Large b behaviour and CGC/saturation approach: the BFKL equation with pion loops.

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EDS Blois 2015, Borgo, Corsica, June 29 - July 4, 2015

Papers: E.L.:

- *"Large* $\mathbf{b}$ *behaviour in the CGC/saturation approach: BFKL equation with pion loops,"* Phys. Rev. D **91** (2015) 5, 054007 [arXiv:1412.0893 [hep-ph]].

Goals:

1. To find a theoretical way to introduce the correct large b behaviour $\propto \exp(-\mu b)$ into CGC/saturation formalism;

2. To study how the suggested procedure will affect the features of the BFKL Pomeron: spectrum, shrinkage of the diffraction peak, large b behaviour and so on;

BFKL Pomeron: large b behaviour

- $$N\left(r_1, r_2; Y, b\right) = \int \frac{d\gamma}{2\,\pi\,i}\,\phi^{(0)}_{in}(\nu)\,e^{\omega(\gamma = \frac{1}{2} + i\nu, 0)\,Y}$$
$$\times \left\{ b_\nu\,(w w^*)^{\frac{1}{2} + i\nu} + b_{-\nu}\,(w w^*)^{\frac{1}{2} - i\nu} \right\} \xrightarrow{\nu \ll 1} \frac{r_1\,r_2}{b^2}\,e^{\omega_0\,Y}$$

- $$w\,w^* = \frac{r_1^2\,r_2^2}{\left(\vec{b} - \frac{1}{2}\left(\vec{r}_1 - \vec{r}_2\right)\right)^2\left(\vec{b} + \frac{1}{2}\left(\vec{r}_1 - \vec{r}_2\right)\right)^2}$$

$$N\left(r_1, r_2; Y, b\right) \le 1 \quad \text{for} \quad b^2 \le r_1\,r_2 e^{\omega_0 Y}$$

Violation of Froissart theorem: (Kovner & Wiedemann, 2002-2003)

$$\int d^2b\,N\left(r_1, r_2; Y, b\right) \propto s^{\omega_0} \gg Y^2 = ln^2 s$$

BFKL Pomeron: shrinkage of the diffraction peak

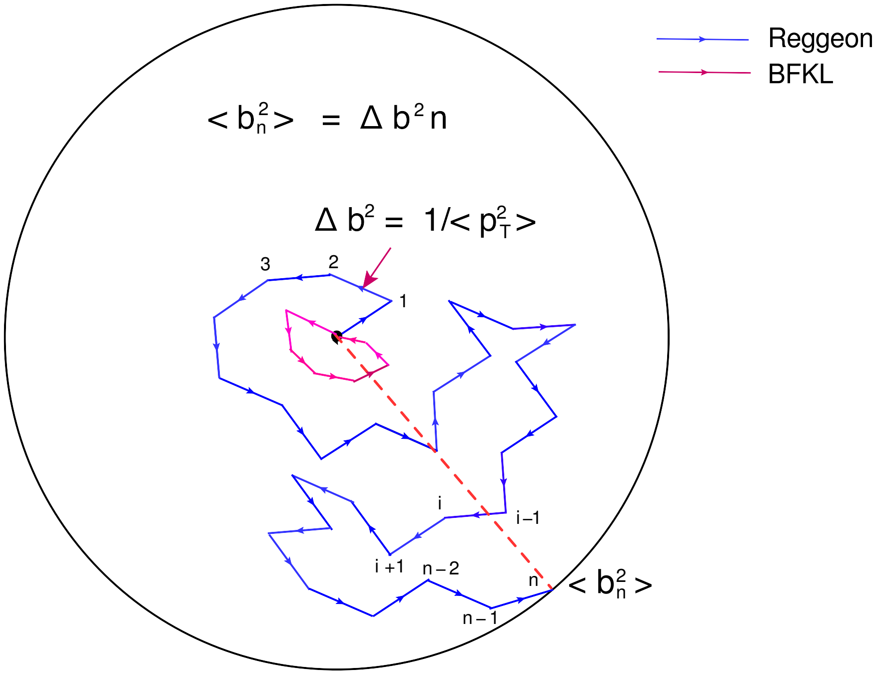
Froissart theorem for Regge pole:

$$g_1 g_2 s^{\alpha_{I\!P}(t)} \rightarrow g_1 g_2 \exp\left(\Delta Y - b^2/(4\alpha'_{I\!P}\,Y)\right):$$

$$N\,(Y,b) \leq 1 \quad \text{for} \quad b^2 \leq 4\alpha'_{I\!P}\,\Delta\,Y^2$$

$$\int d^2b\,N\,(Y,b) \propto \alpha'_{I\!P}\Delta Y^2 \qquad Y^2 = ln^2 s$$

BFKL Pomeron: $\alpha'_{I\!P} = 0$ since $\langle p_T \rangle \propto s^{\lambda}$



