

# Particle Production at NLO in Hybrid Formalism

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[ T. A. , N. Armesto, G. Beuf, A. Kovner, M. Lublinsky, *Phys.Rev. D91 (2015) 9, 094016* ]



- Motivation
- What is new in this approach?
- A little tasting of the calculation: the quark channel
- Summary

## Single inclusive hadron production in pA scattering:

- 1 **" $k_T$ -factorized" approach** : Kovchegov & Tuchin
  - Both the projectile and the target are at very small- $x$  (very high energy) $\Rightarrow$  Color Glass Condensate (CGC) is applicable to both!
- 2 **"Hybrid" formalism** : Dumitru, Hayashigaki & Jalilian-Marian
  - The wave function of the projectile proton is treated in the spirit of collinear factorization (an assembly of partons with zero intrinsic transverse momenta)
  - Perturbative corrections to this wave function are provided by the usual QCD perturbative splitting processes.
  - Target is treated as distribution of strong color fields which during the scattering event transfer transverse momentum to the propagating partonic configuration. (CGC like treatment)

# Particle Production at NLO within "Hybrid" formalism

T.A., A. Kovner - 2011

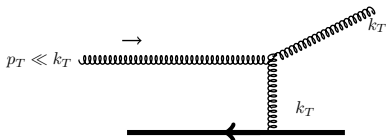
DOES LO "HYBRID" FORMULA TAKE INTO ACCOUNT ALL CONTRIBUTIONS AT HIGH  $k_{\perp}$  ?

The single inclusive gluon spectrum :

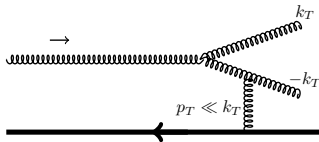
$$\frac{dN}{d^2kd\eta d^2b} \propto \left[ \frac{dN}{d^2kd\eta} \right]_{elastic} + \left[ \frac{dN}{d^2kd\eta} \right]_{inelastic}$$

In the limit of large transverse momentum of the produced gluon  $k \gg Q_s, \Lambda_{QCD}$  there are two dominant contributions:

"Elastic Scattering" (LO)



"Inelastic Scattering" (NLO)



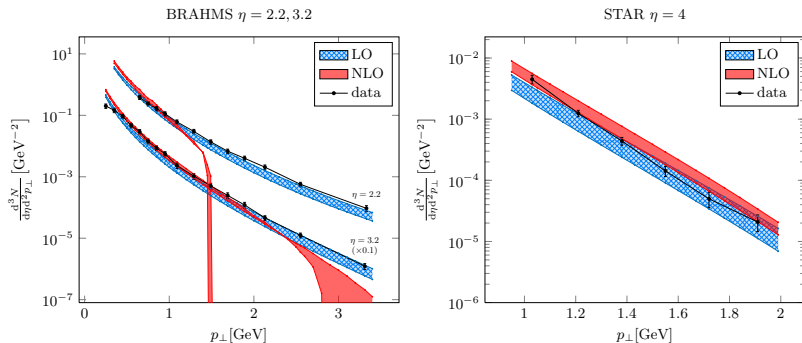
# Particle Production at NLO within "Hybrid" formalism

G.A. Chirilli, B.W. Xiao, F. Yuan - 2012

Full NLO calculation...

A.M. Stasto, B.W. Xiao, D. Zaslavsky - 2013

Numerical analysis...



Comparison of BRAHMS ( $h^-$ ) and STAR ( $\pi^0$ ) yields in dAu collisions to results of the numerical calculation with rcBK gluon distribution, both at LO and with NLO corrections included.

