

# Parton Distributions for **LO** Monte Carlo Generators

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PDF4LHC08

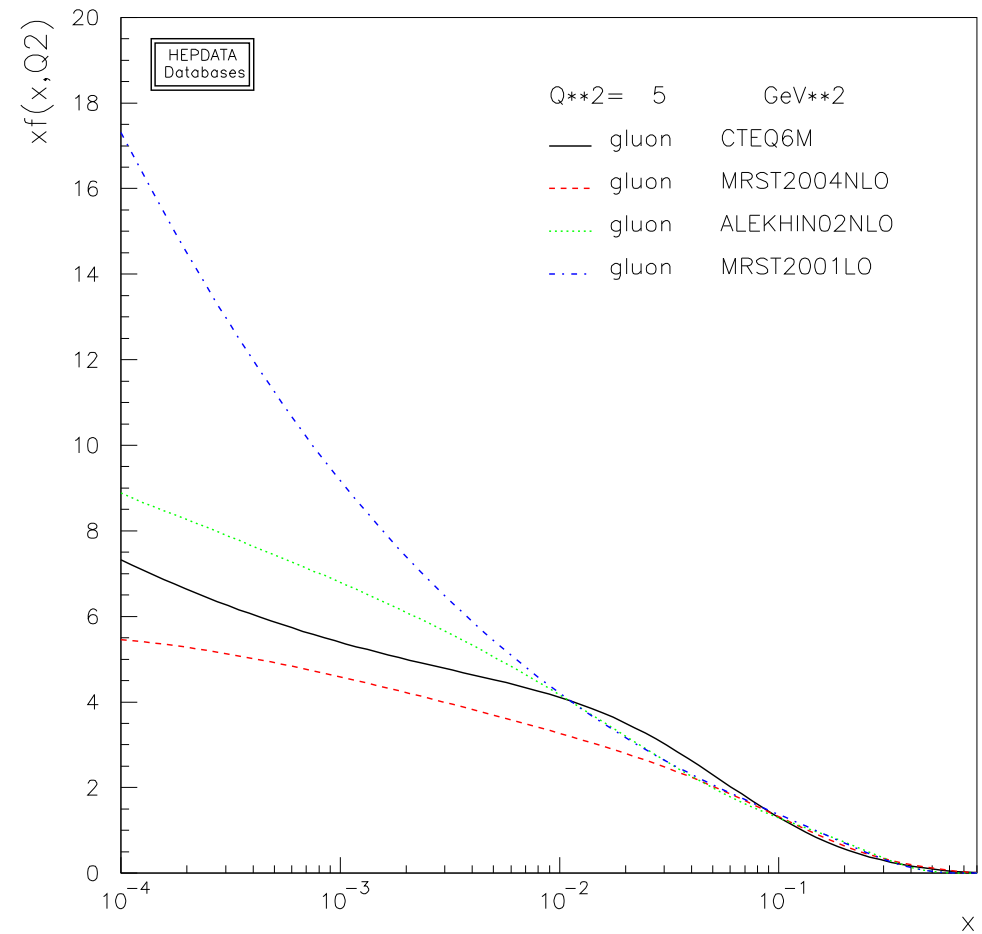
Which order of partons should be used in **LO** Monte Carlo generators.

Enormous change in partons, especially gluon when going from **LO** → **NLO**.

**LO** partons are the usual one used with many **LO** Monte Carlo programs.

All such results should be treated with care.

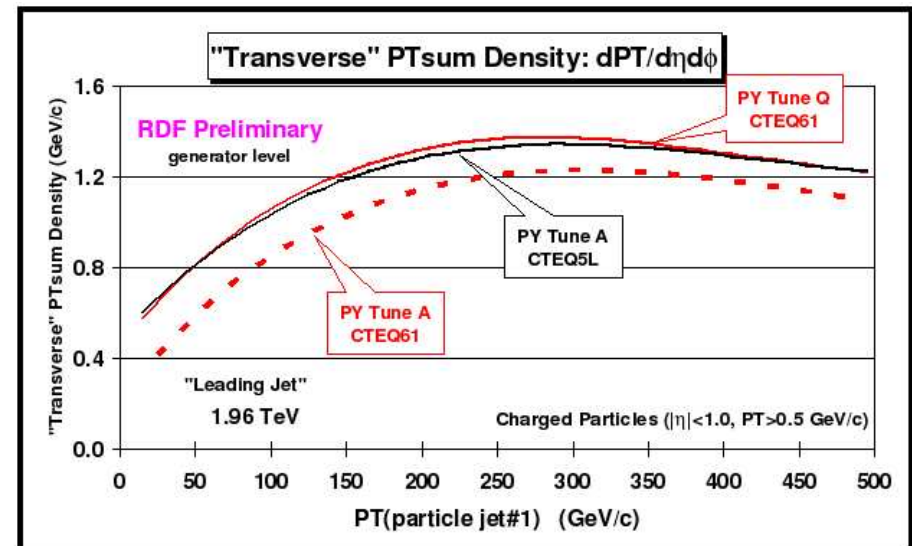
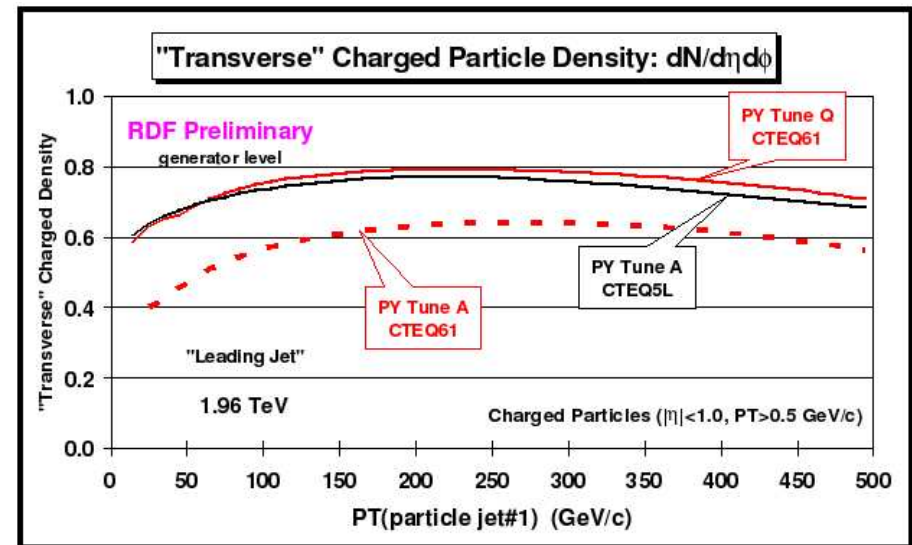
Not **NLO** partons? Not a trivial issue.



Already investigated in terms of tuning for underlying event ([Field](#)). See big difference between using [CTEQ6L](#) and [CTEQ6.1M](#) partons, mainly due to gluon.

