Passive Mitigation

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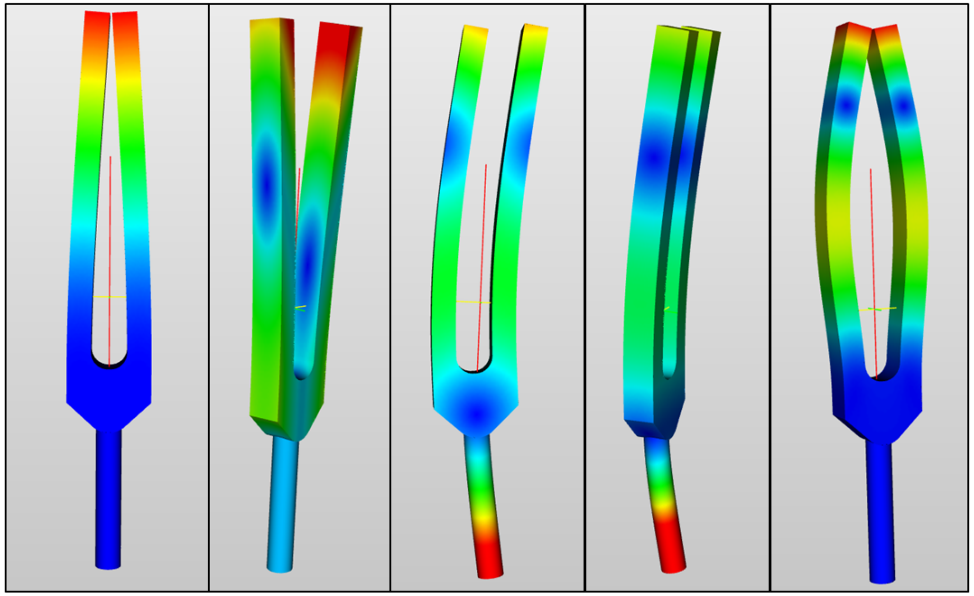
Eigenmodes

$$A\vec{x} = \lambda\vec{x}$$

eigenvalue → λ

eigenmode → $\vec{x}$



Eigenmodes of a tuning fork.
Pure tone at eigenfrequencies.

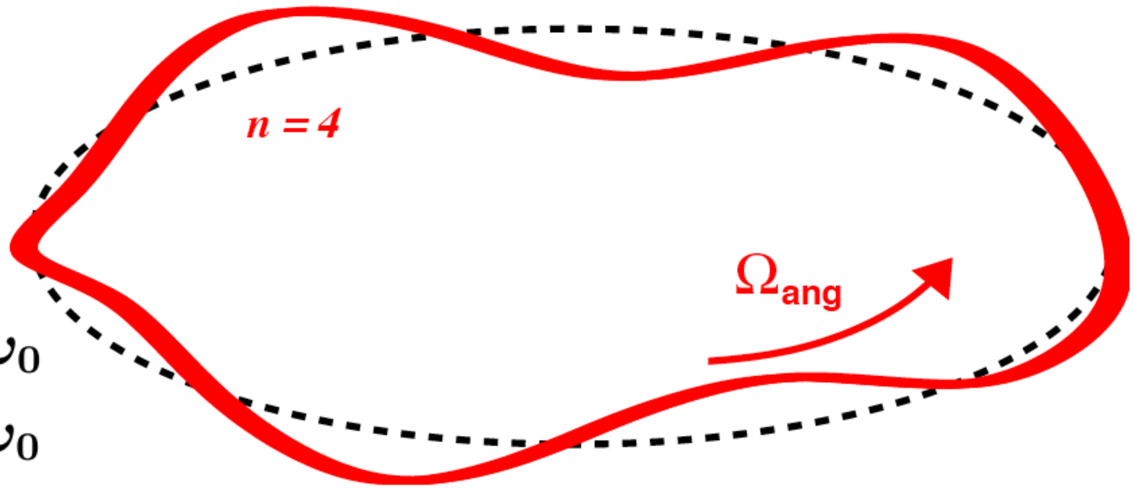
We often talk about the shift:

$$\Delta\Omega = \Omega - \Omega_{\mathrm{eigenfrequency}}$$

Eigenmodes

Transverse eigenmodes of a coasting beam

$$x(s,t) = x_0 e^{ins/R - i\Omega t}$$



Eigenfrequencies:

$$\begin{aligned} \text{slow wave} \quad \Omega_{\mathrm{s}} &= (n - Q_\beta)\omega_0 \\ \text{fast wave} \quad \Omega_{\mathrm{f}} &= (n + Q_\beta)\omega_0 \end{aligned}$$

$$\Omega_{\mathrm{ang}} = \left(1 - \frac{Q_\beta}{n}\right)\omega_0$$

With a driving impedance, a mode has a complex shift:

$$\Omega = \Omega_{\mathrm{s}} + \kappa\ \mathrm{Im}Z^{\perp} + i\ \kappa\ \mathrm{Re}Z^{\perp}$$

With a damping mechanism,

$$\Omega = \Omega_{\mathrm{s}} + i\gamma_{\mathrm{damping}} \quad \gamma_{\mathrm{damping}} < 0$$

Passive Mitigation

Basic consideration of a passive mitigation

$$\Delta\Omega = \Delta\Omega_{\mathrm{Re}} + i\gamma_{\mathrm{drive}} + i\gamma_{\mathrm{damping}}$$

change the parameters and the source of the driving mechanism (γ_{drive})

use and enhance the intrinsic damping mechanism ($\gamma_{\mathrm{damping}}$)

$\gamma_{\mathrm{drive}} + \gamma_{\mathrm{damping}}$	> 0	Instability
$\gamma_{\mathrm{drive}} + \gamma_{\mathrm{damping}}$	< 0	Stabilized mode
γ_{drive}	> 0	Driven (unsuppressed) mode
γ_{drive}	< 0	Mode suppressed by its drive

γ_{drive}–mitigation

Adjusting the components of the instability

- tunes Q_x , Q_y , tune split
- chromaticities, coupling
- synchrotron tune Q_s

Beam parameters:

- beam sizes, emittance, momentum spread

The driving sources

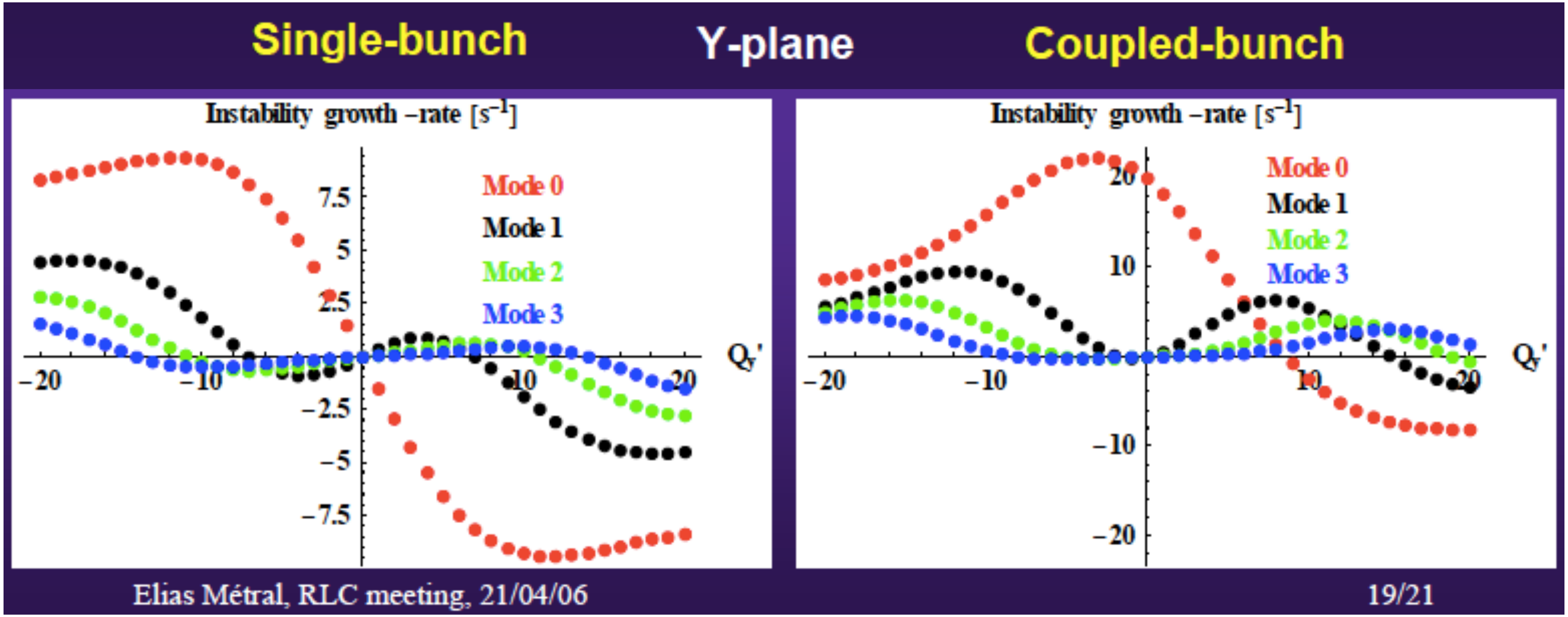
- Impedances

…… Very long list of possibilities ………

Every instability (eigenmode), if well understood, has a lot of adjustable parameters

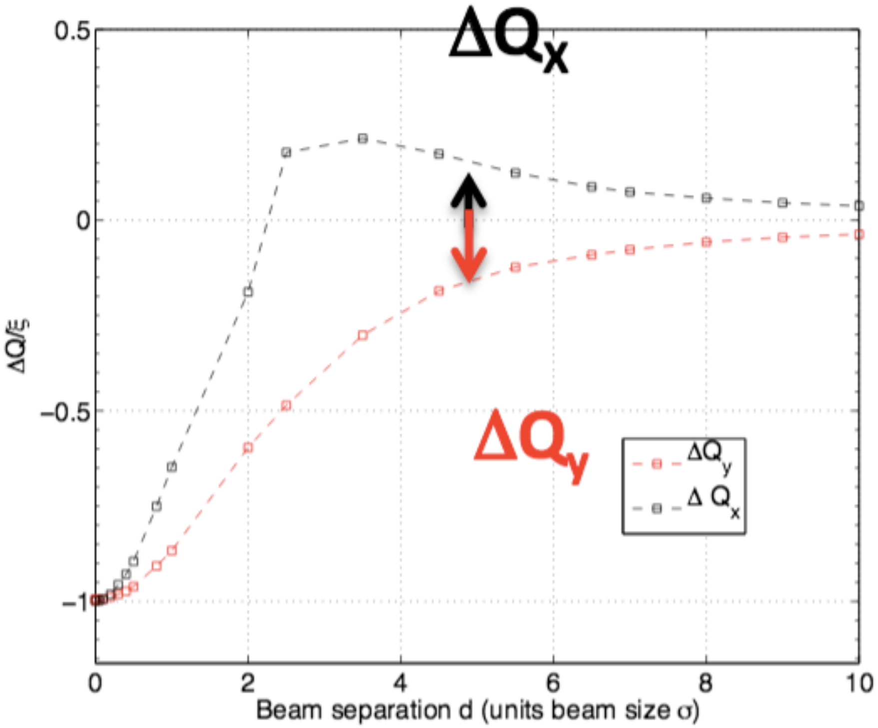
Examples γ_{drive}–mitigation

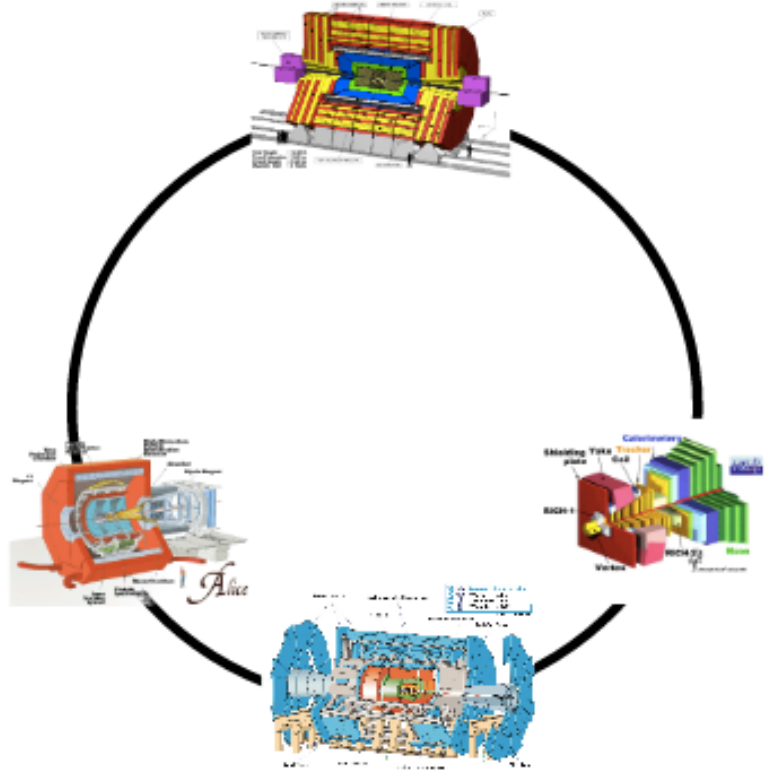
Predictions for LHC injection energy



Examples γ_{drive}–mitigation

Long-range Beam-Beam compensatiop in LHC
(T.Pieloni, this CAS)