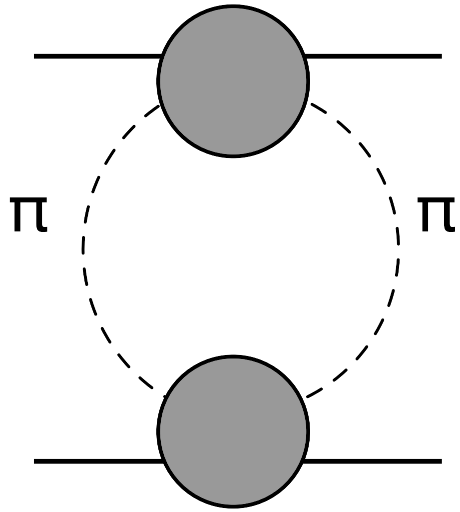
$\Longleftarrow$ **Gribov's diffusion**

$$< b^2 > \propto \frac{1}{< p_T^2 >} \ln s$$

However, introducing a non-perturbative $< p_T > = \mu$ at Y =0, we can have Gribov's diffusion with this p_T with probability: (E.L. & Ryskin, 1989)

$$P_{\text{BFKL}}(Y, p_t = \mu) \propto \frac{1}{\sqrt{lns}}; \qquad \langle b^2 \rangle \propto P_{\text{BFKL}} \frac{1}{\mu^2} \ln s = \frac{1}{\mu^2} \sqrt{\ln s}$$

We wish to animate these ideas developing a systematic approach



• The pions exchange lead to $\exp\left(-2\mu b\right)$ in the proof of the Froissart theorem

Our approach Is based on:

1. the assumption that the BFKL Pomeron takes into account short distances $\propto 1/m_G$, where m_G is the mass of the lightest glueball);

2. the long distance contribution can be described by the exchange of pions $(1/2\mu \gg 1/m_G)$.

BFKL Pomeron

- $\text{Im}\, G_{I\!P}\,(Y, t, r_1, r_2) = r_1\, r_2 \int_{\epsilon - i\infty}^{\epsilon + i\infty} \frac{d\omega}{2\pi i}\, e^{\omega\, Y}\; G_{I\!P}\,(\omega, Q_T; r_1, r_2)$

(Lipatov, 1986)

- $G_{I\!P}\,(\omega, Q_T; r_1, r_2)\,\Big|_{\substack{\omega \to \omega_0 \\ Q_T \to 0}} = \frac{\pi}{\kappa_0} \left\{ \left(\frac{r_1}{r_2}\right)^{\kappa_0} + \left(\frac{r_2}{r_1}\right)^{\kappa_0} - 2\, \left(Q_T^2\, r_1\, r_2\right)^{\kappa_0} \right\}$
- $\omega = 2\,\bar{\alpha}_S\,\left(\psi(1) - Re\,\psi\left(\frac{1}{2} - \kappa_0\right)\right) = \omega_0 + D\,\kappa_0^2 + \mathcal{O}\left(\kappa_0^3\right)$
- $\kappa_0 = \sqrt{\frac{(\omega - \omega_0)}{D}}$ **with** $\omega_0 = 4\,\ln 2\,\bar{\alpha}_S$ **and** $D = 14\,\zeta\,(3)\;\bar{\alpha}_S$

