Forward-Backward Correlation between Mean Event Transverse Momenta in String Fusion Model

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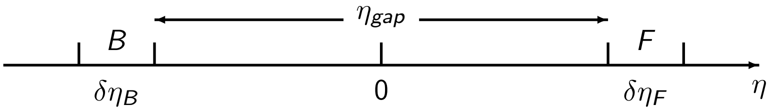
Forward-Backward Rapidity Correlations



$\langle B\rangle_F = f(F)$ - the FB correlation function

$\langle B\rangle_F = a + b_{BF}F$ - the linear regression

The correlation coefficient:

$$b_{BF} = \frac{\langle FB\rangle - \langle F\rangle\langle B\rangle}{\langle F^2\rangle - \langle F\rangle^2} = \frac{cov(F,B)}{D_F}$$

$$b_{BF} = \left.\frac{d\langle B\rangle_F}{dF}\right|_{F=\langle F\rangle}$$

Observables

B, F:

n_B, n_F - the extensive variables $\Rightarrow$ b_{nn}

p_{tB}, p_{tF} - the intensive variables $\Rightarrow$ $b_{p_t p_t}$

$$p_{tB} = \frac{1}{n_B} \sum_{i=1}^{n_B} |\mathbf{p}^i_{tB}| \qquad p_{tF} = \frac{1}{n_F} \sum_{i=1}^{n_F} |\mathbf{p}^i_{tF}|$$

p_{tB}, n_F - the combination of the variables $\Rightarrow$ $b_{p_t n}$

A. Capella and A. Krzywicki, Phys.Rev.D**18**, 4120 (1978)

b_{nn} — the Long-Range FB Correlations (LRC) at large η_{sep}

The locality of strong interaction in rapidity?

Event-by-event variance in the number of cut pomerons (strings).

String fusion effects

But event-by-event fluctuation in the number of cut pomerons (strings) (the "volume" fluctuation) do not give rise to the correlation between the intensive variables, e.g. the p_{tB}-p_{tF} correlation ($b_{p_t p_t}$).

p_{tB}-p_{tF} correlation can indicate the fluctuations in "quality" of sources.

pp $\longrightarrow$ pA $\longrightarrow$ AA - the increase of the string density in transverse plain
M.A. Braun,C. Pajares, Phys.Lett. **B287**, 154 (1992);
Nucl. Phys. **B390**, 542 (1993).
$\Rightarrow$ Reduction of multiplicity, increase of transverse momenta.

N.S. Amelin, N. Armesto, M.A. Braun, E.G. Ferreiro, C. Pajares, Phys.Rev.Lett. **73**, 2813 (1994).
$\Rightarrow$ The influence on the Long-Range FB Correlations (LRC).

Various versions of string fusion

local fusion (overlaps)

M.A. Braun, C. Pajares Eur.Phys.J. **C16**, 349, (2000)

$$\langle n\rangle_k = \mu_0\sqrt{k}\ S_k/\sigma_0\ , \qquad \langle p_t^2\rangle_k = p_0^2\sqrt{k}\ , \qquad k = 1,2,3,... \qquad (1)$$

global fusion (clusters)

M.A. Braun, F. del Moral, C. Pajares, Phys.Rev. **C65**, 024907, (2002)

$$\langle p_t^2\rangle_{cl} = p_0^2\sqrt{k_{cl}}\ , \qquad \langle n\rangle_{cl} = \mu_0\sqrt{k_{cl}}S_{cl}/\sigma_0\ , \qquad k_{cl} = k\sigma_0/S_{cl} \quad (2)$$

