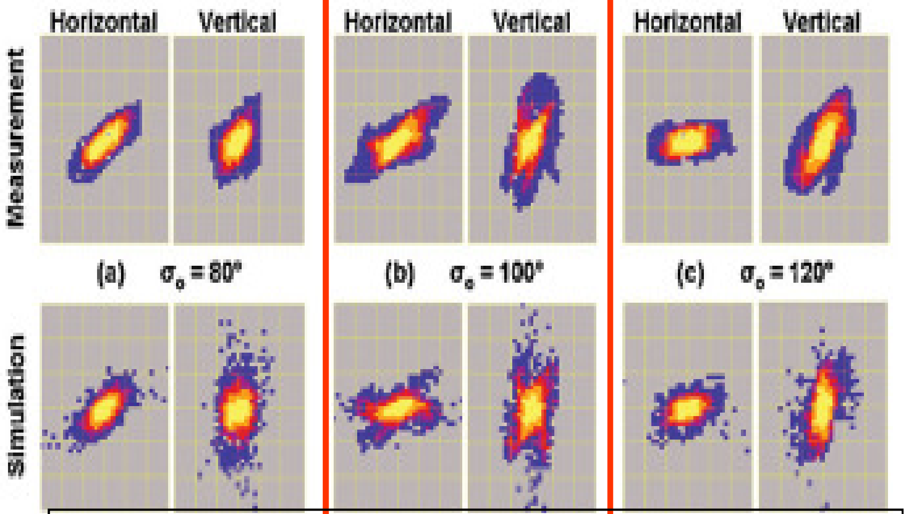
$2k_x \sim 180^0$ = envelope instability

$4k_x \sim 360^0$ = fourth order resonance (driven by space charge octupole in non-uniform beam)

both may occur! - *experiment should decide which one dominates!*



confirmed by 3 major codes!



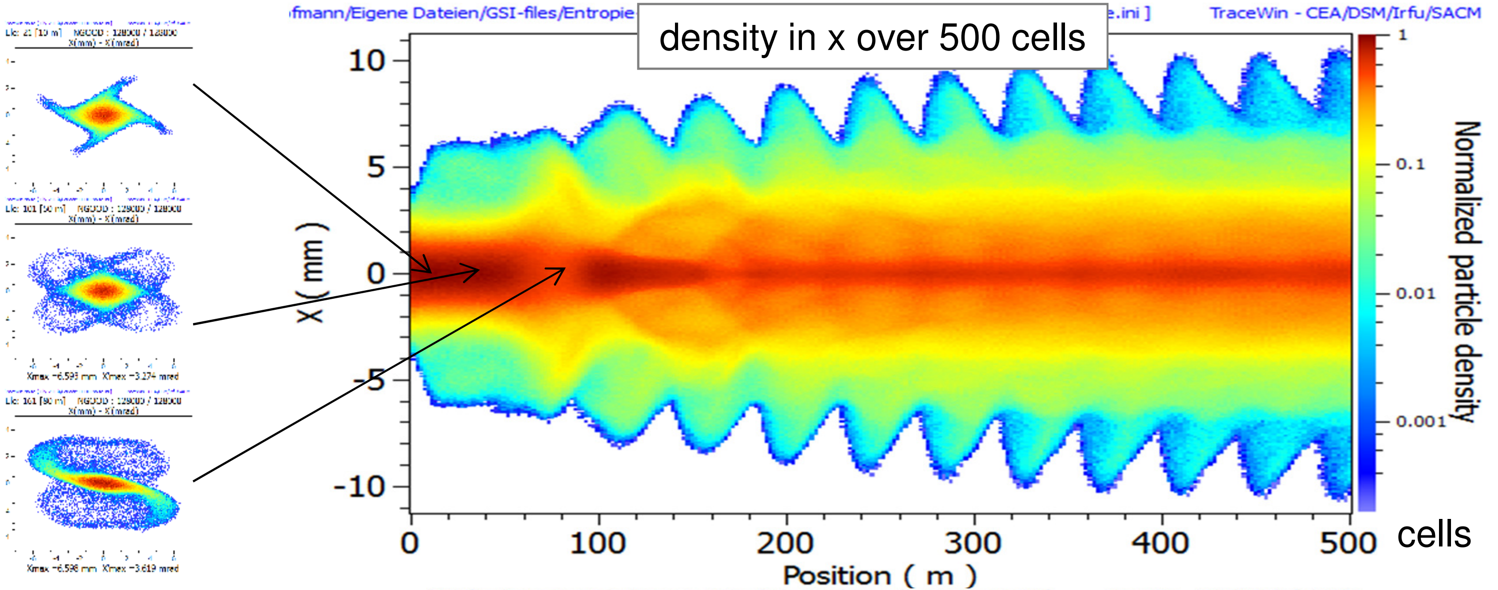
4-th order!

evidence that dominance of 4th order resonance over 2nd order instability?