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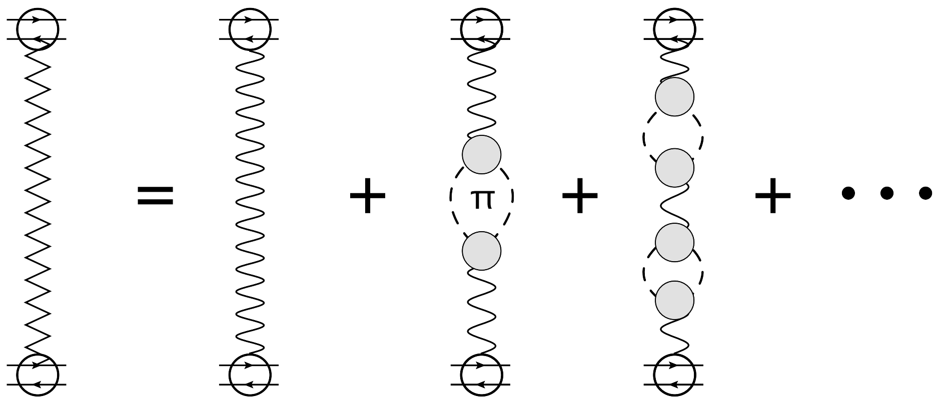
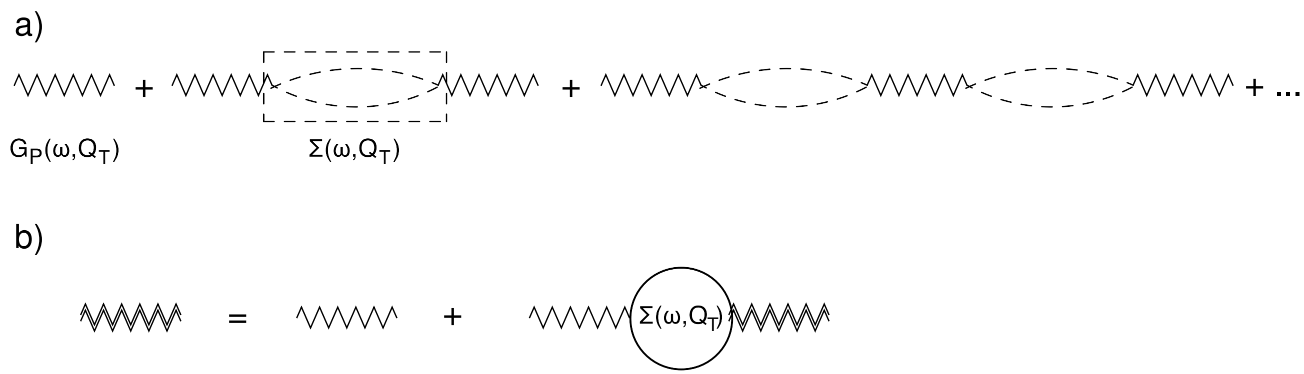
- **If** $|(\omega - \omega_0)\;\ln^2\left(Q_T^2 r_1 r_2\right)| \gg 1$, $G_{I\!P}\,(\omega, Q_T; r_1, r_2) \to 1/\sqrt{\omega - \omega_0}$
- **If** $|(\omega - \omega_0)\;\ln^2\left(Q_T^2 r_1 r_2\right)| \sim 1$, $G_{I\!P}\,(\omega, Q_T; r_1, r_2) \to \sqrt{\omega - \omega_0}$
- $A\,(Y, Q_T) = \int d^2 r'_1 \int d^2 r'_2\; |\Psi\,(r'_1)\,|^2\, |\Psi\,(r'_2)\,|^2\; \text{Im}\, G_{I\!P}\,\left(Y, t = -Q_T^2; r'_1, r'_2\right)$

Summing pion loops





$$G_{I\!P}\left(\omega, Q_T; R_\pi, R_\pi\right) = G_{I\!P}\left(\omega, Q_T; R_\pi, R_\pi\right) + G_{I\!P}\left(\omega, Q_T; R_\pi, R_\pi\right)\,\Sigma\left(\omega, Q_T\right) G_{I\!P}\left(\omega, Q_T; R_\pi, R_\pi\right)$$

$$G_{I\!P}\left(\omega, Q_T; R_\pi, R_\pi\right) = 1/\left(G^{-1}_{I\!P}\left(\omega, Q_T; R_\pi, R_\pi\right) - \Sigma\left(\omega, Q_T\right)\right)$$