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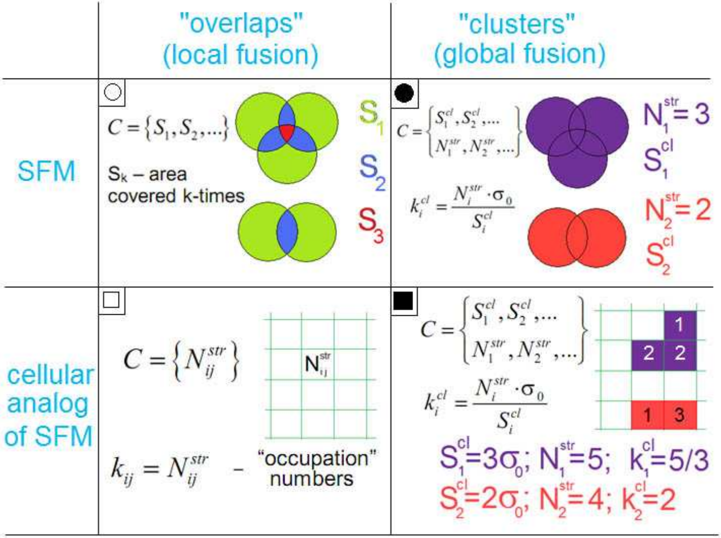
the version of SFM with the finite lattice in transverse plane

Braun M.A., Kolevatov R.S., Pajares C., V.V. Eur.Phys.J. C32 (2004) 535.

V.V., Kolevatov R.S. Phys.of Atom.Nucl. **70** (2007) 1797; 1858.

Various versions of string fusion



General formulae for LR correlations

Braun M.A., Pajares C., V.V. Phys.Lett. B493 (2000) 54.

LRC:

$$P(F,B) = \sum_C P(C)\, P_C(F)\, P_C(B)\ . \qquad (3)$$

$$P(F) = \sum_C P(C)\, P_C(F)\ . \qquad (4)$$

Correlation function:

$$\langle B\rangle_F = \frac{\sum_C \langle B\rangle_C\, P(C)\, P_C(F)}{P(F)} = \frac{\sum_C \langle B\rangle_C\, P(C)\, P_C(F)}{\sum_C P(C)\, P_C(F)} \qquad (5)$$

$$\langle B\rangle = \sum_C P(C) \langle B\rangle_C\ , \qquad \langle F\rangle = \sum_C P(C) \langle F\rangle_C \qquad (6)$$

Recall the correlation coefficient:

$$b_{BF} \equiv = \frac{\langle F\rangle}{\langle B\rangle}\ \frac{d\langle B\rangle_F}{dF}\Big|_{F=\langle F\rangle}\ . \qquad (7)$$

Description of configurations

String configuration (η_i - the number of string centers in i-th cell):

$$C_\eta = \{\eta_1, ..., \eta_M\} \ , \tag{8}$$

General configuration (n_i-the number of particles produced from i-th cell):

$$C = \left\{C_\eta, C_n^F, C_n^B\right\} \ , \quad C_n^F = \left\{n_1^F, ..., n_M^F\right\} \ , \quad C_n^B = \left\{n_1^B, ..., n_M^B\right\} \ . \tag{9}$$

$$P(C) = P(C_\eta) P_{C_\eta}(C_n^F) P_{C_\eta}(C_n^B) \ , \qquad P_C(F) = P_{C_\eta C_n^F}(F) \ . \tag{10}$$

Monte-Carlo Simulations:

$$\sum_C P(C) \ ... \Rightarrow \frac{1}{n_{sim}} \sum_{sim} \ ...$$

E.g. the similar MC approach was applied for the calculations of anisotropic azimuthal flows in String Fusion Model:

M.A.Braun and C.Pajares, *Eur. Phys. J. C* **71** (2011) 1558.
M.A. Braun, C. Pajares, V.V., *Nucl. Phys. A* **906** (2013) 14.
M.A. Braun, C. Pajares, V.V., *Eur. Phys. J. A* **51** (2015) 44.

