Improved effective TMD factorization for forward dijets in pA collisions

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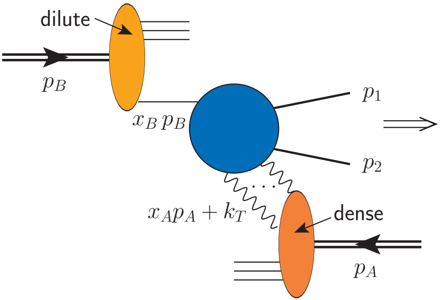
Dijets in dilute-dense collision at high energy

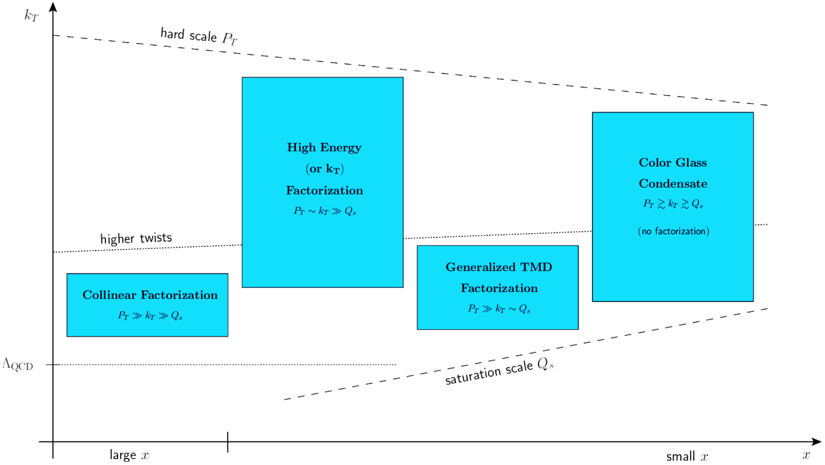


forward dijets with transverse momentum imbalance

$$\left|\vec{p}_{T1} + \vec{p}_{T2}\right| = \left|\vec{k}_T\right| = k_T$$

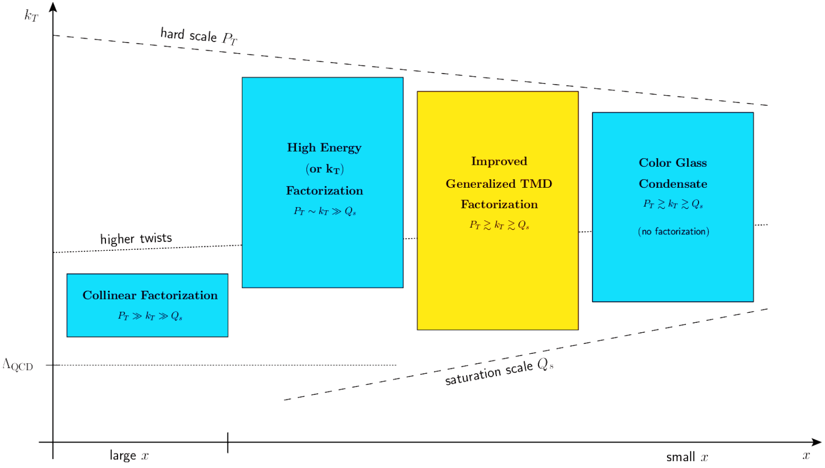
- Hybrid approach: $x_B \gg x_A \Rightarrow$ parton in hadron B is 'collinear'
- Three scales involved:
  - hard scale P_T (of the order of the average transverse momentum of jets)
  - transverse momentum imbalance k_T
  - saturation scale Q_s (increasing with energy)

Regimes and factorization approaches



- All-order QCD theorem for factorization exists only for collinear case.
- No TMD factorization due to violation of universality (other problems at higher orders may occur).
- However, at small x and large N_c the universality is recovered leading to an 'effective' Generalized TMD factorization. [F. Dominguez, C. Marquet, B-W. Xiao, F. Yuan Phys.Rev. D 83 (2011) 105005]

Regimes and factorization approaches



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Factorization formulae

Collinear Factorization

$$d\sigma_{AB} = \sum_{a,b} \int dx_A\, dx_B\, f_{a/A}\left(x_A, \mu^2\right) f_{b/B}\left(x_B, \mu^2\right) d\sigma_{ab}\left(x_A, x_B, \mu^2\right)$$

The hard partonic cross section σ_{ab} is computed with partons a, b on-shell.
The off-shellness $k_T \ll \mu$ is integrated over inside the PDF.