We found double response on 90^0 stopband

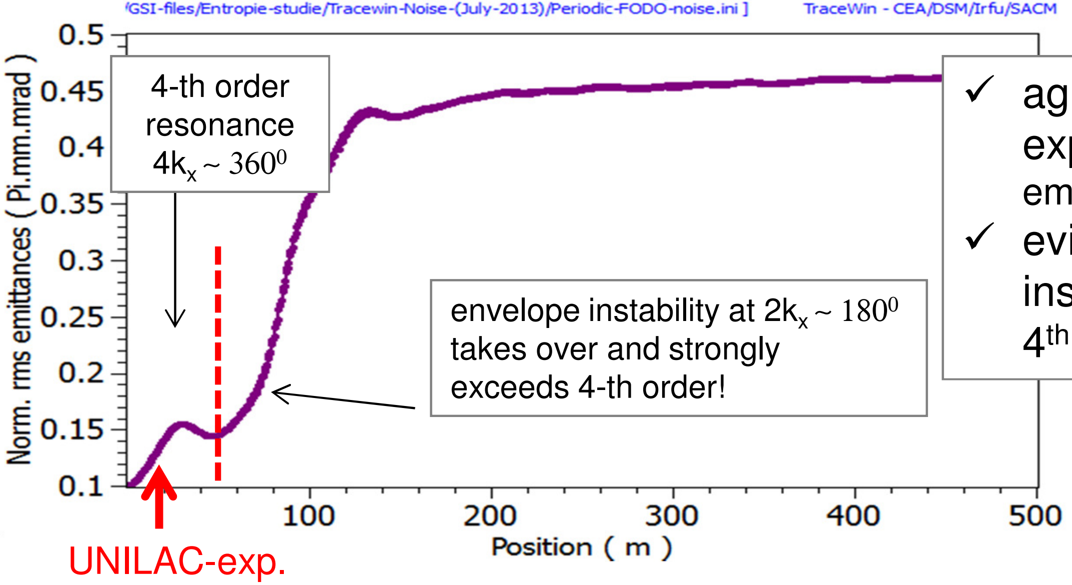
TRACEWIN 3D bunched beam simulation

to be published in PRL, Nov. 2015



density in x over 500 cells

k_{0xy} =95^0
k_{xy} =80^0
Gaussian bunch



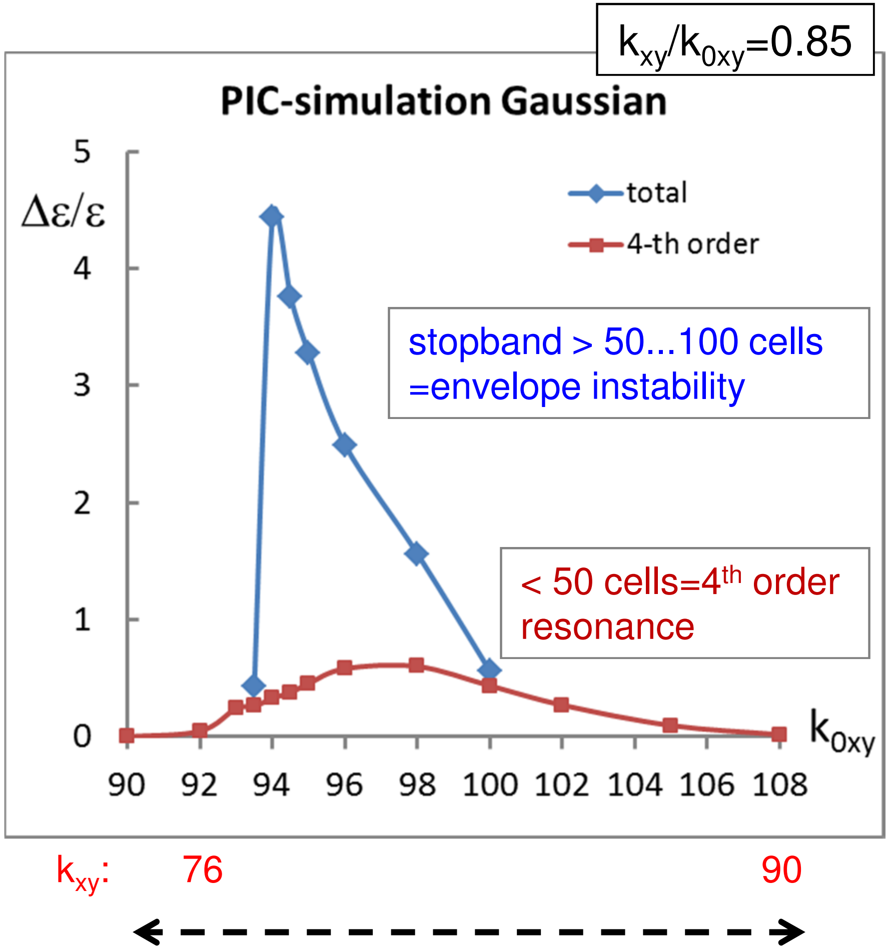
4-th order resonance $4k_x \sim 360^0$

envelope instability at $2k_x \sim 180^0$ takes over and strongly exceeds 4-th order!

UNILAC-exp.

- ✓ agrees well with UNILAC-experiment (~30% rms emittance growth over 16 cells)
- ✓ evidence that envelope instability can dominate over 4^{th} order in a longer system

Complete stopbands show higher complexity than could be concluded from UNILAC-experiment



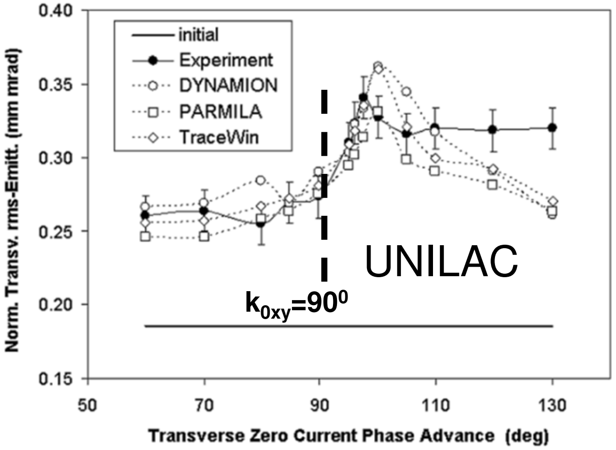
k_{xy}/k_{0xy}=0.85

stopband > 50...100 cells =envelope instability

< 50 cells=4th order resonance

k_{xy}: 76 90

stop-band width ~ Δk (space charge tune shift)



$k_{0xy}=90^0$

UNILAC

2^{nd} criterion for linac design: $k_{0x} < 90^0$

For $k_{ox} < 90^0$ avoid:

- ➢ envelope instability as well as
- ➢ 4th order resonance

✓ Envelope instability

- a real instability growing exponentially from small initial perturbation
- no effect for $k_{xy}=90^0$, requires $k_{0xy}>90^0$ and $k_{xy}<90^0$ → shifted from single particle resonance condition!

✓ Fourth order resonance

- driven by “space charge octupole”
- stopband partially overlapping with envelope instability

Might be observable (which one?) also in SIS 18 (12 Sup-Per) for Q_y → 3 → k_{0xy} → 90^0 (possibly by bunch compression with $Q_{0y}=3.2$)

Structural instability – resonance

in connection with space charge (only)

Instabilities require:

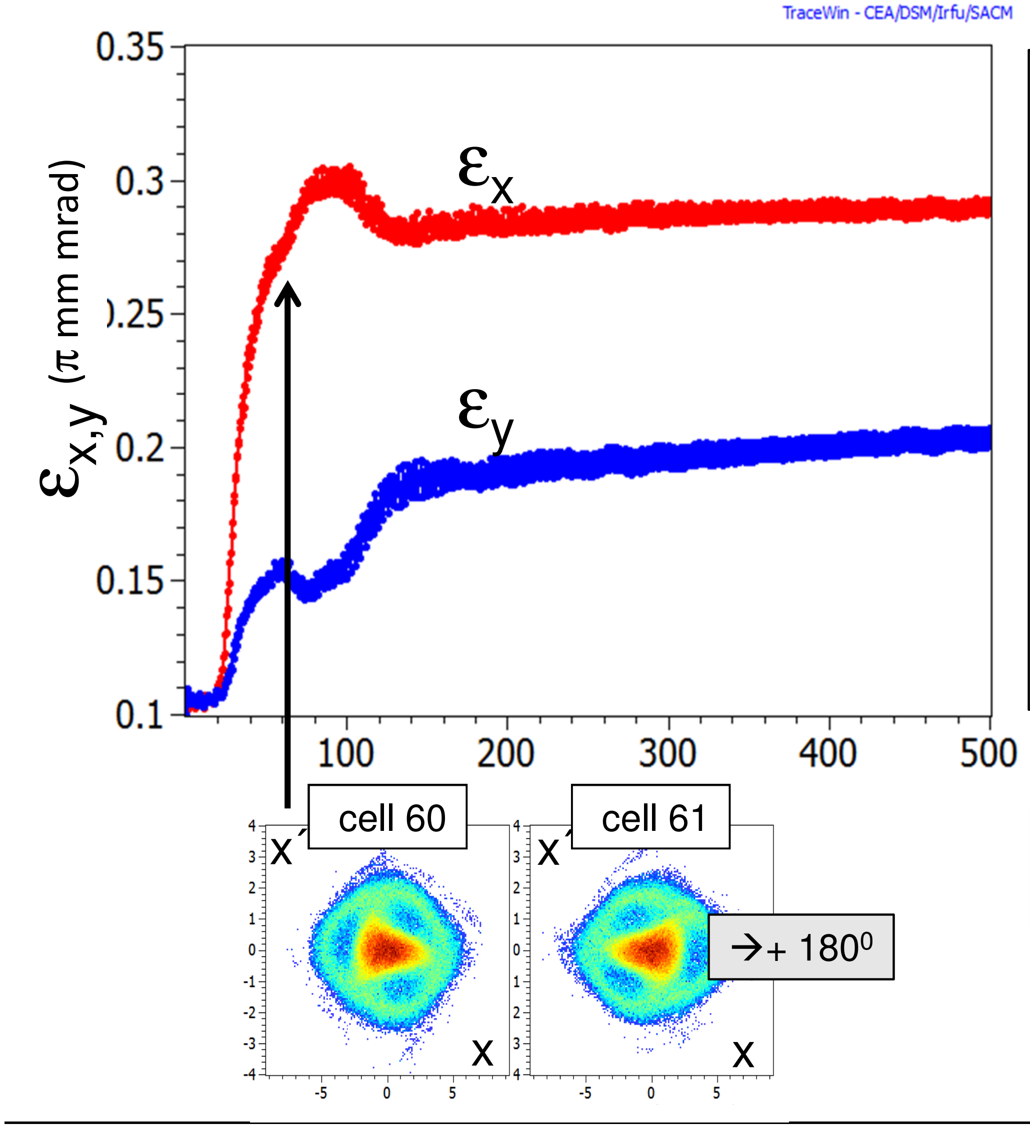
- driving force (space charge multipole):
  - absent initially – seeded only on noise level!
  - grows with instability going on
  - feedback leads to exponential growth
- normally resonance condition needed → resonant instability
- theoretically they exist in all orders – practically may be limited (mixing)
- no justification on usual resonance diagram

Resonances:

- for space charge multipoles present initially with non-uniform density
- multipole might grow further – self-consistent treatment - → a mix of resonance and instability
- theoretically in all orders – mixing!