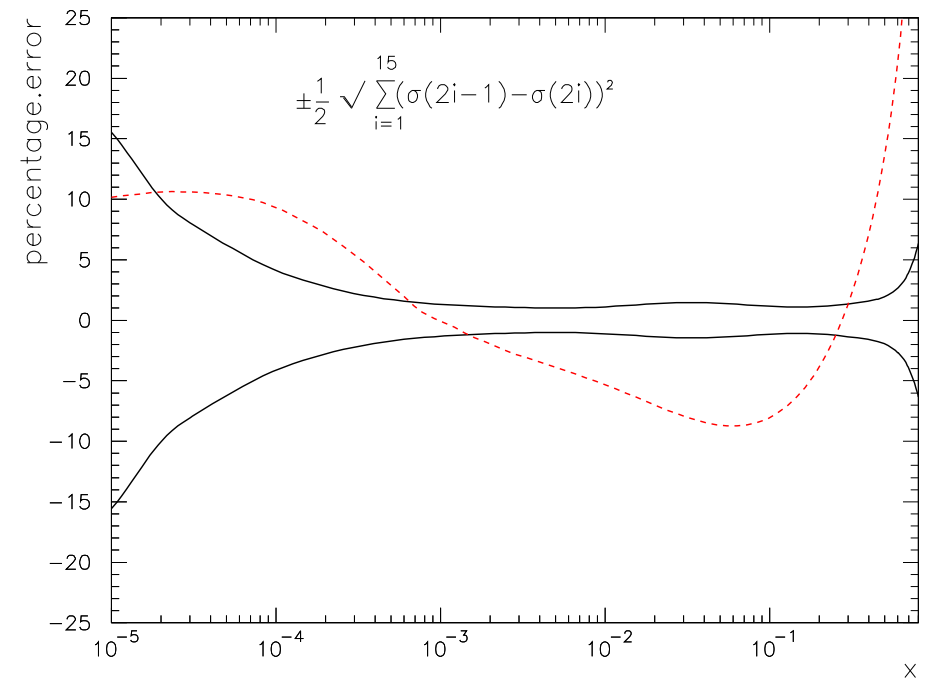
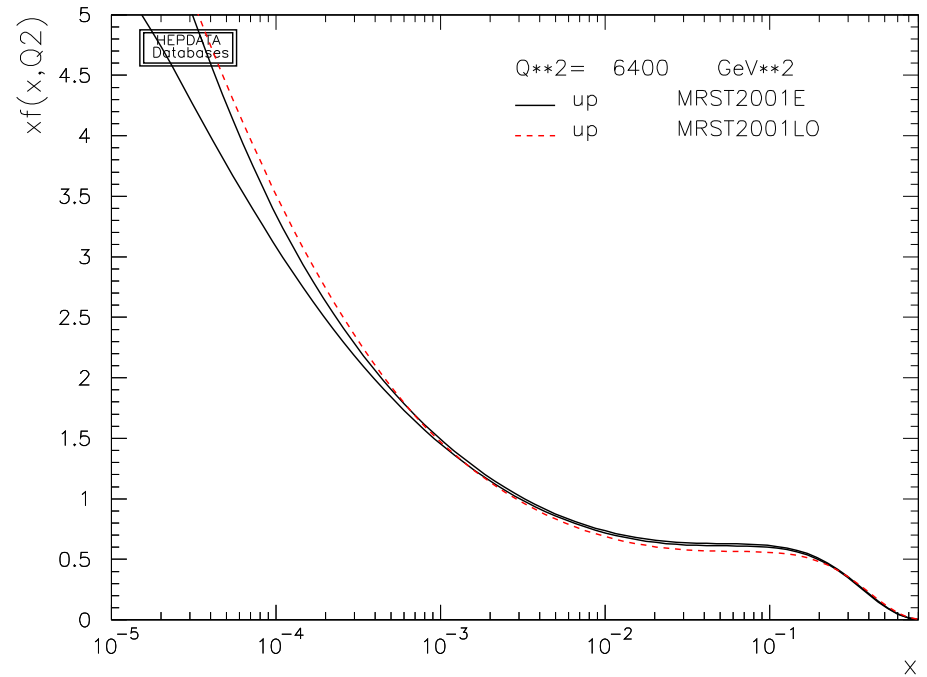
Agreement can be reached by retuning. Will affect predictions for other quantities. Want universality.

In order to investigate this look at indications from well-understood (simple) processes.



First note that the **LO** quarks over wide region of smaller  $x$  qualitatively smaller than **NLO**. Lack of additional quark evolution at **NLO**.

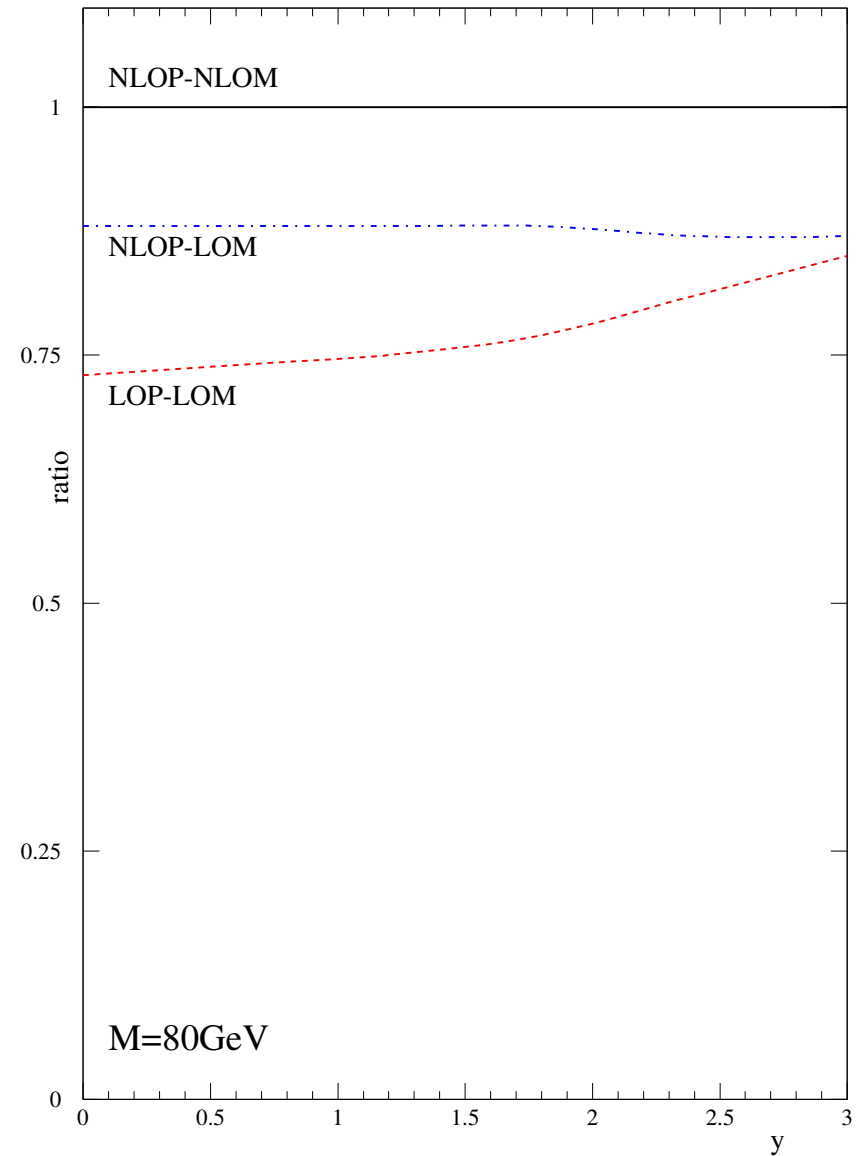
At high  $x$   $\ln(1 - x)$  terms in **NLO** matrix elements lead to **NLO** quarks being smaller.



NLO partons lead to best shape for inclusive fixed order heavy boson production at the LHC.

Has lead to the proposal that NLO partons should always be used.

Drell-Yan Cross-section at LHC for 80 GeV with Different Orders



Small  $x$  counter-example. Consider production of charm in DIS. All charm produced in final state (FFNS).