High Energy Factorization

[L.V. Gribov, E.M. Levin, M.G. Ryskin, Phys.Rept. 100 (1983) 1-150]
[S. Catani, M. Ciafaloni, F. Hautmann, Nucl.Phys. B366 (1991) 135-188]
[J.C. Collins, R.K. Ellis, Nucl.Phys. B360 (1991) 3-30]
hybrid → [M. Deak, F. Hautmann, H. Jung, K. Kutak, JHEP 0909 (2009) 121]

$$d\sigma_{AB} = \sum_{b} \int dx_A\, dx_B \int dk_T^2\, \mathcal{F}\left(x_A, k_T^2, \mu^2\right) f_{b/B}\left(x_B, \mu^2\right) d\sigma_{g^*b}\left(x_A, x_B, k_T^2, \mu^2\right)$$

The partonic cross section σ_{g^*b} is computed (in a gauge-invariant way) with off-shell incoming gluon.
$\mathcal{F}\left(x_A, k_T^2, \mu^2\right)$ – Unintegrated Gluon Distribution evolving via BFKL (or similar).
⋆ Can be derived from CGC in the dilute limit.

Generalized (small x) TMD Factorization

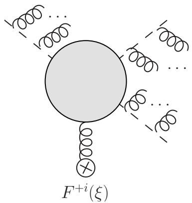
[F. Dominguez, C. Marquet, B-W. Xiao, F. Yuan Phys.Rev. D 83 (2011) 105005]

$$d\sigma_{AB} = \sum_b \int dx_A dx_B \, f_{b/B}\left(x_B, \mu^2\right) \sum_i \int dk_T^2 \phi_{bg}^{(i)}\left(x_A, k_T^2, \mu^2\right) d\sigma_{gb}^{(i)}\left(x_A, x_B, k_T^2, \mu^2\right)$$

The hard partonic cross sections $\sigma_{gb}^{(i)}$ are computed with partons a, b on-shell.
$\phi_{bg}^{(i)}\left(x_A, k_T^2, \mu^2\right)$ – TMD Gluon Distributions depending on the process.
★ Can be also derived from CGC in the correlation limit.

$$\phi_{bg}\left(x, k_T\right) = 2 \int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_A p_A^- \xi^+ - i\vec{k}_T \cdot \vec{\xi}_T} \langle p_A | \, \mathrm{Tr} \left\{ F^{+i}\left(\xi\right) \left[\xi, 0\right]_{C_1} F^{+i}\left(0\right) \left[0, \xi\right]_{C_2} \right\} | p_A \rangle$$

The Wilson lines $[\xi, 0]_{C_i}$ depend on the process-dependent paths.

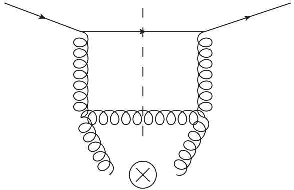


Wilson lines come from resummation of collinear gluons attached to the external lines. Then they are 'glued' by the color structure of the hard process to give $[\xi, 0]_{C_i}$.

Generalized (small x) TMD Factorization (cont.)

Example: TMD for a particular diagram

[C.J. Bomhof, P.J. Mulders, F. Pijlman, Eur.Phys.J.C. 47, 147 (2006)]



$$\langle p_A|\,\mathrm{Tr}\{F(\xi)\,\mathcal{U}^{[+]\dagger}F(0)\left[\frac{\mathrm{Tr}\,\mathcal{U}^{[\square]\dagger}}{N_c}\mathcal{U}^{[+]}+\mathcal{U}^{[-]}\right]\}\,|p_A\rangle$$

with the following definitions of Wilson lines and loops:

$$\mathcal{U}^{[\pm]}=U(0,\pm\infty;0_T)U(\pm\infty,\xi^+;\xi_T)\quad \mathcal{U}^{[\square]}=\mathcal{U}^{[+]}\mathcal{U}^{[-]\dagger}=\mathcal{U}^{[-]}\mathcal{U}^{[+]\dagger}$$

where $U(a,b;x_T)=\mathcal{P}\exp\left[ig\int_a^b dx^+A_a^-(x^+,x_T)t^a\right]$.

