



# CTEQ pdf parametrization\*

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\*not an expert on this subject, just filling in, so this will be brief

# CTEQ parametrization



- PDF's ( $f_a(x, Q)$ ) are parametrized with a flexible form motivated by physics considerations (Regge behavior, spectator counting, for example) at fixed small  $Q_0$  (1.3 GeV for CTEQ) and then evolved for  $Q > Q_0$  by DGLAP
  - ◆ assume for most of the general analyses that the c and b distributions are zero at scales below their masses and are generated by QCD evolution above
- Parametrization of parton distributions at  $Q_0$  used to obtain the CTEQ5 and CTEQ6 parton distributions contained 5 shape parameters (apart from normalization) for each flavor
  - ◆ global analysis data sets not sufficiently constraining to determine all of the parameters, so a number are frozen at some particular (motivated) values
  - ◆ 20 free parameters for CTEQ6.1/6.5 (22 for CTEQ6.6 (see next slide))
- For CTEQ6.5/6.6, adopt a simpler form with 4 shape parameters for the valence quarks  $u_v(x)$ ,  $d_v(x)$  and the gluon  $g(x)$

$$f(x) = a_o x^{a_1} (1-x)^{a_2} e^{a_3 x + a_4 x^2}$$

- ◆ a reasonable generalization of the conventional minimal form

$$f(x) = a_o x^{a_1} (1-x)^{a_2}$$

- ◆ which combines Regge behavior at  $x \rightarrow 0$  and spectator counting at  $x \rightarrow 1$
- Both forms above are positive definite and have simplified logarithmic derivatives

# CTEQ parametrization



- Is this form flexible enough?
- Remember the lesson of Tevatron jets, where low  $x$  and high  $x$  can easily be (artificially) tied together through the parametrization
- We find that significantly better fits cannot be achieved by introducing additional parameters or changing the functional form
  - ◆ NB: prior to CTEQ6.6, the analysis generally assumed
$$s(x) = \bar{s}(x) \propto \bar{d}(x) + \bar{u}(x)$$
  - ◆ that ansatz has been dropped in CTEQ6.6

# W/Z cross sections at the LHC



- CTEQ6.1 and MRST2004 NLO predictions in good agreement with each other
- NNLO corrections are small and negative
- NNLO mostly a K-factor; NLO predictions adequate for most predictions at the LHC

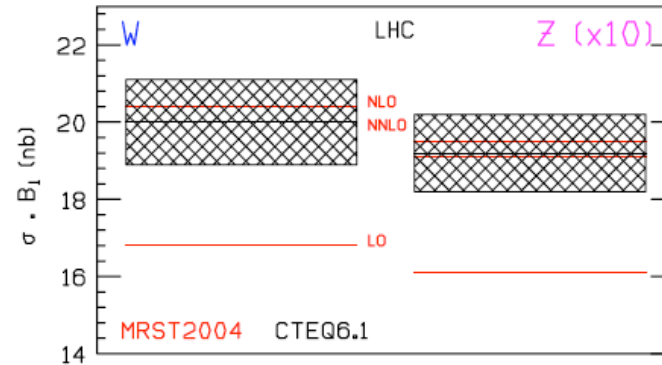
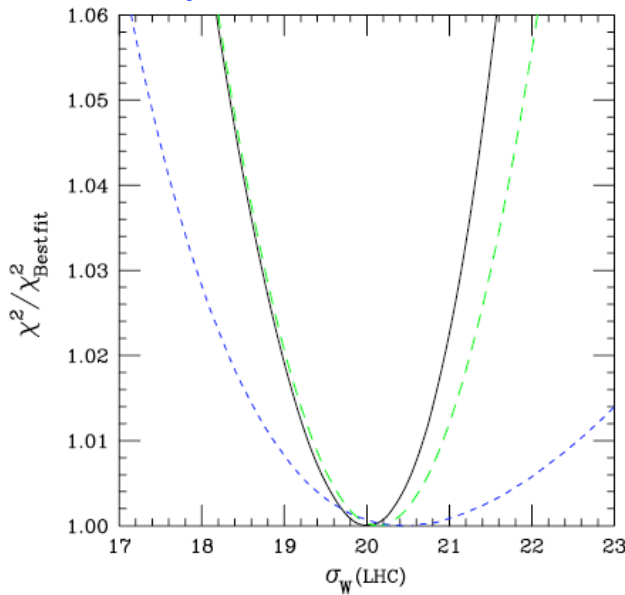
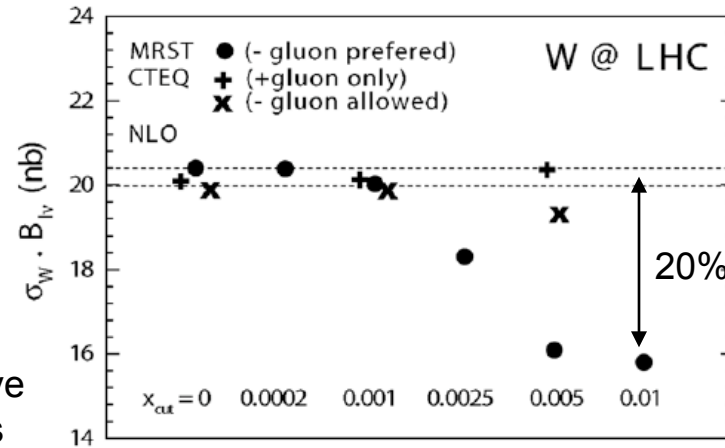