# What are the missing pieces of the puzzle?

## (1) The choice of simplest frame: PROJ frame

It is convenient to work in the frame where most energy of the process is carried by the target. In this frame :

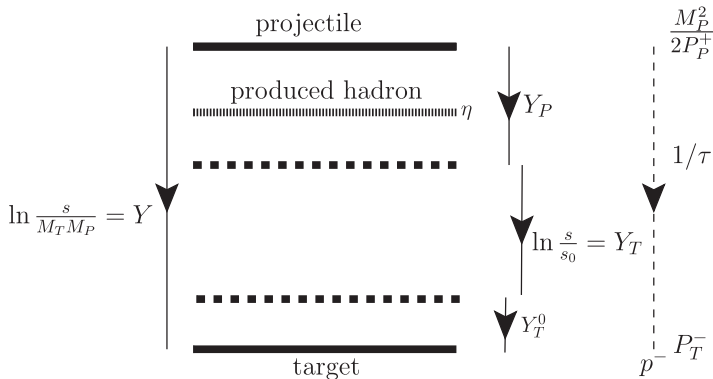
- The target moves fast and carries almost all the energy of the process.
- The projectile moves fast enough to be able to accommodate partons with momentum fraction  $x_p$  but not so fast that it develops a large low- $x$  tail.

In PROJ frame:

- $P_{P, PROJ}^+ = \frac{s}{2P_{T, PROJ}^-}$  ,  $P_{P, PROJ}^+ = const.$  ,  $P_{T, PROJ}^- \propto s$
- Target is evolved to  $s$  from an initial  $s_0$  via BK evolution.
- To get to  $s_0$ :
  - projectile is boosted to rapidity  $Y_P$  from its rest frame.
  - target is boosted to rapidity  $Y_T^0$  from its rest frame.

$$s_0 = 2P_{P, PROJ}^+ P_T^{-0}; \quad P_{P, PROJ}^+ = \frac{M_P}{\sqrt{2}} e^{Y_P};$$
$$P_T^{-0} = \frac{M_T}{\sqrt{2}} e^{Y_T^0}; \quad P_T^- = \frac{M_T}{\sqrt{2}} e^{Y_T^0 + Y_T}$$

# Rapidity Balance



# What are the missing pieces of the puzzle?

## (2) What scatters? The Ioffe Time Restriction

The Ioffe Time Restriction provides a consistent description on what will be resolved by the target and what not!

- Only the pairs whose coherence time (Ioffe time) is greater than the propagation time through the target can be resolved by the target!
- Ioffe time is related with the size of the target at initial energy  $s_0$ .

## (3) The rapidity to which eikonal scattering amplitudes have to be evolved?

$Y_T$  vs  $Y_g$

- $Y_g = \ln \frac{1}{x_g}$  &  $x_g = e^{-\eta} \frac{p_{\perp}}{\sqrt{2s}}$   
for a dense target projectile parton undergoes multiple scattering.  
the momentum transfer  $p^-$  is not from a single gluon but from several.  
 $x_g$  is an upper bound on the momentum fraction of the target gluon  $\Rightarrow Y_g$   
gives a lower bound on the rapidity up to which the target wave function has to be evolved!
- $Y_T = \ln \frac{s}{s_0}$  ✓



# The quark channel

The parton level production cross section at LO :

$$\frac{d\sigma^q}{d^2p_\perp d\eta} = \frac{1}{(2\pi)^2} \int d^2x d^2y e^{ip_\perp(x-y)} s_{Y_T}(x, y)$$

fundamental dipole scattering amplitude

$$s(x, y) = \frac{1}{N_c} \text{tr} [S_F(x) S_F^\dagger(y)]$$

At NLO the quark splits in the projectile wave function with probability of order  $\alpha_s$  into a quark-gluon configuration.

The dressed quark state :

$$\begin{aligned} |(q)_{XB} P^+, k_\perp, \alpha, s\rangle_D &= \int_x e^{ik_\perp x} \left\{ A^q |(q)_{XB} P^+, x, \alpha, s\rangle \right. \\ &+ g \int_{\xi, yz} F_{(qg)}(XB P^+, \xi, y-x, z-x)_{s\bar{s};j} t_{\alpha\beta}^a \\ &\left. |(q)_{y, p^+} = (1-\xi)_{XB} P^+, \beta, \bar{s}; (g)_{z, q^+} = \xi_{XB} P^+, a, j\rangle \right\} \end{aligned}$$

- $A^q$  is of order  $g^2$  and needed to preserve the normalisation of the state at order  $\alpha_s$ .
- $F_{(qg)}$  is the function that defines the splitting of a quark into a quark-gluon pair.

# The quark channel

The dressed quark scatters on the target and produces final state particles.