Generalized (small x) TMD Factorization (cont.)

All TMD structures for dijets

$$\mathcal{F}_{qg}^{(1)} \sim \langle p_A | \, \mathrm{Tr}\{F(\xi)\, \mathcal{U}^{[-]\dagger} F(0)\, \mathcal{U}^{[+]}\} \, | p_A \rangle \quad \mathcal{F}_{qg}^{(2)} \sim \langle p_A | \, \mathrm{Tr}\{F(\xi)\, \frac{\mathrm{Tr}\mathcal{U}^{[\square]}}{N_c} \mathcal{U}^{[+]\dagger} F(0)\, \mathcal{U}^{[+]}\} \, | p_A \rangle$$

$$\mathcal{F}_{gg}^{(1)} \sim \langle p_A | \, \mathrm{Tr}\{F(\xi)\, \frac{\mathrm{Tr}\mathcal{U}^{[\square]}}{N_c} \mathcal{U}^{[-]\dagger} F(0)\, \mathcal{U}^{[+]}\} \, | p_A \rangle \quad \mathcal{F}_{gg}^{(2)} \sim \frac{1}{N_c} \langle p_A | \, \mathrm{Tr}\left\{F(\xi)\, \mathcal{U}^{[\square]\dagger}\right\} \mathrm{Tr}\left\{F(0)\, \mathcal{U}^{[\square]}\right\} | p_A \rangle$$

$$\mathcal{F}_{gg}^{(3)} \sim \langle p_A | \, \mathrm{Tr}\left\{F(\xi)\, \mathcal{U}^{[+]\dagger} F(0)\, \mathcal{U}^{[+]}\right\} | p_A \rangle \quad \mathcal{F}_{gg}^{(4)} \sim \langle p_A | \, \mathrm{Tr}\left\{F(\xi)\, \mathcal{U}^{[-]\dagger} F(0)\, \mathcal{U}^{[-]}\right\} | p_A \rangle$$

$$\mathcal{F}_{gg}^{(5)} \sim \langle p_A | \, \mathrm{Tr}\left\{F(\xi)\, \mathcal{U}^{[\square]\dagger} \mathcal{U}^{[+]\dagger} F(0)\, \mathcal{U}^{[\square]} \mathcal{U}^{[+]}\right\} | p_A \rangle \quad \mathcal{F}_{gg}^{(6)} \sim \langle p_A | \, \mathrm{Tr}\left\{F(\xi)\, \mathcal{U}^{[+]\dagger} F(0)\, \mathcal{U}^{[+]}\right\} \frac{\mathrm{Tr}\mathcal{U}^{[\square]}}{N_c} \frac{\mathrm{Tr}\mathcal{U}^{[\square]}}{N_c} | p_A \rangle$$

In the large N_c limit $\mathcal{F}_{qg}^{(1)}, \mathcal{F}_{gg}^{(3)}$ are the only necessary TMDs:

- $\mathcal{F}_{gg}^{(3)} = xG_1$ is the Weizsacker-Williams gluon distribution with parton number interpretation (calculable in CGC but decoupled from most simple processes; appears also in dijet production in DIS)
- $\mathcal{F}_{qg}^{(1)} = xG_2$ is so-called 'dipole' gluon distribution probed in inclusive or semi-inclusive processes at small x

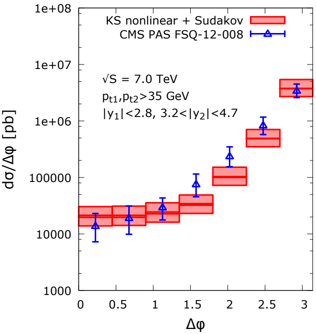
High Energy Factorization and Azimuthal Decorrelations

- The previous approach is valid in the correlation limit $k_T \ll P_T$.
- The decorrelation region for inclusive dijets is nicely described by the High Energy Factorization.

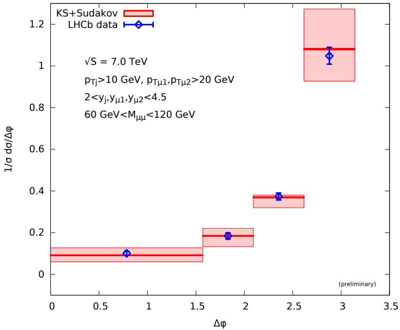
[A. van Hameren, PK, K. Kutak, S. Sapeta, Phys.Lett. B737 (2014) 335-340]

[A. van Hameren, PK, K. Kutak, arXiv:1505.02763]

forward-central dijets



forward Z_0 + jet



- Try to improve TMD factorization by introducing off-shellness to the hard factors.

