Twist analysis of proton structure functions in BFKL and BK framework

Mariusz Sadzikowski

Jagiellonian University

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Outline

- Higher twists: motivation.
- Possible signs of higher twists in diffractive and inclusive DIS – experiment.
- Estimation of the higher twist contributions within the dipole framework based on BFKL equation.
- Comparison between Golec-Biernat – Wusthoff saturation model and BFKL approach.
- Implication for twist content of Balitsky – Kovchegov cross-section.

Based on Leszek Motyka, MS, Acta Phys.Polon. **B45** (2014), 2079.

Introduction (1)

- The standard description of proton hard interactions, based on the leading twist-2 term in OPE is very precise.

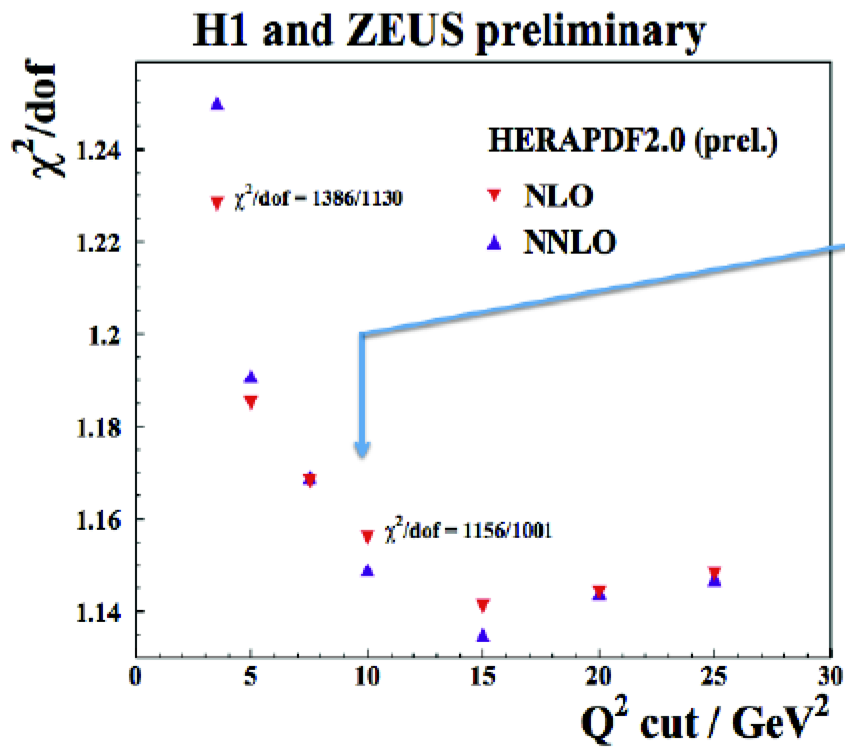
- However, the higher twist terms, although power suppressed by the proces hard scale, may provide important corrections at moderate scales especially at small x.

- Good understanding of higher twists:
  - broadening of QCD applicability,
  - better precision, qualitative determination of DGLAP limitations,
  - better determination of parton densities,
  - novel observables in proton structure.

Introduction (2)

- First-principle theory of higher twists: highly involved, few studies done within decades, not complete.
- To provide reliable predictions: a lot of input from measurements is necessary – missing so far.
- Therefore, adopt at first a simplified picture: QCD guided model of rescattering with unitarity constraints (parton saturation).
- Most advanced QCD studies of multipple scattering carried out so far in the high energy limit, in kT-factorisation approach and small-x resummations (of log(1/x))

Higher twists in DIS?