Higher order instabilities / resonances?

discussed in 2015 PRL paper



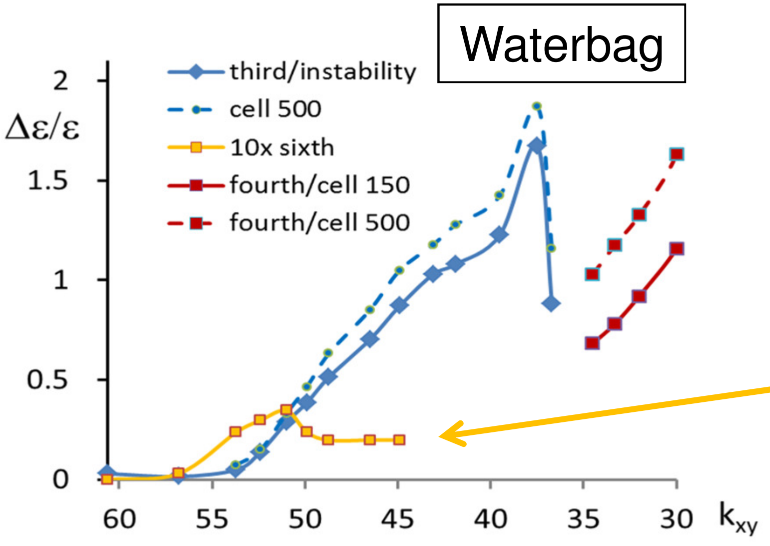
Found "third order instability"

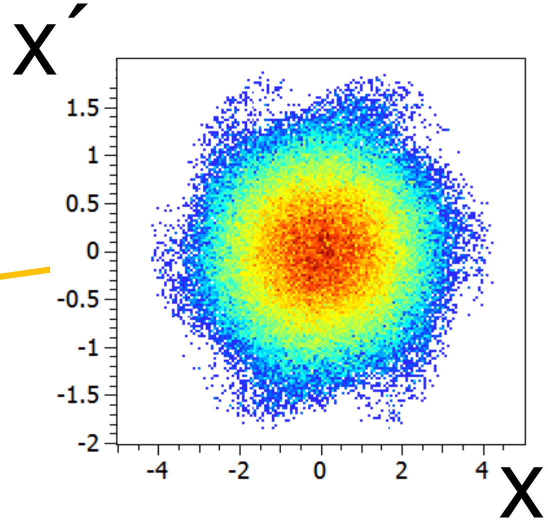
- $k_{0xy}=90^0$ (> 60^0 !) and $k_{xy}=42^0$ (< 60^0 !)
- analogous to 2nd order envelope instability
  - 2 periods per lattice period
  - "180^0" parametric 2:1 instability
- driven by space charge pseudo-sextupole
  - not a priori present in beam
  - → grows with exponential growth from noise level
  - essentially different from a 3rd order $3Q_{xy}$~n (n =1,2, ...) in a circular machine !

	Linac	Circular machine
3rd order instability	$3k_{xy}$~180^0	$3Q_{xy}$~½

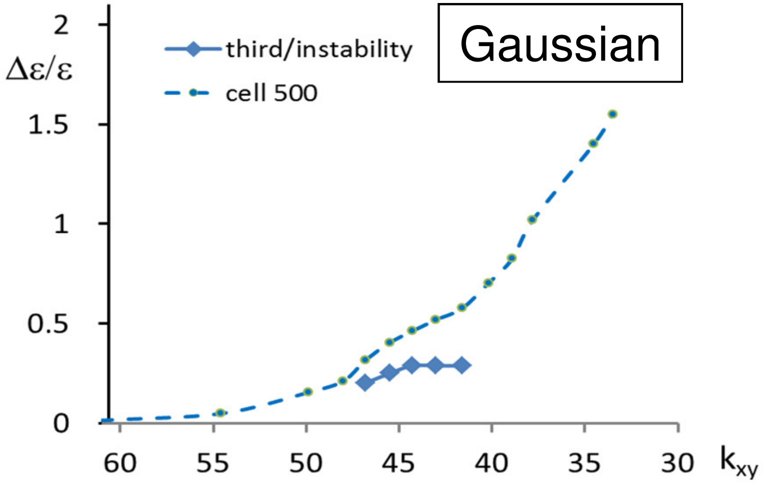
3^{rd} order instability + 6^{th} etc. resonances

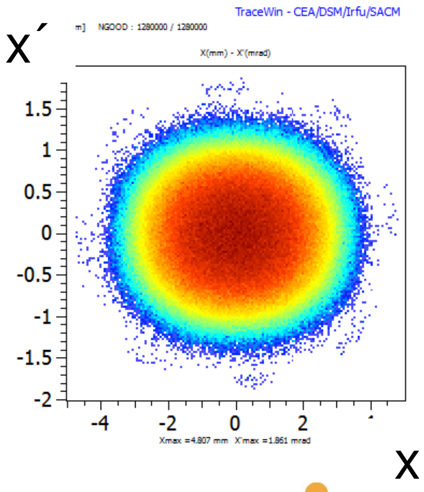
< few % effect - negligible





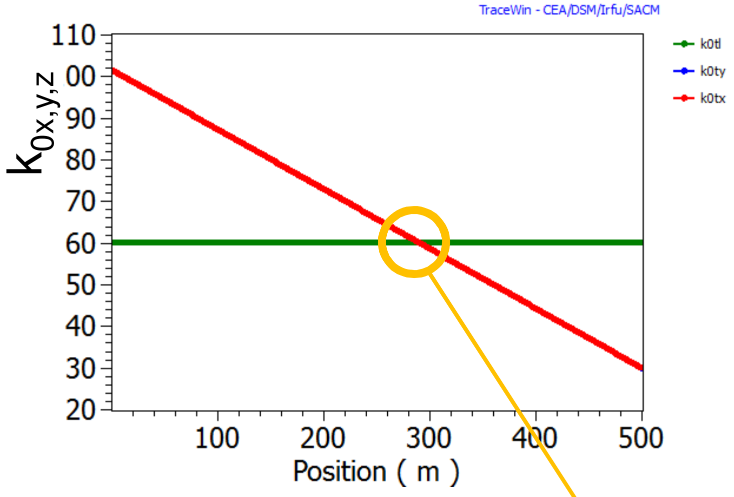
$6k_{xy}$~360^0
$8k_{xy}$~360^0





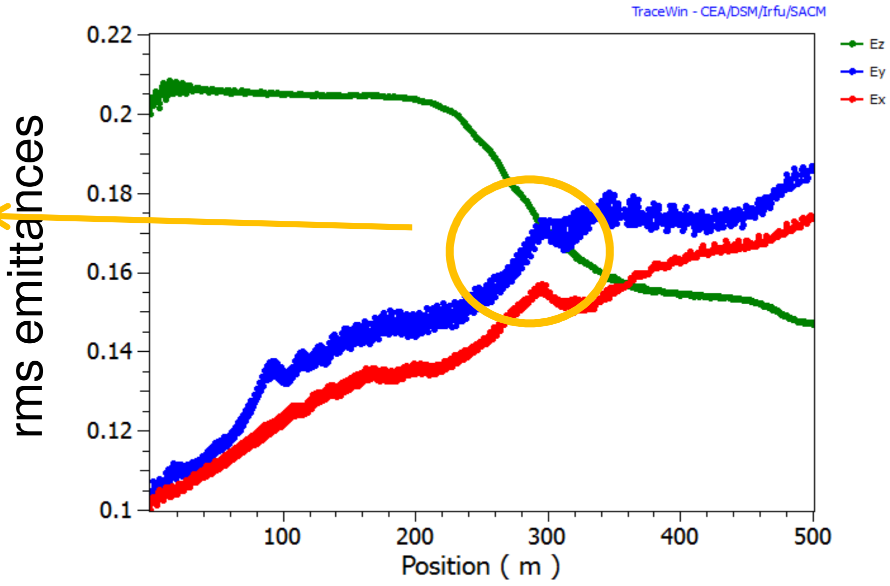
Much weaker for Gaussian distribution
- ignorable
- due to Landau damping?

Third candidate (in “demo-lattice”):



	Linac	**Circular machine**
Coupling resonance	$2k_{xy}-2k_z\sim0$	$2Q_x-2Q_y\sim0$ (“Montague”)

coupling resonance with exchange of transverse – longitudinal emittances



Emittance exchange, how?

How can emittance exchange happen?

1. collisions - too slow in linac → no
2. nonlinear forces between particles (FPU)? → no
3. nonlinear potential
    - due to magnet nonlinearities? → no
    - due to space charge modes? → yes!