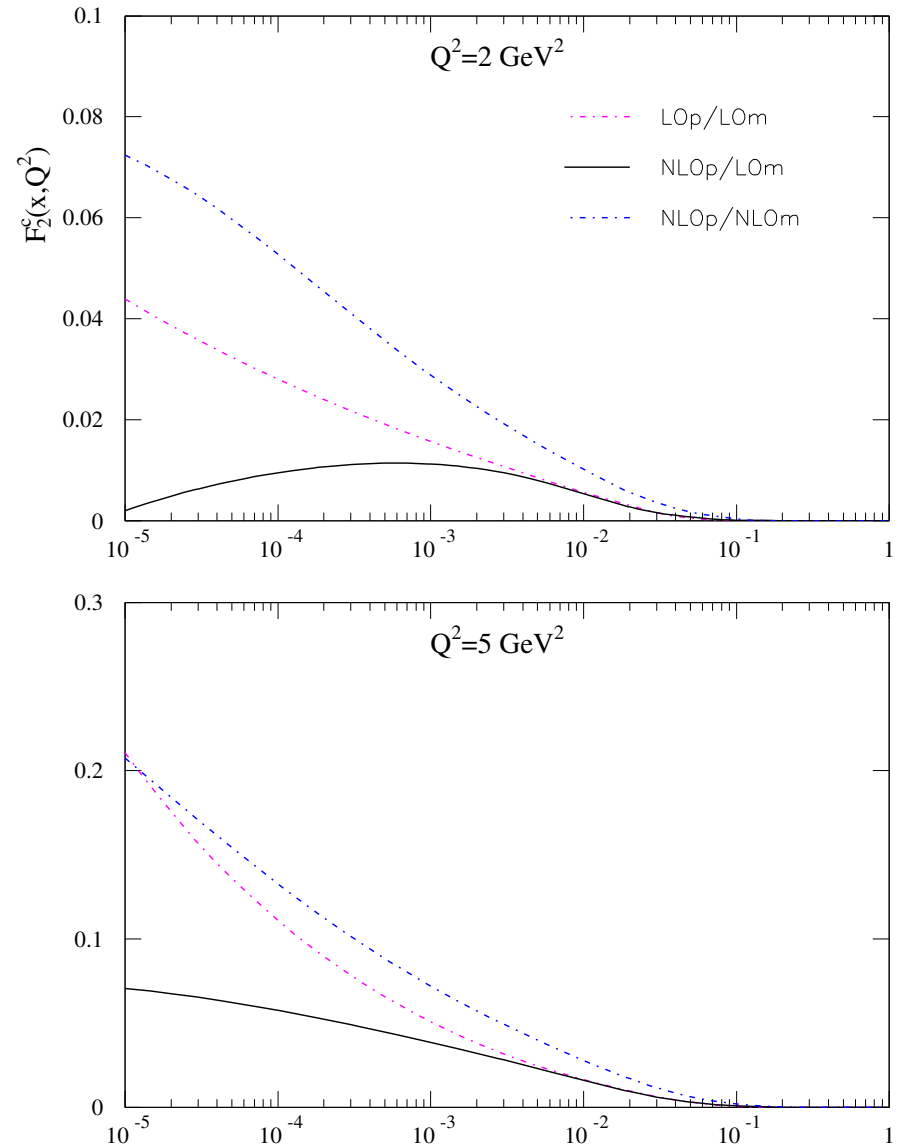
NLO matrix element contain divergence at small  $x$  not present at LO.

Same issues in heavy flavour hadro-production.

Using NLO partons the LO matrix element result is well below the truth at low scales. Shape totally wrong.

LO gluon is very large at small  $x$  since it has been extracted with missing enhancements at small  $x$ .

LO partons and LO matrix element more sensible. compensation between failings in both.



Sometimes **NLO** partons better to use if only **LO** matrix elements are known. Can get significant problems with shape if **LO** partons used.

**But** can be completely wrong at small  $x$  using **NLO** partons due to *zero*-counting of  $\ln(1/x)$  terms.

At **LO** compared to **NLO** (and higher orders) missing terms in  $\ln(1-x)$  and  $\ln(1/x)$  in coefficient functions and/or evolution.

→ partons at **LO** bigger at  $x \rightarrow 1$  and at  $x \rightarrow 0$  in order to compensate.

From momentum sum rule not enough partons to go around – leads to bad global fit at **LO** – partially compensated by large  $\alpha_S(M_Z^2)$ .

However, Relaxing momentum sum rule at **LO** could make **LO** partons rather more like **NLO** partons where they are normally too small but would still be bigger than **NLO** where necessary.

Also useful to use **NLO** definition of coupling constant.

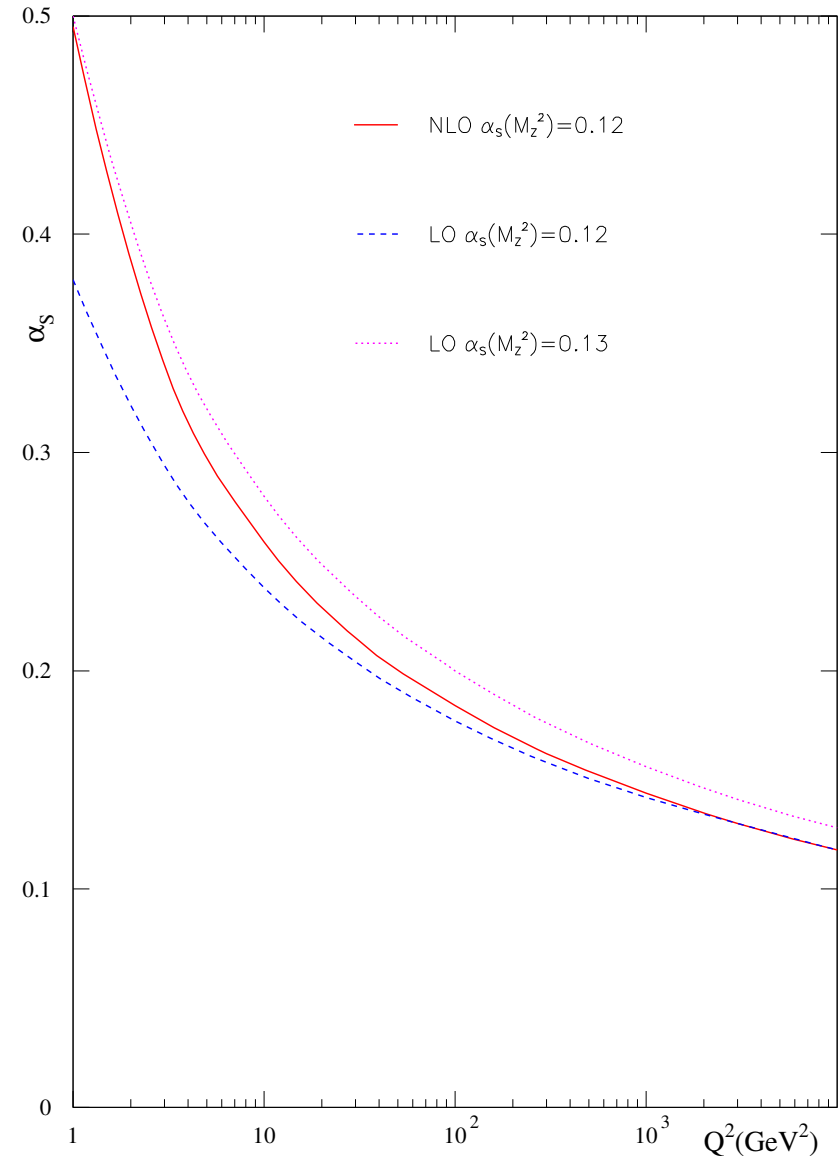
Because of quicker running at **NLO** couplings with same value of  $\alpha_S(M_Z^2)$  very different at lower scales where **DIS** data exists.

Near  $Q^2 = 1\text{GeV}^2$  **NLO** coupling with  $\alpha_S(M_Z^2) = 0.120$  similar to **LO** coupling with  $\alpha_S(M_Z^2) = 0.130$ .