At Q^2_{min}= 10 is when the fit stabilises with respect of chi2/dof vs Q^2cut

For Q^2_{min}= 3.5 GeV^2

Chi2/dof (NLO) = 1386/1130

Chi2/dof(NNLO)= 1414/1130

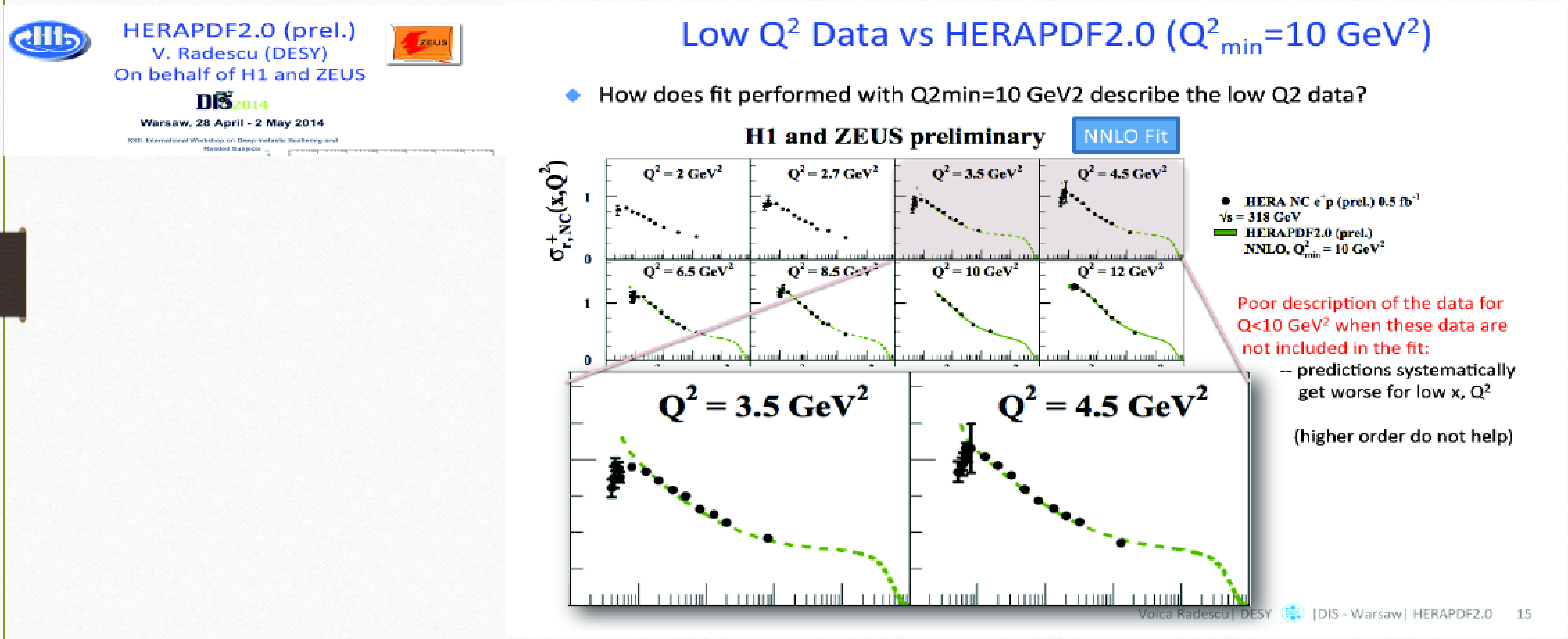
For Q^2_{min}= 10 GeV^2

Chi2/dof (NLO) = 1156/1001

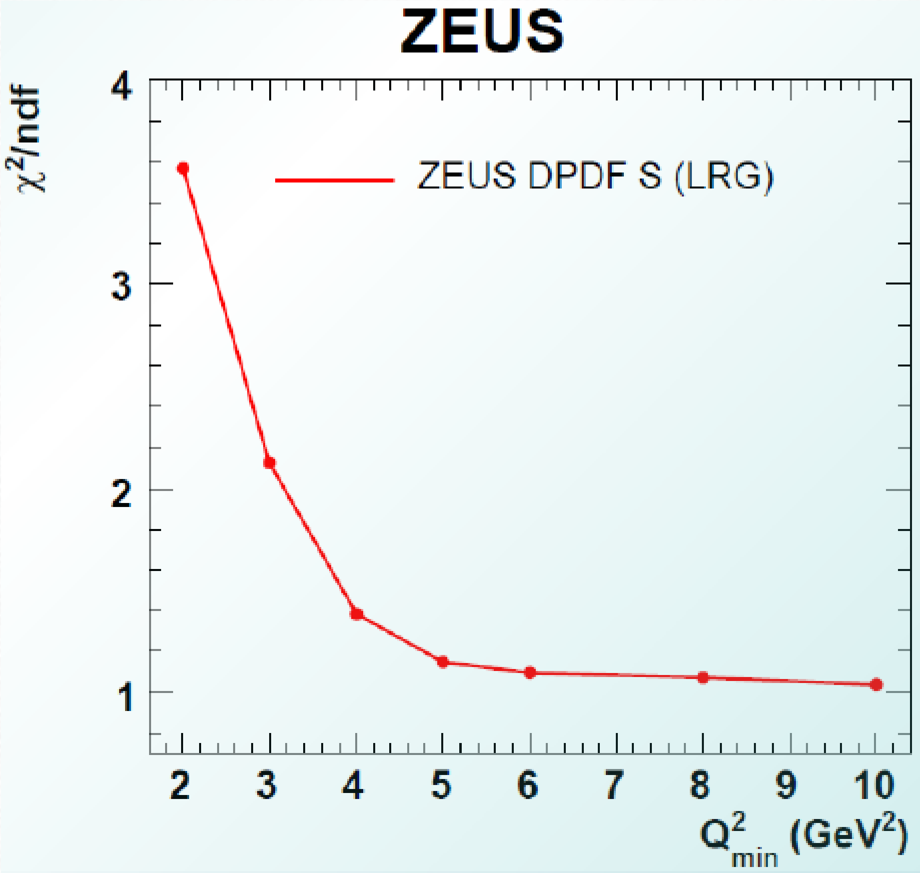
Chi2/dof(NNLO)= 1150/1001

Voica Radescu| DESY |DIS - Warsaw| HERAPDF2.0 13

Higher twists in DIS?