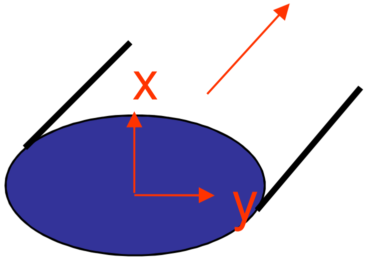
$$\cancel{dT_\perp = -\nu\left(T_\perp - T_z\right)dt}$$

$$\cancel{\nu \propto n \times Z^4 / A^{1/2} / T_z^{3/2}}$$

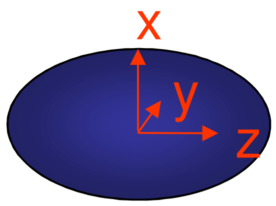
$$\boxed{T \equiv \frac{T_z}{T_x} \approx \frac{\varepsilon_z k_z}{\varepsilon_x k_x}}$$

T=1:

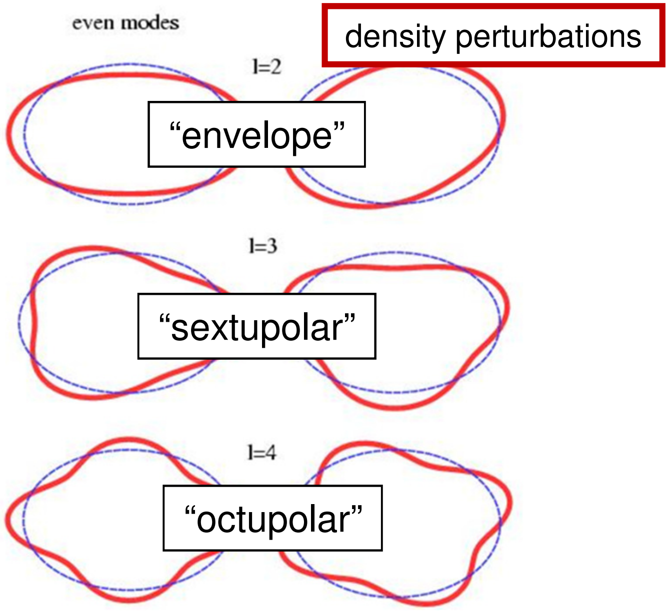
"Equipartitioned beam":



2D: Vlasov-theory + PI



3D: PIC-simulations



Selfconsistent perturbation theory of space charge modes in anisotropic KV-beam

requires Vlasov-Poisson equations:

$$\frac{df}{dt}=\frac{\partial f}{\partial t}+\sum_{i=1}^{2}\left(\dot{x}_i\frac{\partial f}{\partial x_i}+\dot{p}_i\frac{\partial f}{\partial p_i}\right)=0$$

$$\nabla\cdot E=\frac{q}{\varepsilon_0}\iint f(x_1,x_2,p_1,p_2,t)\,dp_1dp_2$$

perturbation analysis $f = f_0 + f_1$
around anisotropic KV - beam :

$$f_0 \sim \delta(p_x^2+\nu_x^2x^2+T(p_y^2+\nu_y^2y^2)-1)$$

$$\nabla^2\Phi_1=-\frac{q}{\varepsilon_0}n_1-\frac{q}{\varepsilon_0}\int f_1dp_1dp_2$$

n^{th} order :

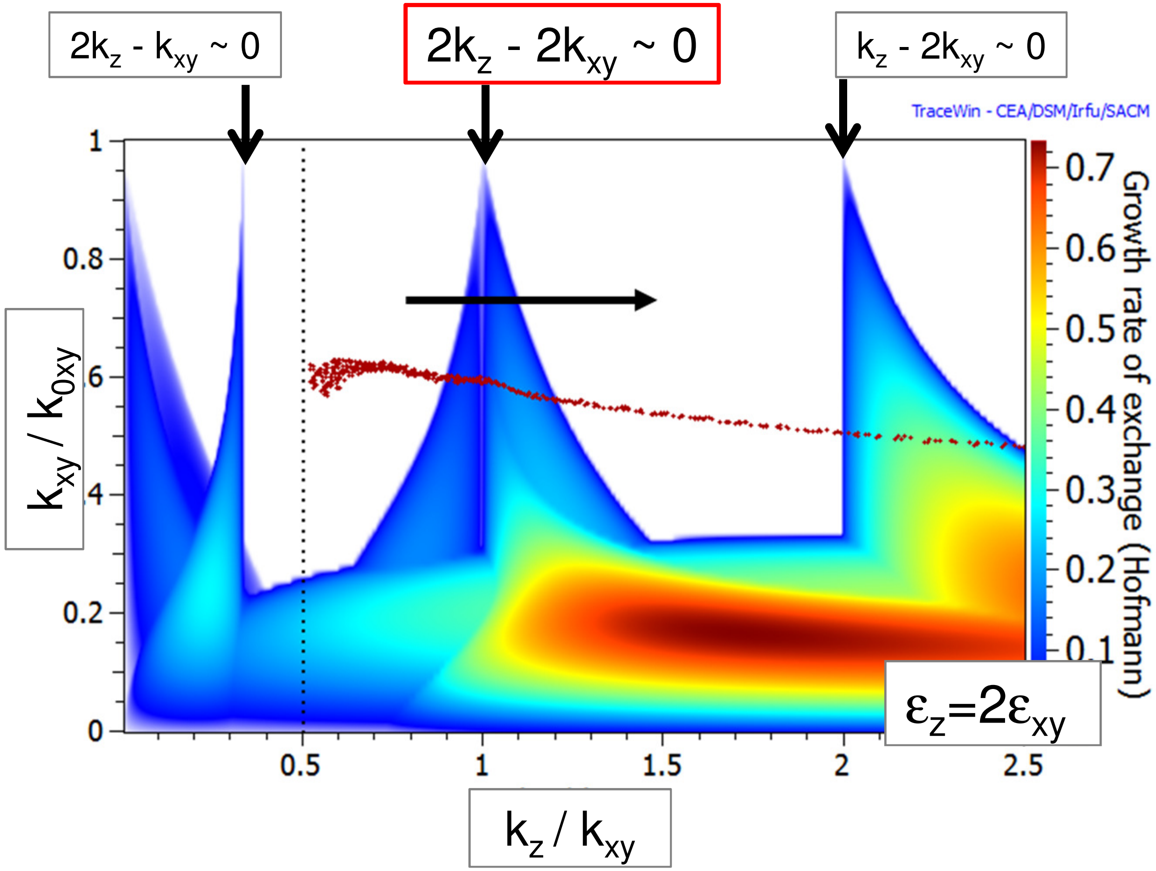
$$\Phi_1=\Phi_1\,(\mathrm{x}^{\mathrm{n}}+ax^{n-1}y+....)e^{i\omega t}$$

→ analytical dispersion relations
for orders n=2, 3, 4

Theory see:
Hofmann, Phys. Rev. E 57, p.56 (1998)

→ Stability chart as tool for linac design

TRACEWIN: plot tune footprint along linac - here “demo-lattice”



stop-band width:

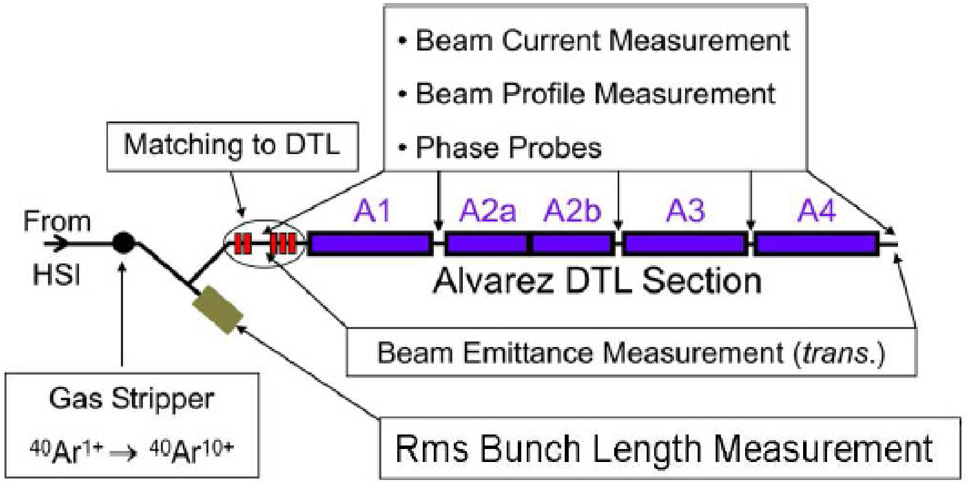
$$\Theta = \frac{3}{2}\left(\sqrt{\frac{\varepsilon_z}{\varepsilon_x}}-1\right)\Delta k_z$$