Use of **NLO** coupling helps alleviate discrepancy between different orders.

**NLO** coupling already used in **CTEQ** **LO** partons and in Monte Carlo generators.

Comparison of  $\alpha_S$  at LO and NLO





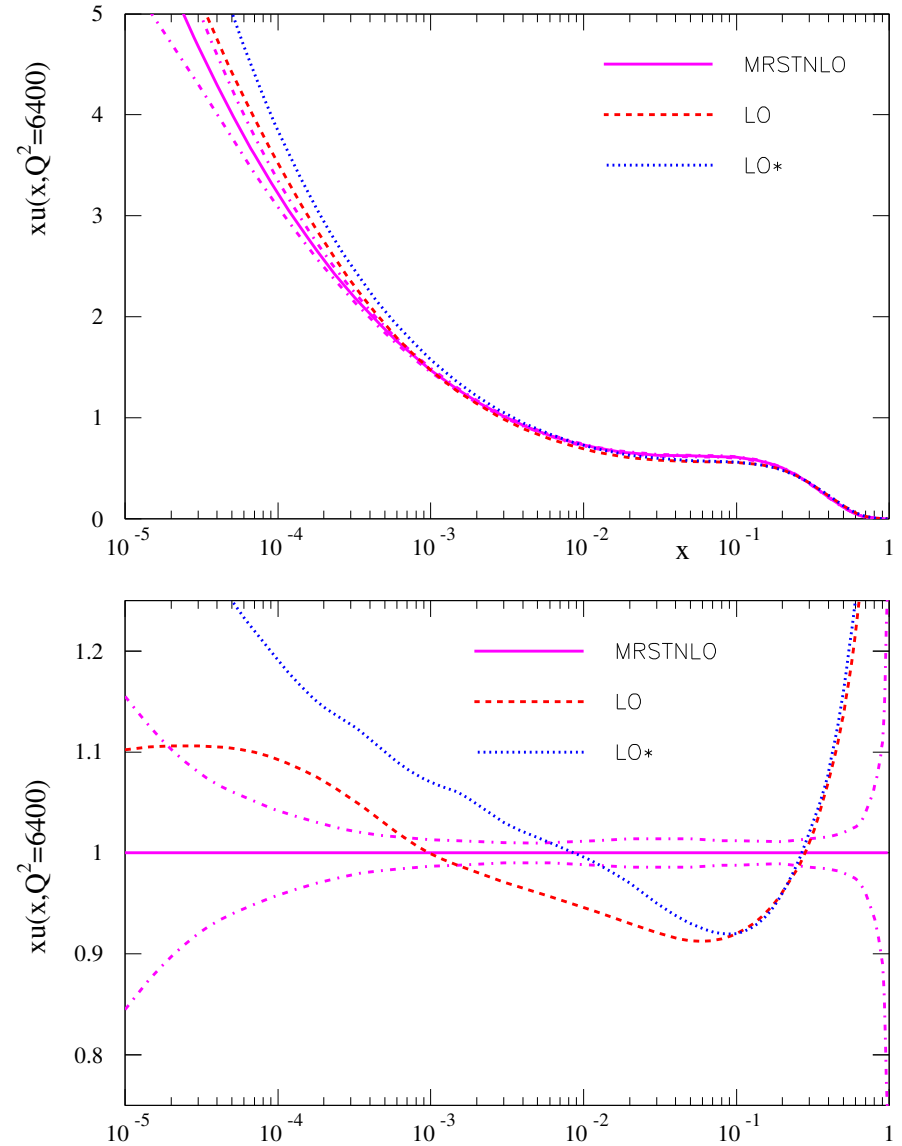
Relaxing momentum violation and allowing **NLO** definition of coupling does dramatically improve quality of **LO** global fit.

Momentum carried by input partons goes up to **113%**.

Using **NLO** definition  $\alpha_S(M_Z^2) = 0.121$ .

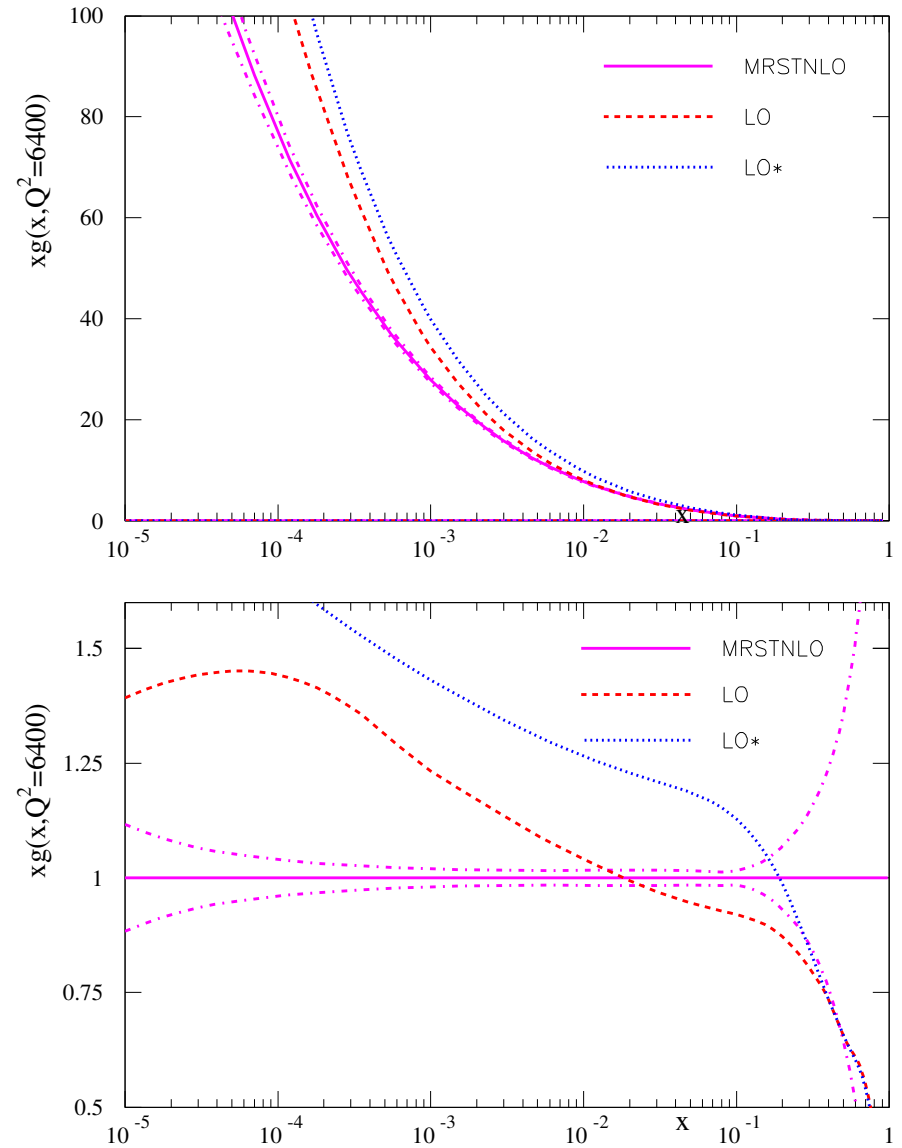
The **LO\*** and **NLO** partons are more similar in this case, particularly for  $x \sim 0.001-0.01$ . (**LO\*** often bigger – compensates for smaller cross-section at **LO**).

Full details of study in [A. Shertsnev and R.S. Thorne, e-Print: arXiv:0711.2473 \[hep-ph\]](#).