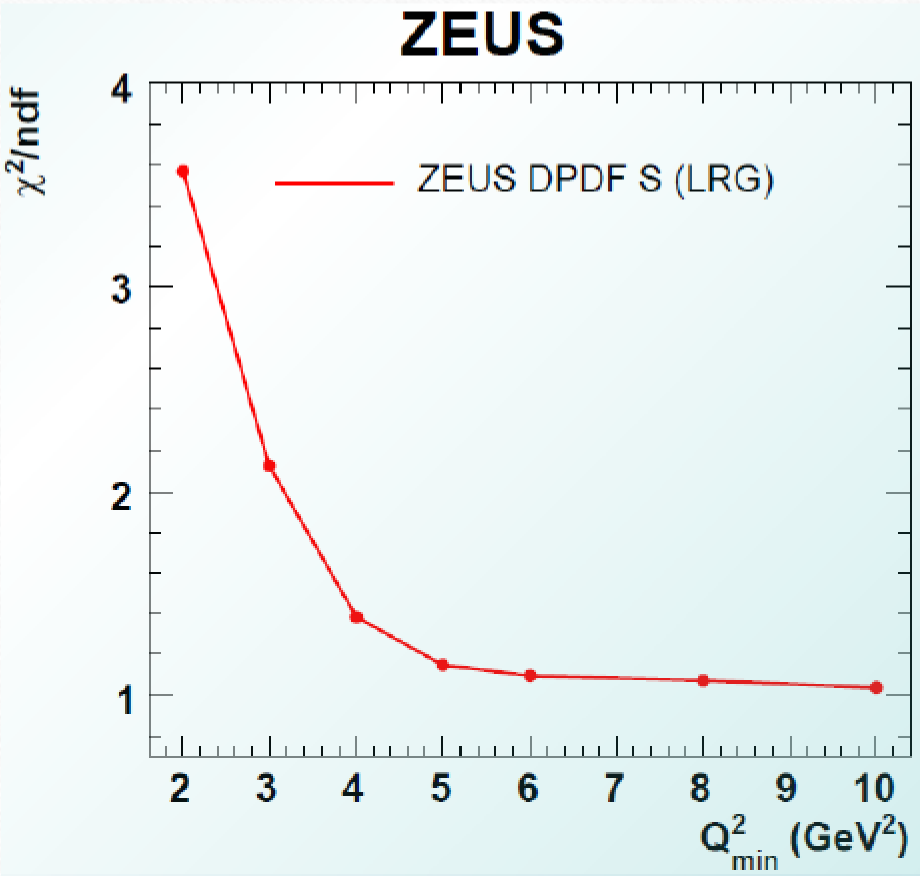


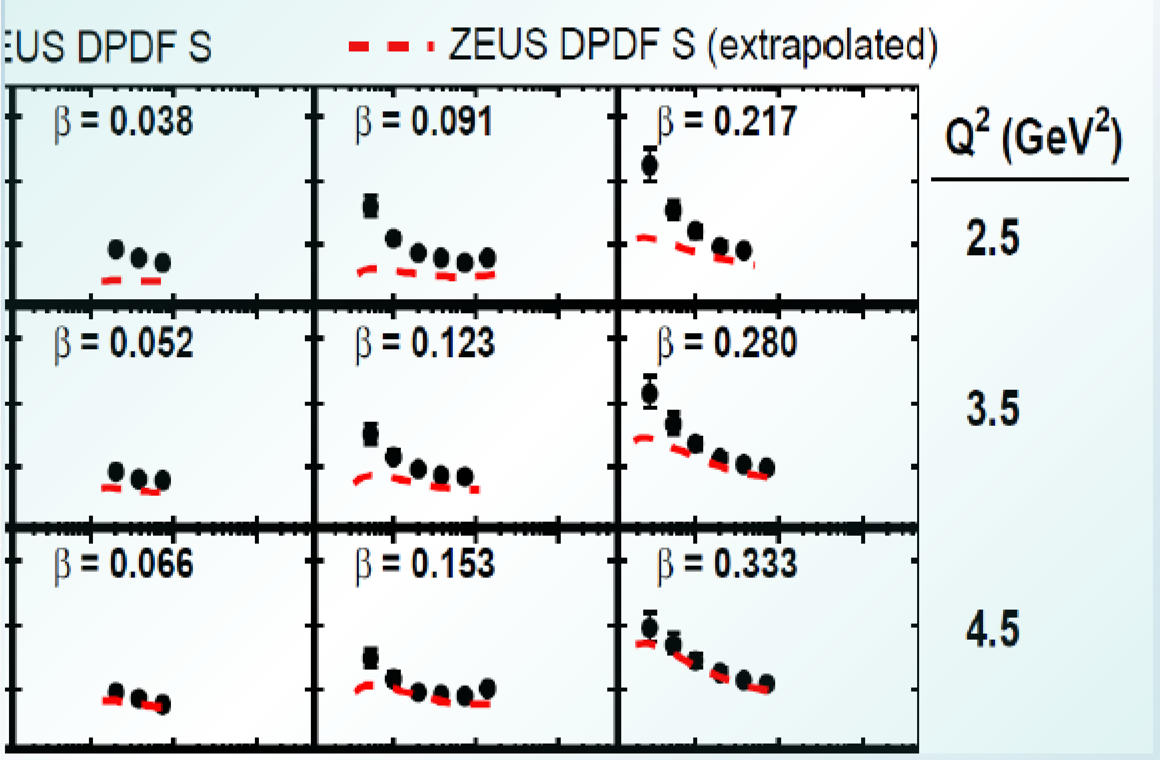
Higher twits in diffractive DIS



- Extensions of DGLAP fits below 5 GeV^2 fail
- Strong DGLAP breaking effects below 3 GeV^2
- Rapid growth with desreasing Q^2

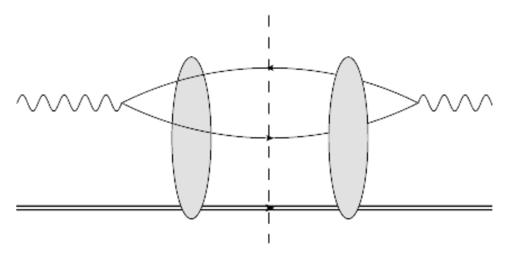
Higher twits in diffractive DIS

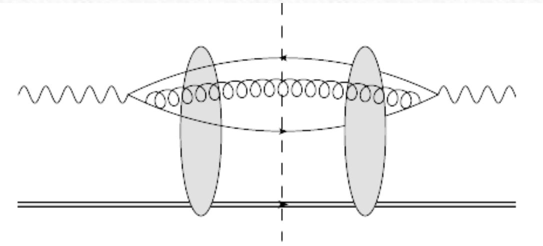


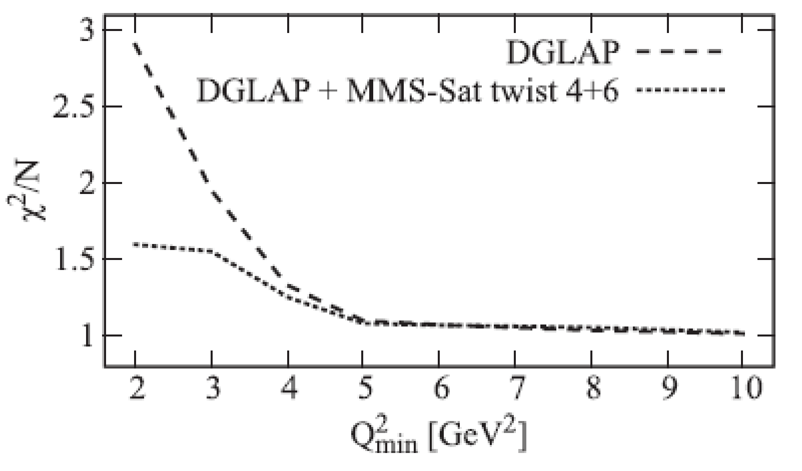


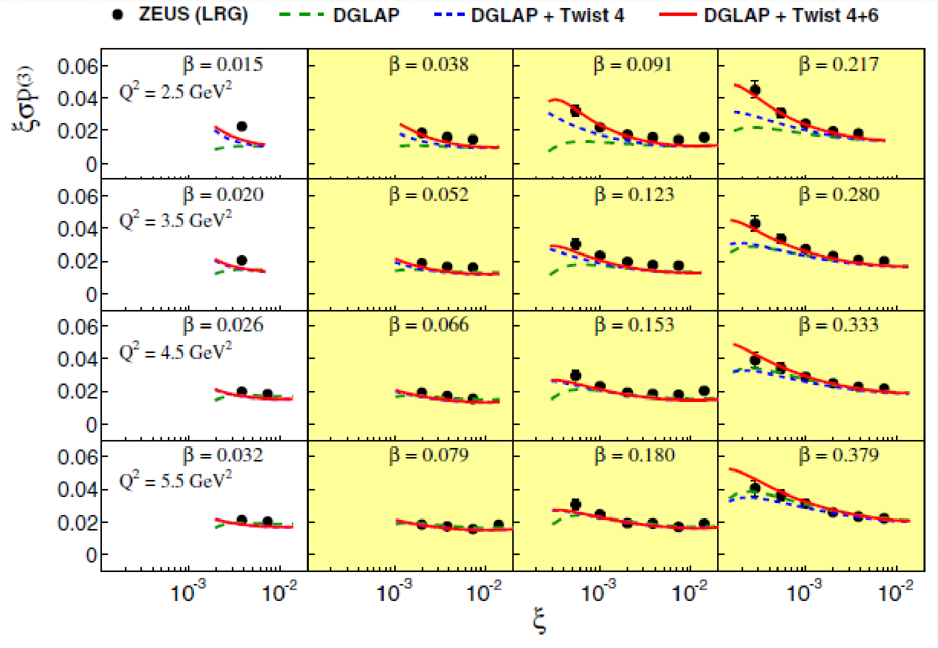
•Extensions of DGLAP fits below 5 GeV2 fail
•Strong DGLAP breaking effects below 3 GeV2
•Rapid growth with desreasing Q2

HT and DDIS – higher twists?