CMS Horizontal crossing angle

ATLAS Vertical crosing angle

Examples γ_{drive}–mitigation

reduce the the source of the
driving mechanism:
the impedance

SPS impedance reduction 2001
(E.Shaposhnikova, this CAS)

year	$\mathrm{Im}\, Z_x$ MΩ/m	$\mathrm{Im}\, Z_y$ MΩ/m
2000	-0.9 ± 1.8	26 ± 3
2001	-0.35 ± 0.53	18.4 ± 0.5

Existence of Landau damping

In any accelerator, there are many Re(Z) sources

In any beam, there are many unsuppressed eigenmodes

$$\mathrm{Im}(\Delta Q_{\mathrm{coh}}) = \frac{\lambda_0 r_p}{\gamma Q_0} \frac{\mathrm{Re}(Z^{\perp})}{Z_0/R}$$

the driving dipole impedance here

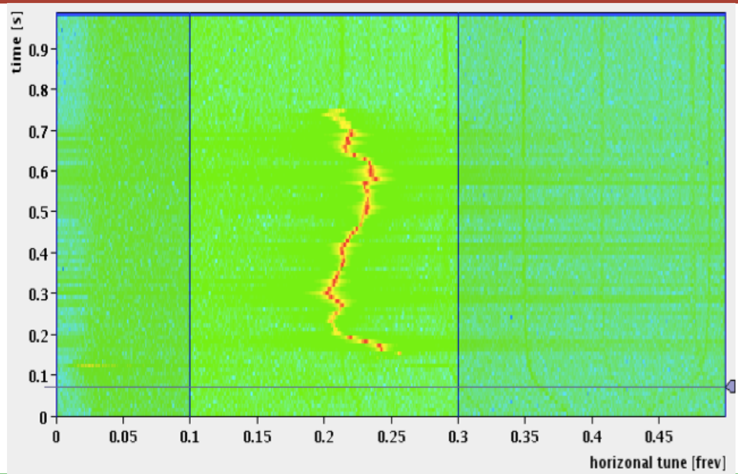
Still, the beams are often stable without an active mitigation

There must be a fundamental damping mechanism in beams

Existence of Landau damping

Additionally to Re(Z), deliberate excitation is often applied (tune measurements, optics controll, …)

Energy is directly transferred to the beam, mostly at the beam resonant frequencies

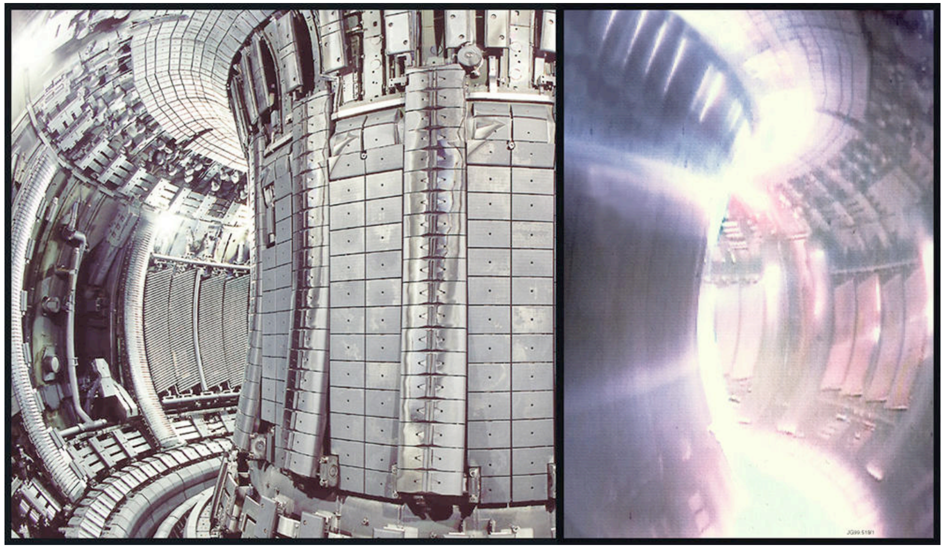


Tune measurements at PS, kick every 10 ms.

The beams are stable and absorb some energy

There must be a fundamental damping mechanism in beams

Plasma



Plasma in the JET tokamak

Plasma is a quasi-neutral gas of unbound ions and electrons.

Waves in plasma: collective propagating oscillations of particles and E-M fields

Some waves can be damped.

“Friction” in plasma is collisions.

In plasma, a collisionless damping has been discovered by L.Landau, 1946: **Landau damping**.

Plasma Wave

A basic plasma oscillation: Langmuir wave

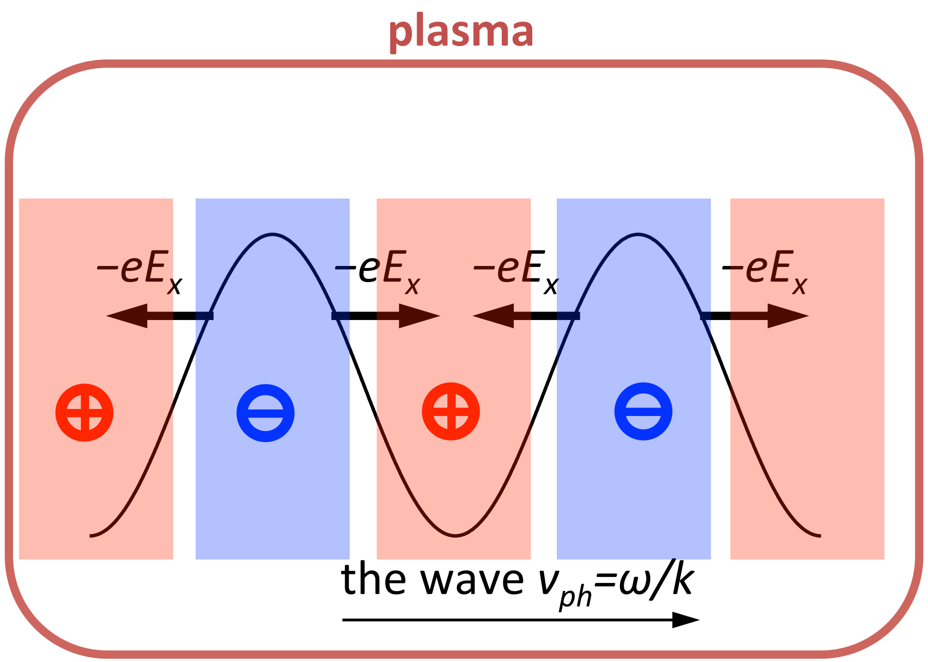
Wave number $k=2\pi/\lambda$

The phase velocity $v_{ph} = \omega/k$

There are resonant particles $v_x \approx v_{ph}$

The plasma frequency

$$\omega_p^2 = \frac{n_e e^2}{m_e \epsilon_0}$$



The dispersion relation

$$\frac{\omega_p^2}{k^2}\int \frac{\partial \hat{f}_0/\partial v_x}{v_x - \omega/k} dv_x = 1$$

has a singularity

Landau Damping In Plasma

The wave frequency is complex

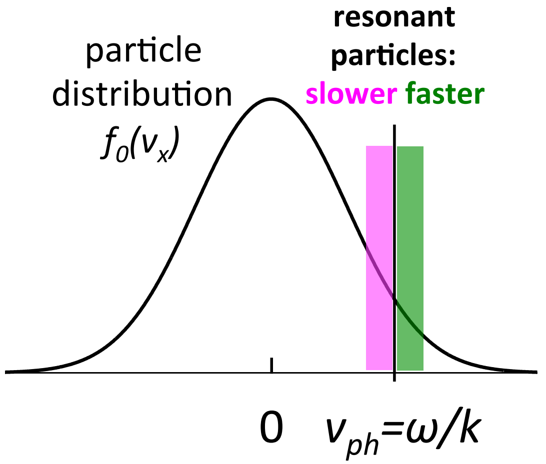
$$\omega = \omega_r + i\omega_i$$

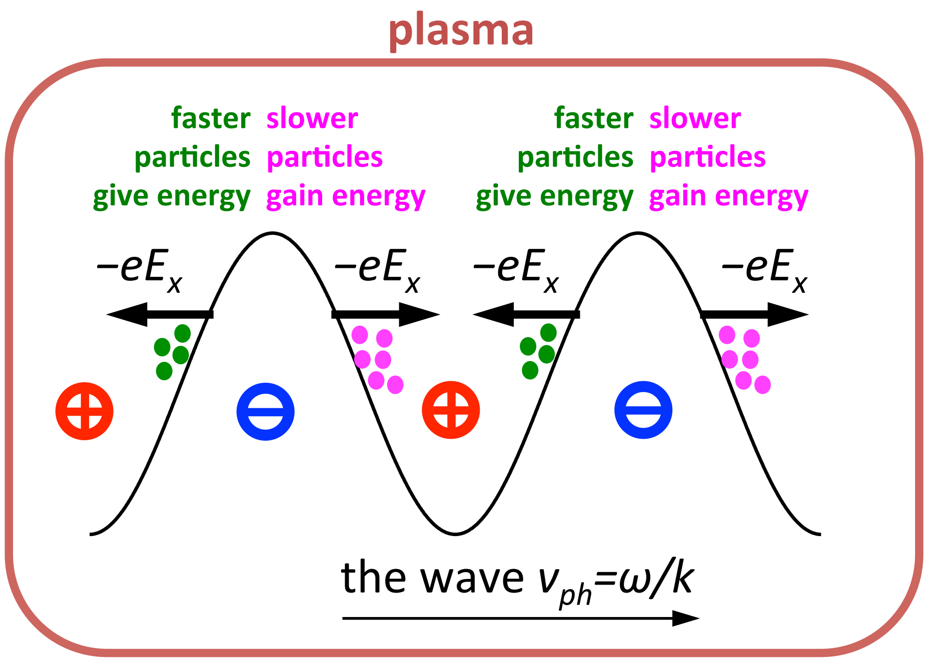
The dispersion relation can be solved,
the integral is calculated as PV + residue

$$\frac{\omega_p^2}{k^2}\left[\mathrm{PV}\int \frac{\partial \hat{f}_0/\partial v_x}{v_x - \omega/k} dv_x + i\pi \frac{\partial \hat{f}_0}{\partial v_x}\bigg|_{v_x=\frac{\omega}{k}}\right] = 1$$

$$\begin{aligned} \omega_r^2 &= \omega_p^2 + 3k^2 v_{th}^2 \\ \omega_i &= -\frac{\pi\omega_r}{2}\frac{\omega_p^2}{k^2}\frac{\partial \hat{f}_0}{\partial v_x}\bigg|_{v_x=\frac{\omega}{k}} \end{aligned}$$

Landau Damping In Plasma

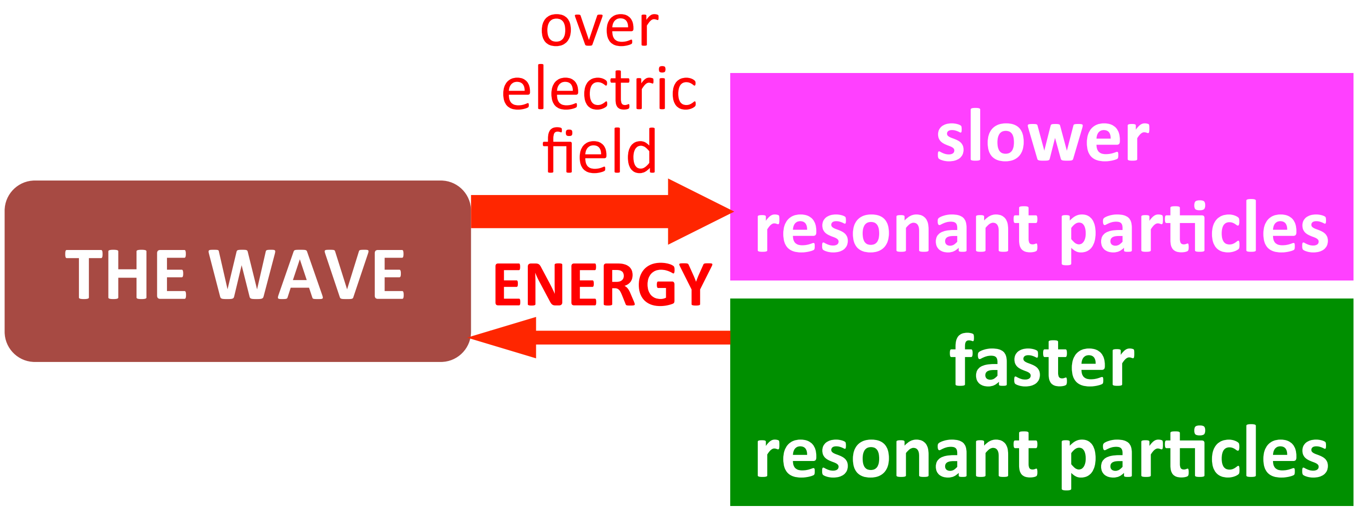




negative $f_0(v_x)$ slope: $N_{gain} > N_{give}$ → the wave decays, **damping**

positive $f_0(v_x)$ slope: $N_{gain} < N_{give}$ → the wave grows, **instability**

Landau Damping In Plasma



$$\left.\frac{\partial f_0}{\partial v}\right|_{v=\frac{\omega}{k}}$$

Main ingredients of Landau damping:

- wave−particle collisionless interaction. Here this is the electric field.
- energy transfer: the wave ↔ the (few) resonant particles.

