HERA PDF parametrisations

PDF4LHC 14/7/2008

Use 3 different types of parametrization

But each of these can have more or less parameters

Choice of numbers of parameters by ‘saturation of chisq’

What if we don’t do this?- double minimum in gluon shape.

All variants have acceptable chisq – incorporate as part of model error?

Model errors matter because experimental errors are now small after HERA combination procedure

Different parametrizations give different estimates for error bands

Central parametrization

Chosen form of the PDF parametrization at Q_0^2

$$xf(x) = Ax^B(1-x)^C(1+Dx+Ex^2+Fx^3\ldots)$$

	A	B	C	D	E
gluon	sum rule	[blue]	[blue]		
u_v	sum rule	[blue]	[blue]	[red]	[red]
d_v	sum rule	$= B(u_v)$	[blue]		
U_{bar}	Lim x→0 $\bar{u}/\bar{d}$ →1	[blue]	[blue]		
D_{bar}	[blue]	$= B(\bar{U})$	[blue]		

The number of parameters for each parton has been optimized

Optimization means starting with only BLUE parameters and adding D, E,F parameters until there is no further $\chi 2$ advantage

PDFs fitted: gluon, u_v, d_v, $U_{bar}=u_{bar}+c_{bar}$, $D_{bar}=d_{bar}+s_{bar}+b_{bar}$

Sea flavour break-up at Q_0: s = fs*D, c=fc*U , $AU_{bar}=(1-fs)/(1-fc)AD_{bar}$ Lim x→0 u_{bar}/d_{bar} →1

fs = 0.33D (s=0.5d), fc = 0.15U consistent with dynamical generation

mc=1.4 GeV mass of charm quark   mb=4.75 GeV mass of beauty quark

Zero-mass variable flavour number heavy quark scheme (for now)

Q_0^2 = 4 GeV^2 input scale Q^2_{min} = 3.5 GeV^2 minimum Q^2 of input data

$\alpha_s(Mz)$ = 0.1176 PDG2006 value

Renormalization and factorization scales = Q^2

Choices of mb, mc, fs, fc, Q20, Q2min are varied as part of model error

Variation of alpha_s is also considered

Two alternaive parametrizations are considered

$$xf(x) = Ax^B(1-x)^C(1 + Dx + Ex^2 + Fx^3 \ldots)$$

Alternative form of PDF parametrization: H1 style

	A	B	C	D	E	F
gluon	sum rule					
U	$\lim_{x\to 0} \bar{u}/\bar{d} \to 1$			sum rule		
D		$= B(U)$		sum rule		
U_{bar}	$= A(U)$	$= B(U)$				
D_{bar}	$= A(D)$	$= B(U)$				

Strong assumptions on low-x valence behaviour

PDFs: gluon, U=u+c, $U_{bar}=u_{bar}+c_{bar}$, D=d+s+b, $D_{bar}=d_{bar}+s_{bar}+b_{bar}$

Sea flavour break-up at Q_0: s = fs*D, c=fc*U   AU=(1-fs)/(1-fc)AD

Alternative form of PDF parametrization: ZEUS style

	A	B	C	D	E!
gluon	From Sum Rule				0.
u_v	From Sum Rule				
d_v	From Sum Rule	$= B_{uv}$			0.
$u_{bar} - d_{bar}$	from Z S 11 fit	from Z S 11 fit	from Z S 11 fit	0.	0.
Sea				0.	0.

Strong assumptions on dbar-ubar

PDFs: gluon, u_v, d_v, Sea= $u_{sea}+u_{bar}+d_{sea}+d_{bar}+s+s_{bar}+c+c_{bar}$

Sea flavour break-up at Q_0: $s_{bar} = (d_{bar}+u_{bar})/4$, charm dynamically generated,

$d_{bar}-u_{bar}$ fixed to fit E866 data

Choice of parameterization

All three forms have good $\chi 2$
our choice has the best

Further motivations are:

- Less model dependence on B parameters than in H1 param.
- No need for an additional input (ubar-dbar) x distribution as in ZEUS-Jet param
- Most conservative errors.
- It is inspired by both H1 and ZEUS parameterizations.