Within "hybrid" formalism, the scattering of the  $qg$  pair is treated as a completely coherent process  $\Rightarrow$  each parton picks an eikonal phase during the interaction with the target.

**THIS IS ONLY POSSIBLE** if the coherence time (Ioffe Time) of the configuration is greater than the propagation time through the target.

$$\text{coherent scattering} \Rightarrow \frac{2(1-\xi)\xi x_B P^+}{k_{\perp}^2} > \tau$$

$\tau \equiv$  a fixed time scale determined by the longitudinal size of the target.

It enters to calculation via initial energy  $P^+/\tau = s_0/2$ .

The Ioffe time restriction is in fact given in terms of initial energy  $s_0$ !

The pairs that do not exist long enough are not resolved! Those pairs:

- have large  $k_{\perp}$  and have small transverse size.
- scattering and particle production from those pairs are **indistinguishable** from single parent quark.

# The quark channel

The standard eikonal paradigm for propagation of the initial dressed quark with vanishing transverse momentum through the target leads to the final state

$$\begin{aligned}
 |\text{out}, \alpha, s\rangle = & \int_x \left\{ S_{\alpha\beta}^F(x) |(q)x, \beta, s\rangle_D \right. \\
 & + \frac{g^2}{2\pi} \int d\text{LPS} \int_{y,z} \left[ t_{\alpha\beta}^a S_{\beta\gamma}^F(y) S_{ab}^A(z) - S_{\alpha\beta}^F(x) t_{\beta\gamma}^b \right] \bar{F}_{(qg)}^2(\xi, x_p, y-x, z-x) \\
 & \quad t_{\gamma\delta}^b |(q) x, \delta, s\rangle_D \\
 & + \frac{g}{2\pi} \int d\text{LPS} \int_{y,z} F_{(qg)}(\xi, x_p, y-x, z-x)_{s,\bar{s},i} \left[ t_{\alpha\beta}^a S_{\beta\gamma}^F(y) S_{ab}^A(z) - S_{\alpha\beta}^F(x) t_{\beta\gamma}^b \right] \\
 & \quad \left. |(q) y, (1-\xi), \gamma, \bar{s}; (g) z, \xi, b, i\rangle_D \right\}
 \end{aligned}$$

LPS  $\equiv$  **Longitudinal Phase Space**

The function  $F_{(qg)}$  is written as

$$F_{(qg)} = \frac{i}{\sqrt{2\xi x_B P^+}} \left\{ \delta_{s\bar{s}} \delta_{ij} (2-\xi) - i \epsilon_{ij} \sigma_{s\bar{s}}^3 \xi \right\} \delta^2 \left( x - [(1-\xi)y + \xi z] \right) A_{\xi, x_B}^i(y-z)$$

Modified Weizacker-Williams field



# The quark channel

The modified Weizsacker-Williams field is defined as

$$\begin{aligned} A_{\xi, x_B}^i(y-z) &= -i \int_{l_{\perp}^2 < 2\xi(1-\xi)x_B \frac{p^+}{\tau}} \frac{d^2 l_{\perp}}{(2\pi)^2} \frac{l_{\perp}^i}{l_{\perp}^2} e^{i l_{\perp} \cdot (y-z)} \\ &= -\frac{1}{2\pi} \frac{(y-z)^i}{(y-z)^2} \left[ 1 - J_0 \left( |y-z| \sqrt{2\xi(1-\xi) \frac{x_B p^+}{\tau}} \right) \right] \end{aligned}$$

with transverse momentum  $l_{\perp}$  is

$$l_{\perp} = \xi p_{\perp} - (1-\xi) q_{\perp}$$

- The loffe time constraint is implemented on the phase space  $\{k_{\perp}, \xi\}$  in the definition of  $F_{(qg)}(y-x, z-x)$  rather than in the integral over  $\xi$ .
- Neglecting the loffe time constraint on  $l_{\perp}$ , one gets the Fourier transform of the standard Weizsacker-Williams field.
- With the loffe time constraint, the relative contribution of short distances are suppressed.  $F_{(qg)}$  at small  $z-x$  becomes reduced.