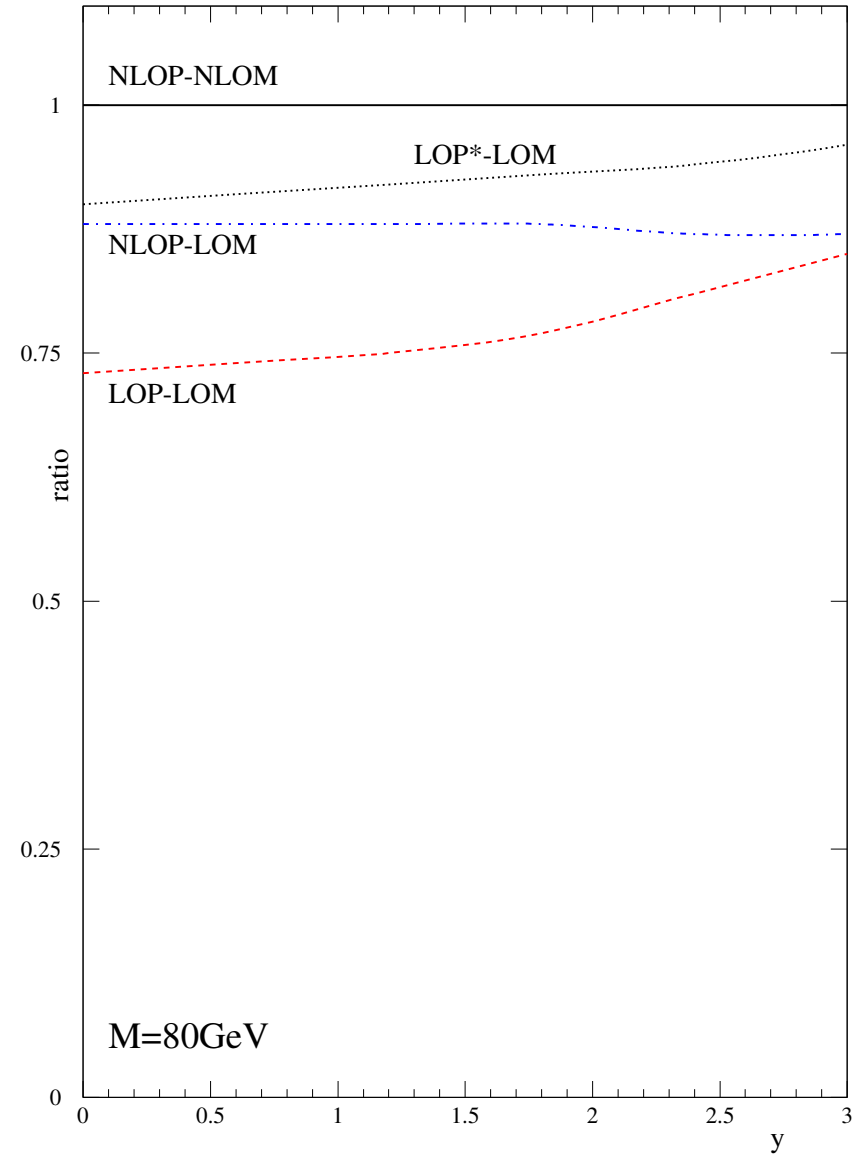
Similarly  $g(x, Q^2)$  is significantly bigger at  $\text{LO}^*$  than at  $\text{LO}$ , and much bigger than  $\text{NLO}$  at small  $x$ .

Should do better for gluon-gluon initiated processes (e.g. Higgs production where  $K$ -factors are often much greater than unity).

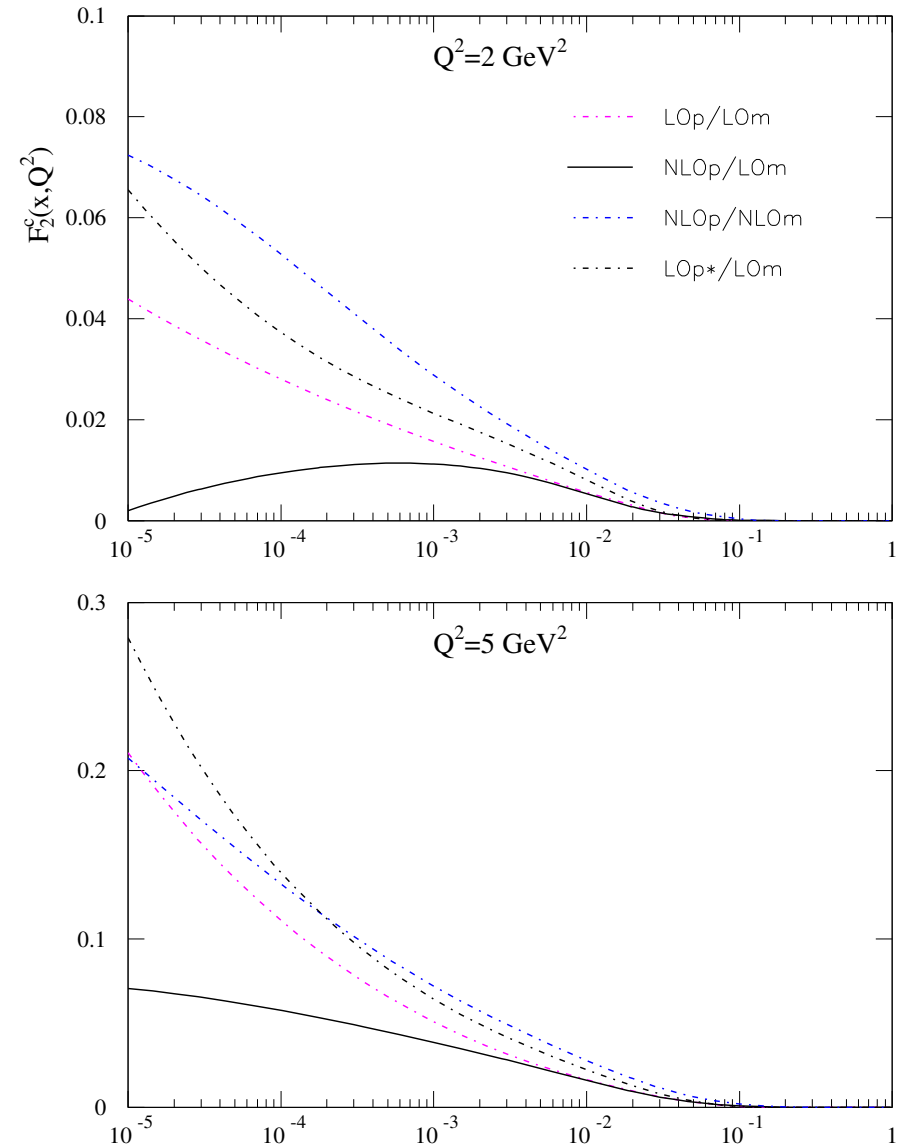


For LHC  $LO^*$  partons lead to shape of comparable quality as NLO partons. Normalization better.

Drell-Yan Cross-section at LHC for 80 GeV with Different Orders



For charm structure function comparing all possibilities  $\text{LO}^*$  partons and  $\text{LO}$  matrix element is indeed nearest to *truth* at low scales.



These are for totally inclusive, strictly fixed order calculations. Consider using generators (work with/by [A Sherstnev](#)) and include parton showering (i.e. use [MC@NLO](#) at [NLO](#)).

Consider first  $Z \rightarrow \mu^+ \mu^-$  production at the LHC with  $p_T > 10\text{GeV}$  and  $|\eta| < 5$

$$\text{NLO}(\text{ME}) \otimes \text{NLO}(\text{pdf}) = 2.40\text{nb}.$$

$$\text{LO}(\text{ME}) \otimes \text{LO}(\text{pdf}) = 1.85\text{nb}.$$

$$\text{LO}(\text{ME}) \otimes \text{NLO}(\text{pdf}) = 1.98\text{nb}.$$

$$\text{LO}(\text{ME}) \otimes \text{LO}^*(\text{pdf}) = 2.19\text{nb}.$$

With very similar relative results for  $W \rightarrow \nu\mu$ , i.e.