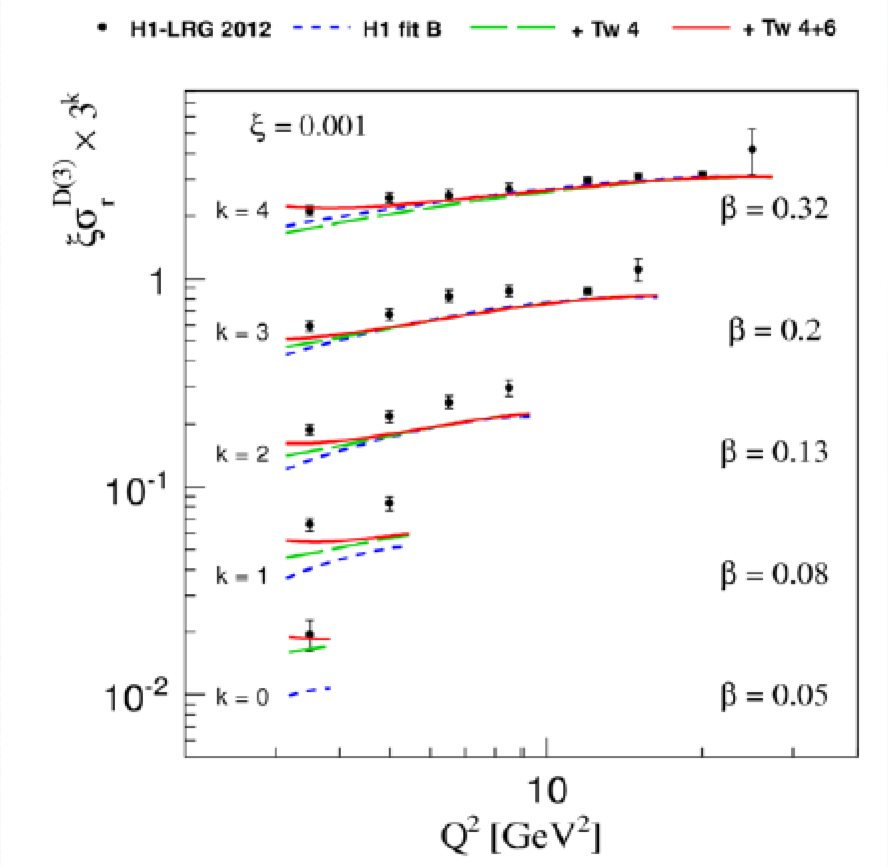






L. Motyka, MS, and W. Słomiński
Phys. Rev. **D86**, 111501(R)

HT and DDIS – higher twists?



Break-down of DGLAP description visible in H1 data

Applied: the model of higher twist terms tunned to ZEUS data

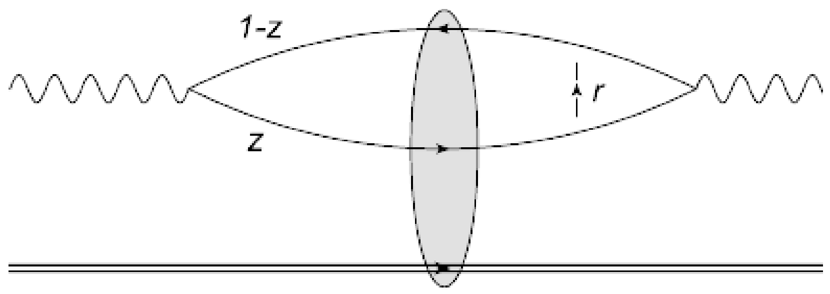
Inclusion of higher twists improves description of data

However, weaker effect in combined data of H1 and ZEUS

Colour dipole picture

•High energy limit of QCD = leading contributions in 1/s expansion
•Leads (at LO) to a simple picture in target rest frame
  - long-living fluctuations: colour dipoles
  - short interaction time
  - parton energy – z is conserved
  - parton transverse position do not change
•Simple factorized form of cross-section in position space:

Perturbative object:
virtual photon wave-function



Universal object: dipole cross-section

$$\sigma^{\gamma^* p}_{\mathrm{T,L}}(x, Q^2) = \int d^2\boldsymbol{r} \int_0^1 dz \left|\psi_{\mathrm{T,L}}(z, r, Q^2)\right|^2 \sigma_{q\bar{q}}(x, \boldsymbol{r})$$

In Mellin space:

$$\sigma^{\gamma^* p}_{\mathrm{T,L}}(x, Q^2) = \frac{1}{2\pi i} \int_{C_f} ds \left(\frac{Q_0^2}{Q^2}\right)^{-s} H_{\mathrm{T,L}}(-s)\tilde{\sigma}_{q\bar{q}}(s, Y)$$

Saturation model (Golec-Biernat and Wusthoff)

Simple idea: interpolation between QCD single scattering cross-section and unitarity (total absorption) limit, based on assumption of independence of multiple scatterings

$$\sigma(x,r) = \sigma_0 \left[1 - \exp\left(-\frac{r^2}{4R_0^2(x)}\right)\right]$$

$$R_0(x) = \frac{1}{Q_0}\left(\frac{x}{x_0}\right)^{\lambda/2}, \qquad \lambda \simeq 0.3$$

$$\tilde{\sigma}(x,s) = \sigma_0 \int_0^\infty dr^2 (r^2)^{s-1}\{1 - \exp(-r^2 Q_{\rm sat}^2(x)/4)\}$$
$$= -\sigma_0 \left(\frac{Q_{\rm sat}^2}{4}\right)^{-s} \Gamma(s).$$

$$\sigma_T^{(\tau=2)} = \frac{\alpha_{\rm em}\sigma_0}{\pi}\langle e^2\rangle \frac{Q_{\rm sat}^2}{Q^2}\{\log(Q^2/Q_{\rm sat}^2) + \gamma_E + 1/6\}$$

$$\sigma_L^{(\tau=2)} = \frac{\alpha_{\rm em}\sigma_0}{\pi}\langle e^2\rangle \frac{Q_{\rm sat}^2}{Q^2}$$

$$\sigma_T^{(\tau=4)} = \frac{3}{5}\frac{\alpha_{\rm em}\sigma_0}{\pi}\langle e^2\rangle \frac{Q_{\rm sat}^4}{Q^4}$$

$$\sigma_L^{(\tau=4)} = -\frac{4}{5}\frac{\alpha_{\rm em}\sigma_0}{\pi}\langle e^2\rangle \frac{Q_{\rm sat}^4}{Q^4} \times \{\log(Q^2/Q_{\rm sat}^2) + \gamma_E + 1/15\}$$

J. Bartels, K. Golec-Biernat, K. Peters, Eur. Phys. J. **C17** (2000) 121;
J. Bartels, K. Golec-Biernat, L. Motyka Phy. Rev. **D81**, (2010) 054017.

Twist decomposition from BFKL equation

Dipole cross-section: $\sigma_{q\bar{q}}(x, r) = 2\int d^2b\, N(x, r, b) \equiv 2\pi R_p^2 N(Y, r)$

imaginary part of the forward dipole-nucleon scattering amplitude

$$N(x, r) = \frac{1}{2\pi i}\int_{C_f} ds\, r^{-2s} C(s) e^{\bar{\alpha}_s \chi(s) Y}$$

$$\chi(s) = 2\psi(1) - \psi(-s) - \psi(1+s)$$

$$\bar{\alpha}_s = \frac{N_c \alpha_s}{\pi}$$

LL BFKL characteristic function

C(s) depends on initial conditions:

$$N(Y = 0, r) = 1 - \exp\left(-\frac{r^2 Q_0^2}{4}\right)$$

Dipole cross-section in Mellin space:

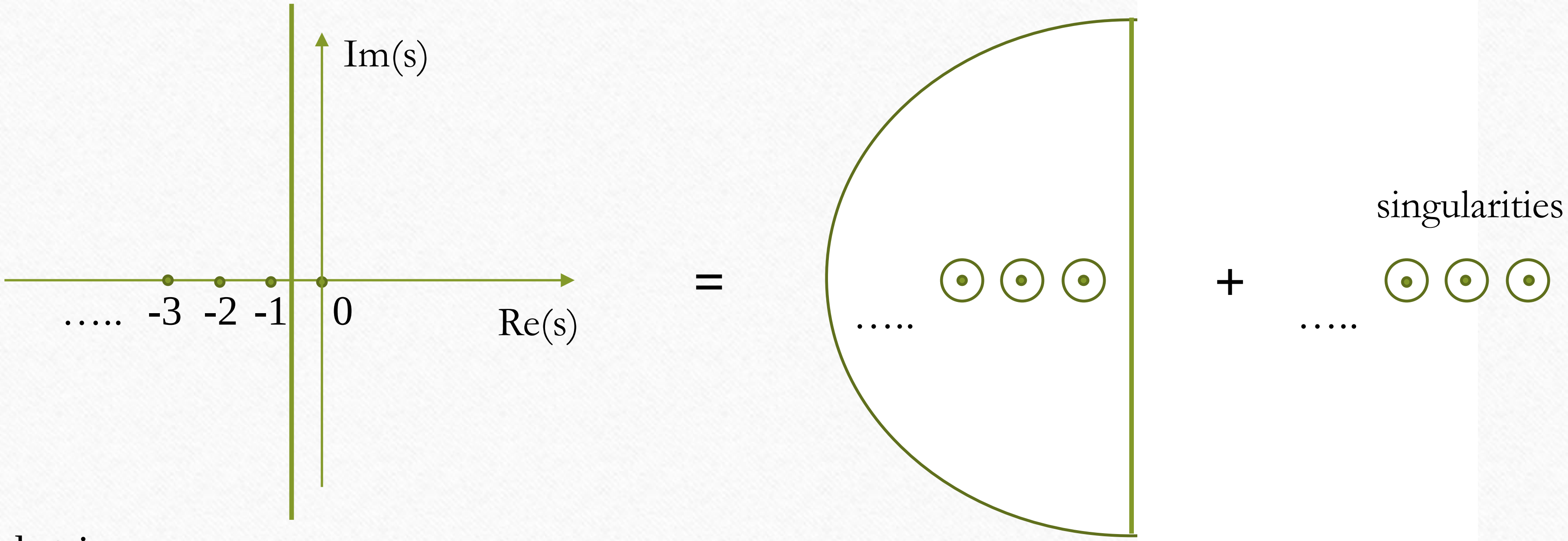
$$\tilde{\sigma}_{q\bar{q}}(s, Y) = -2\pi R_p^2 \Gamma(s) e^{\bar{\alpha}_s \chi(s) Y}$$

χ possess simple poles at integer s

Twist decomposition from BFKL equation

$$\sigma^{\gamma^* p}_{\mathrm{T,L}}\left(x,Q^2\right) = \frac{1}{2\pi i}\int_{C_f} ds \left(\frac{Q_0^2}{Q^2}\right)^{-s} H_{\mathrm{T,L}}(-s)\tilde{\sigma}_{q\bar{q}}(s,Y)$$

$$\tilde{\sigma}_{q\bar{q}}(s,Y) = -2\pi R_p^2 \Gamma(s) e^{\bar{\alpha}_s \chi(s) Y}$$



Fundamental strip:

$C_f = (-1/2 - i\infty, -1/2 + i\infty)$

$$\boxed{\sigma^{\gamma^* p}_{\mathrm{T,L}}\left(x,Q^2\right) = \sum_{n=1}^{\infty} \sigma^{(2n)}_{\mathrm{T,L}}\left(x,Q^2\right)}$$

Twist decomposition from BFKL equation

$$\sigma^{(2n)}_{\mathrm{T,L}}\left(x,Q^2\right) = -R_p^2 e^{-nt}\int\limits_0^{2\pi} d\theta h^{(n)} \exp\left(\epsilon t e^{i\theta} + \frac{\bar{\alpha}_{\mathrm{s}}Y}{\epsilon}e^{-i\theta}\right)$$

$$h^{(n)} = A_0 \sum_{m=-1}^{\infty} a_m^{(2n)\mathrm{T,L}}\left(\epsilon e^{i\theta}\right)^m$$

$$\int\limits_0^{2\pi} d\theta \exp\left(\epsilon t e^{i\theta} + \frac{\bar{\alpha}_{\mathrm{s}}Y}{\epsilon}e^{-i\theta} + im\theta\right) = \left(\frac{\bar{\alpha}_{\mathrm{s}}Y}{\epsilon^2 t}\right)^{m/2} I_{|m|}\left(2\sqrt{\bar{\alpha}_{\mathrm{s}}Y t}\right)$$

$$m = -1, 0, 1, 2, \ldots ,$$

$$\boxed{\sigma^{(2n)}_{\mathrm{T,L}}\left(x,Q^2\right) \;=\; -2\pi R_p^2 A_0 \left(\frac{Q_0}{Q}\right)^{2n} \sum_{m=-1}^{\infty} a_m^{(2n)\mathrm{T,L}}\left(\frac{\bar{\alpha}_{\mathrm{s}}Y}{t}\right)^{m/2} I_{|m|}\left(2\sqrt{\alpha_{\mathrm{s}}Y t}\right)}$$

$$t = \ln Q^2/Q_0^2 \qquad A_0 \;=\; N_c \alpha_{\mathrm{em}} \textstyle\sum_f e_f^2/\pi$$

Twist decomposition from BFKL equation

Expansion coefficients:

$$
\begin{aligned}
a_{-1}^{(2)\mathrm{T}} &= -\frac{1}{3}, \qquad a_0^{(2)\mathrm{T}} = -\frac{1+6\gamma_E}{18}, \\
a_1^{(2)\mathrm{T}} &= -\frac{112-3\pi^2+6\gamma_E(1+3\gamma_E)}{108} - \frac{2}{3}\zeta(3)\bar{\alpha}_s Y, \\
a_2^{(2)\mathrm{T}} &= -\frac{124-3\pi^2+6\gamma_E\left(112+3\gamma_E+6\gamma_E^2-3\pi^2\right)+72\zeta(3)}{648} \\
&\quad -\frac{1}{9}(1+6\gamma_E)\zeta(3)\bar{\alpha}_s Y, \\
a_{-1}^{(2)\mathrm{L}} &= 0, \qquad a_0^{(2)\mathrm{L}} = -\frac{1}{3}, \qquad a_1^{(2)\mathrm{L}} = -\frac{-4+3\gamma_E}{9}, \\
a_2^{(2)\mathrm{L}} &= -\frac{148-3\pi^2+6\gamma_E(3\gamma_E-8)}{108} - \frac{2}{3}\zeta(3)\bar{\alpha}_s Y,
\end{aligned}
$$