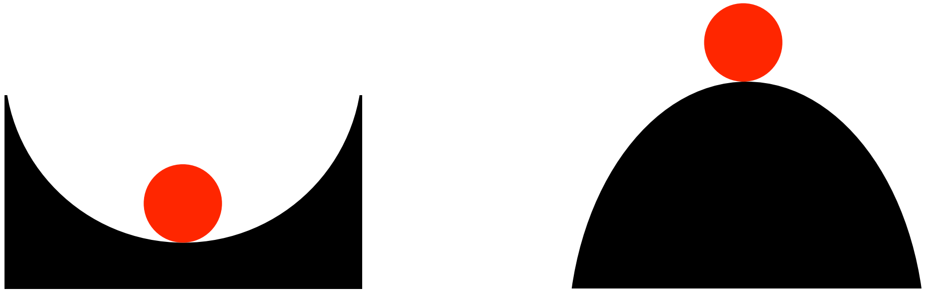
The result is the exponential decay of a small perturbation.

Landau damping is a fundamental mechanism in plasma physics.
Extensively studied in experiment, simulations and theory.

Stability: the basic idea

Stable System

Unstable System



This is Landau Damping

small perturbation → Mechanism to suppress the perturbation (DAMPING) → perturbation decays

or $-\gamma_{damping} > \gamma_{drive}$

small perturbation → Mechanism to reinforce the perturbation (INSTABILITY) → perturbation grows

or $\gamma_{drive} > -\gamma_{damping}$

Landau Damping in Beams

K.Y.Ng, Physics of Intensity Dependent Beam Instabilities, 2006
A.Hofmann, Proc. CAS 2003, CERN-2006-002
A.Chao, Phys. Coll. Beam Instab. in High Energy Acc. 1993

Beam Transfer Function

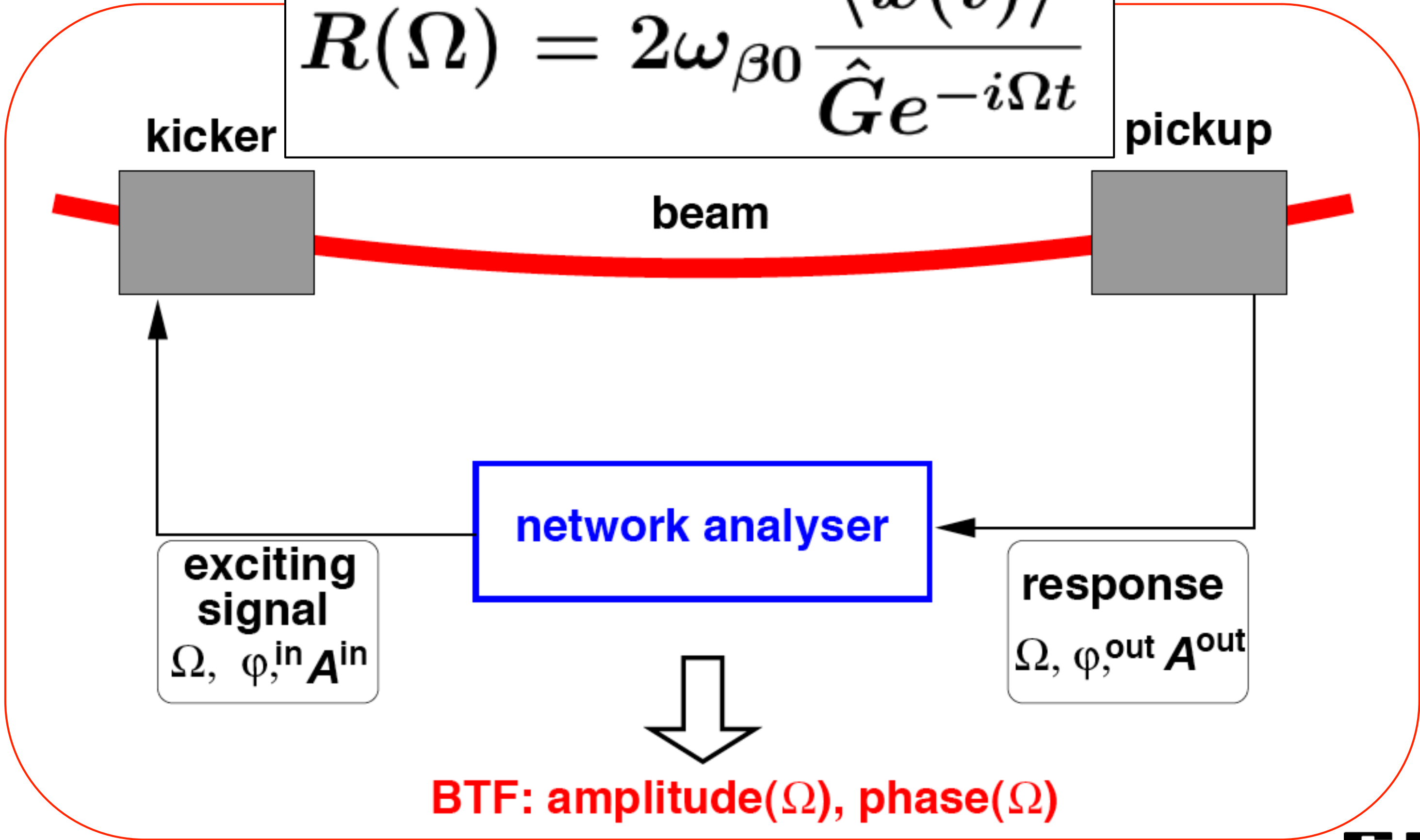
an excitation:

$$x'' + \omega_{\beta i}^2 x = \hat{G} e^{-i\Omega t}$$

beam forced response:

$$\langle x \rangle = A\ e^{-i\Omega t + \Delta\phi}$$

$$R(\Omega) = 2\omega_{\beta 0} \frac{\langle x(t) \rangle}{\hat{G} e^{-i\Omega t}}$$

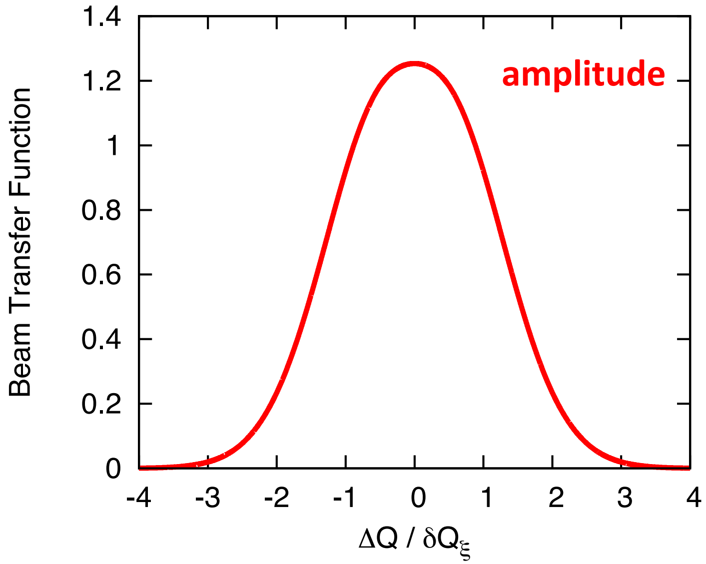


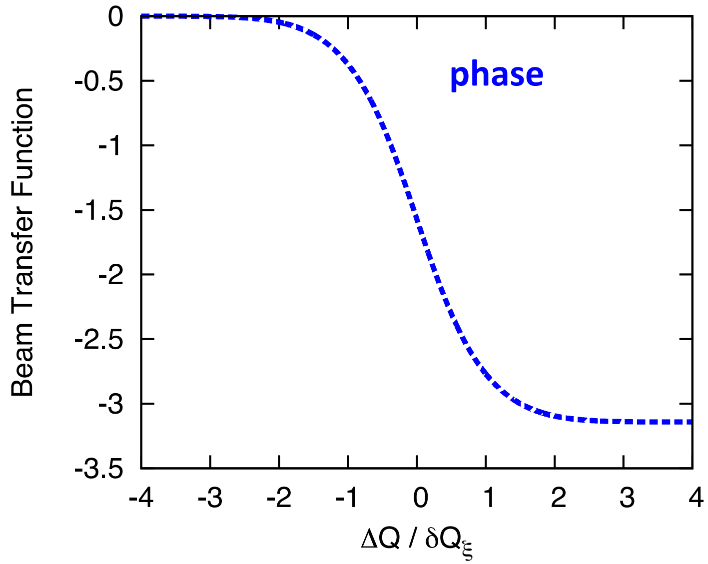
Beam Transfer Function

BTF is:

- Useful diagnostics; gives the tune, δp, chromaticity, beam distribution
- A fundamental function in the beam dynamics
- Necessary to describe the beam signals and Landau damping

$$R(\Omega) = \mathrm{PV} \int \frac{f(\omega)d\omega}{\omega - \Omega} + i\pi f(\Omega)$$





$\Delta Q=(\Omega-(m\pm Q_f)f_0)/f_0$

$\delta Q_\xi=|m\eta\pm(Q_{f\eta}\eta-Q_0\xi)|\delta p/p$

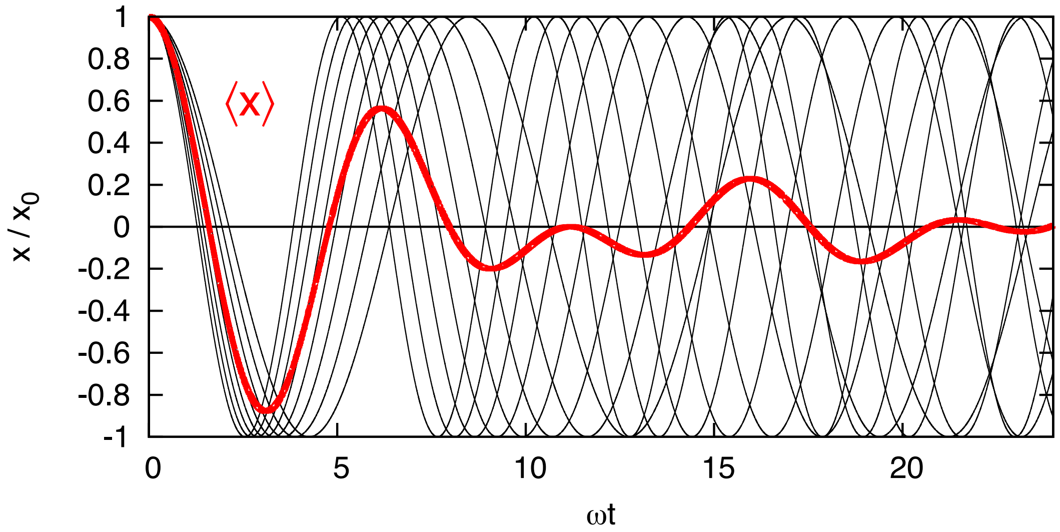
J.Borer, et al, PAC1979

D.Boussard, CAS 1993, CERN 95-06, p.749

A.Chao, Phys. Coll. Beam Instab. in High Energy Acc. 1993

Handbook of Acc. Physics and Eng. 2013, 7.4.17

Pulse Response



8 particles with different frequencies

Betatron oscillations: frequency spread

$$\delta\omega = Q_0 \xi \omega_0 \delta_p$$

$$g(t) = \frac{\langle x(t) \rangle}{x_0}$$

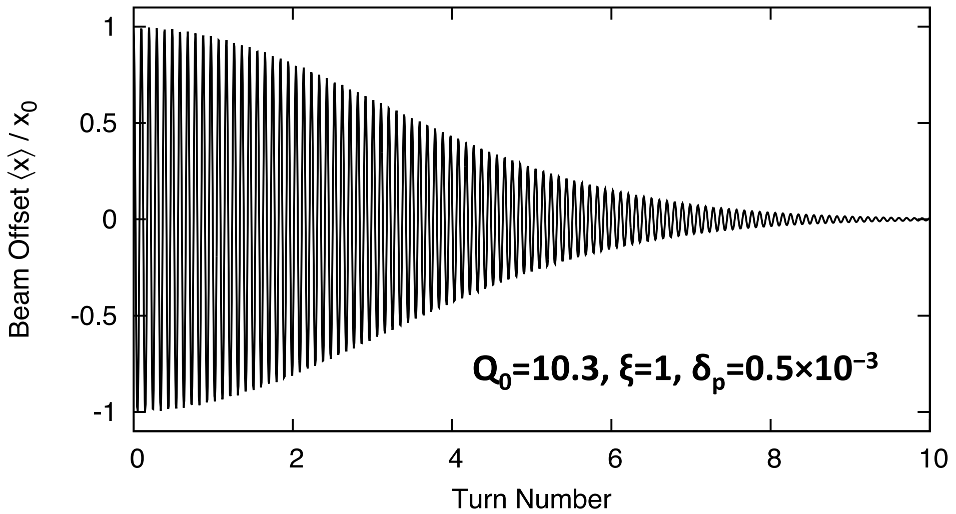
$$g(t) = \int f(\omega) \cos(\omega t) d\omega$$

$$g(t) = \text{Fourier}^{-1}\left\{R(\omega)\right\} = \frac{1}{2\pi} \int R(\omega) e^{-i\omega t} d\omega$$