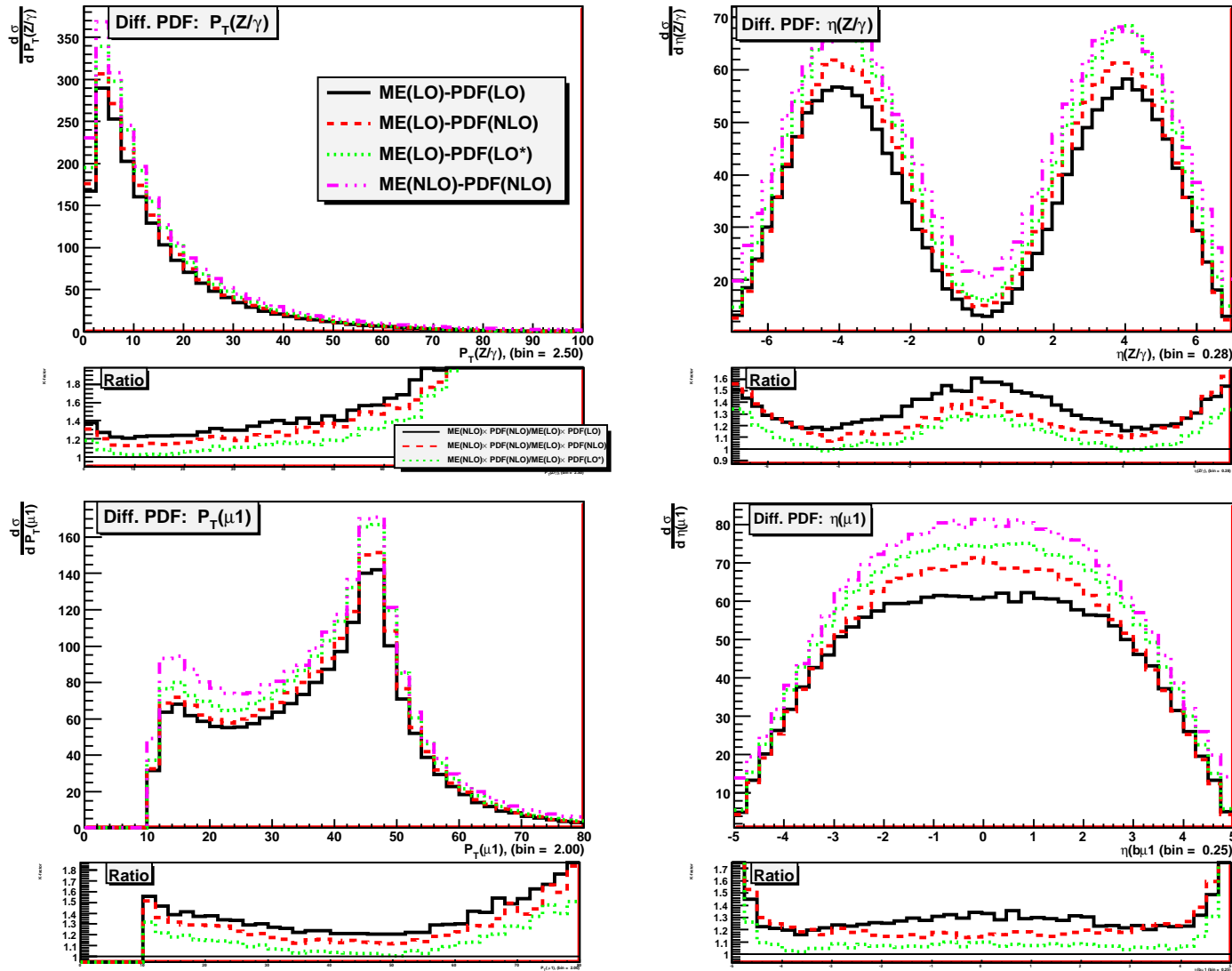
$$\text{NLO}(\text{ME}) \otimes \text{NLO}(\text{pdf}) = 21.1\text{nb}.$$

$$\text{LO}(\text{ME}) \otimes \text{LO}(\text{pdf}) = 17.5\text{nb}.$$

$$\text{LO}(\text{ME}) \otimes \text{NLO}(\text{pdf}) = 18.6\text{nb}.$$

$$\text{LO}(\text{ME}) \otimes \text{LO}^*(\text{pdf}) = 20.6\text{nb}.$$

Also look at distributions for  $Z$  boson and final state muon.



Results using  $LO^*$  partons clearly best. No parton can account for details of  $p_T$ -distribution due to hard emissions at  $NLO$ .

Consider Higgs ( $130\text{GeV}$ ) production from  $gg$  fusion at the LHC.

$$\text{NLO}(\text{ME}) \otimes \text{NLO}(\text{pdf}) = 38.0\text{pb}.$$

$$\text{LO}(\text{ME}) \otimes \text{LO}(\text{pdf}) = 22.4\text{pb}.$$

$$\text{LO}(\text{ME}) \otimes \text{NLO}(\text{pdf}) = 20.3\text{pb}.$$

$$\text{LO}(\text{ME}) \otimes \text{LO}^*(\text{pdf}) = 32.4\text{pb}.$$

Similar for  $t\bar{t}$  production.

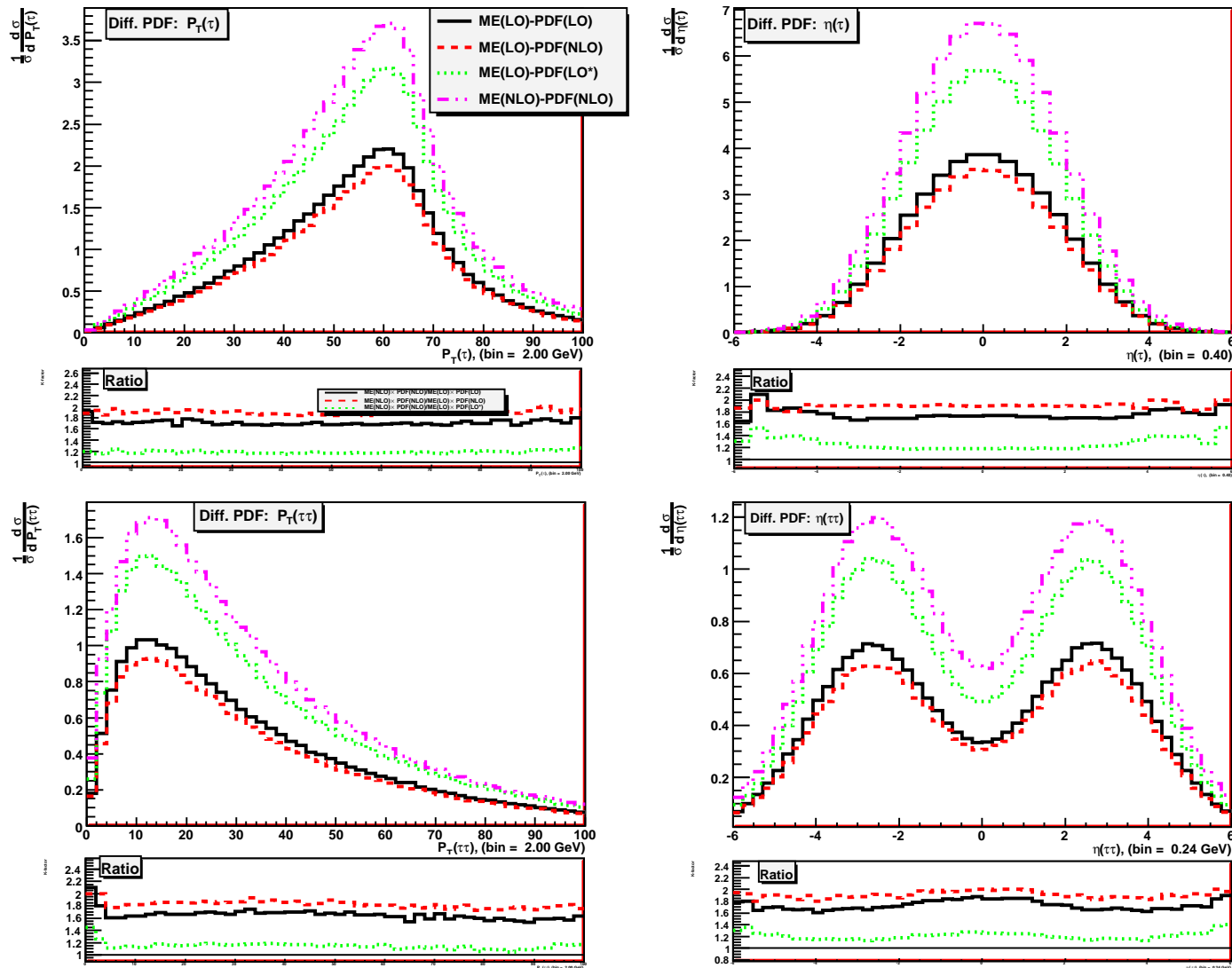
$$\text{NLO}(\text{ME}) \otimes \text{NLO}(\text{pdf}) = 813\text{pb}.$$