Gaussian approximation

$$\langle B\rangle_F = \frac{1}{P(F)} \sum_{C_\eta} P(C_\eta) \left[\sum_{C_n^B} P_{C_\eta}(C_n^B) \langle B\rangle_{C_\eta C_n^B} \right] \left[\sum_{C_n^F} P_{C_\eta}(C_n^F) P_{C_\eta C_n^F}(F) \right] . \qquad (11)$$

For p_{tB}-p_{tF} correlation: $F = p_{tF} \equiv p_F$ and $B = p_{tB} \equiv p_B$:

$$p_F \equiv \frac{1}{n_F} \sum_{i=1}^{M} \sum_{j=1}^{n_i^F} p_i^{jF} \ , \quad n_F = \sum_{i=1}^{M} n_i^F \ , \text{ and the same for } p_B \text{ and } n_B. \qquad (12)$$

$$P(C_\eta) = \prod_{i=1}^{M} \frac{1}{\sqrt{2\pi D_{\eta_i}}} \exp\left\{ -\frac{(\eta_i - \overline{\eta}_i)^2}{2D_{\eta_i}} \right\}, \qquad (13)$$

$$D_{\eta_i} \equiv \overline{\eta_i^2} - \overline{\eta}_i^2 \sim \overline{\eta}_i \gg 1 \ . \qquad (14)$$

$$\sum_{C_\eta} = \prod_{i=1}^{M} \sum_{\eta_i=0}^{\infty} \Rightarrow \prod_{i=1}^{M} \int_0^\infty d\eta_i \qquad (15)$$

Gaussian approximation

$$P_{C_\eta}(C_n^F) = \prod_{i=1}^{M} \frac{1}{\sqrt{2\pi D_{n_i^F}(\eta_i)}} \exp\left\{ -\frac{[n_i^F - \overline{n}_i^F(\eta_i)]^2}{2D_{n_i^F}(\eta_i)} \right\} . \qquad (16)$$

$$D_{n_i^F}(\eta_i) \sim \overline{n}_i^F(\eta_i) \gg 1 \ . \qquad (17)$$

$$P_{C_\eta C_n^F}(p_F) = \frac{1}{\sqrt{2\pi D_{p_F}(C_\eta, C_n^F)}} \exp\left\{ -\frac{[p_F - \langle p_F \rangle_{C_\eta C_n^F}]^2}{2D_{p_F}(C_\eta, C_n^F)} \right\} , \qquad (18)$$

$$D_{p_F}(C_\eta, C_n^F) = \frac{1}{n_F^2} \sum_{i=1}^{M} n_i^F D_{p_i}(\eta_i) \ . \qquad (19)$$

$$\langle p_F \rangle_{C_\eta C_n^F} = \frac{1}{n_F} \sum_{i=1}^{M} n_i^F \overline{p}_i(\eta_i) \qquad (20)$$

Saddle point calculations

$$\langle p_B \rangle_{p_F} = \frac{1}{P(F)} \prod_{i=1}^{M} \int_0^\infty d\eta_i \, dn_i^F \, N(C_\eta, C_n^F) \, e^{-\Phi(p_F, C_\eta, C_n^F)}$$

$$\times \prod_{i=1}^{M} \int_0^\infty dn_i^B e^{-\Psi(C_\eta, C_n^B)} \langle p_B \rangle_{C_\eta C_n^B} \;, \qquad (21)$$

where

$$\Phi(p_F, C_\eta, C_n^F) = \sum_{i=1}^{M} \frac{(\eta_i - \overline{\eta}_i)^2}{2D_{\eta_i}} + \sum_{i=1}^{M} \frac{[n_i^F - \overline{n}_i^F(\eta_i)]^2}{2D_{n_i^F}(\eta_i)} + \frac{[p_F - \langle p_F \rangle_{C_\eta C_n^F}]^2}{2D_{p_F}(C_\eta, C_n^F)} \qquad (22)$$

and

$$\Psi(C_\eta, C_n^B) = \sum_{i=1}^{M} \frac{[n_i^B - \overline{n}_i^B(\eta_i)]^2}{2D_{n_i^B}(\eta_i)} \;. \qquad (23)$$