# The quark channel

The quark production cross section is given by the expectation value of the dressed quark number in the outgoing state, multiplied by the number of dressed quarks in the incoming wave function:

$$\frac{d\sigma^q}{d^2p_\perp d\eta} = x_p f_{\mu^2}^D(x_p) \langle \text{out} | D^\dagger(k_\perp, x) D(k_\perp, x) | \text{out} \rangle$$

For the quark production we find

$$\frac{d\sigma^q}{d^2p_\perp d\eta} = \underbrace{\frac{1}{(2\pi)^2} x_p f_{\mu^2}^D(x_p) \int_{x,y} e^{ip_\perp(x-y)} s_{Y_T}(x,y)}_{\text{LO}} + \underbrace{\frac{d\sigma_1^q}{d^2p_\perp d\eta}}_{\text{NLO}}$$

The quark production cross section at NLO :

$$\frac{d\sigma_1^q}{d^2p_\perp d\eta} = p^+ \frac{d\sigma_1^q}{d^2p_\perp dp^+} = p^+ \frac{d\sigma_1^{q \rightarrow q,r}}{d^2p_\perp dp^+} + p^+ \frac{d\sigma_1^{q \rightarrow q,v}}{d^2p_\perp dp^+}$$

# The quark channel

The NLO cross section contain collinear divergences.

$$I_1^r = \frac{g^2}{(2\pi)^3} C_F \int dx_B f_{\mu^2}^{D,q}(x_B) \int_0^{1-x_p} d\xi \frac{x_p}{1-\xi} \delta\left(x_B - \frac{x_p}{1-\xi}\right) \left[\frac{1+(1-\xi)^2}{\xi}\right] \\ \times C_{\mu^2}(\xi, x_B) \int_{y\bar{y}} e^{ip_{\perp}(y-\bar{y})} s[y, \bar{y}]$$
$$I_2^r = \frac{g^2}{(2\pi)^3} C_F \int dx_B f_{\mu^2}^{D,q}(x_B) \int_0^{1-x_p} d\xi \frac{x_p}{1-\xi} \delta\left(x_B - \frac{x_p}{1-\xi}\right) \left[\frac{1+(1-\xi)^2}{\xi}\right] \\ \times (1-\xi)^2 C_{\mu^2}(\xi, x_B) \int_{y\bar{y}} e^{ip_{\perp}(y-\bar{y})} s[(1-\xi)y, (1-\xi)\bar{y}]$$
$$I^v = -(1+1) \frac{g^2}{(2\pi)^3} C_F \int dx_B f_{\mu^2}^{D,q}(x_B) x_p \delta(x_B - x_p) \int_0^1 d\xi \left[\frac{1+(1-\xi)^2}{\xi}\right] C_{\mu^2}(\xi, x_B) \\ \times \int_{y\bar{y}} e^{ip_{\perp}(y-\bar{y})} s[y, \bar{y}]$$

where the integral over  $z$  up to “factorization scale”  $\mu$  can be defined for example as

$$C_{\mu^2}(\xi, x_B) = \int_z A_{\xi, x_B}^i(z) A_{\xi, x_B}^i(z) \theta(z^2 \mu^2 - 1)$$

# PDF's and Fragmentation functions

$f^D$  that appears in the LO term is the number of “dressed quarks” in the proton. Part of the  $O(\alpha_s)$  terms complete it to the NLO pdf of bare quarks (DGLAP of PDFs and FFs).

$$f_{\mu^2}^q(x_p) = f_{\mu^2}^D(x_p) + \frac{g^2 C_F}{2\pi} \int_0^{1-x_p} \frac{d\xi}{1-\xi} f_{\mu^2}^D\left(\frac{x_p}{1-\xi}\right) \frac{1+(1-\xi)^2}{\xi} C_{\mu^2}\left(\xi, \frac{x_p}{1-\xi}\right) - \frac{g^2 C_F}{2\pi} f_{\mu^2}^D(x_p) \int_0^1 d\xi \frac{1+(1-\xi)^2}{\xi} C_{\mu^2}(\xi, x_p)$$