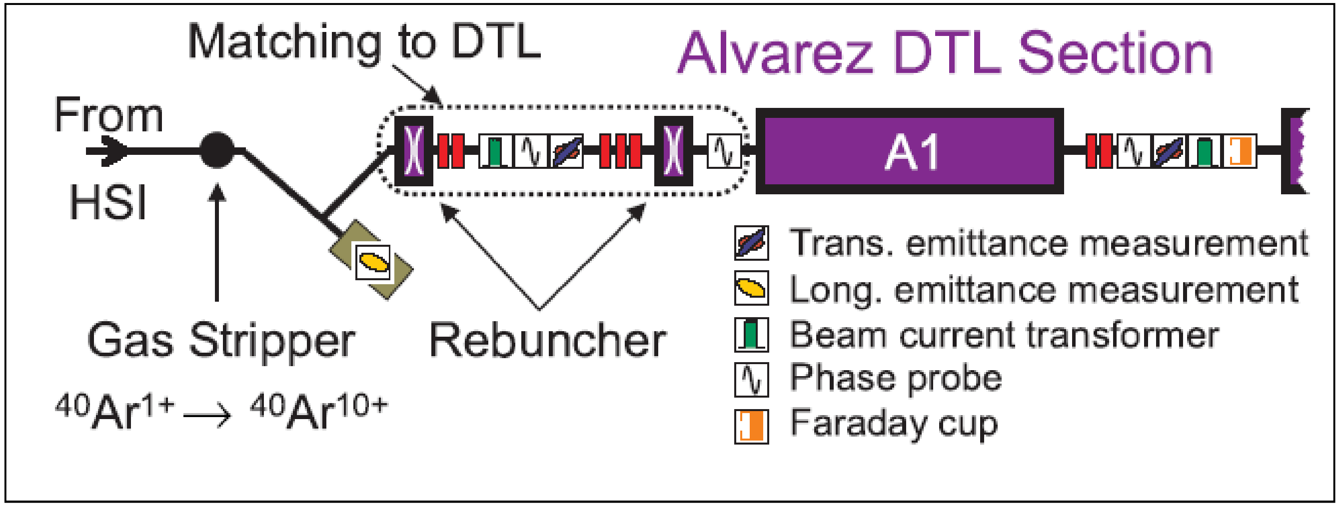
N_{ex}= # of betatron periods needed for exchange

$$N_{ex}^{-1} \approx \left(\sqrt{\frac{\varepsilon_z}{\varepsilon_x}}-1\right)\cdot\frac{\Delta\sigma_z}{360}$$

Experimental verification at UNILAC (2009)

European HIPPI Project (2003-08)
(High Intensity Pulsed Proton Injector)
Strengthen basis for future high intensity linacs
(CERN-SPL, FAIR p injector...)





Experimental verification cont´d

Experimental Evidence of Space Charge Driven Emittance Coupling in High Intensity Linear Accelerators

L. Groening et al. PRL 103, 224801 (2009)

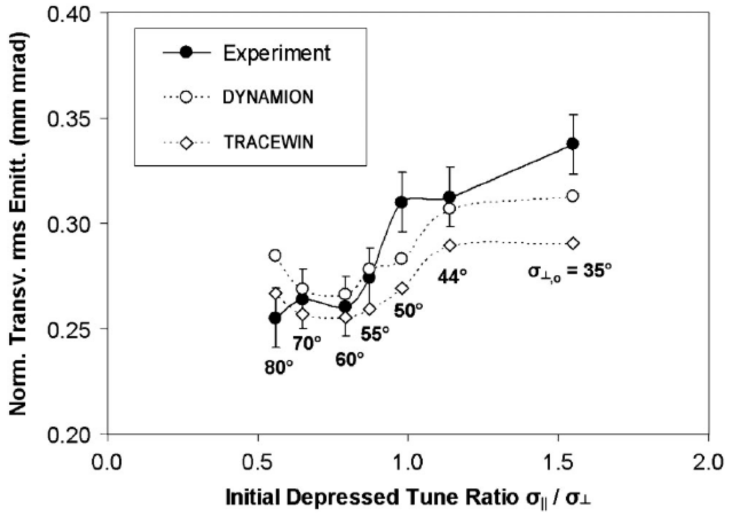
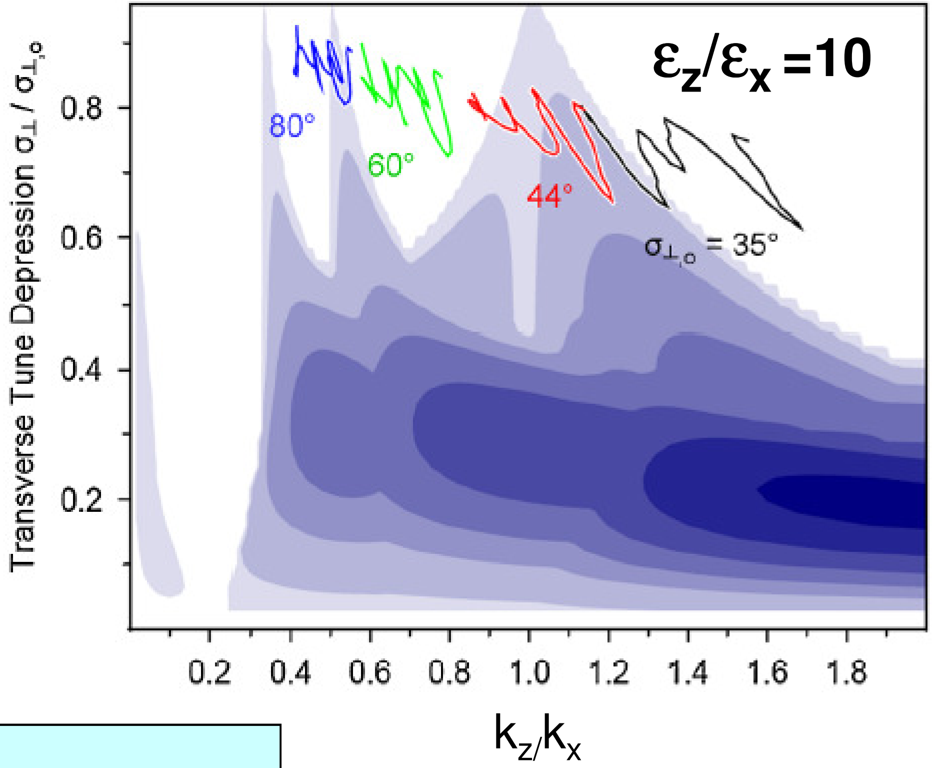


FIG. 4. Mean of horizontal and vertical rms emittance at the DTL exit as a function of the initial ratio of depressed longitudinal and transverse tune $\eta = \sigma_\parallel/\sigma_\perp$.



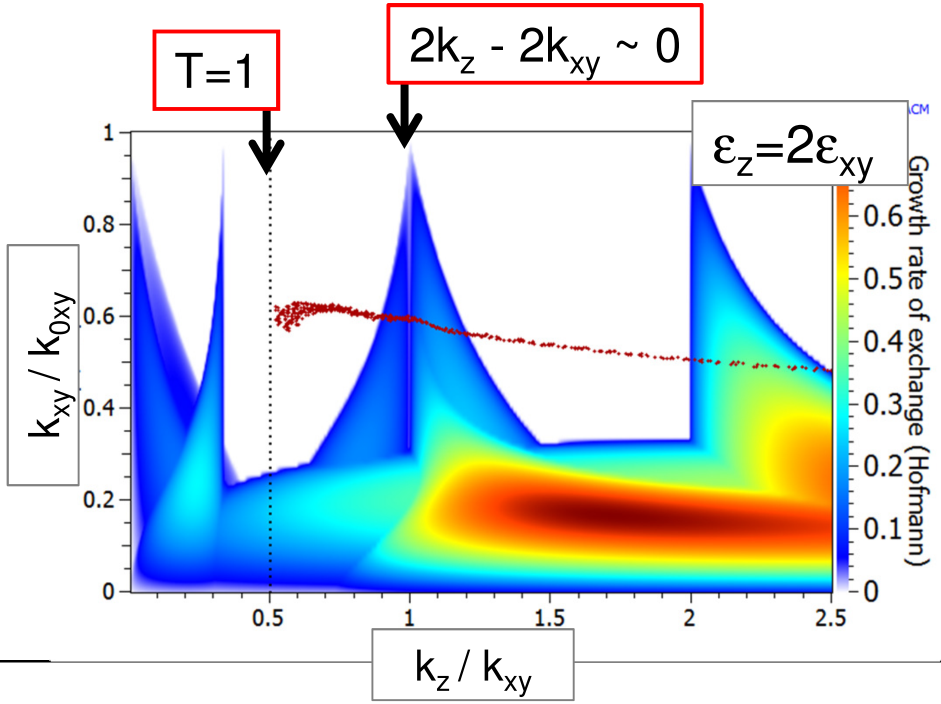
- far from equipartition
- driven by large energy anisotropy $\varepsilon_{lo}\sigma_{lo} \sim 10\ \varepsilon_{tr}\sigma_{tr}$
- observed in transverse plane (growth)