Saddle point method for integrations over n_i^B leads to the simple substitution: $n_i^B \Rightarrow \overline{n}_i^B(\eta_i)$ in the remainder integrand. Then

Correlation function

$$\langle p_B \rangle_{p_F} = \frac{1}{P(F)} \prod_{i=1}^{M} \int_0^{\infty} d\eta_i \, dn_i^F \, N(C_\eta, C_n^F) \, e^{-\Phi(p_F, C_\eta, C_n^F)} \langle p_B \rangle_{C_\eta C_{\overline{n}}^B} \; , \qquad (24)$$

Saddle point method for integrations over n_i^F and η_i leads to the nontrivial saddle point $\eta_i = \eta_i^*(p_F)$ and $n_i^F = n_i^{*F}(p_F)$, determining by the conditions:

$$\frac{\partial \Phi(p_F, C_\eta, C_n^F)}{\partial \eta_i} = 0 \; , \qquad \frac{\partial \Phi(p_F, C_\eta, C_n^F)}{\partial n_i^F} = 0 \; , \qquad (25)$$

which position depends on p_F. In this approximation we find

$$\langle p_B \rangle_{p_F} = \langle p_B \rangle_{C_{\eta^*} C_{\overline{n}}^B} \; , \qquad (26)$$

where by (20) we have

$$\langle p_B \rangle_{C_{\eta^*} C_{\overline{n}}^B} = \frac{1}{\overline{n}_B} \sum_{i=1}^{M} \overline{n}_i^B(\eta_i^*) \overline{p}_i(\eta_i^*) \; , \qquad \overline{n}_B = \sum_{i=1}^{M} \overline{n}_i^B(\eta_i^*) \; , \qquad (27)$$

Correlation coefficient

The p_t-p_t correlation coefficient in this approximation:

$$b_{p_t p_t} \equiv \left. \frac{d\langle p_B \rangle_{p_F}}{d p_F} \right|_{p_F=\langle p_F \rangle} = \sum_{i=1}^{M} \frac{\partial}{\partial \eta_i^*} \left[\langle p_B \rangle_{C_{\eta^*} C^B_{\overline{n}}} \right] \cdot \left. \frac{d\eta_i^*}{d p_F} \right|_{p_F=\langle p_F \rangle} . \quad (28)$$

Example with homogeneous string distribution in transverse plane:

$$\overline{\eta}_i = \overline{\eta} \equiv \eta, \qquad D_{\eta_i} = D_\eta, \qquad \omega_\eta = D_\eta / \overline{\eta} \ . \quad (29)$$

In String Fusion Model:

M.A. Braun,C. Pajares, Nucl. Phys. **B390**, 542 (1993).

M.A.Braun, C.Pajares, Phys.Rev.Lett. **85**, 4864 (2000).

$$\overline{n}_i^F(\eta_i) = \overline{n}_i^B(\eta_i) = \mu_F \sqrt{\eta_i} \ , \qquad \overline{p}_i(\eta_i) = p_0 \sqrt[4]{\eta_i} \quad (30)$$

$$D_{n_i^F}(\eta_i) = \omega_\mu \overline{n}_i^F(\eta_i) \quad (31)$$

and the same for $n_i^B(\eta_i)$.

The ω_η and ω_μ describe the deviations from poissonian distributions.

Transverse momentum distribution

$$D_{p_i}(\eta_i) = \gamma \overline{p}_i^{\,2}(\eta_i) \ . \tag{32}$$

For transverse momentum distributions with one dimensional parameter, $\tilde{p}$, usually applied, this relation is always true due to dimensional reasons. Wherein $\tilde{p} \sim \sqrt[4]{\eta_i}$ and $\gamma \sim 1$ depends only of the shape of the distribution.