BTF is the Fourier image of the pulse response

Decoherence

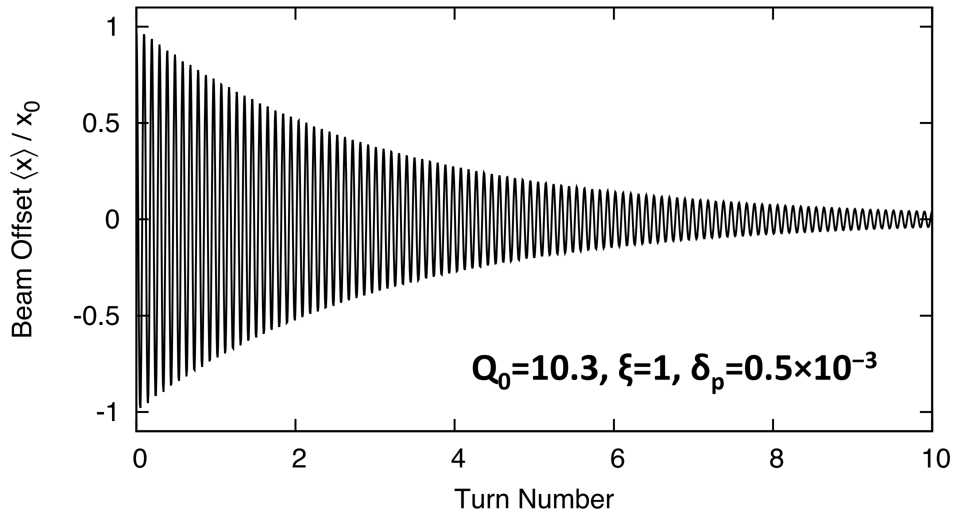
Gaussian Distribution



$$f(\omega_\beta) = \frac{1}{\sqrt{2\pi}\delta\omega^2} e^{-\omega_\beta^2/2\delta\omega^2}$$

$$g(t) = e^{-\delta\omega^2 t^2/2} \cos(\omega_\beta t)$$

Lorentz Distribution



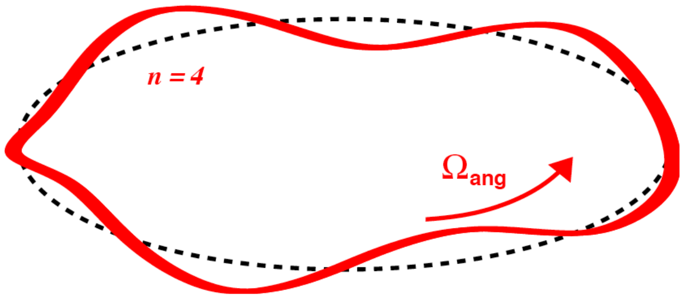
$$f(\omega_\beta) = \frac{1}{\pi\ \delta\omega} \frac{1}{1 + \omega_\beta^2/\delta\omega^2}$$

$$g(t) = e^{-\delta\omega\ t} \cos(\omega_\beta t)$$

This is the case without any collective interactions: phase-mixing of non-correlated particles

Oscillation without damping

small initial perturbation ⟨***x***⟩



produces →

excitation force

$$G \propto Z^{\perp}_{\text{ext}} I_0 \langle x \rangle$$

↓ forced oscillations (eigenmode)

the perturbation is amplified

⟨***x***⟩ × ***(1+Δ)***

reinforcing mechanism

The result is ΔQ_{coh} and the exponential growth: instability

$$\langle x \rangle (t) = x_0 e^{\text{Im}(\Omega) t}$$

Coherent Oscillations

An easy derivation of the dispersion relation

the external drive is INTENSITY × IMPEDANCE × PERTURBATION

$$G = \frac{\langle F_x \rangle}{m\gamma} = \frac{q\beta}{m\gamma C} i Z_{\rm ext}^{\perp} I_0 \langle x \rangle$$

the no-damping complex coherent tune shift is
INTENSITY × IMPEDANCE

$$\Delta Q_{\rm coh} = \frac{I_0 q_{\rm ion}}{4\pi\gamma m c Q_0 \omega_0} i Z_{\rm ext}^{\perp}$$

only the dipole
impedance here,
no incoherent effects

thus, the external drive is

$$G = 2\omega_{\beta 0}\omega_0 \Delta Q_{\rm coh} \langle x \rangle$$

Dispersion Relation

An easy derivation of the dispersion relation

the external drive is IMPEDANCE TUNE SHIFT × PERTURBATION

$$G = 2\omega_{\beta 0}\omega_0 \Delta Q_{\mathrm{coh}} \langle x \rangle$$

the beam response is the BTF

$$\langle x \rangle = \frac{G}{2\omega_{\beta 0}\sigma_\omega} R(u)$$

combined: the DISPERSION RELATION

$$\Delta Q_{\mathrm{coh}} R(\Omega) = 1$$

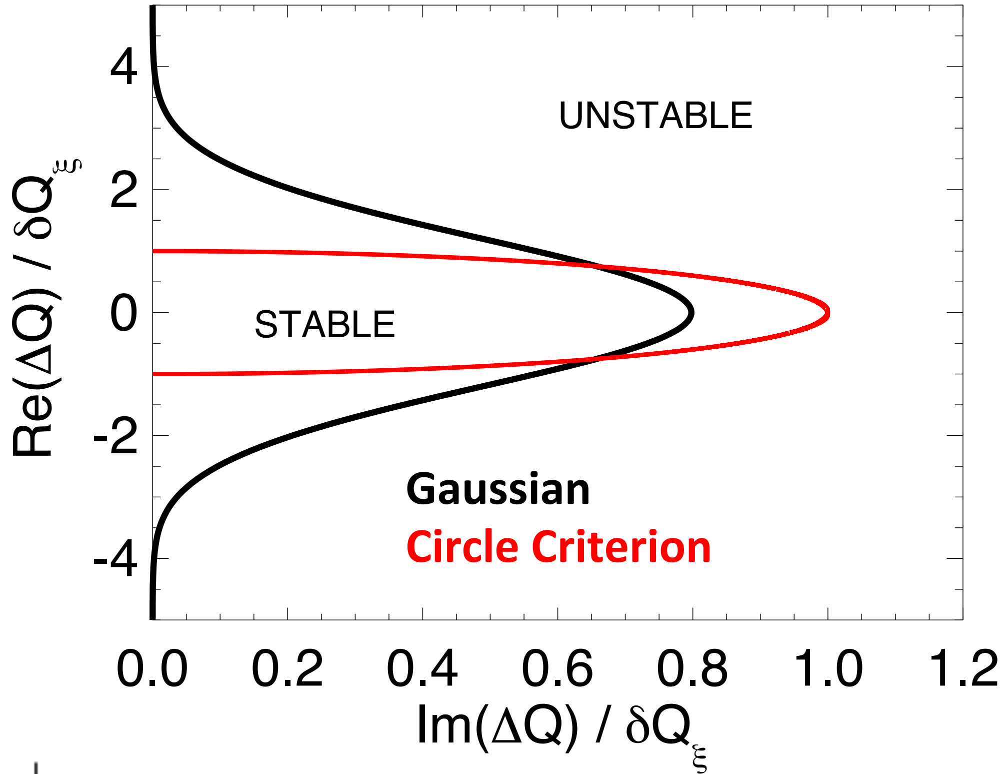
provides the resulting Ω for the given impedance and beam

Stability Diagram

the resulting Ω for the given impedance and beam

$$\Delta Q_{\mathrm{coh}} R(\Omega) = 1$$

$$\Delta Q_{\mathrm{coh}} \omega_0 \int \frac{f(\omega) d\omega}{\omega - \Omega} = 1$$



Re(Z)>0: the slow wave

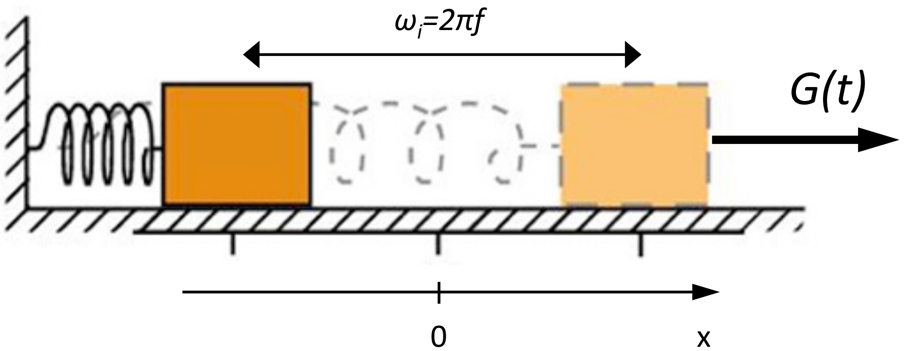
$$\omega_{\mathrm{s}} = (n - Q_0)\omega_0$$

$$\delta Q_\xi = \left|\eta(n - Q_0) + Q_0 \xi\right| \delta_p$$

$$\frac{|\Delta Q|}{\delta Q_\xi} = 1$$

Circle Criterion, E.Keil, W.Schnell, CERN ISR-TH-RF/69-48 (1969)

Driven Harmonic Oscillator



$$x'' + \omega_i^2 x = \hat{G} e^{-i\Omega t}$$

The solution	=	homogeneous solution (pulse response) initial conditions	+	particular solution (forced oscillations)

Off-resonance ($\Omega \neq \omega_i$) and at resonance ($\Omega = \omega_i$), different particular solutions.
Zero initial conditions.

$$x_G(t) = \frac{2\hat{G}}{\omega_i^2 - \Omega^2} \sin\left(\frac{\omega_i - \Omega}{2} t\right) \sin\left(\frac{\omega_i + \Omega}{2} t\right)$$

$$x_G(t) = \frac{\hat{G}}{2\Omega}\, t\, \sin(\Omega t)$$

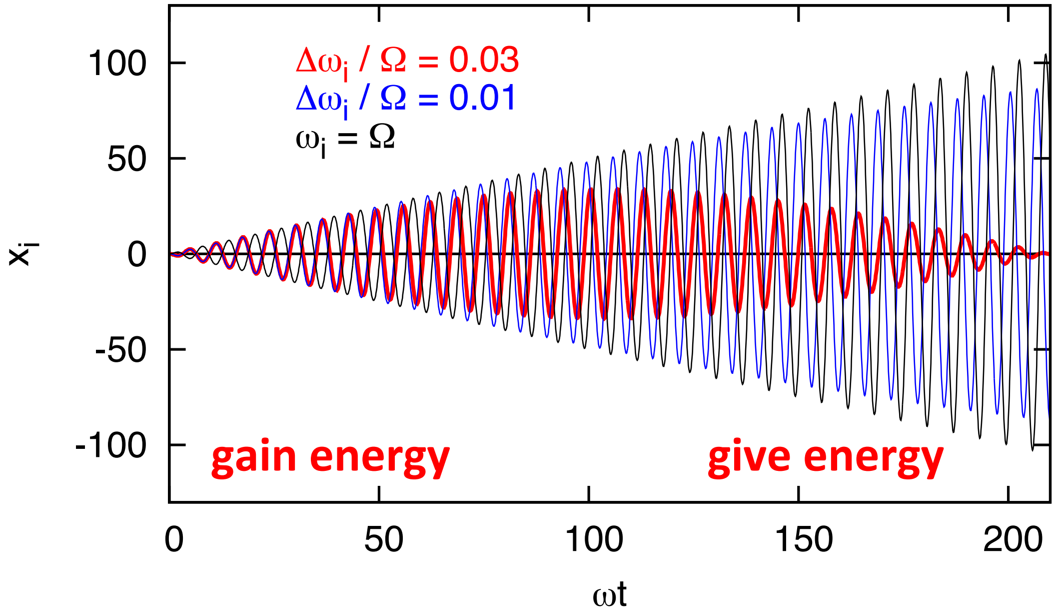
Driven Harmonic Oscillator

off-resonant beating solution

$$x_G(t) = \frac{2\hat{G}}{\omega_i^2 - \Omega^2} \sin\left(\frac{\omega_i - \Omega}{2} t\right) \sin\left(\frac{\omega_i + \Omega}{2} t\right)$$