3^{rd} criterion for linac design: avoid $k_z / k_{xy} \sim 1$

$$T \equiv \frac{T_z}{T_x} \approx \frac{\varepsilon_z k_z}{\varepsilon_x k_x}$$

$$\mathrm{EP}: \quad \mathrm{T} = 1$$

- no need to design linac "equipartitioned" with T=1
  - unnecessary constraint on design freedom
- just avoid $k_z / k_{xy} \sim 1$ exchange resonance
  - all "white" zones "good"
  - helps avoid exchange between ε_z and ε_{xy} (intensity dependent design uncertainty!)
  - avoids a danger of halo coupling



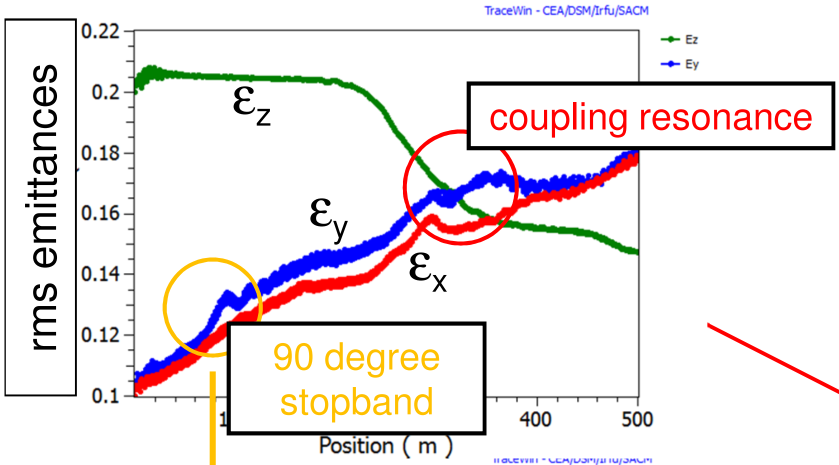
T=1

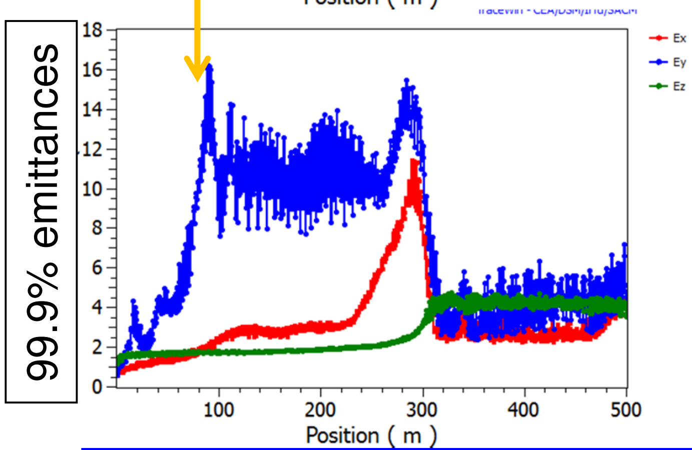
$2k_z - 2k_{xy} \sim 0$

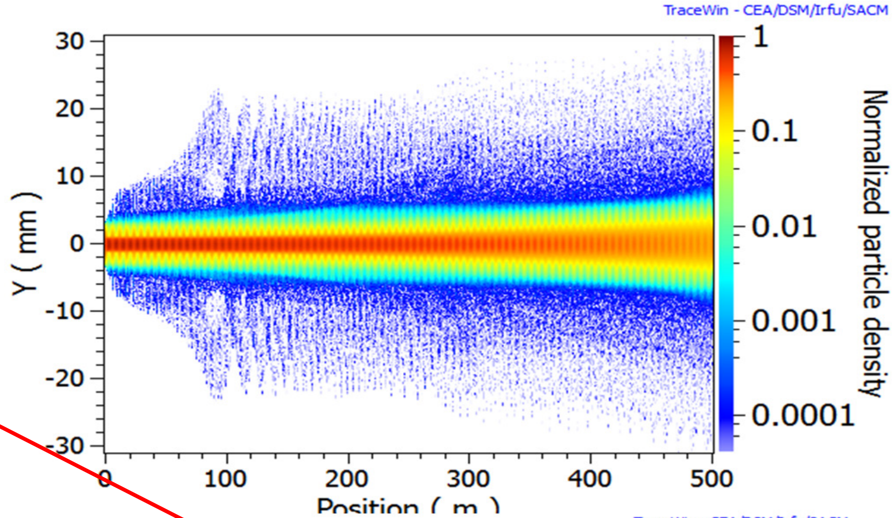
$\varepsilon_z = 2\varepsilon_{xy}$

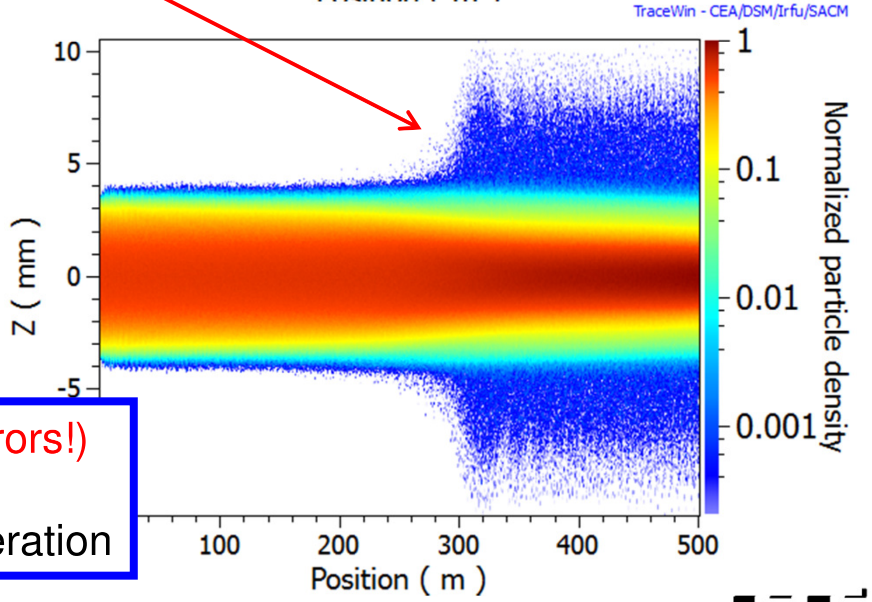
Beam halo coupling x-y → z under $2k_z - 2k_{xy} \sim 0$

a possible risk - might be even more dangerous!