$$\text{LO}(\text{ME}) \otimes \text{LO}(\text{pdf}) = 561\text{pb}.$$

$$\text{LO}(\text{ME}) \otimes \text{NLO}(\text{pdf}) = 531\text{pb}.$$

$$\text{LO}(\text{ME}) \otimes \text{LO}^*(\text{pdf}) = 699\text{pb}.$$

Also look at distributions with  $H \rightarrow \tau^+\tau^-$  for single  $\tau$  and  $\tau^+\tau^-$  pair.



Results using LO\* partons clearly best in normalization. All reasonable in shape.



Consider instead single top production with  $t \rightarrow \mu + \nu + b$  production at the LHC.  
Now a  $t$ -channel process.

$$\text{NLO}(\text{ME}) \otimes \text{NLO}(\text{pdf}) = 259 \text{pb}.$$

$$\text{LO}(\text{ME}) \otimes \text{LO}(\text{pdf}) = 238 \text{pb}.$$

$$\text{LO}(\text{ME}) \otimes \text{NLO}(\text{pdf}) = 270 \text{pb}.$$

$$\text{LO}(\text{ME}) \otimes \text{LO}^*(\text{pdf}) = 297 \text{pb}.$$

Similar for other  $t$ -channel process vector boson production of Higgs + two jets using  
NLO code VBFNLO (Zeppenfeld *et al*).

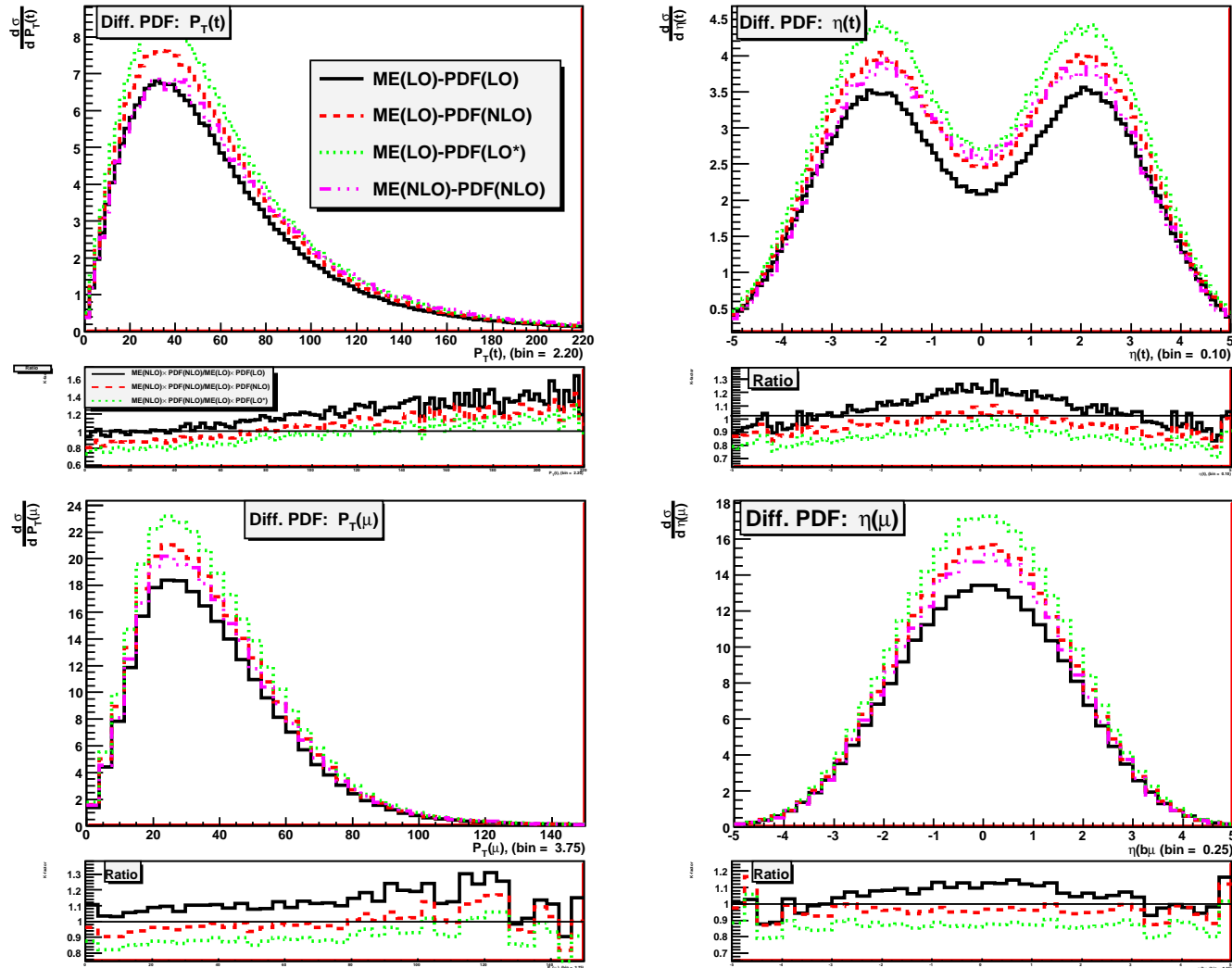
$$\text{NLO}(\text{ME}) \otimes \text{NLO}(\text{pdf}) = 4.52 \text{pb}.$$

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$$\text{LO}(\text{ME}) \otimes \text{LO}^*(\text{pdf}) = 4.95 \text{pb}.$$

Also look at distributions for  $t$  and final state muon.



Results using **LO\*** partons a bit high in normalization, but a better shape than **LO** which has the best normalization. No parton can account completely for details of  $p_T$ -distribution due to hard emissions at **NLO**. Similar for Higgs via **VBF**.