$\varphi(p)$	γ
$\sim \exp(-p^2/\tilde{p}^2)$	$\sqrt{\frac{4-\pi}{\pi}}$
$\sim \exp(-p/\tilde{p})$	$\frac{1}{\sqrt{2}}$
$\sim [1/(1+p/\tilde{p})]^m$	$\sqrt{\frac{m-1}{2(m-4)}}$

Saddle point for uniform distribution

For uniform string distribution in transverse plane it is convenient to introduce the relative variables:

$$z_i^4 = \frac{\eta_i}{\overline{\eta}} \;, \qquad \rho_i = \frac{n_i^F}{\overline{n}_i^F(\overline{\eta})} \;, \qquad f = \frac{p_F}{\langle p_F \rangle} \;. \tag{33}$$

$$\overline{n}_i^F(\overline{\eta}) = \mu_F \sqrt{\overline{\eta}} \;, \qquad \langle p_F \rangle = \langle p_F \rangle_{C_{\overline{\eta}} C_{\overline{n}}^F} = \overline{p}_i(\overline{\eta}) = p_0 \sqrt[4]{\overline{\eta}} \;. \tag{34}$$

Then the equations for the position of symmetric saddle point are as follows

$$4z^{10} + 4z^6 + a(z^4 - \rho^2) + b\,\rho(z - f)f = 0 \;, \tag{35}$$
$$2a(\rho - z^2) + b(z - f)^2 = 0 \;,$$

where

$$a = \frac{\omega_\eta \, \mu_F}{\omega_\mu \sqrt{\overline{\eta}}} \;, \qquad b = \frac{\omega_\eta \, \mu_F}{\gamma \sqrt{\overline{\eta}}} \;. \tag{36}$$

Correlation function for uniform distribution

Important that the position of saddle point do not depends on M, whereas the function $\Phi(p_F, C_\eta, C_n^F)$ in exponent (24) is proportional to M:

$$\Phi(p_F, C_\eta, C_n^F) = M\overline{\Phi}(f, z, \rho) \ , \tag{37}$$

what justifies the saddle point calculations at $M \gg 1$. So in relative variables we find

$$\langle p_B \rangle_{p_F} = \langle p_B \rangle \cdot z^*(f) \ , \qquad f = p_F / \langle p_F \rangle \ , \tag{38}$$

where $z = z^*(f)$ is given by (35) and

$$b_{p_t p_t} = \left. \frac{dz^*(f)}{df} \right|_{f=1} = z^{*\prime}(1) \ . \tag{39}$$

Correlation coefficient for uniform distribution

The correlation coefficient can be calculated explicitly. Really we see that at $f=1$ the equations (35) have the solution $z=1$ and $\rho=1$, i.e. $z^*(1)=1$ $\rho^*(1)=1$.
Linearizing then the equations in vicinity of this point, we have

$$16z^{*\prime}(1)+2a[2z^{*\prime}(1)-\rho^{*\prime}(1)]+b[z^{*\prime}(1)-1]=0, \qquad a[\rho^{*\prime}(1)-2z^{*\prime}(1)]=0\,, \tag{40}$$

what leads to

$$b_{p_t p_t}=\frac{b}{b+16}\,, \tag{41}$$

which does not depend on a. Recall that

$$a=\frac{\omega_\eta\,\mu_F}{\omega_\mu\sqrt{\overline{\eta}}}\,, \qquad\qquad b=\frac{\omega_\eta\,\mu_F}{\gamma\sqrt{\overline{\eta}}}\;. \tag{42}$$

Analytical asymptotics of the LR correlation coefficients

So in version of SFM with the finite lattice in transverse plane at large multiplicities and large homogeneous mean string overlapping, $\eta \equiv \langle k \rangle >> 1$ (for the central AA interactions at high energy), we find:

$$b_{p_t p_t} = \frac{\omega_\eta \, \mu_F}{\omega_\eta \, \mu_F + 16\gamma^2 \sqrt{\eta}} \; . \tag{43}$$