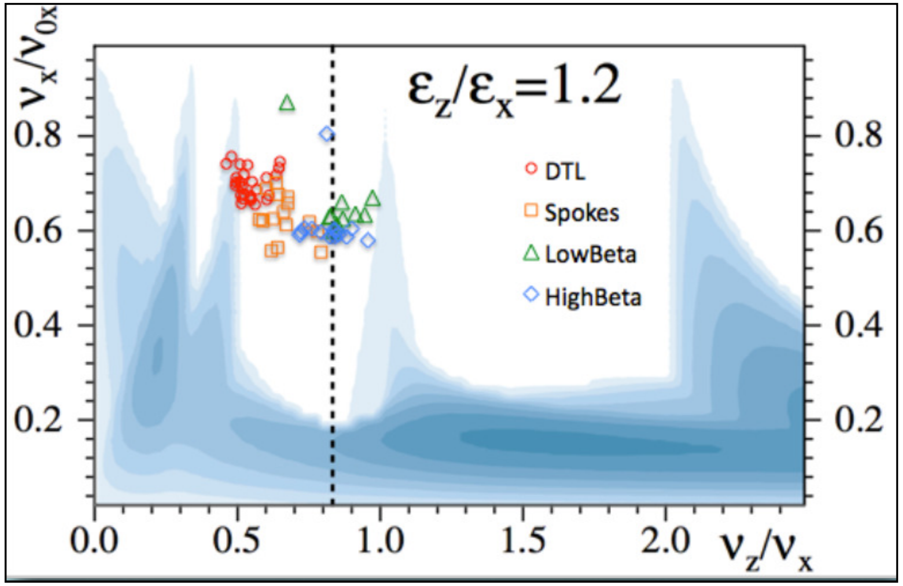




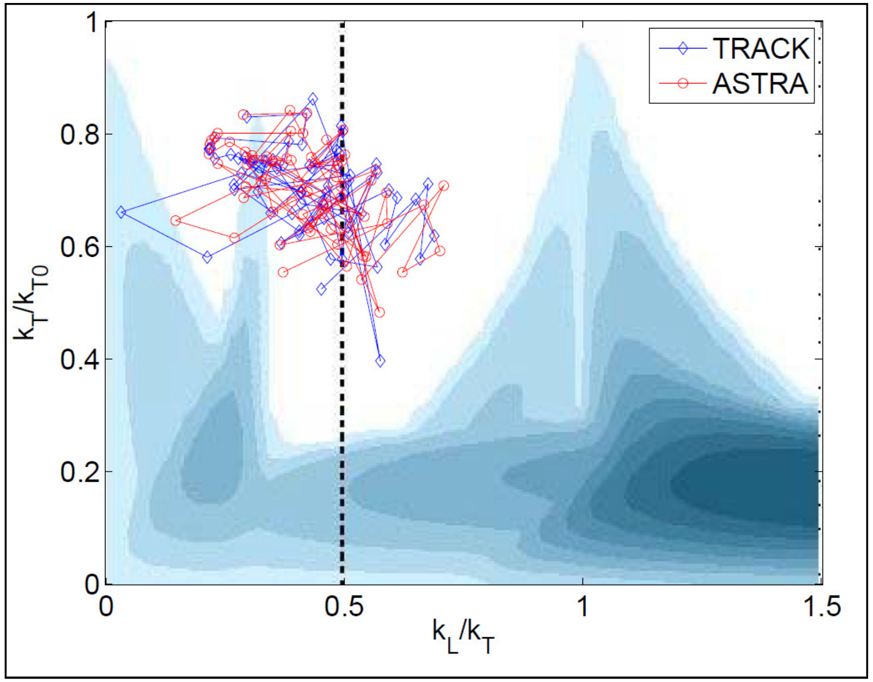


x-y-halo from 90^0 stopband (or from errors!)
→ couples into longitudinal plane
→ risk **loss out of bucket** during acceleration

2 examples “avoiding” ε-exchange



M. Eshraqi, HB2010

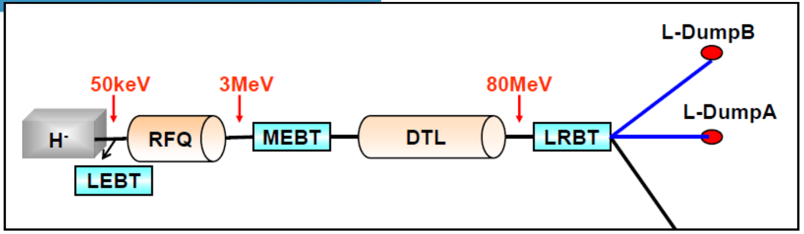


Project X, P. Ostroumov. 2008

Another example: CSNS - DTL

CSNS CHINESE ACADEMY OF SCIENCES

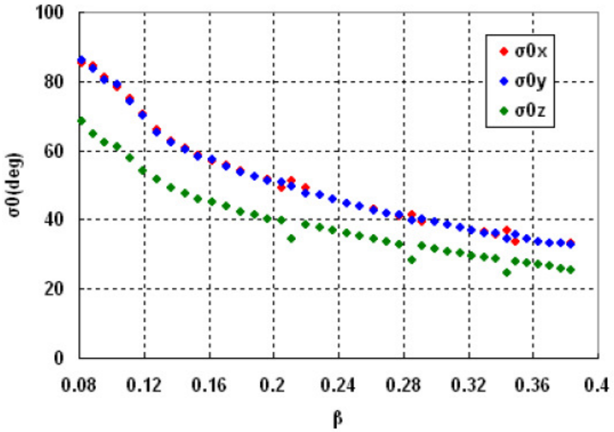
散裂中子源
China Spallation Neutron Source



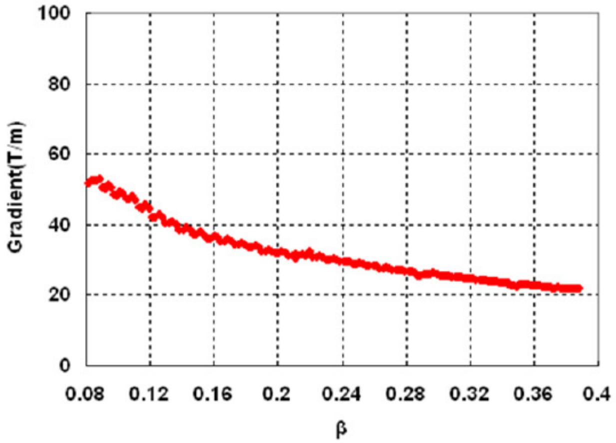
Transverse focusing

- σ_{t0}, σ_{z0}<90°
- $\sigma_{0t} \neq n\sigma_{0l}/2$,for n=1, 3…
- Equipartitioning require:

$$\frac{k_{t0}}{k_{z0}} = \sqrt{\frac{3}{2}\frac{\varepsilon_{nz}}{\varepsilon_{nt}} - \frac{1}{2}}$$



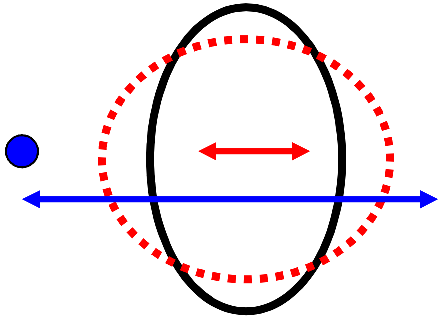
Zero-Current Phase Advance per Period for FFDD lattice



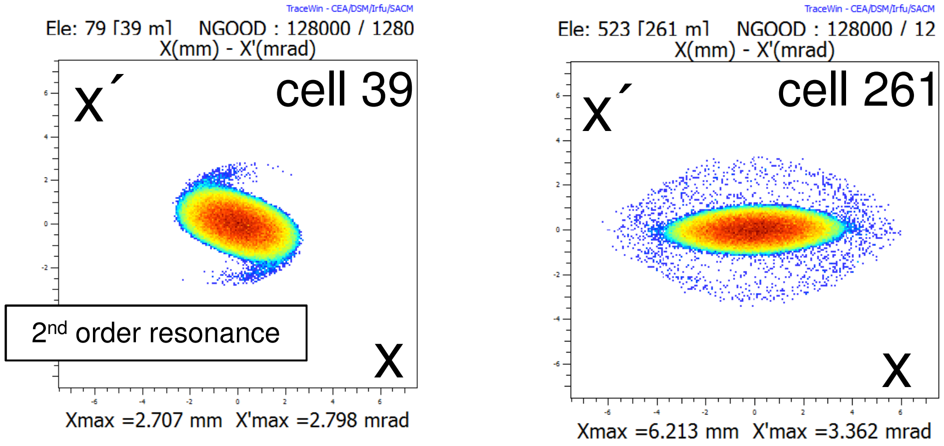
Quad gradient

H.C. Liu, HB2014

General rule: minimize rms mismatch + lattice errors
source of halo formation + beam loss

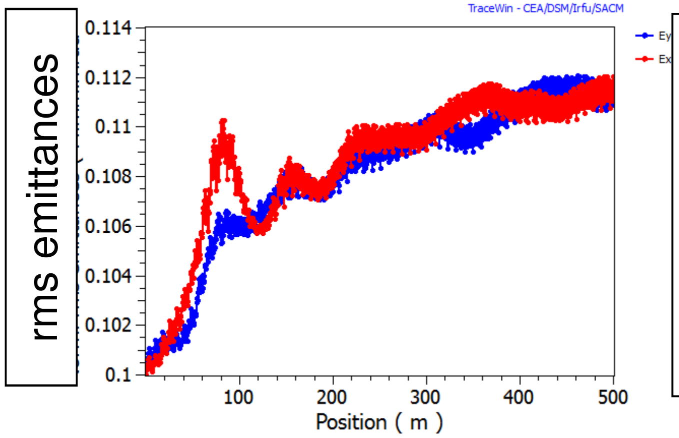


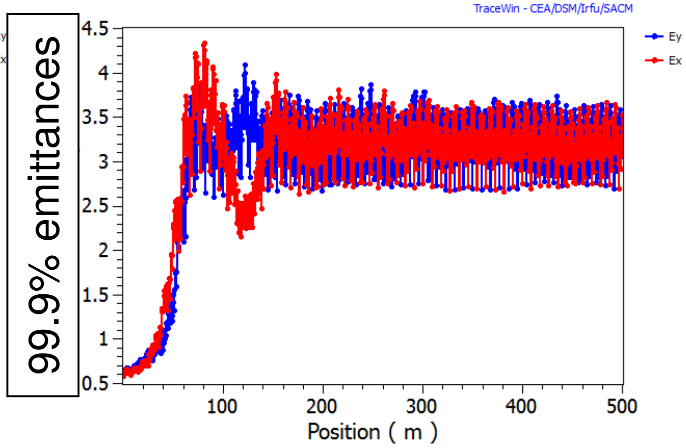
2:1 resonance
core:particle



mismatch- factor MM=1.3

- modest (12%) effect on rms emittance
- **large** (500%) effect on **99.9%** emittance (halo)