The fragmentation function of the “dressed quark”:

$$D_{H,\mu^2}^{D,q}(\zeta) = D_{H,\mu^2}^q(\zeta) + \frac{g^2}{2\pi} C_F D_{H,\mu^2}^q(\zeta) \int_0^1 d\xi \frac{1+(1-\xi)^2}{\xi} C_{\mu^2}\left(\xi, \frac{x_p}{\zeta}\right) - \frac{g^2}{2\pi} C_F \int_0^{1-\zeta} \frac{d\xi}{1-\xi} D_{H,\mu^2}^q\left(\frac{\zeta}{1-\xi}\right) \frac{1+(1-\xi)^2}{\xi} C_{\mu^2}\left(\xi, \frac{x_p}{\zeta}\right)$$

# Relating our factorization scheme to $\overline{MS}$

The collinear factorization scheme that we use, does not coincide with the standard  $\overline{MS}$  scheme.

In order to find the relation between the two schemes:

- we have calculated the d-dimensional generalisation of our collinear subtraction term  $C_{\mu^2}$ .
- use the fact that single inclusive cross section is scheme independent.
- compare our result (scheme X) with the  $\overline{MS}$  one.

$$f_X^q(x_B; \mu_F^2) = f_{\overline{MS}}^q(x_B; R^2 \mu_F^2)$$

$$D_{H,X}^q(\zeta; \mu_{frag}^2) = D_{H,\overline{MS}}^q(\zeta; R^2 \mu_{frag}^2)$$

with the rescaling factor  $R = 2e^{\psi(1)} \approx 1.1229$ .



# The Final Result

Adding all the pieces together, we have the final expression for the quark channel:

$$\begin{aligned} p^+ \frac{d\sigma^{q \rightarrow H}}{d^2 p_\perp d p^+} &= \frac{1}{(2\pi)^2} \int \frac{d\zeta}{\zeta^2} D_H^q(\zeta) \frac{x_p}{\zeta} f_{p_\perp}^q \left( \frac{x_p}{\zeta} \right) \int_{y\bar{y}} e^{i p_\perp (y-\bar{y})} s_{Y_T}[y, \bar{y}] \\ &+ \int \frac{d\zeta}{\zeta^2} D_H^q(\zeta) \frac{d\bar{\sigma}^q}{d^2 p_\perp d\eta} \left( \frac{p_\perp}{\zeta}, \frac{x_p}{\zeta} \right) \end{aligned}$$

The quark production cross section has two parts:

- a piece that is independent of loffe time restriction which coincides with the existing results in the literature.
- a piece that carries the loffe time restriction:

$$\begin{aligned} \frac{g^2}{(2\pi)^3} N_{c x_p} f_{\mu^2}^q(x_p) \int_0^1 \frac{d\xi}{\xi} \int_{y\bar{y}z} e^{i p_\perp (y-\bar{y})} [A_\xi^i(y-z) - A_\xi^i(\bar{y}-z)]^2 \\ \times [s(y,z)s(z,\bar{y}) - s(y,\bar{y})] \end{aligned}$$

# What about the evolution?

- The way we set up the problem, the dipole scattering amplitude is evolved up to rapidity  $Y_T = \ln \frac{s}{s_0}$  starting with an initial condition provided at  $Y_T^0$ .
- The final result should not care which  $s_0$  we choose if we evolve the dipole cross section appropriately.
- The dependence on  $s_0$  enters explicitly through the cutoff on the phase space and through the dependence of the scattering amplitude on the amount of evolution  $Y_T$ . Therefore

$$s_0 \frac{d}{ds_0} \left[ \frac{d\sigma}{d^2 p_\perp d\eta} \right] = \left[ s_0 \frac{\partial}{\partial s_0} - \frac{ds_{Y_T}}{dY_T} \frac{\delta}{\delta s_{Y_T}} \right] \frac{d\sigma}{d^2 p_\perp d\eta} = 0$$

and

$$s_0 \frac{\partial}{\partial s_0} \left[ \frac{d\sigma}{d^2 p_\perp d\eta} \right] = -\frac{\alpha_s N_c}{\pi} x_p f(x_p) \int_{y, \bar{y}, z} \frac{1}{(2\pi)^3} e^{ip_\perp(y-\bar{y})} \frac{(y-\bar{y})^2}{(y-z)^2(\bar{y}-z)^2} \times [s(y, \bar{y}) - s(y, z)s(z, \bar{y})]$$

⇒ the dipole amplitude evolves according to the BK equation...