$$
\begin{aligned}
a_{-1}^{(4)\mathrm{T}} &= 0, \qquad a_0^{(4)\mathrm{T}} = -\frac{1}{5}e^{-2\bar{\alpha}_s Y}, \\
a_1^{(4)\mathrm{T}} &= -e^{-2\bar{\alpha}_s Y}\left(\frac{37+30\gamma_E}{150} - \frac{2}{5}\zeta(3)\bar{\alpha}_s Y\right), \\
a_2^{(4)\mathrm{T}} &= -e^{-2\bar{\alpha}_s Y}\left(\frac{2144+30\gamma_E(37+15\gamma_E)-75\pi^2}{4500}\right. \\
&\quad \left. -\frac{67+30\gamma_E-30\zeta(3)}{75}\bar{\alpha}_s Y + \frac{2}{5}(\bar{\alpha}_s Y)^2\right), \\
a_{-1}^{(4)\mathrm{L}} &= \frac{4}{15}e^{-2\bar{\alpha}_s Y}, \qquad a_0^{(4)\mathrm{L}} = e^{-2\bar{\alpha}_s Y}\left(\frac{4(1+15\gamma_E)}{225} - \frac{8}{15}\bar{\alpha}_s Y\right), \\
a_1^{(4)\mathrm{L}} &= e^{-2\bar{\alpha}_s Y}\left(\frac{949+30\gamma_E(2+15\gamma_E)-75\pi^2}{3375}\right. \\
&\quad \left. -\frac{8(16+15\gamma_E-15\zeta(3))}{225}\bar{\alpha}_s Y + \frac{8}{15}(\bar{\alpha}_s Y)^2\right).
\end{aligned}
$$

Series is quickly convergent in a broad kinematical range: $2 < Y < 7$, $1 < t < 10$

Twist decomposition from BFKL equation

Lowest order approximation gives a saddle point approximation of the integral:

$$
\begin{aligned}
\sigma_{\mathrm{T}}^{(2)} &= R_p^2 A_0 \sqrt{\pi} \left(\frac{Q_0}{Q}\right)^2 \frac{t^{1/4} e^{2\sqrt{\bar{\alpha}_s Y t}}}{3(\bar{\alpha}_s Y)^{3/4}} , \\
\sigma_{\mathrm{L}}^{(2)} &= R_p^2 A_0 \sqrt{\pi} \left(\frac{Q_0}{Q}\right)^2 \frac{e^{2\sqrt{\bar{\alpha}_s Y t}}}{3(\bar{\alpha}_s Y t)^{1/4}} , \\
\sigma_{\mathrm{T}}^{(4)} &= R_p^2 A_0 \sqrt{\pi} \left(\frac{Q_0}{Q}\right)^4 \frac{e^{2\sqrt{\bar{\alpha}_s Y t} - 2\bar{\alpha}_s Y}}{5(\bar{\alpha}_s Y t)^{1/4}} , \\
\sigma_{\mathrm{L}}^{(4)} &= -R_p^2 A_0 \sqrt{\pi} \left(\frac{Q_0}{Q}\right)^4 \frac{4 t^{1/4} e^{2\sqrt{\bar{\alpha}_s Y t} - 2\bar{\alpha}_s Y}}{15(\bar{\alpha}_s Y)^{3/4}}
\end{aligned}
$$

$\exp(-n\alpha_s Y)$

Such factor is present in the higher twist-*2n* coefficients. This term is responsible for the suppression of higher twist terms at small *x* values, contrary to expectations of the eikonal approach.

Twist decomposition from BFKL equation

The dipole cross section depends on 4 parameters: the fixed strong coupling constant, the initial saturation scale assumed at x=xin, and the effective proton radius.

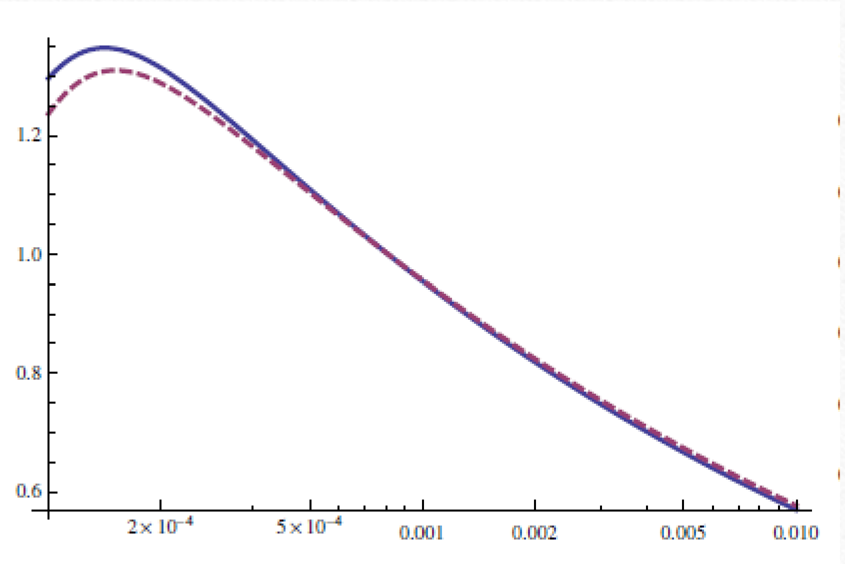
$\sigma_0' = 2\pi R_p^2 = 17.04$ mb, $Q_0 = 0.67$ GeV and the strong coupling constant $\bar{\alpha}_s = 0.083$ which corresponds to the intercept $\lambda = \bar{\alpha}_s 4 \ln 2 = 0.23$ (around 80% of the GBW value). For numerical studies, we assume $x_{in} = 0.1$. Ad-

$$\sigma_r = F_2 - \frac{y^2}{Y_+} F_L\,, \qquad Y_+ = 1 + (1-y)^2\,, \qquad y = \frac{Q^2}{xs}$$