Model uncertainties: to be added into the total PDF uncertainty

- mc 1.3 → 1.55 GeV variation of mass of c quark
- mb 4.3 → 5.0 GeV variation of mass of b quark
- fs 0.25→ 0.40 variation of strange sea fraction at Q_0
- fc 0.10→ 0.20 variation of charm sea fraction at Q_0
- Q_0^2 2.0 →6.0 GeV^2 variation of starting scale
- Q_{min}^2 2.5→5.0 GeV^2 variation of cuts on the data included

Model variations: to be compared with our results

Variation of $\alpha_S(Mz)$ 0.1156 → 0.1196

Variation of form of parametrization

But how do we chose the number of terms in the polynomial?

An additional parameter is considered only when its introduction significantly improves the χ^2, which is data sets dependent.

1) Start point: N_0 parameters

2) Add one term to the polynomial(order: $1/2, 1, 3/2, 2, 3, 4$ ...)

3) Choose the one which improves χ^2 most significantly

4) New start point, goto 2, until the introduction of new term has no **significant improvement** of χ^2.

5) New parameterization: $N(> N_0)$ parameters

Example using H1 style parametrization

Start point:8 parameters				
	A	B	C	D
gluon	sum rule			
U	$\lim_{x \to 0} \bar{u}/\bar{d} \to 1$			sum rule
D		$= B(U)$		sum rule
U_{bar}	$= A(U)$	$= B(U)$		
D_{bar}	$= A(D)$	$= B(U)$		

depart from 8 parameters 529.814/(573-8)					
	$P_g = 1$	$P_U = 1 + x$	$P_D = 1 + x$	$P_{\bar{U}} = 1$	$P_{\bar{D}} = 1$
$\sqrt{x}$	520.804	515.714	525.361	518.565	521.312
x	513.057	——	——	522.416	510.778
$x^{3/2}$	517.499	528.428	526.596	527.081	518.626
x^2	516.844	529.183	528.106	529.406	513.757
x^3	518.213	**495.576**	514.596	529.276	529.807
x^4	520.578	528.309	514.678	524.481	529.021

An x3 term in U makes a significant difference

depart from 9 parameters 495.576/(573-9)					
	$P_g = 1$	$P_U = 1 + x + x^3$	$P_D = 1 + x$	$P_{\bar{U}} = 1$	$P_{\bar{D}} = 1$
$\sqrt{x}$	490.706	——	492.716	486.429	490.241
x	482.228	——	——	493.900	484.895
$x^{3/2}$	481.857	——	495.527	495.285	490.998
x^2	482.306	479.063	495.517	495.256	491.285
x^3	482.165	——	495.413	495.565	492.739
x^4	481.541	484.492	495.275	494.675	493.557

Given the x3 term in U the next most significant difference is an x2 term in U

depart from 10 parameters 479.063/(573-10)					
	$P_g = 1$	$P_U = 1 + x + x^2 + x^3$	$P_D = 1 + x$	$P_{\bar{U}} = 1$	$P_{\bar{D}} = 1$
$\sqrt{x}$	478.322	——	478.638	474.848	478.908
x	478.228	——	——	479.048	479.010
$x^{3/2}$	479.063	——	478.811	477.833	478.746
x^2	480.742	——	478.917	477.472	478.580
x^3	479.056	——	478.986	478.369	478.177
x^4	479.063	477.977	479.061	479.014	477.978

529.814/(573 − 8)	0.9377
495.576/(573 − 9)	0.8787
479.063/(573 − 10)	0.8509
474.040/(573 − 11)	0.0449

- The largest $\Delta\chi^2$ is about 4.2, not a significant improvement, stop at 10 parameters.
- We also tried with only integer polynomial up to 5th order, and reached the same result.

After that nothing much significant happens

So what was that about ambiguity in the form of the gluon? - Well if we don't optimize the number of parameters but allow more parameters in the gluon

Resolution of an old discrepancy

For each of the parametrizations, if a non-zero D parameter for the gluon is used, there are two minima: 'straight' gluon and 'humpy' gluon solution.

These look rather like the published ZEUS and H1 gluons respectively!

For the H1/ZEUS combined data set the χ2 of the straight solution is always lower by about 10 χ2 points. But whereas the humpy solutions are disfavoured by χ2 they are still acceptable fits

We compare the humpy and straight solutions for our chosen parametrization here. These parametrizations are very consistent.