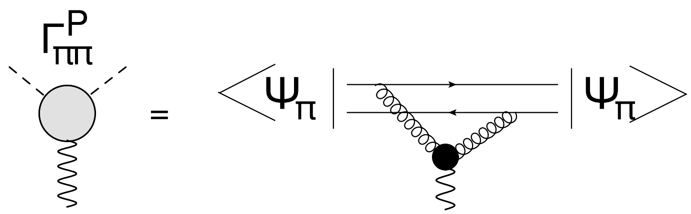
$\Sigma\left(\omega, Q_T\right)$

t-channel unitarity: (Anselm & Gribov , 1972)

$$i\{G_{I\!P}\left(\omega, t+i\epsilon; R_\pi, R_\pi\right) - G_{I\!P}\left(\omega, t-i\epsilon; R_\pi, R_\pi\right)\} = \rho\left(\omega, t\right) G_{I\!P}\left(\omega, t+i\epsilon; R_\pi, R_\pi\right)\, G_{I\!P}\left(\omega, t-i\epsilon; R_\pi, R_\pi\right)$$

$$\rho\left(\omega, t\right) = \left(t - 4\mu^2\right)^{\frac{3}{2}+\omega} / \sqrt{t}$$

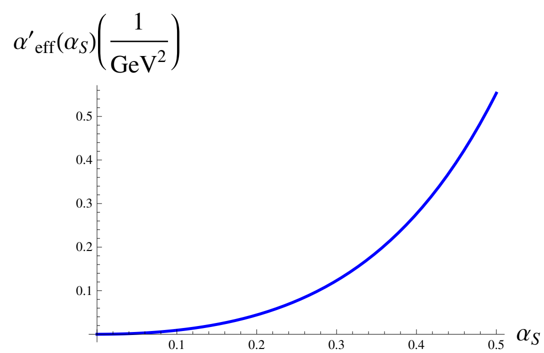
- $$\mathrm{Im}_t \Sigma\left(\omega, t\right) = \frac{1}{2}\rho\left(\omega, t\right)\left(\Gamma^{I\!P}_{\pi\pi}\right)^2 \frac{1}{\omega+2}$$



$$\Gamma^{I\!P}_{\pi\pi} = \frac{8}{9}\bar{\alpha}_S \int d^2r\, r\, |\Psi_\pi\left(r\right)|^2 = \frac{8}{9}\,\bar{\alpha}_S\, R_\pi$$

- $$\Sigma\left(\omega, t\right) = \frac{t}{\pi}\int \frac{dt'\, \mathrm{Im}_t \Sigma\left(\omega, t'\right)}{t'\left(t' - t\right)} = \frac{3}{2}\,\frac{t}{\omega+2}\,\frac{\left(\Gamma^{I\!P}_{\pi\pi}\right)^2}{\pi}\int_{4\mu^2}^{m_G^2} \frac{dt'\, \rho\left(\omega, t'\right)}{t'\left(t' - t\right)}$$

- $\Sigma\left(\omega, t\right)\Big|_{Q_t \to 0} = 0$;(E.L.,Lipatov & Siddikov, 2014)
- $\Sigma\left(\omega, t\right)\Big|_{Q_t \to 0} = -\,\alpha'_{eff}\left(\bar{\alpha}_S\right)\, Q_T^2$
- $G_{I\!P}\left(\omega, Q_T; R_\pi, R_\pi\right) = \dfrac{\pi\sqrt{D}}{\sqrt{\omega - \omega_0} + \alpha'_{eff}\pi\sqrt{D}\, Q_T^2}$



Shrinkage of the diffraction peak:

- $$B = 2\frac{d\,\mathrm{Im}\,G_{I\!P}(Y,Q_T;R_\pi,R_\pi)\Big/ dQ_T^2|_{Q_T=0}}{\mathrm{Im}\,G_{I\!P}(Y,Q_T;R_\pi,R_\pi)|_{Q_T=0}} = 2\,\frac{\alpha'_{eff}\sqrt{D}\pi\int_C \frac{d\,\delta\omega}{2\pi i}\,e^{\omega_0\,Y}\,e^{\delta\omega\,Y}\frac{\pi\sqrt{D}}{\delta\omega}}{\int_C \frac{d\,\delta\omega}{2\pi i}\,e^{\omega_0\,Y}\,e^{\delta\omega\,Y}\frac{\pi\sqrt{D}}{\sqrt{\delta\omega}}}$$
- $$= 2\,\alpha'_{eff}\sqrt{D}\sqrt{Y} \equiv 2\,\alpha'_{eff}\sqrt{D}\sqrt{\ln s}$$