Q^2=10 GeV^2

BFKL – solid line

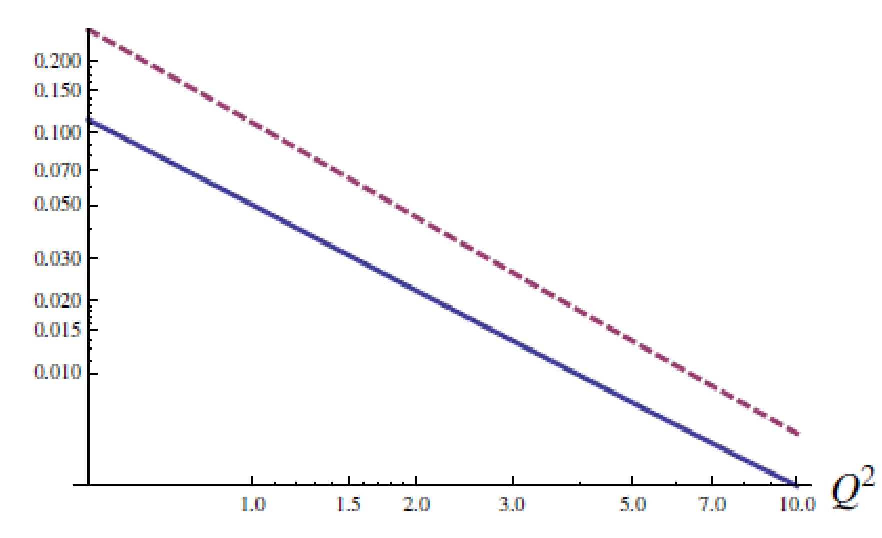
GBW – dashed line

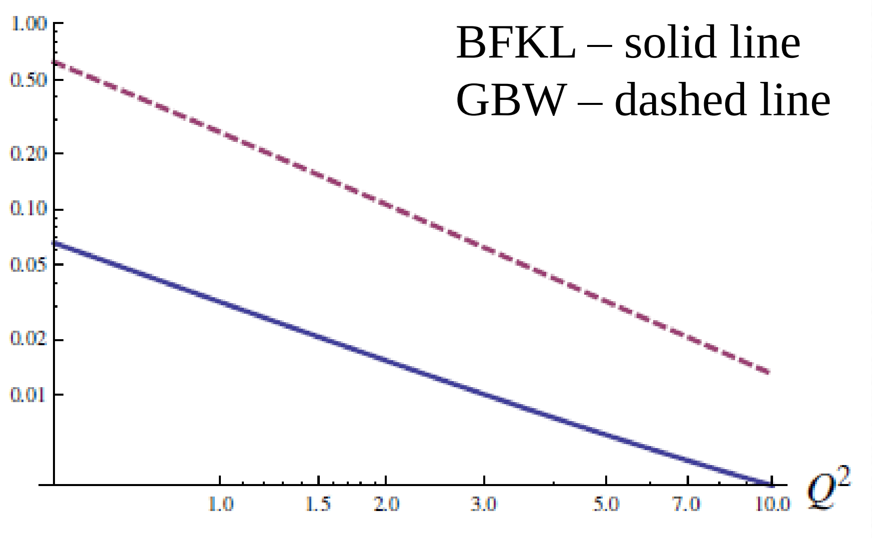


Comparison between BFKL and GBW approach

$$R^{(2,\dots,2k)}_{\mathrm{T,L}} = 1 - \frac{\sum_{i=1}^{k} \sigma^{(2i)}_{\mathrm{T,L}}}{\sigma^{\gamma^* p}_{\mathrm{T,L}}}$$

$R_T^{(2)}$





BFKL – solid line
GBW – dashed line

x=0.01, Q_0^2=0.36 GeV^2

x=0.001, Q_0^2=0.71 GeV^2

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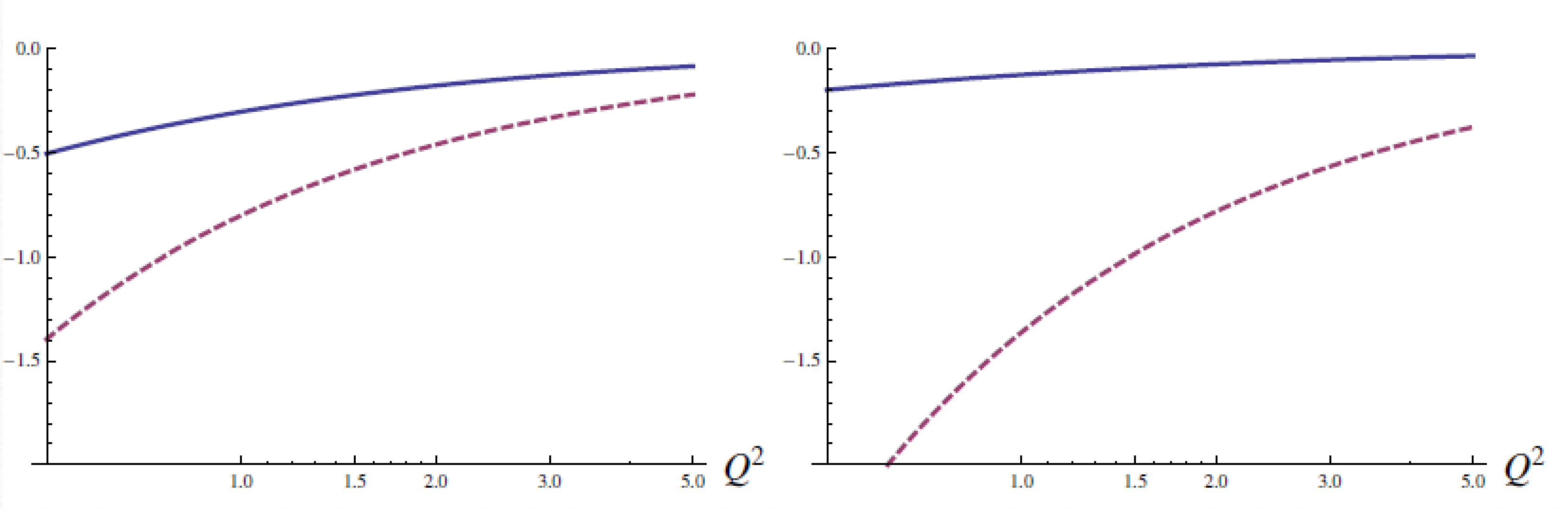
GBW – dashed line

$R_L^{(2)}$



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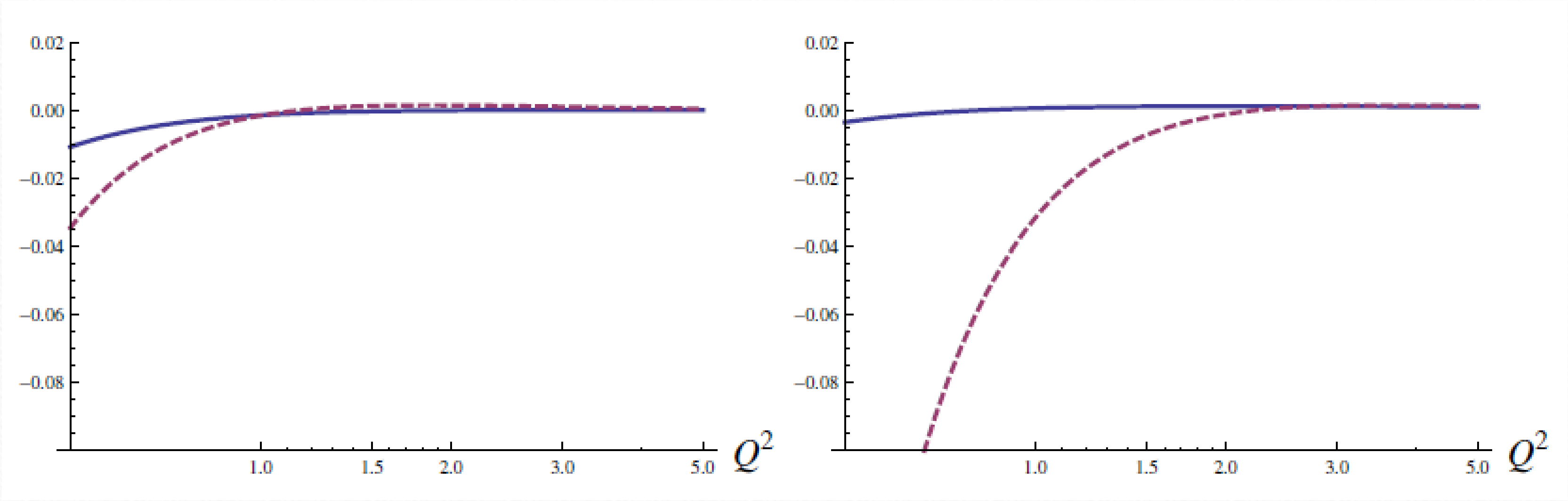
$R_T^{(2,4)}$