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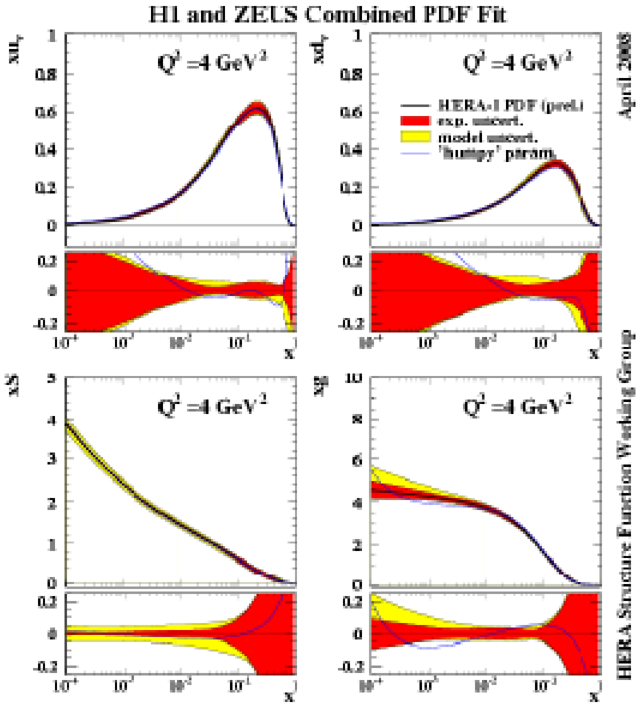
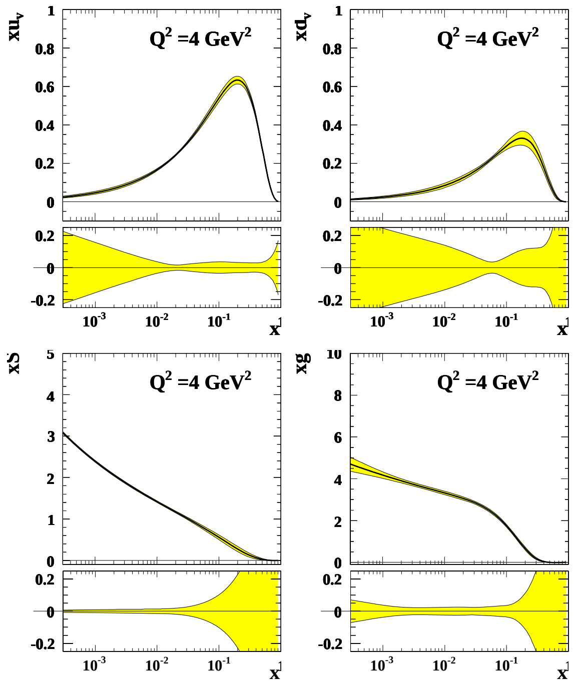


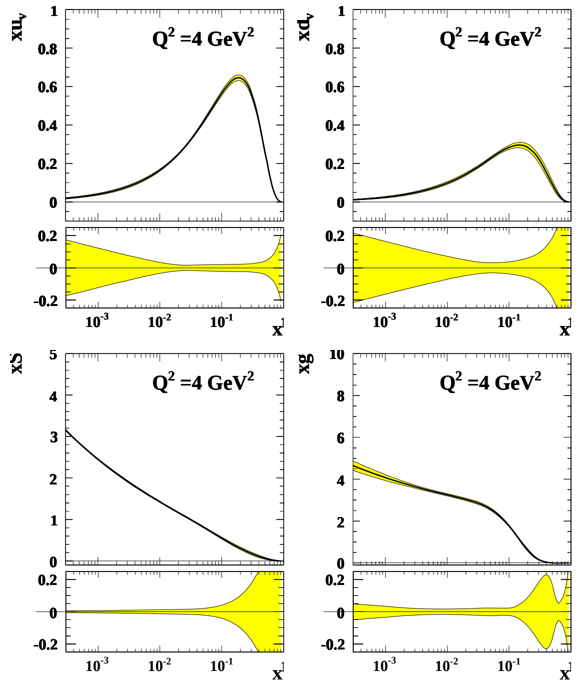
But they can obviously become different outside the measurable range.

It is also true that the form of parametrization can affect the size of the uncertainty bands.