Compare with earlier results:

$$b_{nn} = \frac{\omega_\eta \, \mu_F}{\omega_\eta \, \mu_F + 4\omega_{\mu_F} \sqrt{\eta}} \; , \tag{44}$$

$$b_{p_t n} = \frac{1}{2} \cdot \frac{\omega_\eta \, \mu_F}{\omega_\eta \, \mu_F + 4\omega_{\mu_F} \sqrt{\eta}} \; , \tag{45}$$

Here

$$\mu_F = \mu_0 \delta\eta_F \qquad \omega_{\mu_F} = \frac{D_{\mu_F}}{\mu_F} \qquad \omega_\eta = \frac{D_\eta}{\eta} \tag{46}$$

μ_F - mean particle density from one initial string in the forward window.

M - scaling

The same M - scaling in $b_{p_t p_t}$as in $b_{nn}$and $b_{p_t n}$is not trivial. It take place only for correlation between Mean Event Transverse Momenta, as defined above. For the LR part of p_t-p_t correlation coefficient between transverse momenta of two particles, defined as

$$\overline{b}_{p_t p_t} \equiv \frac{I_2(p_{t1}, p_{t2})}{I(p_{t1})I(p_{t2})} - 1$$

where $I(p_{t1})$ and $I_2(p_{t1}, p_{t2})$ are single and double inclusive cross sections, it was shown that

M.A.Braun, R.S.Kolevatov, C.Pajares, V.V., Eur.Phys.J.C32, 535 (2004).

$$\overline{b}_{p_t p_t} \sim \frac{1}{Nsources} \sim \frac{1}{M}$$

and hence is very small e.g. for PbPb interactions at LHC energy in which the number the sources (strings) is of order of few thousand.

ω_{μ_F} and $\mu_F/\sqrt{\eta}$ - scaling

For $b_{p_t p_t}$ we have additional ω_{μ_F} - scaling compared with b_{nn} and $b_{p_t n}$ asymptotics.

ω_{μ_F} characterizes the fluctuations in the number of particles produced from a string. (We have supposed that for fused strings the value of ω_{μ_F} remains the same as for initial ones.)

Instead of ω_{μ_F} - dependence for $b_{p_t p_t}$ we have γ - dependence.

γ - characterize transverse momentum distribution from one initial string. For one dimensional parameter distribution it does not depends on fusion.

$$\text{then } \gamma = \frac{\sigma_{p_t}}{\langle\langle p_t \rangle\rangle} \text{ - can be found from data}$$

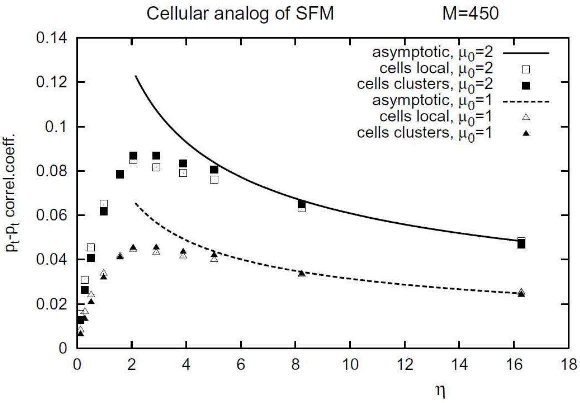
Note that the $\langle\langle ... \rangle\rangle$ means averaging over tracks from all events.

$$\sigma_{p_t}^2 \equiv \langle\langle p_t^2 \rangle\rangle - \langle\langle p_t \rangle\rangle^2 \neq \langle {p_F}^2 \rangle - \langle p_F \rangle^2 \equiv D_{p_F}$$

$$\langle p_F \rangle \approx \langle\langle p_t \rangle\rangle \qquad D_{p_F} \approx \frac{\sigma_{p_t}^2}{\langle n_F \rangle}$$