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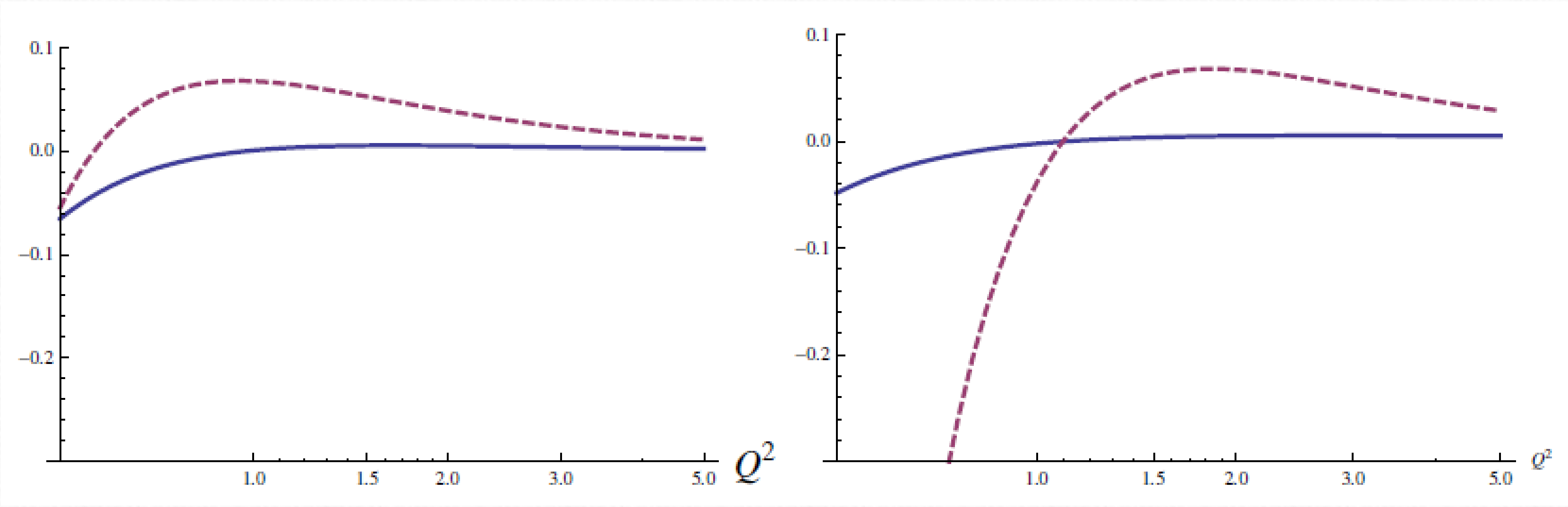
$R_L^{(2,4)}$



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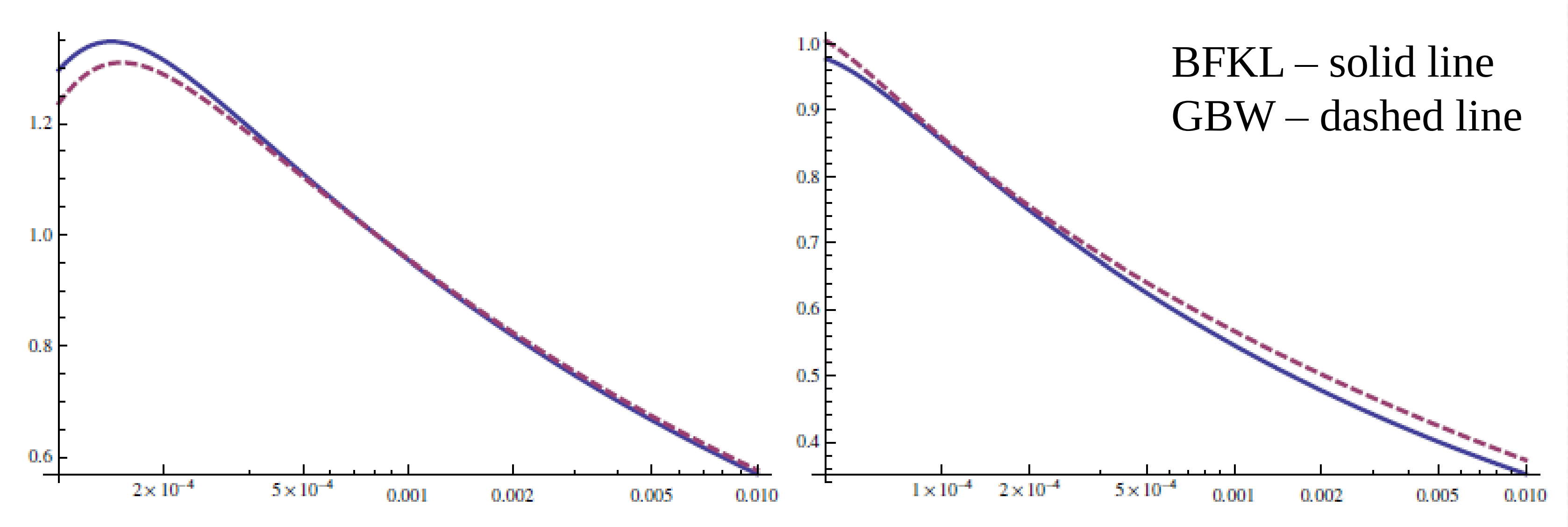

Q^2=10 GeV^2 Q^2=2 GeV^2

Twist-2 reduced cross-section of BFKL found to be closed to twists-2+4 of GBW

BFKL vs BK

- The basis: Balitsky-Fadin-Kuraev-Lipatov equation: resumation of leading log(1/x)

- Unitarity: change of the number of t-channel gluons possible: J. Bartels, C. Ewerz, I. I. Balitsky, Y. V. Kovchegov

- Double logarithmic limit – triple pomeron vertex vanishes: J. Bartels, K. Kutak

- Consistent with colinear quasi-multipartonic evolution by Bukhvostov, Frolov, Lipatov, Kuraev, 1985.

- Twist evolution in BK reduces to BFKL/DGLAP evolution at LLA accuracy.

- In this limit saturation effects appear in inputs for twist evolution

  $\longrightarrow$ **the obtained higher twist rapidity dependence applies to BK as well.**

Conclusions

- Higher twist contributions are sensitive to multi-parton configurations/correlations: more information on proton structure and corrections to DGLAP/parton densities.

- Analysis of twist content of DIS cross-section was performed in BFKL/BK framework

- New parametrisations of higher twist terms is proposed.

- There is a clear difference between BFKL/BK and GBW in magnitude of the higher twist corrections. The higher twists are strongly suppressed in BFKL/BK approach with decreasing x in contrast to GBW model and eikonal picture in general.

- The dipole model provides a reasonable approximation for DIS/DDIS cross-sections. With better data the higher twists analysis can discriminate between eikonal and BFKL/BK pictures of multiple scattering and parton saturation.