Improved Generalized TMD Factorization

[P.K., K. Kutak, C. Marquet, E. Petreska, S. Sapeta, A. van Hameren, arXiv:150342]

1. We revise the calculation of TMDs using color decomposition of amplitudes

$$\mathcal{M}^{a_1 \dots a_N}\left(\varepsilon_1^{\lambda_1}, \dots, \varepsilon_N^{\lambda_N}\right) = \sum_{\sigma \in S_{N-1}} \mathrm{Tr}\left(t^{a_1} t^{a_{\sigma_2}} \dots t^{a_{\sigma_N}}\right) \mathcal{M}\left(1^{\lambda_1}, \sigma_2^{\lambda_{\sigma 2}} \dots, \sigma_N^{\lambda_{\sigma N}}\right)$$

a_i- color indices, $\varepsilon_i^{\lambda_i}$ - polarization vectors with helicity λ_i, S_{N-1} - set of noncyclic permutations.

We conclude that there are only two independent TMDs $\Phi^{(i)}$, $i = 1, 2$ (being a combination of $\mathcal{F}_{qg}^{(1)}$'s) needed for each channel.

2. We calculate off-shell color-ordered helicity amplitudes needed to construct a hard factor for each $\Phi^{(i)}$.

Methods for gauge invariant off-shell amplitudes:

[E. Antonov, L. Lipatov, E. Kuraev, I. Cherednikov, Nucl.Phys. B721 (2005) 111-135]

[A. van Hameren, PK, K. Kutak, JHEP 1212 (2012) 029; JHEP 1301 (2013) 078]

[A. van Hameren, JHEP 1407 (2014) 138], [PK, JHEP 1407 (2014) 128]

[A. van Hameren, M. Serino, arXiv:1504.00315]

Improved Generalized TMD Factorization (cont.)

In spinor formalism, the non-zero gauge invariant off-shell helicity amplitudes have the form of the MHV amplitudes with certain modification of spinor products:

[A. van Hameren, PK, K. Kutak, JHEP 1212 (2012) 029]

$$\mathcal{M}_{g^*g\to gg}\left(1^*,2^-,3^+,4^+\right) = 2g^2\,\rho_1\,\frac{\langle 1^*2\rangle^4}{\langle 1^*2\rangle\langle 23\rangle\langle 34\rangle\langle 41^*\rangle}$$

$$\mathcal{M}_{g^*g\to gg}\left(1^*,2^+,3^-,4^+\right) = 2g^2\,\rho_1\,\frac{\langle 1^*3\rangle^4}{\langle 1^*2\rangle\langle 23\rangle\langle 34\rangle\langle 41^*\rangle}$$

$$\mathcal{M}_{g^*g\to gg}\left(1^*,2^+,3^+,4^-\right) = 2g^2\,\rho_1\,\frac{\langle 1^*4\rangle^4}{\langle 1^*2\rangle\langle 23\rangle\langle 34\rangle\langle 41^*\rangle}$$

where $\langle ij\rangle = \langle k_i - | k_j + \rangle$ with spinors defined as $|k_i \pm\rangle = \frac{1}{2}\left(1 \pm \gamma_5\right) u\left(k_i\right)$. Modified spinor products involve only longitudinal component of the off-shell momentum $\langle 1^* i\rangle = \langle p_A\, i\rangle$. Similar expressions can be derived for quarks.

We construct hard factors $K^{(i)}$ corresponding to $\Phi^{(i)}$. The final factorization formula reads:

$$\frac{d\sigma_{AB}}{dP.S.} \sim \sum_{i=1,2}\left[f_{q/B}\otimes\Phi^{(i)}_{gq\to qg}\otimes K^{(i)}_{gq\to qg} + f_{g/B}\otimes\left(\Phi^{(i)}_{gg\to q\bar{q}}\otimes K^{(i)}_{gg\to q\bar{q}} + \Phi^{(i)}_{gg\to gg}\otimes K^{(i)}_{gg\to gg}\right)\right]$$

Improved Generalized TMD Factorization (cont.)