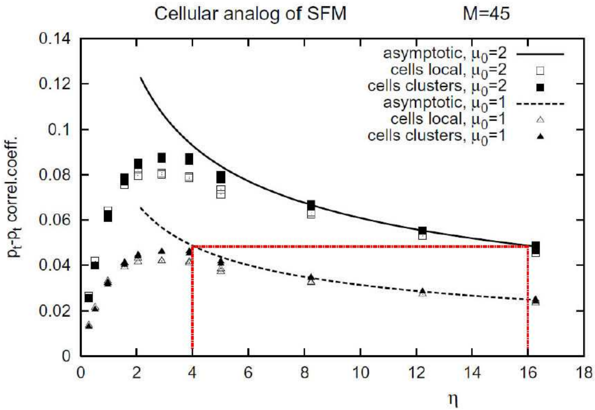
Also the same $\mu_F/\sqrt{\eta}$ - scaling for $b_{p_t p_t}$ asymptotic as in b_{nn} and $b_{p_t n}$.

Comparing Asymptotic with MC Simulations



MC results for the correlation coefficient $b_{p_t p_t}$ at $M = 450$ with poissonian distributions, $\omega_\eta = \omega_\mu = 1$, are taken from *V.V., Kolevatov R.S.* Phys.of Atom.Nucl. **70** (2007) 1858.

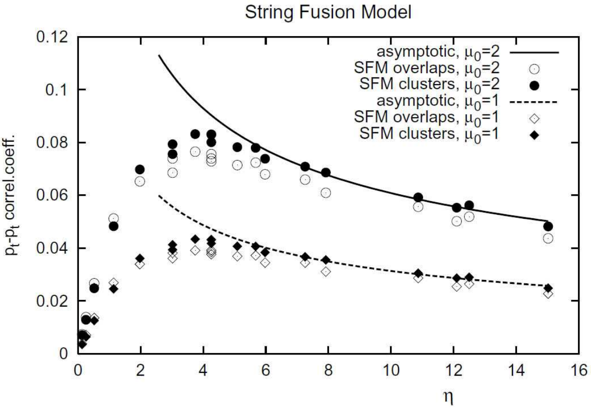
Comparing Asymptotic with MC Simulations



M-scaling, $\overline{N} = M\overline{\eta}$

Red dash lines illustrate $\frac{\mu_F}{\sqrt{\overline{\eta}}}$-scaling

Comparing Asymptotic with MC Simulations

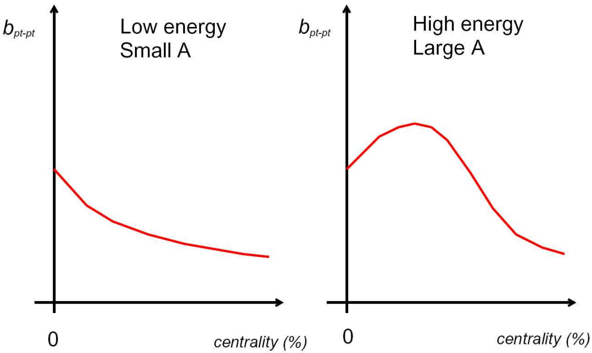


The points simulated with $\overline{N}$ =1000, 2000 and 8000 strings

M-scaling $\longrightarrow$ $\overline{N}$-scaling, $\overline{\eta} = \sigma_0 \overline{N}/S$

$$\eta_{max} = 3 \div 4$$

Behavior with centrality



$\eta_{max} = 3 \div 4$
RHIC Au+Au at 200 GeV (centrality 0-10%) $\overline{\eta}$=2.88±0.09
LHC Pb+Pb at 2.76 TeV (centrality 0-5%) $\overline{\eta}$=10.56±1.05
J. Dias de Deus, A.S. Hirsch, C. Pajares, R.P. Scharenberg, B.K. Srivastava Eur.Phys.J. C **72** (2012) 2123.