Large impact parameter behaviour of the scattering amplitude:

- $$\mathrm{Im}\,G_{I\!P}\,(Y,b;R_\pi,R_\pi) = 2\pi\,R_\pi^2\int_0^\infty Q_T dQ_T J_0\,(Q_T b)\int_C \frac{d\,\delta\omega}{2\pi i}\,e^{\omega_0\,Y}\,e^{\delta\omega\,Y}\frac{\pi\sqrt{D}}{\sqrt{\delta\omega}\;+\;\alpha'_{eff}\pi\sqrt{D}\,Q_T^2}$$

$$(\delta\omega = \omega - \omega_0) = \frac{2\pi R_\pi^2}{\alpha'_{eff}}\int_C \delta\omega\,e^{\omega_0\,Y}\,e^{\delta\omega\,Y}\;K_0\left(\sqrt{\frac{\sqrt{\delta\omega}}{\alpha'_{eff}\,D\,\pi}}\;b\right)$$

- $$\mathrm{Im}\,G_{I\!P}\,(Y,b;R_\pi,R_\pi)\;\xrightarrow{b^2\gg\alpha'_{eff}}\;\frac{2\pi\,R_\pi^2}{\alpha'_{eff}}\int_C \delta\omega\,e^{\omega_0\,Y}\,e^{\delta\omega\,Y}\;\exp\left(-\sqrt{\frac{\sqrt{\delta\omega}}{\alpha'_{eff}\,D\,\pi}}\;b\right)$$
- $$\mathrm{Im}\,G_{I\!P}\,(Y,b;R_\pi,R_\pi)\;\propto\;e^{\omega_0\,Y}\;\exp\left(-3\left(\frac{b^4}{256\,\alpha'^2_{eff}\,D^2\,\pi^2\,Y}\right)^{\frac{1}{3}}\right)$$

Restoration of the Froissart theorem:

$$\sigma_{tot} = 2\int d^2b\,\mathrm{Im}A\,(s,b) \;=\; 2\int_0^{b_0} d^2b\,\mathrm{Im}A\,(s,b) \;+\; 2\int_{b_0}^{\infty} d^2b\,\mathrm{Im}A\,(s,b)$$

- $b \leq b_0 \quad \mathrm{Im}A\,(s,b) \leq 1$ while for $b \geq b_0 \quad \mathrm{Im}A\,(s,b) \ll 1$

- **Equation for b_0:** $\mathrm{Im}A\,(s,b) = f \ll 1$

- $\mathrm{Im}A\,(s,b) \propto \mathrm{Im}\,G_{I\!P}\,(Y, b; R_\pi, R_\pi) \xrightarrow{b^2 \gg \alpha'_{eff}} e^{\omega_0\,Y}\,\exp\left(-3\left(\frac{b^4}{256\,\alpha'^2_{eff}\,D^2\,\pi^2\,Y}\right)^{\frac{1}{3}}\right) = f$

$$b_0 = \frac{4}{3}\omega_0^{4/3}\sqrt{\alpha'_{eff}\,D\,\pi}\;\;Y$$

- $\sigma_{tot} \leq 4\pi b_0^2 \;+\; 4\,\pi\,f\,\frac{16\,D\,\pi}{3\sqrt{3}}\left(\alpha'_{eff}\,b_0\right)^{2/3} \sim Y^2 = \ln^2 s$

Dependence on the sizes of interacting dipoles:

- $G_{I\!P}(\omega, Q_T; r, R_\pi) = G_{I\!P}(\omega, Q_T, r, R_\pi) + G_{I\!P}(\omega, Q_T; r, R_\pi)\ \Sigma(\omega, Q_T)\ G_{I\!P}(\omega, Q_T; R_\pi, R_\pi)$