And the choice of Q20 affects the size of the uncertainty bands

Comparing Q_0^2=4 (standard) with Q_0^2=2~m_c^2 for the HERAPDF parametrization





uv,dv,Sea, gluon

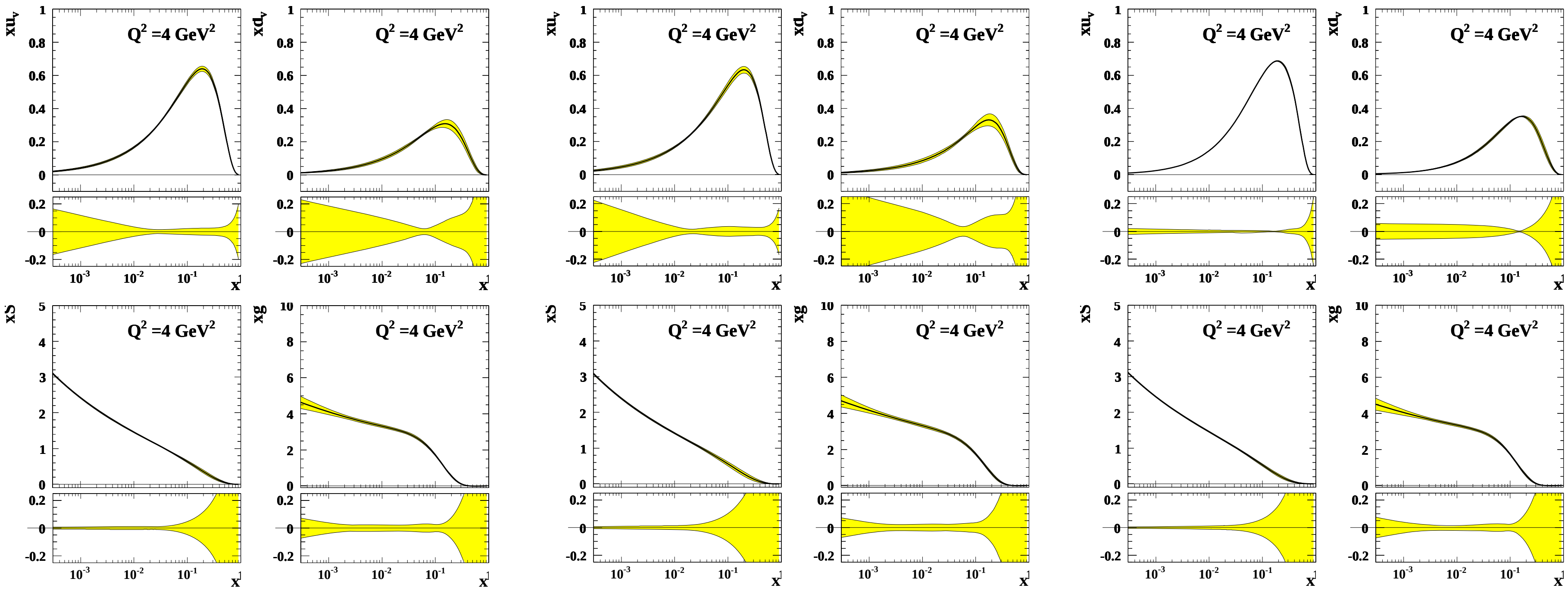
HERAPDF with Q_0^2=4 HERAPDF with Q_0^2=2

Starting at a different Q0 is equivalent to a different parametrization.