# Extra contribution to the evolution?

The finite term that appears in the quark production cross section **looks like** an extra contribution to the evolution:

$$\frac{g^2}{(2\pi)^3} N_c x_p f_{\mu^2}^q(x_p) \int_0^1 \frac{d\xi}{\xi} \int_{y\bar{y}z} e^{ip_{\perp}(y-\bar{y})} [A_{\xi}^i(y-z) - A_{\xi}^i(\bar{y}-z)]^2 \times [s(y,z)s(z,\bar{y}) - s(y,\bar{y})]$$

If we try to write it as an **extra contribution to evolution**:

- change the order of integration:  $\xi$  and F.T. WW field:

$$\int d^2 l_{\perp} \int d^2 m_{\perp} \ln\left(\frac{1}{\xi_{min}}\right) \frac{d}{dY} s(l_{\perp} + p_{\perp}, m_{\perp} - p_{\perp})$$

with

$$\xi_{min} = \max\left\{\frac{l_{\perp}^2}{x_p s_0}, \frac{m_{\perp}^2}{x_p s_0}\right\}$$

- Together with the leading order, this is like an effective evolution by

$$Y_{l_{\perp}} = Y_T + \ln \frac{x_p s_0}{l_{\perp}^2} = \ln \frac{1}{x_g} + \ln \frac{p_{\perp}^2}{l_{\perp}^2}$$

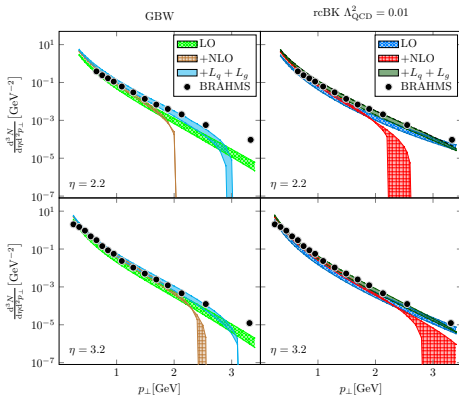
# Remark: a very recent paper

## Implementing the exact kinematical constraint in the saturation formalism

Kazuhiro Watanabe,<sup>1</sup> Bo-Wen Xiao,<sup>1</sup> Feng Yuan,<sup>2</sup> and David Zaslavsky<sup>1</sup>

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# Remark: a very recent paper

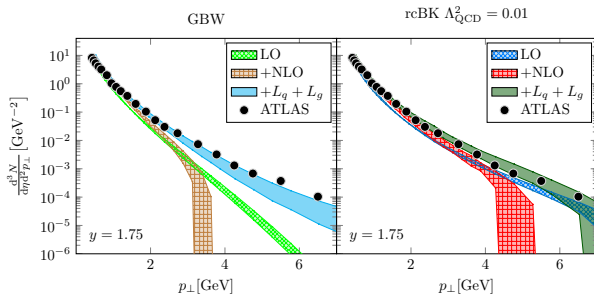


FIG. 6. Comparison of ATLAS forward-rapidity data [21] with the center-of-mass energy of  $\sqrt{s_{NN}} = 5.02$  TeV at  $y = 1.75$  with SOLO results for the GBW and rcBK models. Again, the color scheme is the same as in figure 4. Here the error band shows plots for  $\mu^2 = 10 \text{ GeV}^2$  and  $\mu^2 = 100 \text{ GeV}^2$ . Since the numerical data for these measurements are not published, we have extracted the ATLAS points from Fig. 6 of Ref. [21]. The extraction procedure introduces uncertainties comparable to the size of the points.

# Summary

- By introducing the *Ioffe Time Restriction*, we have defined clearly the limits of coherent scattering and distinguish what will be resolved by the target and what not.
- We have defined the rapidity up to which the scattering amplitude has to be evolved.
- We have shown that how the Balitsky-Kovchegov evolution equation arises as the appropriate tool to evolve the leading order amplitude in this setup.
- Numerical results of Watanabe, Xiao, Yuan, Zaslavsky shows a clear improvement!