Figure 80. Predicted cross sections for W and Z production at the LHC using MRST2004 and CTEQ6.1 pdfs. The overall pdf uncertainty of the NLO CTEQ6.1 prediction is approximately 5%, consistent with figure 77.



removing low x data from global fits increases uncertainty but does not significantly move central answer; negative gluon increases uncertainty even more

Figure 82. Lagrange multiplier results for the W cross section (in nb) at the LHC using a positive-definite gluon. The three curves, in order of decreasing steepness, correspond to three sets of kinematic cuts, standard/intermediate/strong.



tension between low x and high x data?; not a big effect in CTEQ analysis

Figure 81. Predicted total cross section of  $W^+ + W^-$  production at the LHC for the fits obtained in the CTEQ stability study, compared with the MRST results. The overall pdf uncertainty of the prediction is  $\sim 5\%$ , as observed in figure 77.

# Errors in parton distribution functions



- CTEQ/MSTW/HERA provide ways to estimate the error on the central pdf
  - ◆ Hessian methodology enables full characterization of parton parametrization space in neighborhood of global minimum

2-dim (i,j) rendition of d-dim (~16) PDF parameter space

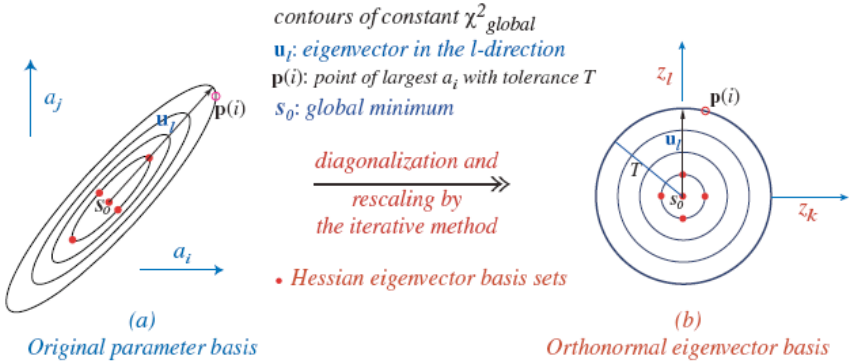


Figure 28. A schematic representation of the transformation from the pdf parameter basis to the orthonormal eigenvector basis.

