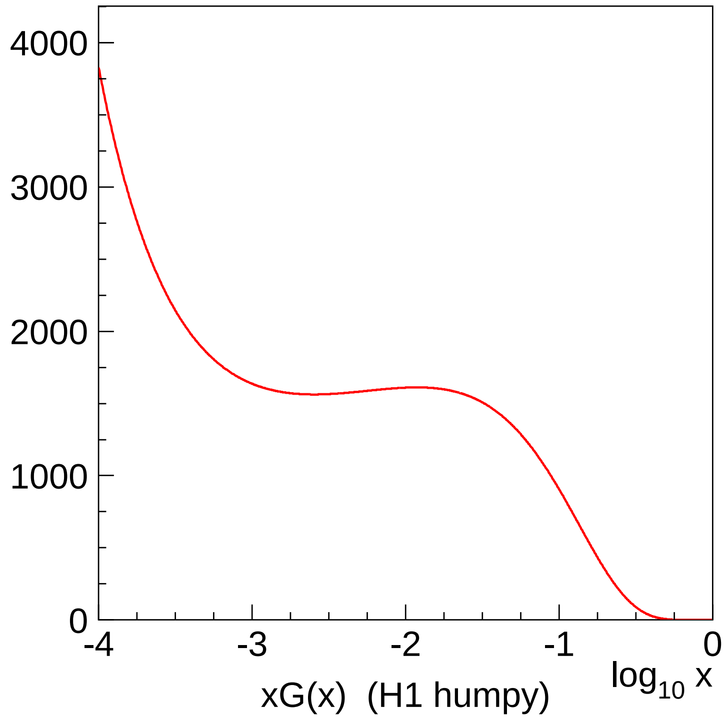
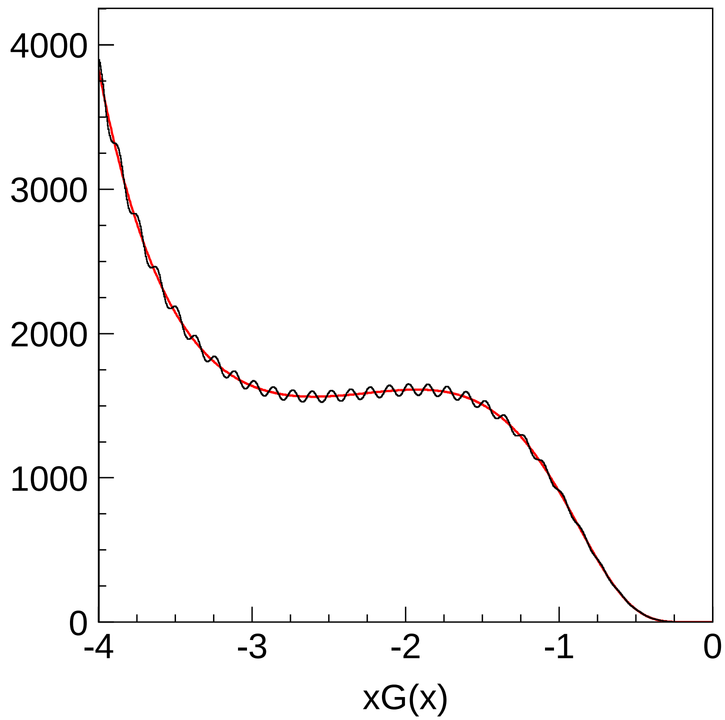
Physical Constraints on PDFs



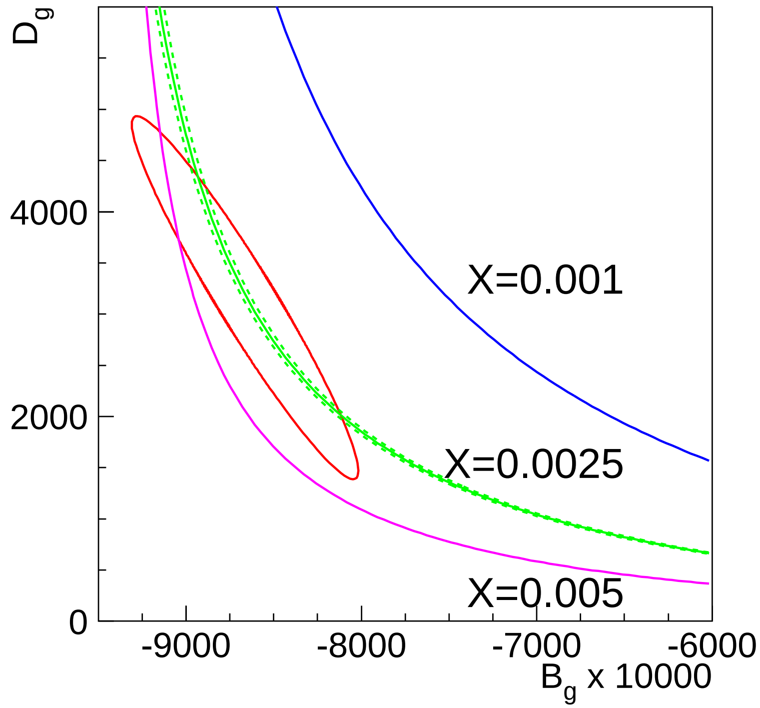
- H1 parameterization, fit to H1 data
- Humpy solution: $xG(x) = Ax^B(1-x)^C(1+Dx)$, $D = 3160 \pm 1773$.
- Large correlation btw D and B: $\rho = -95.5\%$.

Hypothetical unconstraint solution



- Blindly increasing number of parameters may lead to solutions with many minima.
- F_2 shows steady increasing rise vs x at $Q^2 > 2$ GeV^2, large $W^2 \approx Q^2(1-x)/x$.
- $\rightarrow$ Impossible for $xS(x)$ in DIS scheme, probably can be proved impossible for $xG(x)$.

"No extra minima" constraint



For $xG(x) = Ax^B(1-x)^C(1+Dx)$ parameterization, require no extra minima for $x < x_{min}$:

$$\frac{\mathrm{d}xG(x)}{\mathrm{d}x} < 0 \qquad \text{for } x < x_{min}$$

This imposes relation btw B, C, D which is for some x_{min} more restrictive than experimental precision.

Example of working "clever" parameterization

A similar problem: determination of semileptonic form factor $\hat{f}_+(t)$ for $K \to \pi e \nu$ decays. Standard parameterization:

$$\hat{f}_+(t) = 1 + \lambda' t/m_\pi^2 + 0.5\lambda'' t^2/m_\pi^4 + ...$$

does not allow to control phase space integral.

Z-mapping: $t \to z$ (`R. J. Hill, PRD` **`74`** `096006 (2006)`),

$$\hat{f}_+(t) = f_+^z(t_0)\frac{\phi(t_0, t_0, Q^2)}{\phi(t, t_0, Q^2)} \sum_{k=0}^{\infty} a_K(t_0, Q^2) z(t, t_0)^k.$$

For some choice of $\phi(t, t_0, Q^2)$,

$$\sum_{k=0}^{\infty} a_k^2 \leq 170$$

from unitarity arguments. This provides better limit for $k \geq 3$ than experimental data.

Summary

- Can $xF(x)$, $(xF(x))'$, etc be required to have no/finate amount of zeros for certain $0 < x < x_{min}$ and $Q^2 > Q^2_{min}$ kinematic domain and some factorization scheme ?
- If yes, what is the best parameterization to use this ?

Can theory be used to control higher order terms in PDF expansion ?

Ideal parameterization: "low orders" controlled from experiment, "higher orders" limitted by theory.