resonant solution

$$x_G(t) = \frac{\hat{G}}{2\Omega}\, t\, \sin(\Omega t)$$

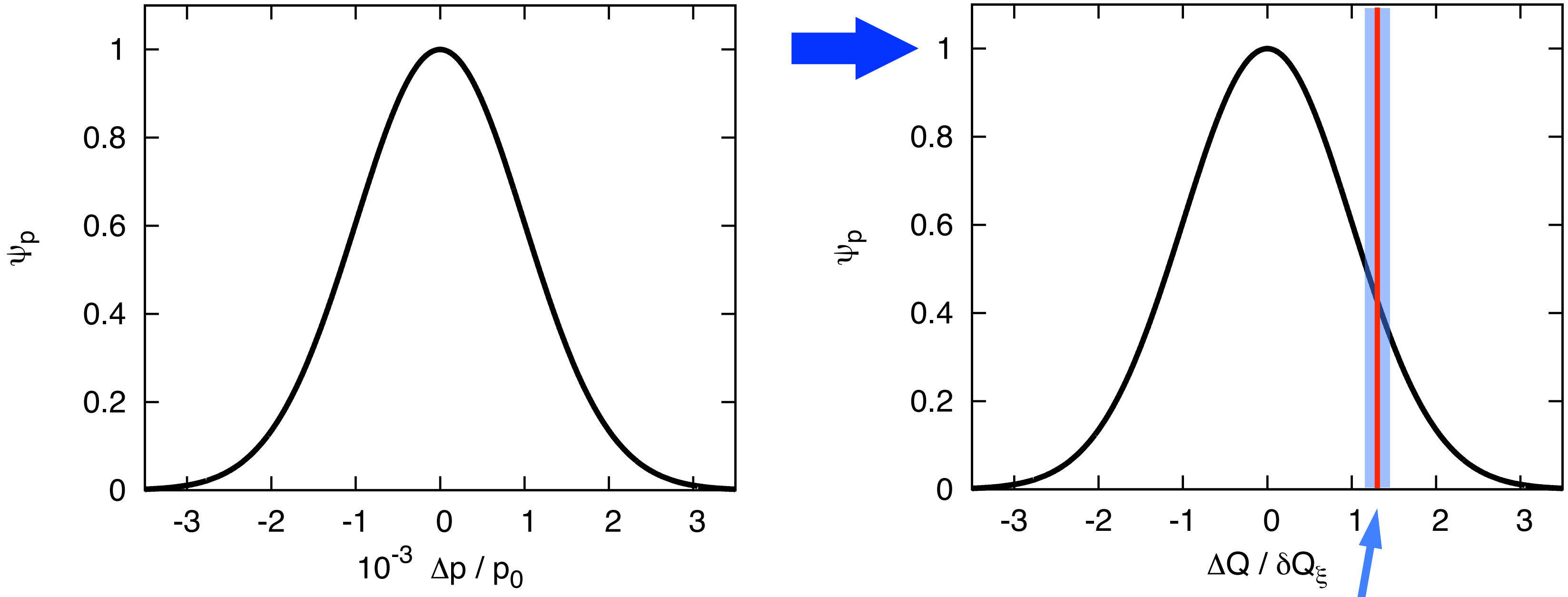


wave↔particle energy transfer takes place

Landau Damping

Momentum spread translates into tune spread

$$\delta Q_\xi = \left|\eta(n - Q_0) + Q_0\xi\right|\delta_p$$

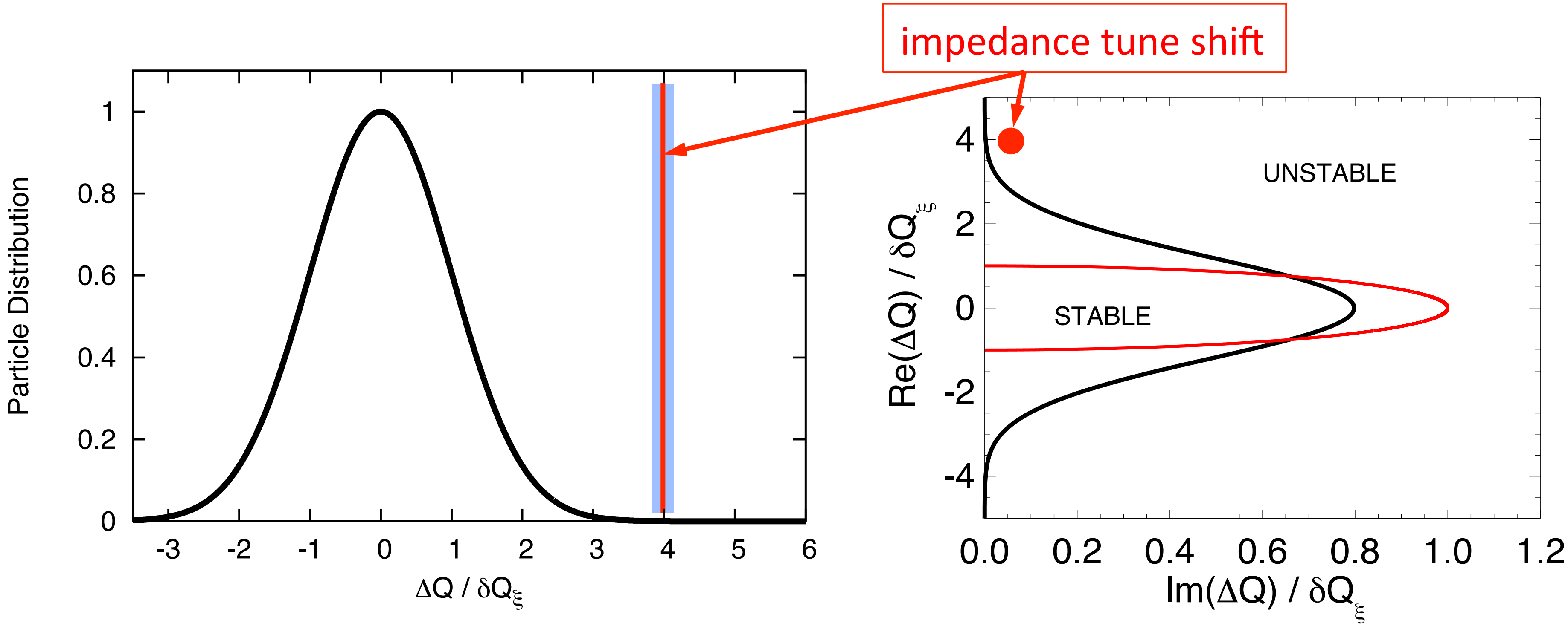


$$\Delta Q_{\mathrm{coh}}\omega_0 \int \frac{f(\omega)d\omega}{\omega - \Omega} = 1$$

resonant particles from both sides contribute to the energy transfer, thus ***f(ω)***

Landau Damping

Loss of Landau damping due to reactive tune shift

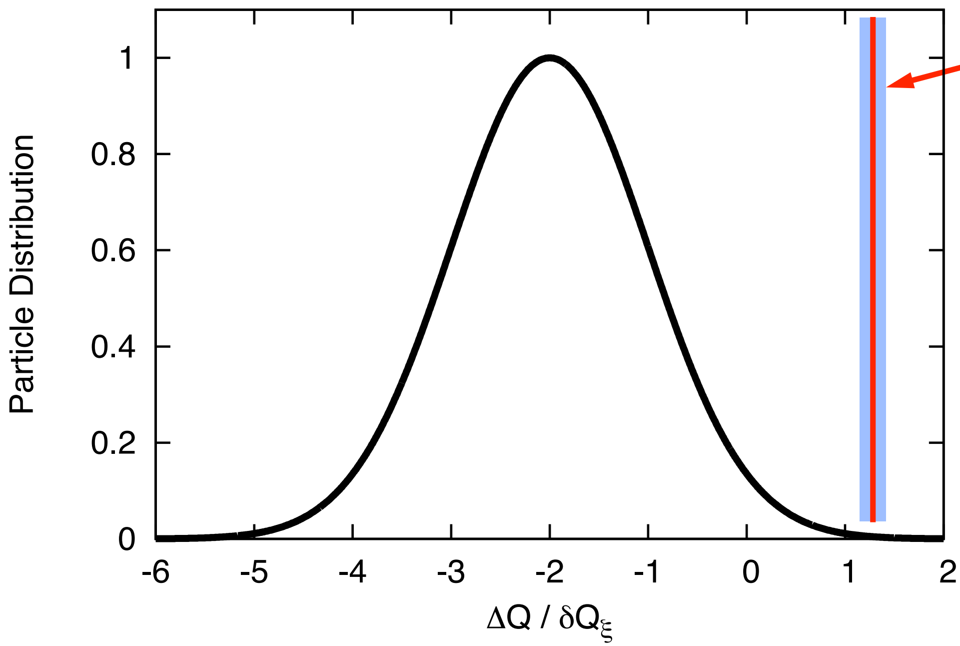


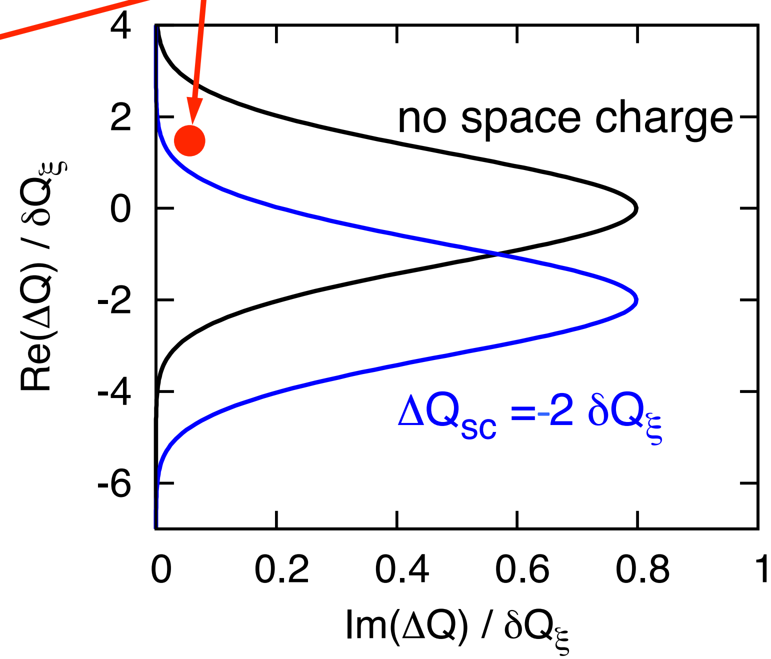
there is still tune spread,
but no resonant particles → no Landau damping

Landau Damping

Loss of Landau damping due to space-charge

impedance tune shift





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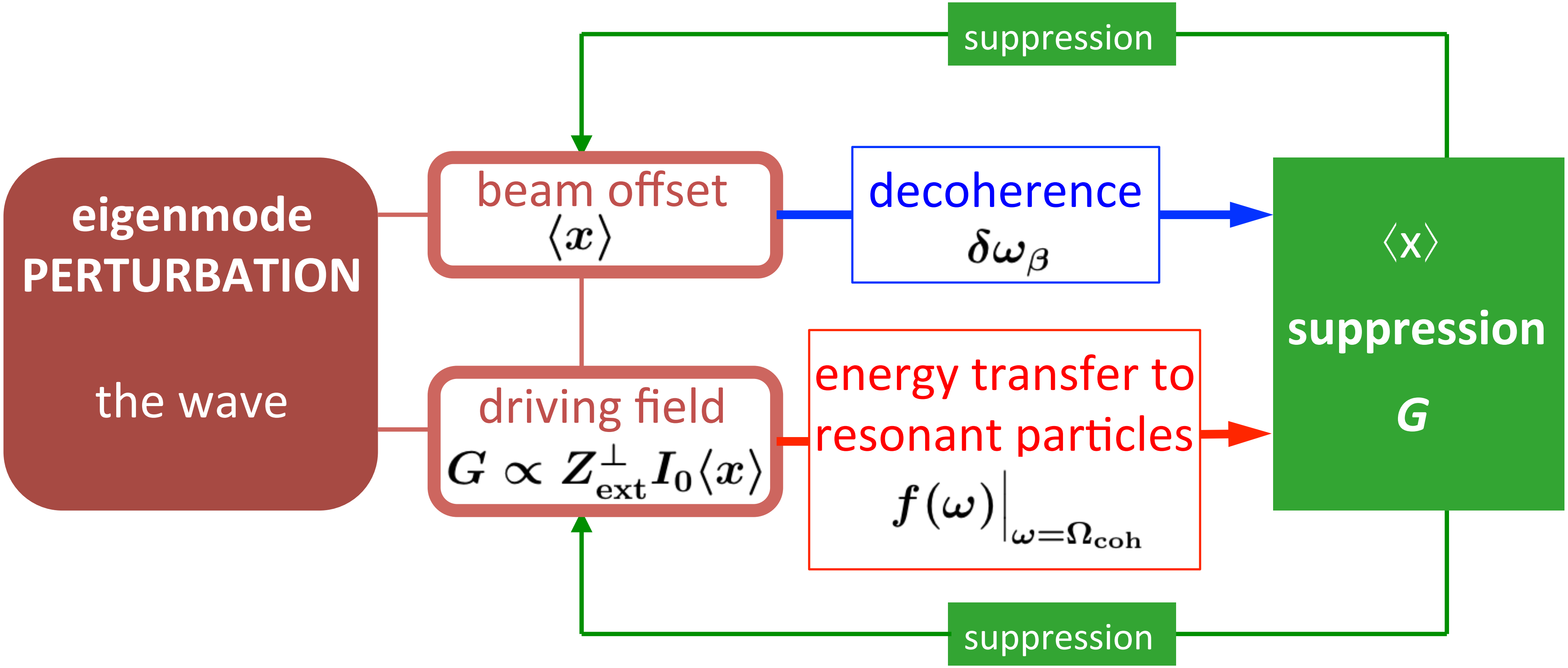
Landau Damping