Hard off-shell factors and corresponding TMDs

$$K^{(1)}_{gq\to gq} = -\frac{\bar{u}\left(\bar{s}^2+\bar{u}^2\right)}{2\bar{t}\hat{t}\hat{s}}\left(1+\frac{\bar{s}\hat{s}-\bar{t}\hat{t}}{N_c^2\,\bar{u}\hat{u}}\right) \qquad \Phi^{(1)}_{gq\to gq} = \mathcal{F}^{(1)}_{qg}$$

$$K^{(2)}_{gq\to gq} = -\frac{C_F}{N_c}\,\frac{\bar{s}\left(\bar{s}^2+\bar{u}^2\right)}{\bar{t}\hat{t}\hat{u}} \qquad \Phi^{(2)}_{gq\to gq} = \frac{1}{N_c^2-1}\left(-\mathcal{F}^{(1)}_{qg}+N_c^2\mathcal{F}^{(2)}_{qg}\right)$$

$$K^{(1)}_{gg\to q\bar{q}} = \frac{1}{2N_c}\,\frac{\left(\bar{t}^2+\bar{u}^2\right)\left(\bar{u}\hat{u}+\bar{t}\hat{t}\right)}{\bar{s}\hat{s}\hat{t}\hat{u}} \qquad \Phi^{(1)}_{gg\to q\bar{q}} = \frac{1}{N_c^2-1}\left(N_c^2\mathcal{F}^{(1)}_{gg}-\mathcal{F}^{(3)}_{gg}\right)$$

$$K^{(2)}_{gg\to q\bar{q}} = \frac{1}{4N_c^2C_F}\,\frac{\left(\bar{t}^2+\bar{u}^2\right)\left(\bar{u}\hat{u}+\bar{t}\hat{t}-\bar{s}\hat{s}\right)}{\bar{s}\hat{s}\hat{t}\hat{u}} \qquad \Phi^{(2)}_{gg\to q\bar{q}} = -N_c^2\mathcal{F}^{(2)}_{gg}+\mathcal{F}^{(3)}_{gg}$$

$$K^{(1)}_{gg\to gg} = \frac{N_c}{C_F}\,\frac{\left(\bar{s}^4+\bar{t}^4+\bar{u}^4\right)\left(\bar{u}\hat{u}+\bar{t}\hat{t}\right)}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}} \qquad \Phi^{(1)}_{gg\to gg} = \frac{1}{2N_c^2}\left(N_c^2\mathcal{F}^{(1)}_{gg}-2\mathcal{F}^{(3)}_{gg}+\mathcal{F}^{(4)}_{gg}+\mathcal{F}^{(5)}_{gg}+N_c^2\mathcal{F}^{(6)}_{gg}\right)$$

$$K^{(2)}_{gg\to gg} = -\frac{N_c}{2C_F}\,\frac{\left(\bar{s}^4+\bar{t}^4+\bar{u}^4\right)\left(\bar{u}\hat{u}+\bar{t}\hat{t}-\bar{s}\hat{s}\right)}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}} \qquad \Phi^{(2)}_{gg\to gg} = \frac{1}{N_c^2}\left(N_c^2\mathcal{F}^{(2)}_{gg}-2\mathcal{F}^{(3)}_{gg}+\mathcal{F}^{(4)}_{gg}+\mathcal{F}^{(5)}_{gg}+N_c^2\mathcal{F}^{(6)}_{gg}\right)$$

$\hat{s},\hat{t},\hat{u}$ – ordinary Mandelstam variables, $\hat{s}+\hat{t}+\hat{u}=k_T^2$

$\bar{s},\bar{t},\bar{u}$ – off-shell momentum $k_A = x_A p_A + k_T$ replaced by $x_A p_A$, $\bar{s}+\bar{u}+\bar{t}=0$

Summary

Generalized ('effective') TMD factorization

Cross section expressed at large N_c in terms of two universal UGDs and a set of on-shell hard factors. Valid only in the correlation limit.

$\Updownarrow$

Improved factorization

Cross section calculated in terms of several UGDs and off-shell hard factors. At large N_c we end up with two UGDs. Valid beyond the correlation limit.

$\Updownarrow$

High Energy (or k_T) factorization

Cross section calculated in terms of single UGD and an off-shell hard factor. Valid beyond the correlation limit but not in saturation regime.

Phenomenology is under development...