Central values fairly similar (d-valence?) error estimates smaller for Q_0^2=2

Compare different parametrizations using the same HERA data set

in terms of uv, dv, Sea, Glue



ZEUS-Jets HERAPDF H1

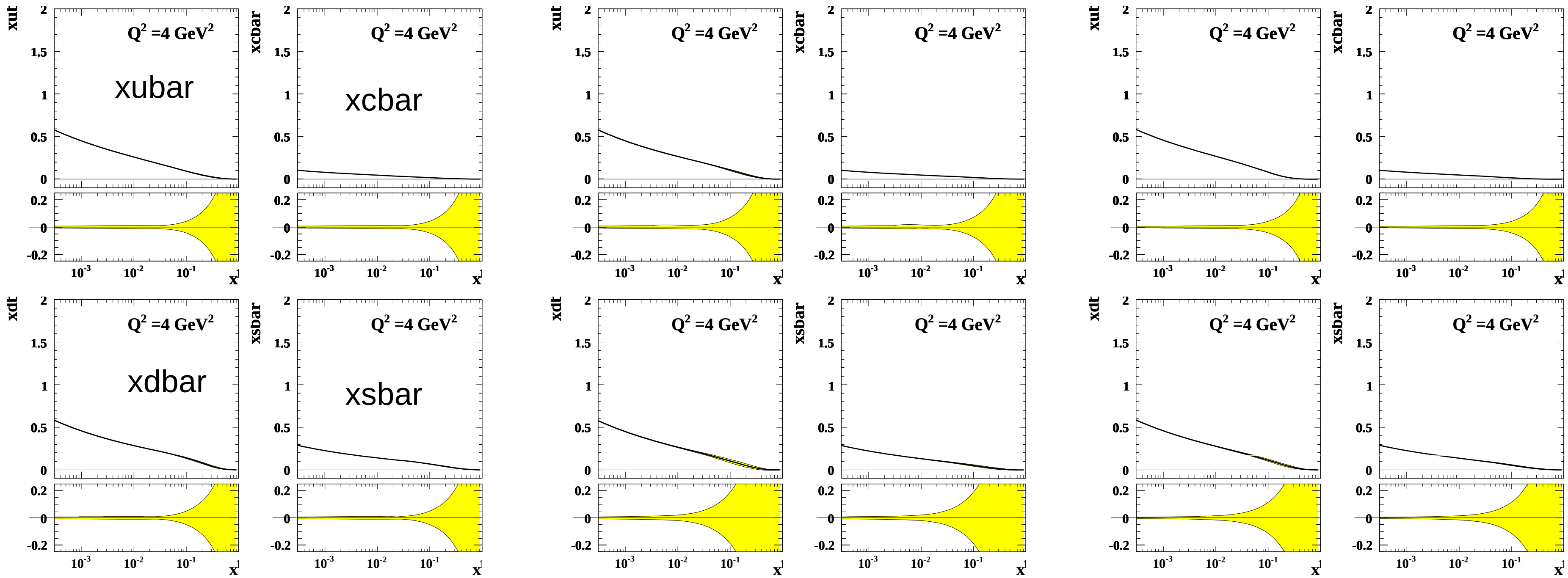
PDFs are really very similar- quite remarkable since ZEUS and H1 parametrizations are not- however the size of errors differs, with the HERAPDF parametrization being the most conservative