Main ingredients of Landau damping:

✓ wave–particle collisionless interaction: Impedance driving field

✓ energy transfer: the wave ↔ the (few) resonant particles

Landau Damping in Beams of 2^{nd} type

Landau Damping of 2nd type

Different situation:
Tune spread due to amplitude-dependent tune shifts

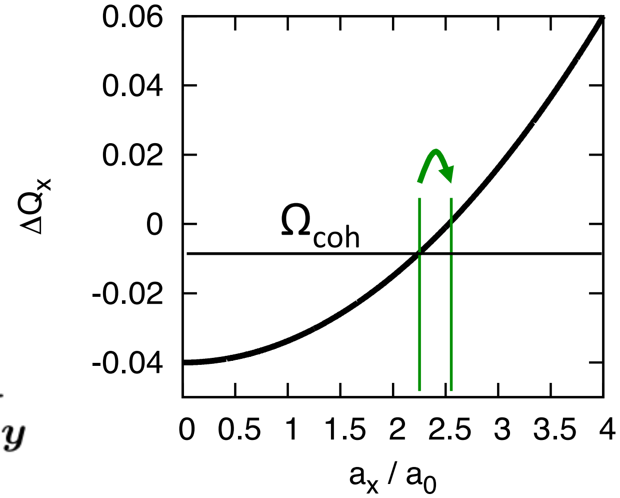
For example, an octupole magnet:

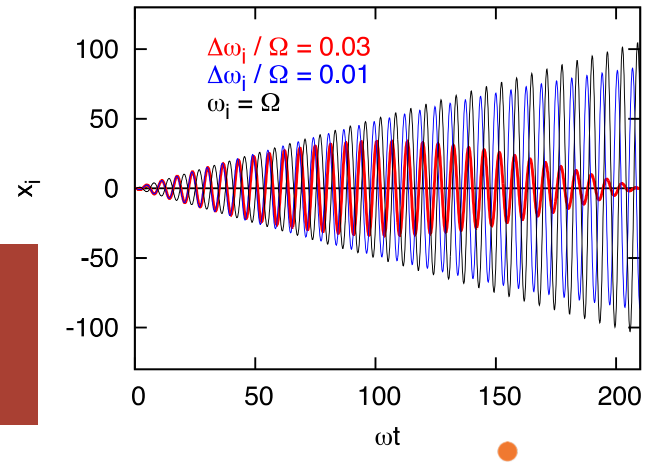
$$B_x = \frac{g}{6}(-y^3 + 3x^2 y), \quad B_y = \frac{g}{6}(x^3 - 3xy^2)$$

Produces amplitude-dependent betatron tune shifts:

$$\Delta Q_x^{\mathrm{oct}} = \Big(\int \frac{K_3 \beta_x^2}{16\pi} \mathrm{d}s\Big) J_x - \Big(\int \frac{K_3 \beta_x \beta_y}{8\pi} \mathrm{d}s\Big) J_y$$

$$x(s) = \sqrt{2 J_x \beta_x(s)} \cos(\phi_x)$$

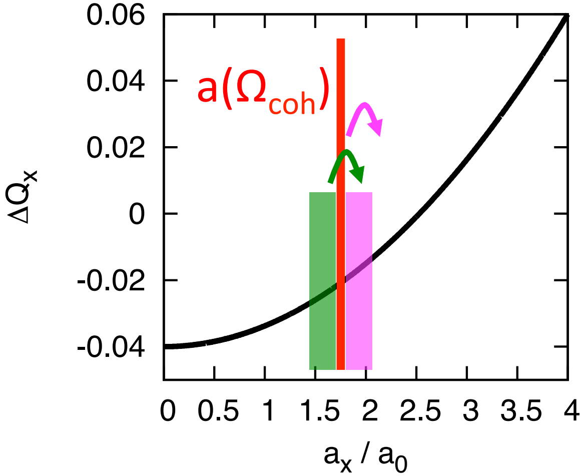


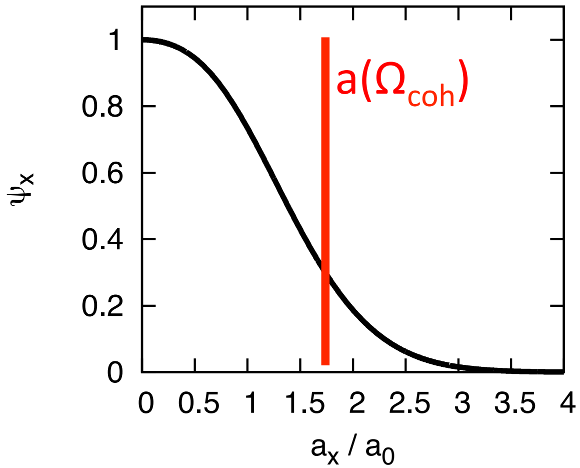


The resonant particles drift away in tune from the resonance as they get excited

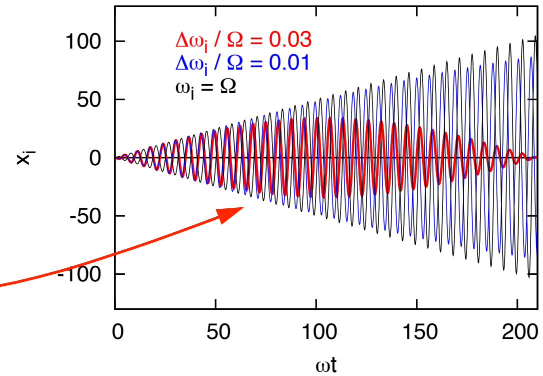
Landau Damping of 2nd type

Particle excitation for amplitude-dependent tune shifts





Once the particle is driven away from the resonance, the energy is transferred back to the wave



We already guess: the distribution slope (*df/da*) might be involved

Landau Damping of 2nd type

The dispersion relation

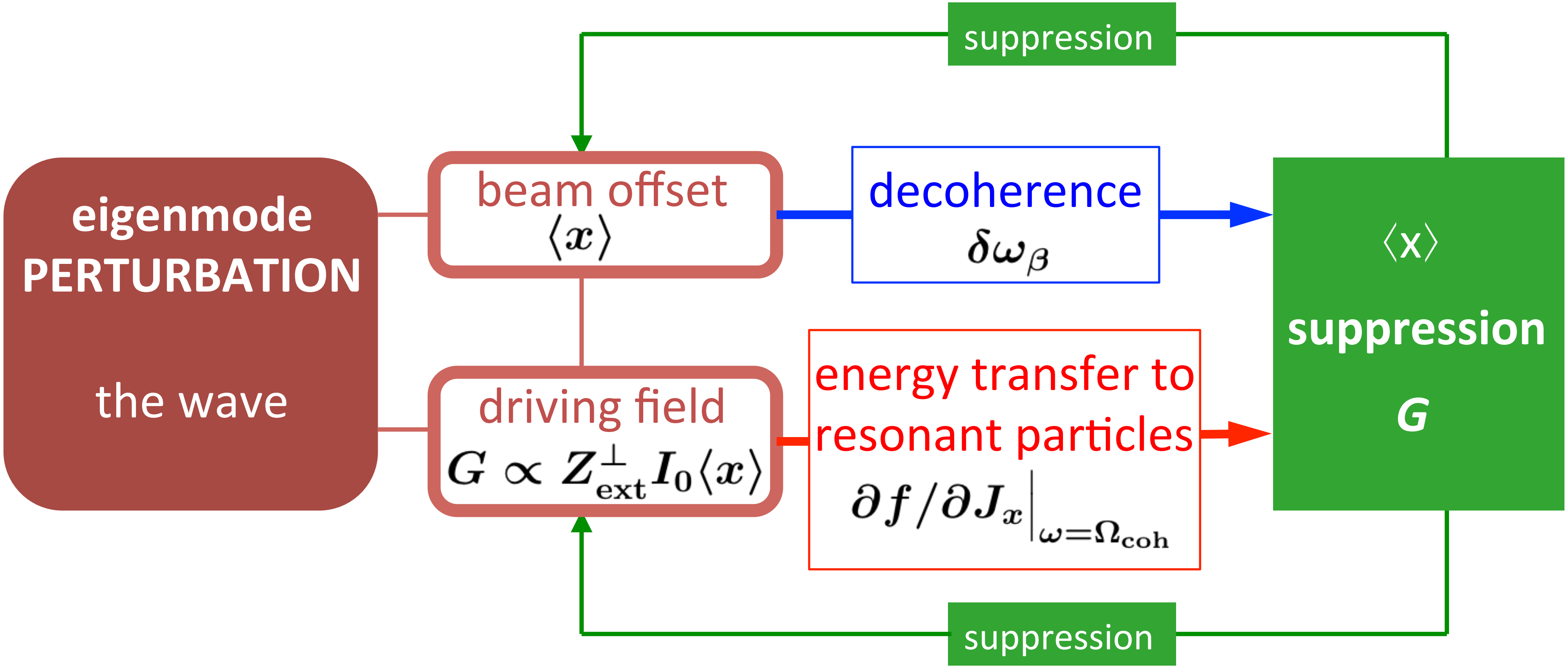
$$\Delta Q_{\rm coh} \int \frac{1}{\Delta Q_{\rm ex} - \Omega/\omega_0} J_x \frac{\partial f}{\partial J_x} dJ_x dJ_y = 1$$

ΔQ_{coh} : coherent no-damping tune shift imposed by an impedance
$\Delta Q_{ex}(J_x,J_y)$: external (lattice) incoherent tune shifts

L.Laslett, V.Neil, A.Sessler, 1965
D.Möhl, H.Schönauer, 1974
J.Berg, F.Ruggiero, CERN SL-96-71 AP 1996

The resulting damping is a complicated 2D convolution of the distribution $\{df(J_x,J_y)/dJ_x\}$ and tune shifts $\Delta Q_{ext}(J_x,J_y)$

Landau Damping of 2nd type

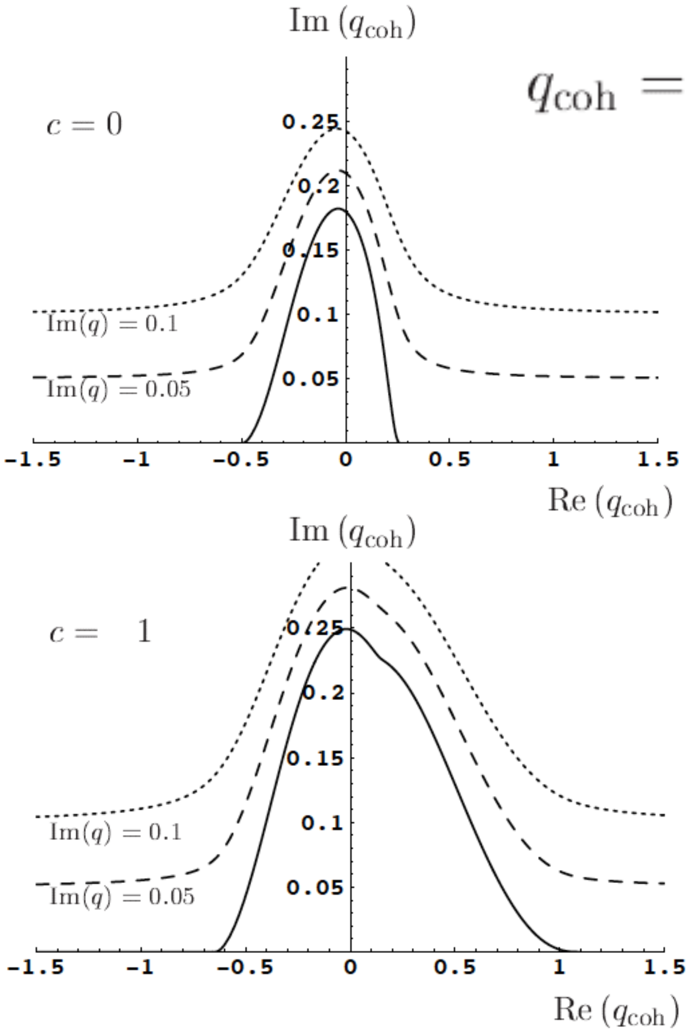


Main ingredients of Landau damping:

✓ wave−particle collisionless interaction: Impedance driving field

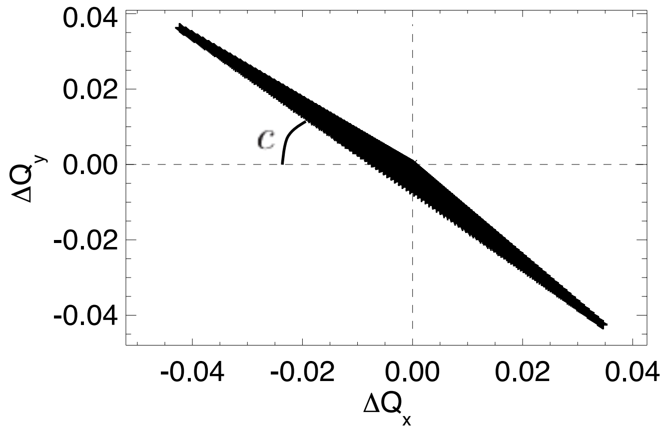
✓ energy transfer: the wave ↔ the (few) resonant particles

Landau Damping of 2nd type



$$q_{coh} = \frac{\Delta Q_{coh}}{S}$$

an octupole tune footprint



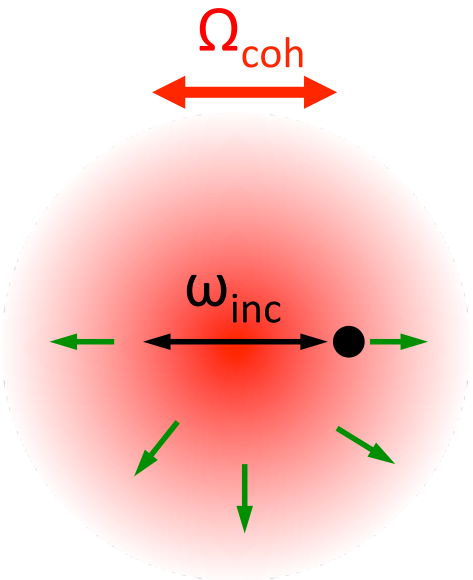
S is the full horizontal tune spread

This has been used for the design of the octupole magnets scheme at LHC. J.Gareyte, J.Koutchuk, F.Ruggiero, LHC Report 91 (1997)

Landau Damping in Beams of 3rd type

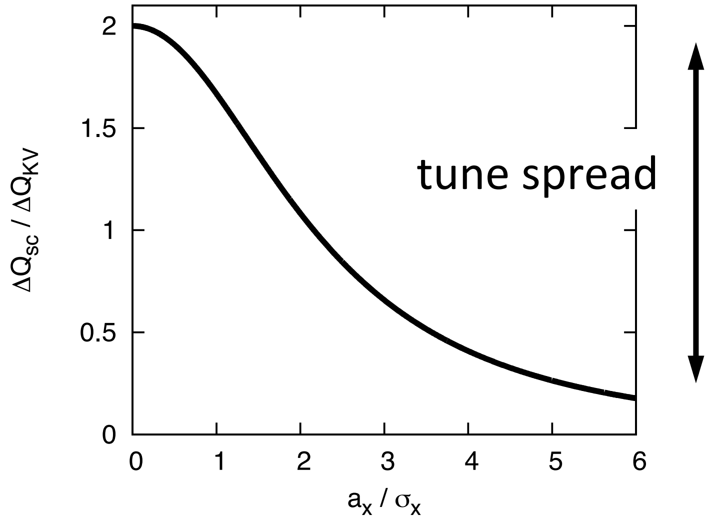
Landau Damping of 3rd type

Different situation:
Wave−Particel interaction is due to space-charge



Electric field
of the self-field
space charge

Space-charge tune shift



For the resonant particles $Q_{inc} \approx Q_{coh}$,
wave↔particles energy transfer should be possible

Landau Damping of 3rd type

The dispersion relation

$$\int \frac{\Delta Q_{\mathrm{coh}} - \Delta Q_{\mathrm{sc}}}{\Delta Q_{\mathrm{ex}} + \Delta Q_{\mathrm{sc}} - \Omega/\omega_0} J_x \frac{\partial f}{\partial J_x} \mathrm{d}J_x \mathrm{d}J_y \mathrm{d}p = 1$$

$f(J_x, J_y, p)$

ΔQ_{coh} : no-damping coherent tune shift imposed

$\Delta Q_{ex}(J_x, J_y, p)$: external (lattice) incoherent tune shift

$\Delta Q_{sc}(J_x, J_y)$: space-charge tune shift

L.Laslett, V.Neil, A.Sessler, 1965
D.Möhl, H.Schönauer, 1974

The resulting damping is a complicated 2D convolution of the distribution $\{df(J_x,J_y)/dJ_x\}$ and tune shifts $\Delta Q_{sc}(J_x,J_y)$, $\Delta Q_{ext}(J_x,J_y)$

Landau Damping of 3rd type

The dispersion relation

$$\int \frac{\Delta Q_{\mathrm{coh}} - \Delta Q_{\mathrm{sc}}}{\Delta Q_{\mathrm{ex}} + \Delta Q_{\mathrm{sc}} - \Omega/\omega_0} J_x \frac{\partial f}{\partial J_x} \mathrm{d}J_x \mathrm{d}J_y \mathrm{d}p = 1$$

$\Delta Q_{ex}=0$: no pole, no damping!

Momentum conservation in a closed system

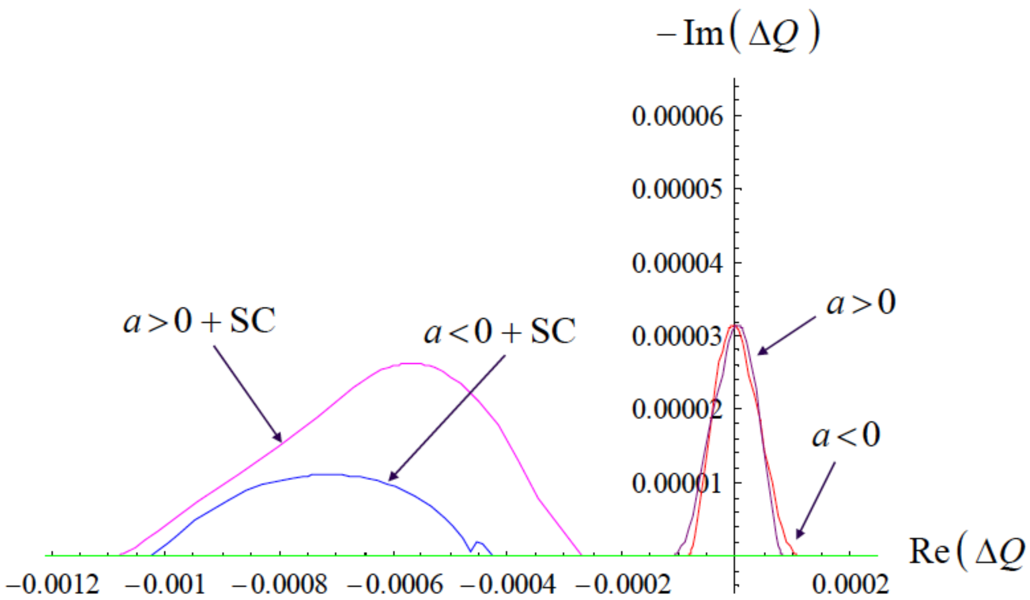
Even if Ω_{coh} is inside the spectrum,
and there are resonant particles $Q_{inc} \approx Q_{coh}$,
there is no Landau damping in coasting beams only due to space-charge

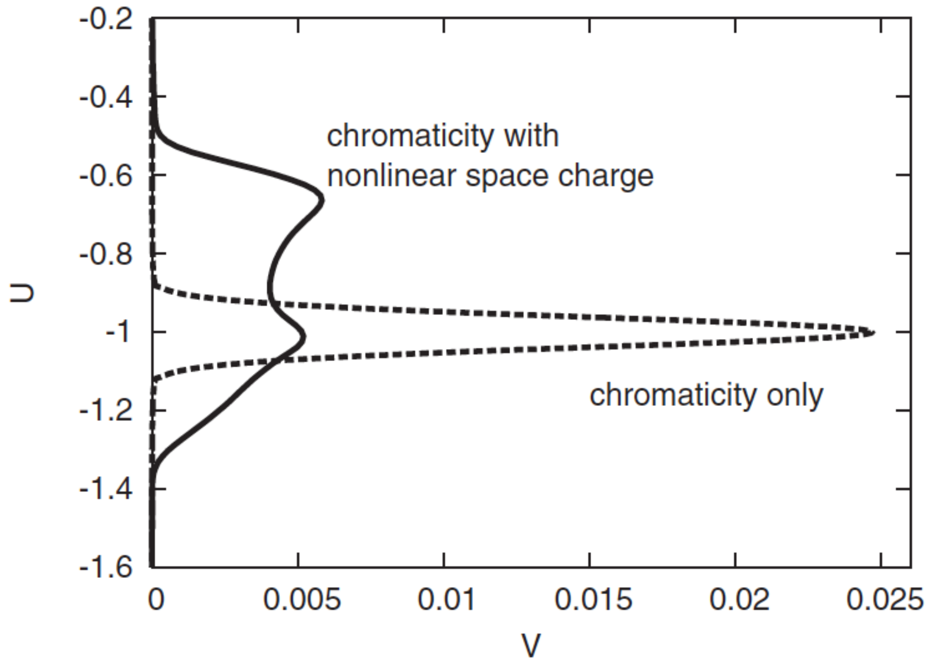
L.Laslett, V.Neil, A.Sessler, 1965
D.Möhl, H.Schönauer, 1974
V.Kornilov, O.Boine-F, I.Hofmann, PRSTAB 11, 014201 (2008)
A.Burov, V.Lebedev, PRSTAB 12, 034201 (2009)

Landau Damping of 3rd type

$$\int \frac{\Delta Q_{\mathrm{coh}} - \Delta Q_{\mathrm{sc}}}{\Delta Q_{\mathrm{ex}} + \Delta Q_{\mathrm{sc}} - \Omega/\omega_0} J_x \frac{\partial f}{\partial J_x} \mathrm{d}J_x \mathrm{d}J_y \mathrm{d}p = 1$$

Solution examples

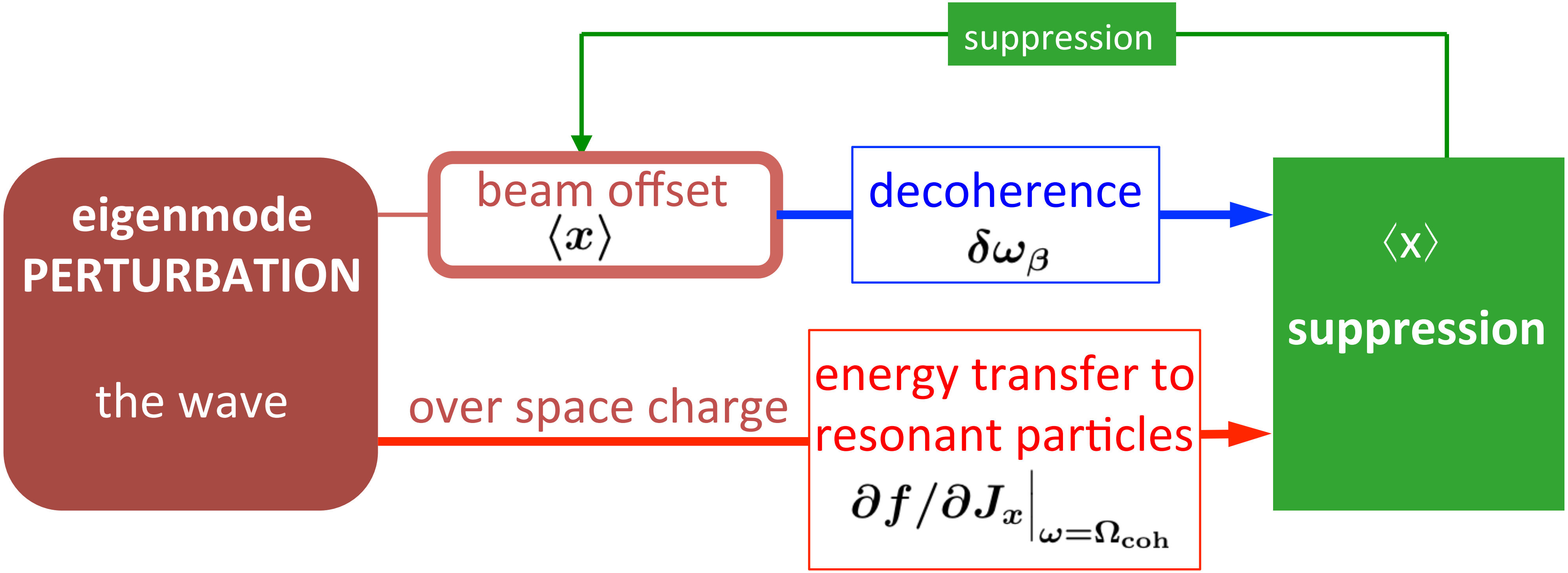




E.Metral, F.Ruggiero, CERN-AB-2004-025 (2004)