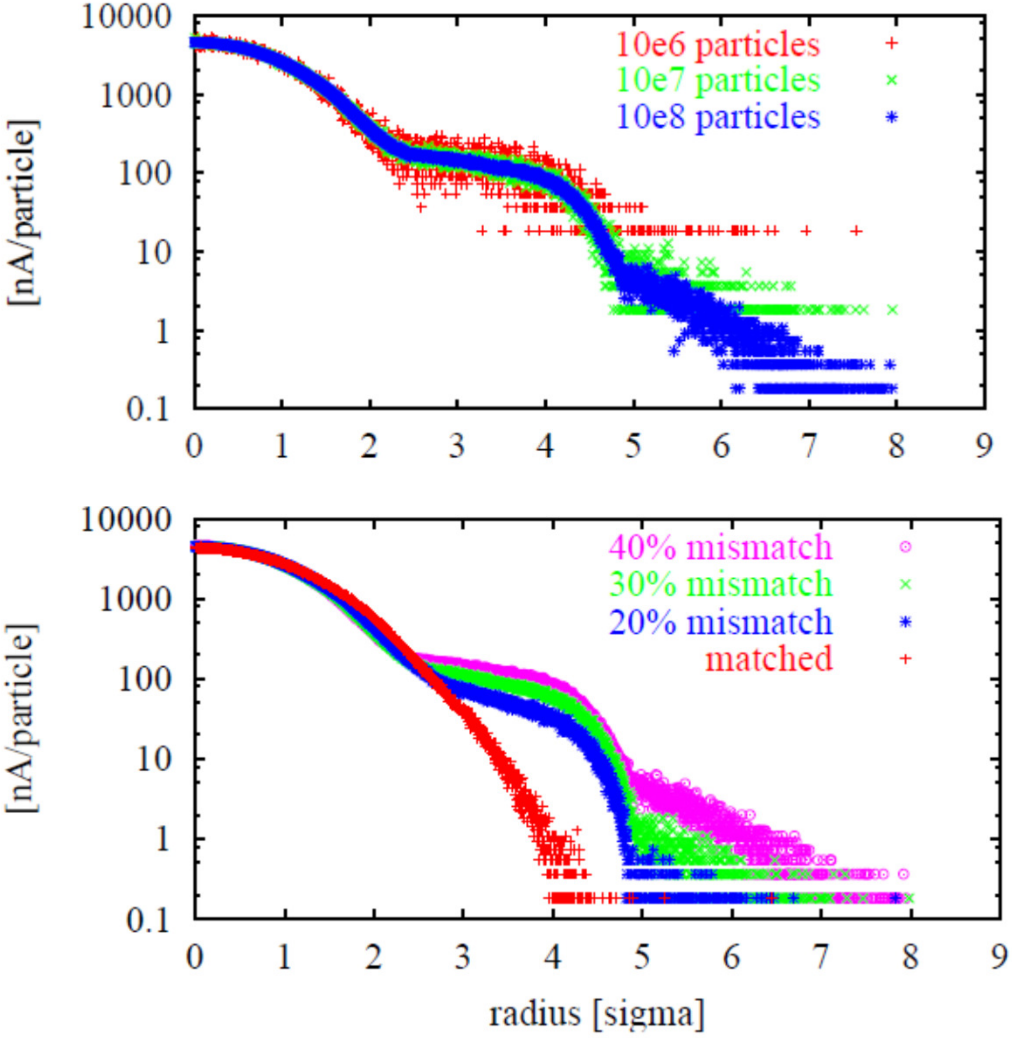


cont'd initial mismatch

F. Gerigk, K. Bongardt and I. Hofmann, Linac02

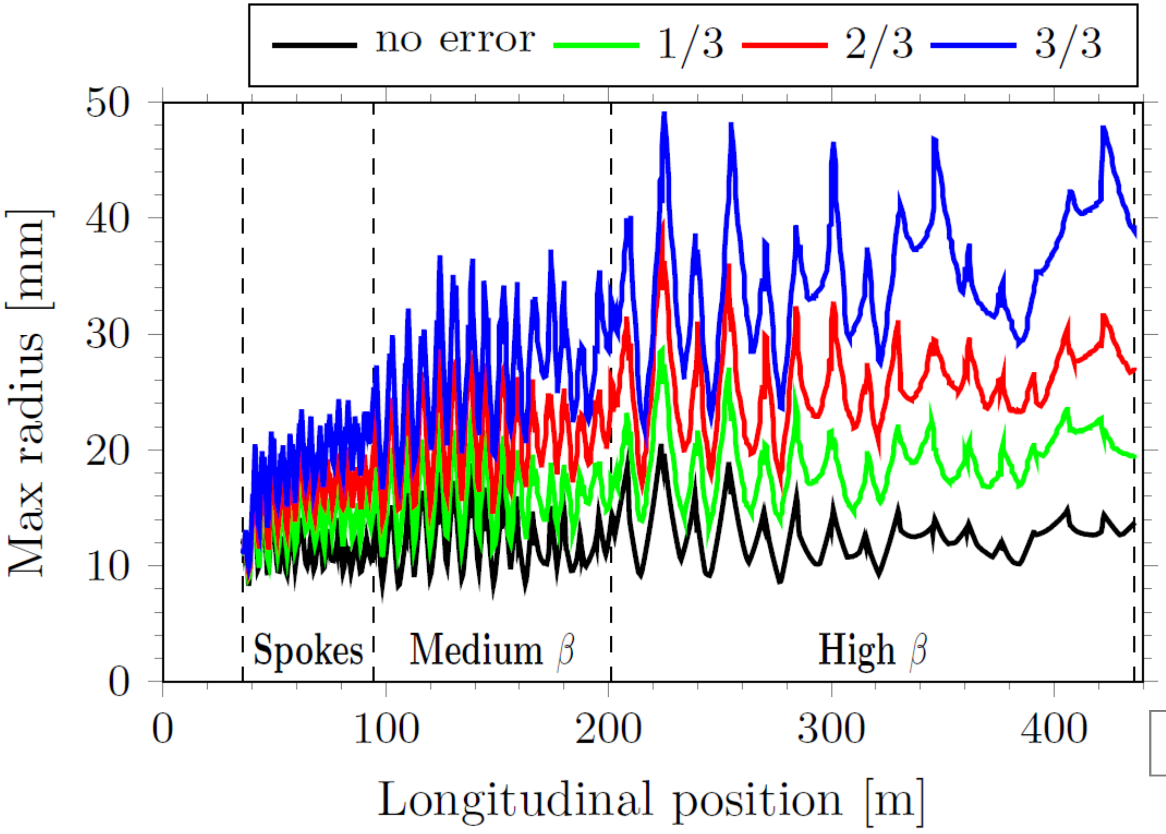
Maximum halo little dependent on

- # simulation particles
- Strength of initial mismatch
- With transitions ~ 11 σ "safe"



Example: European Spallation Neutron Source (ESS) linac:

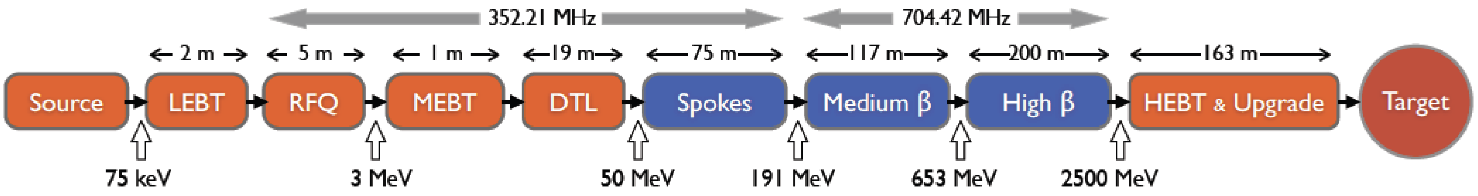
2.5 GeV 50 mA 5 MW (125 MW peak)



Error study:

- 1000 linacs 10^5 particles
- loss tolerance $<10^{-6}$ (< 1W/m) to avoid activation
- confidence level?

source: S. Peggs et al., ESS TDR 2012



Conclusions

- Extensively studied "resonant" mechanisms
  - sources of emittance and halo growth
  - beam dynamics in principle on solid ground
  - in practice very transient situations
- In real linacs try to avoid them
  - often severe impact on design
  - sometimes compromise
- Random errors of linac structure “mix” resonant mechanisms with random effects
  - statistical studies (questions open)
  - more to understand theoretically
- New projects can benefit much from SNS + JPARC experience