## Inclusive jets at the Tevatron

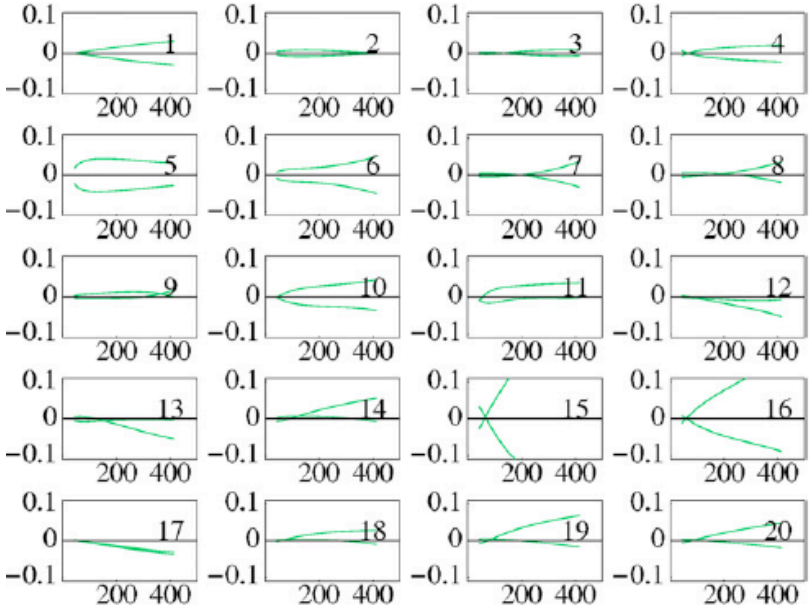


Figure 29. The pdf errors for the CDF inclusive jet cross section in Run 1 for the 20 different eigenvector directions. The vertical axes show the fractional deviation from the central prediction and the horizontal axes the jet transverse momentum in GeV.

- ◆ CTEQ6.1 has 20 free parameters so 20 directions in eigenvector space

40 error pdfs

$$\Delta X_{\max}^+ = \sqrt{\sum_{i=1}^N [\max(X_i^+ - X_0, X_i^- - X_0, 0)]^2}$$

$$\Delta X_{\max}^- = \sqrt{\sum_{i=1}^N [\max(X_0 - X_i^+, X_0 - X_i^-, 0)]^2}$$

- CTEQ6.6 has 22 free parameters so 22 directions in eigenvector space and 44 error pdf's
- Of order of a factor of 1E6 between largest (best determined directions) and smallest (least well-determined directions) eigenvalues

## Eigenvector directions (CTEQ6.1)

Eigenvector 1: primarily  $a_1$  of u valence

Sets	Shape	Parameter	Component
1, 2	BP(	2, 1)	0.057911
1, 2	BP(	2, 2)	-0.022688
1, 2	BP(	2, 3)	0.015496
1, 2	BP(	2, 4)	0.035277
1, 2	BP(	2, 5)	frozen
1, 2	BP(	1, 1)	0.888833
1, 2	BP(	1, 2)	-0.161942
1, 2	BP(	1, 3)	0.118204
1, 2	BP(	1, 4)	0.268405
1, 2	BP(	1, 5)	0.276392
1, 2	BP(	0, 1)	0.038555
1, 2	BP(	0, 2)	-0.006610
1, 2	BP(	0, 3)	frozen
1, 2	BP(	0, 4)	-0.017717
1, 2	BP(	0, 5)	frozen
1, 2	BP(	-1, 1)	-0.007668
1, 2	BP(	-1, 2)	0.012745
1, 2	BP(	-1, 3)	0.001851
1, 2	BP(	-1, 4)	frozen
1, 2	BP(	-1, 5)	0.001004
1, 2	BP(	-2, 1)	0.117517
1, 2	BP(	-2, 2)	-0.008357
1, 2	BP(	-2, 3)	0.006504
1, 2	BP(	-2, 4)	frozen
1, 2	BP(	-2, 5)	frozen

Eigenvector 20: high x sea quark

Sets	Shape	Parameter	Component
39, 40	BP(	2, 1)	0.000248
39, 40	BP(	2, 2)	0.069038
39, 40	BP(	2, 3)	0.173137
39, 40	BP(	2, 4)	-0.029044
39, 40	BP(	2, 5)	frozen
39, 40	BP(	1, 1)	0.000920
39, 40	BP(	1, 2)	-0.001493
39, 40	BP(	1, 3)	0.008380
39, 40	BP(	1, 4)	0.000153
39, 40	BP(	1, 5)	-0.008078
39, 40	BP(	0, 1)	0.003339
39, 40	BP(	0, 2)	-0.010965
39, 40	BP(	0, 3)	frozen
39, 40	BP(	0, 4)	0.008411
39, 40	BP(	0, 5)	frozen
39, 40	BP(	-1, 1)	0.479314
39, 40	BP(	-1, 2)	0.190673
39, 40	BP(	-1, 3)	0.796917
39, 40	BP(	-1, 4)	frozen
39, 40	BP(	-1, 5)	-0.131794
39, 40	BP(	-2, 1)	-0.000408
39, 40	BP(	-2, 2)	0.136504
39, 40	BP(	-2, 3)	0.163995
39, 40	BP(	-2, 4)	frozen
39, 40	BP(	-2, 5)	frozen

# Extrapolations



- How reasonable are extrapolations, say to low  $x$ ?
- Of course, in the absence of data, you may be constrained by the parametrization (and momentum sum rule) and are probably underestimating the uncertainty
- See, for example, Ubiati presentation at La Thuile