where

- $G_{I\!P}(\omega, Q_T; r, R_\pi) \;=\; \frac{\pi}{\kappa_0}\left\{\left(\frac{R_\pi}{r}\right)^{\kappa_0} + \left(\frac{r}{R_\pi}\right)^{\kappa_0} - 2\left(Q_T^2\, r\, R_\pi\right)^{\kappa_0}\right\}$
- Im $G_{I\!P}(Y, Q_T; r, R_\pi) \;=$
- $r\, R_\pi \sqrt{D}\ \int_C \frac{d\,\delta\omega}{2\pi i}\, e^{\omega_0\, Y}\, e^{\delta\omega\, Y} \frac{\left\{\left(\frac{R_\pi}{r}\right)^{\sqrt{\delta\omega/D}} + \left(\frac{r}{R_\pi}\right)^{\sqrt{\delta\omega/D}} - 2\left(Q_T^2\, r\, R_\pi\right)^{\sqrt{\delta\omega/D}}\right\}}{\sqrt{\delta\omega} + \alpha'_{eff}\pi\sqrt{D}\, Q_T^2}$

$Q_T = 0\ \rightarrow$

$$\text{Im}\, G_{I\!P}(Y, Q_T; r, R_\pi) \;=\; 2\, r\, R_\pi \sqrt{\pi\, D}\ \exp\left(\omega_0\, Y \;-\; \frac{\ln^2(R_\pi/r)}{4\, D\, Y}\right)\ \frac{\left(1 + erf\left(i\frac{\ln(R_\pi/r)}{2\sqrt{D\,Y}}\right)\right)}{2\sqrt{Y}}$$

$Q_T^2\, r\, R_\pi \;\ll\; 1\ \rightarrow$

$$\text{Im}\, G_{I\!P}(Y, Q_T; r, R_\pi) \;=\; 2\, r\, R_\pi \sqrt{\pi\, D}\ \exp\left(\omega_0\, Y \;-\; \frac{\ln^2(R_\pi/r)}{4\, D\, Y}\right)\ \frac{1}{1 - \bar{Q}_T^4\, 4\, D\, Y/\ln^2(R_\pi/r)}$$

$Q_T^2\, r\, R_\pi \;\geq\; 1\ \rightarrow$

$$\text{Im}\, G_{I\!P}(Y, Q_T; r, R_\pi) \;=\; 2\, r\, R_\pi \sqrt{\pi\, D}\ \exp\left(\omega_0\, Y \;-\; \frac{\ln^2(R_\pi/r)}{4\, D\, Y}\right)\ \frac{1}{1 - \bar{Q}_T^4\, 4\, D\, Y/\ln^2(R_\pi/r)}$$
$$+\; 2\, r\, R_\pi \sqrt{\pi\, D}\ \exp\left(\omega_0\, Y \;-\; \frac{\ln^2\left(Q_T^2\, r\, R_\pi\right)}{4\, D\, Y}\right)\ \frac{\bar{Q}_T^2\sqrt{\frac{\ln\left(Q_T^2\, r\, R_\pi\right)}{4\, D\, Y}}}{1 - \bar{Q}_T^4\, 4\, D\, Y/\ln^2\left(Q_T^2\, r\, R_\pi\right)}$$

Saturation moment:

$$Q_s^2 \;=\; Q_0^2 e^{\bar{\alpha}_S |\chi'_\gamma(\gamma_{cr})|\, Y} \,\exp\left(-\frac{3}{1-\gamma_{cr}} \left(\frac{b^4}{256\,\alpha'^2_{eff}\, D^2\,\pi^2\, Y} \right)^{\frac{1}{3}} \right)$$

where

- $$\chi\left(\gamma\right) \;=\; 2\psi\left(1\right) \,-\, \psi\left(\gamma\right) - \psi\left(1-\gamma\right)$$

- $$\chi\left(\gamma_{cr}\right) \Big/ \left(1-\gamma_{cr}\right) \;=\; \chi'_\gamma\left(\gamma_{cr}\right)$$

Summary

1. Goal: To find a theoretical way to introduce the correct large b behaviour $\propto \exp(-\mu b)$ into CGC/saturation formalism;

Result: We introduce the steep decrease due to the shrinkage of the diffraction peak but not $\propto \exp(-\mu b)$;

2. Goal: To study how the suggested procedure will affect the features of the BFKL Pomeron: spectrum, shrinkage of the diffraction peak, large b behaviour and so on;

Result: We show that introduced b behaviour restores the Froissart theorem, leads to exponential decrease, but have not studied yet the all consequences of our approach;

3. Goal: To encourage you to look for a new dimensional (non-perturbative) parameter in QCD;

Result: ????????