V.Kornilov, O.Boine-F, I.Hofmann, PRSTAB 11, 014201 (2008)

Landau Damping of 3rd type



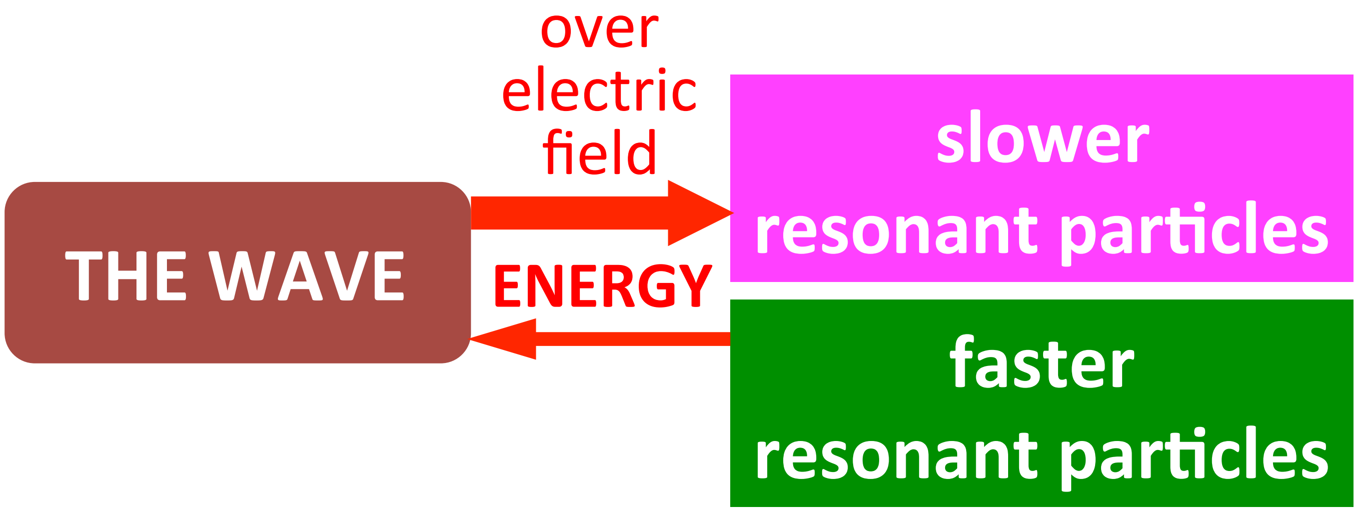
Main ingredients of Landau damping:

✓ wave−particle collisionless interaction: ***E***-field of Space-charge

✓ energy transfer: the wave ↔ the (few) resonant particles

Landau Damping of 3rd type

If simplified, very similar to Landau damping in plasma:



$$\partial f / \partial J_x \Big|_{\omega = \Omega_{\mathrm{coh}}}$$

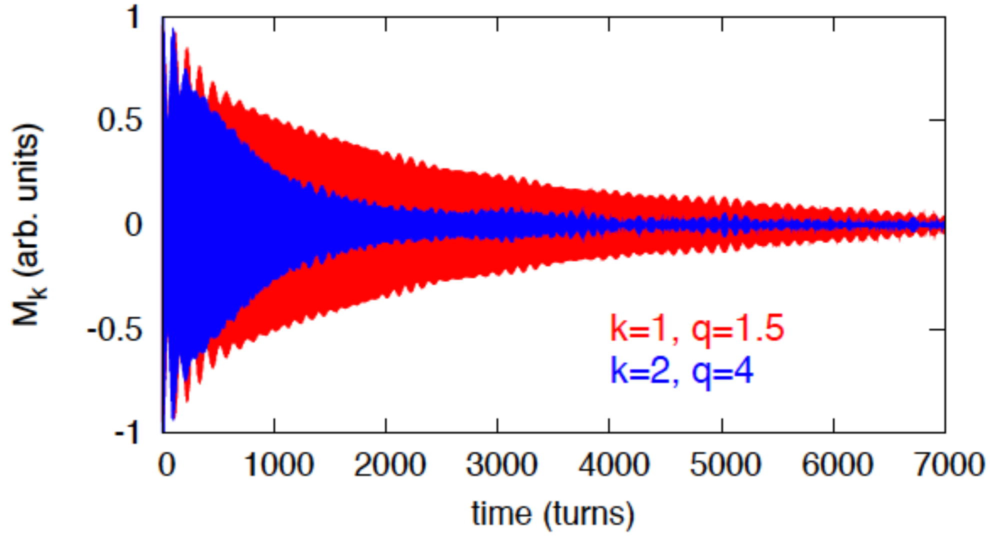
Main ingredients of Landau damping:

✓ wave−particle collisionless interaction:
the electric field of space-charge

✓ energy transfer: the wave ↔ the (few) resonant particles

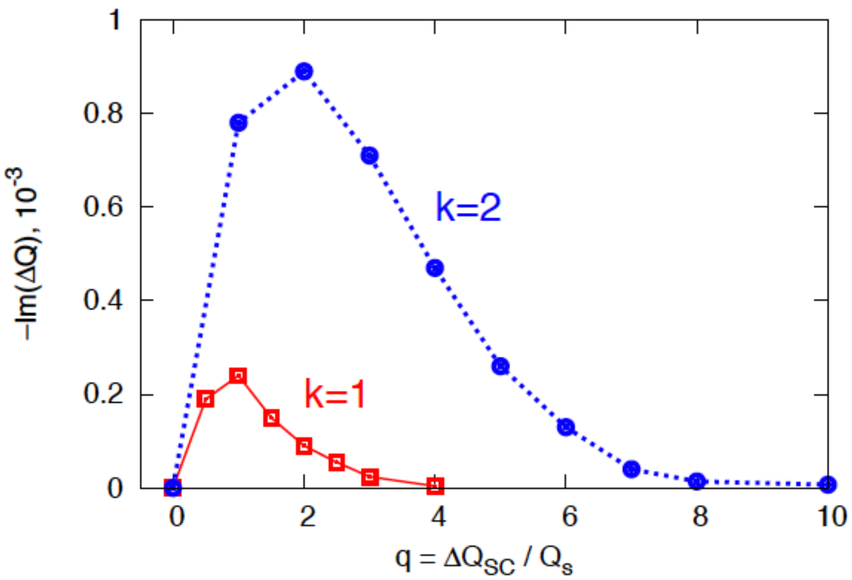
in addition, the decoherence, other $\Delta Q_{ex}(J_x, J_y, p)$, the mix with ***G***

Landau Damping of 3rd type



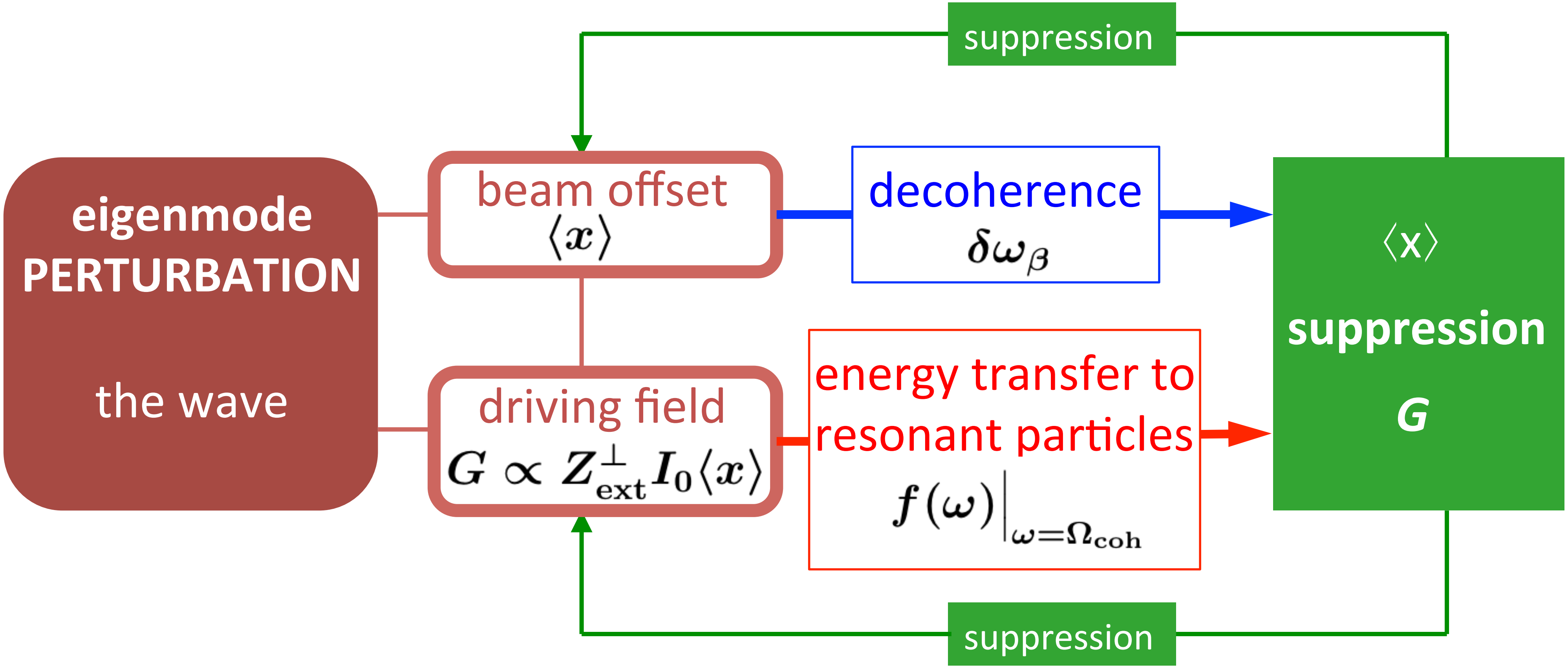
<u>Landau damping in bunches</u>

- There is damping due to only space charge
- Space-charge tune spread due to longitudinal bunch profile



A.Burov, PRSTAB 12, 044202 (2009)
V.Balbekov, PRSTAB 12, 124402 (2009)
V.Kornilov, O.Boine-F, PRSTAB 13, 114201 (2010)

Landau Damping in beams



Main ingredients of Landau damping:

✓ wave–particle collisionless interaction: Impedance driving field

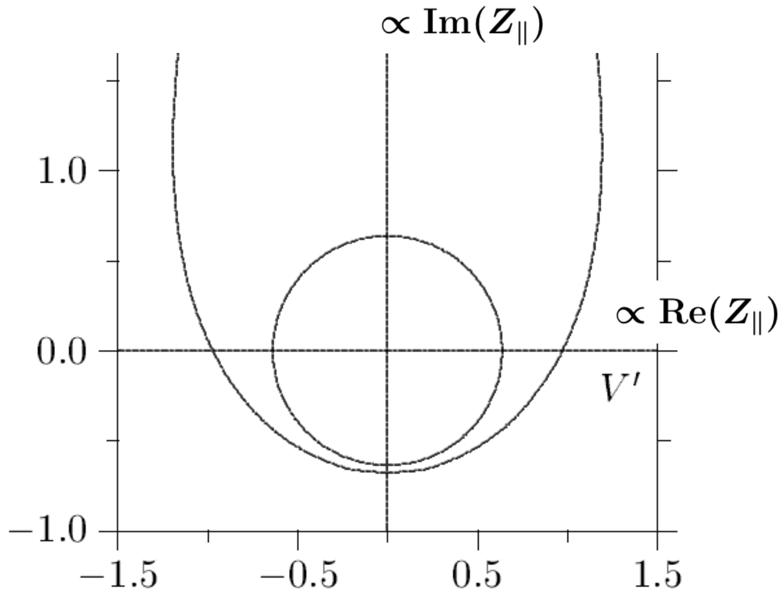
✓ energy transfer: the wave ↔ the (few) resonant particles

Note: the linear Landau damping for infinitesimal amplitudes

Longitudinal Stability

Coasting Beam:
Spread in the revolution frequency

$$\mathcal{A}\, I_0 \frac{Z_{\parallel}(\Omega_{\parallel})}{n} \int \frac{\partial f(\omega_0)/\partial \omega_0}{\omega_0 - \Omega_{\parallel}/n} d\omega_0 = 1$$



$$\left|\frac{Z}{n}\right| \leq 0.6 \frac{2\pi\beta^2 E_0 \eta (\Delta p/p)^2}{e I_0}$$

Bunches beams:

$$\Delta\omega_s^{\mathrm{coh}} \int \frac{f(\omega_s) d\omega_s}{\Omega_{\parallel} - \omega_s} = 1$$

K.Y.Ng, Physics of Intensity Dependent Beam Instabilities, 2006
A.Hofmann, Proc. CAS 2003, CERN-2006-002
E.Keil, W.Schnell, CERN ISR-TH-RF/69-48 (1969)