Conclusions

- Parametrizations need a lot of thought

extras

Now in terms of ubar, dbar, sbar, cbar

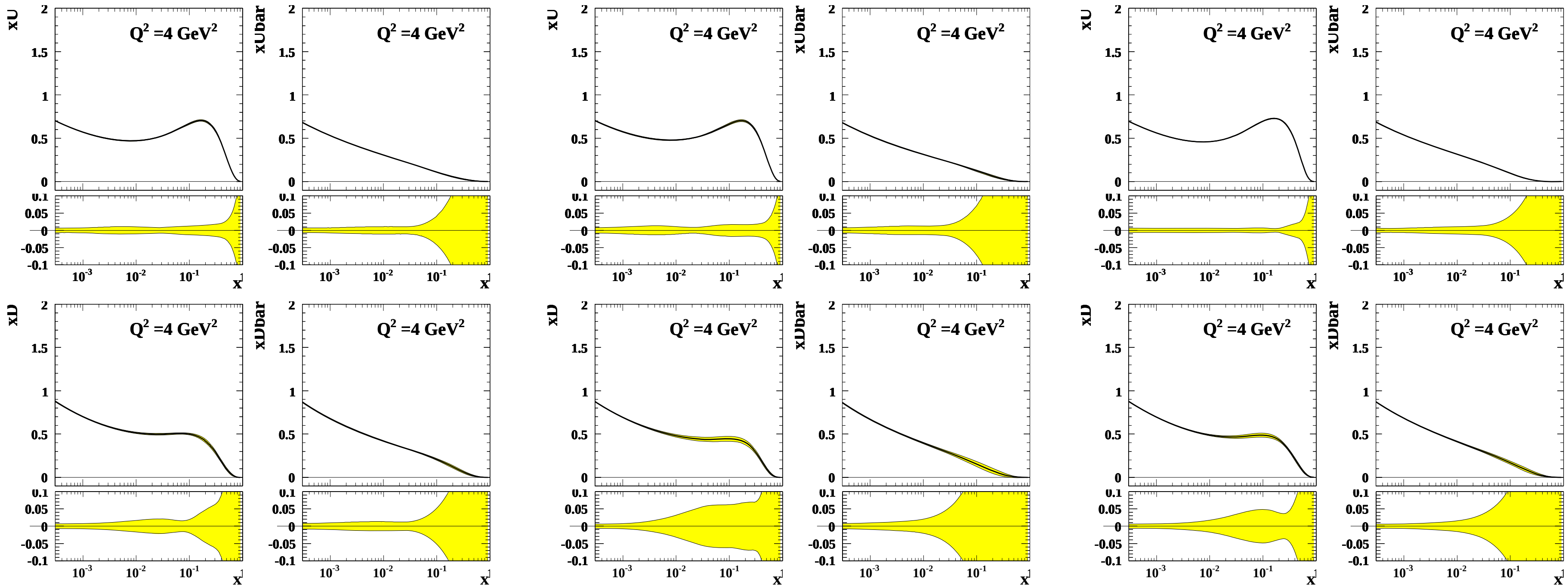


ZEUS-Jets HERAPDF H1

The similarity of these is perhaps even more remarkable given the different treatment of charm- clearly the fixed fraction fc=0.15 is about right compared to dynamical turn on of ZEUS-JETS at at $Q^2=mc^2$

Again the errors of the HERAPDF are the most conservative

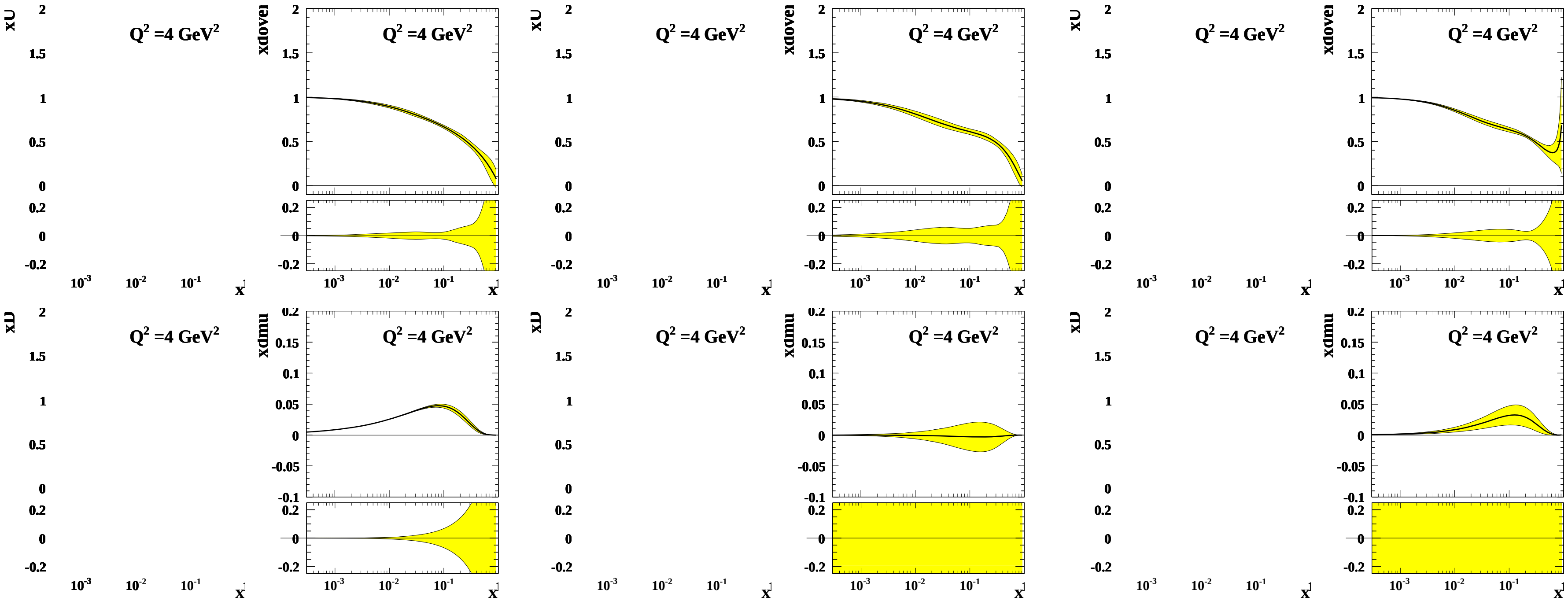
Now in terms of U, D, Ubar, Dbar



ZEUS-Jets HERAPDF H1

PDFs are really very similar- quite remarkable since ZEUS and H1 parametrizations are not- however the size of errors differs, with the HERAPDF parametrization being the most conservative

Finally in terms of d/u and dbar-ubar



ZEUS-Jets	HERAPDF	H1

Here we do see a difference in central values. I like the fact that HERAPDF reflects the fact that we don't have input data to constrain these PDFs