Consider  $b\bar{b}$  production with the included contribution for radiated  $g \rightarrow b\bar{b}$  at the LHC. Noted contribution strictly NLO but vital for  $p_T$ -distribution and included in LO generators. Cuts  $p_t > 20\text{GeV}$ ,  $|\eta(b)| < 5$ ,  $\Delta R(b, b) > 0.5$ .

$$\text{NLO(ME)} \otimes \text{NLO(pdf)} = 2.76\mu b.$$

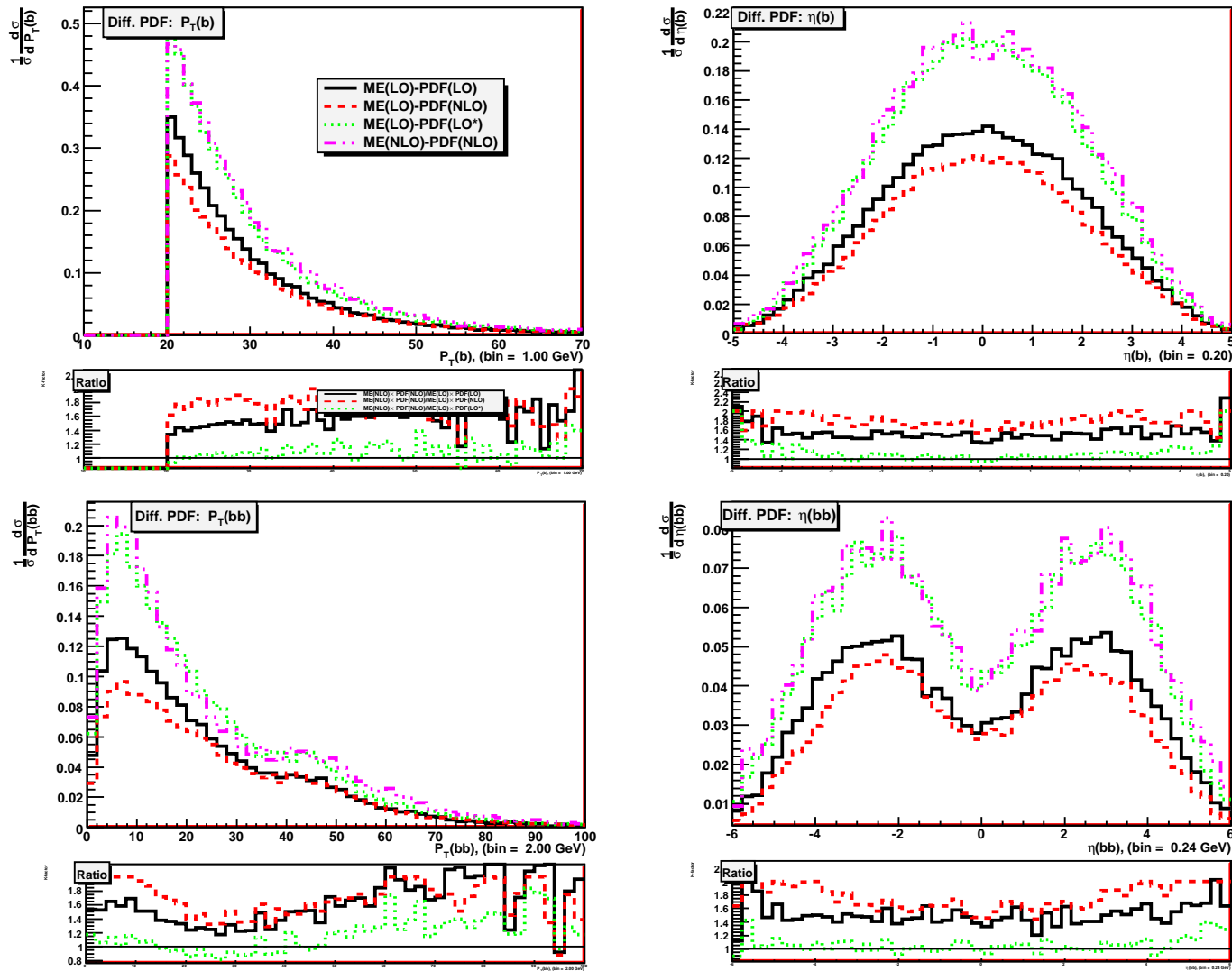
$$\text{LO(ME)} \otimes \text{LO(pdf)} = 1.85\mu b.$$

$$\text{LO(ME)} \otimes \text{NLO(pdf)} = 1.56\mu b.$$

$$\text{LO(ME)} \otimes \text{LO}^*(\text{pdf}) = 2.63\mu b.$$

This process probes the fairly small  $x$  gluon, i.e.  $x \sim 0.001$ , so NLO partons are worst due to small gluon at small  $x$ .

Also look at distributions for single  $b$  and  $b\bar{b}$  pair.



Results using **LO\*** partons clearly best in normalization. **NLO** worst and problems with shape at low scales (i.e. small  $x$ ).

## NLO Corrections

Various reasons why NLO matrix elements may give large corrections.

- $1/z$  divergent terms in matrix elements.
- Large corrections from soft-gluon emissions near the edge of phase space, i.e. large threshold corrections.
- Large correction from analytic continuation from space-like to time-like region, i.e.  $1 + \alpha_S C_F/2$  factor in Drell-Yan production.

$W$ ,  $Z$ , Higgs and  $t\bar{t}$ ,  $b$ -production and jet production (including  $W + j$ ) all have NLO enhancements from at least one of these sources. In each case the enhancement of LO\* partons compared to LO compensates to some extent (often surprisingly well).

$t$ -channel processes do not have these type of large corrections, and for e.g. single  $t$  or Higgs via vector boson fusion the NLO matrix-element correction is small. In these cases LO\* over-compensate.

However, leaves shape of distributions more-or-less unchanged. Unless probe very small- $x$  partons enhancement from LO\* partons is not that big. But these processes do not probe very small- $x$  too much by the nature of processes.

**More recent developments** - change of argument of coupling constant.

Monte Carlo generators use scale  $p_t^2 = Q^2 * (1 - z)$  for the coupling constant in initial state parton branching rather than the standard PDF choice of  $Q^2$ . Automatically incorporates leading log corrections at high  $z$ .

Incorporated this scale in  $P_{qq}$  splitting function (by far most important effect at high  $z$  and  $x$ ) in a parton number conserving manner - nonsinglet evolution still conserves number of valence quarks.

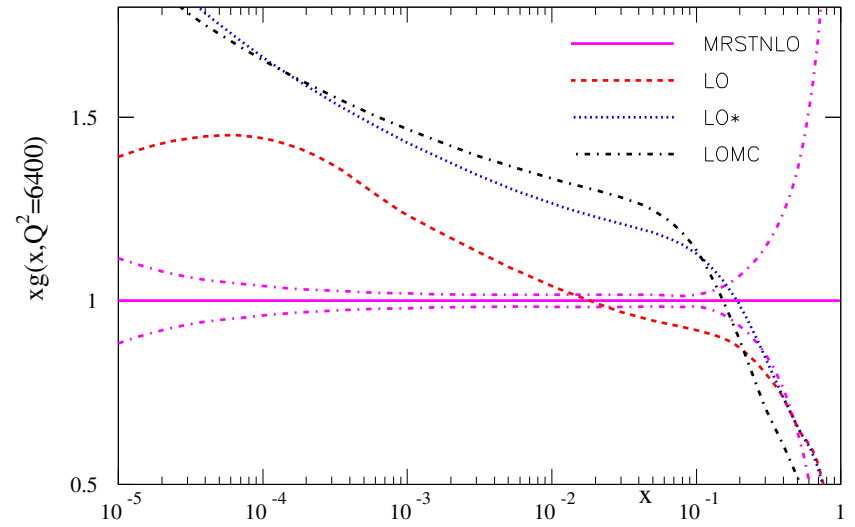
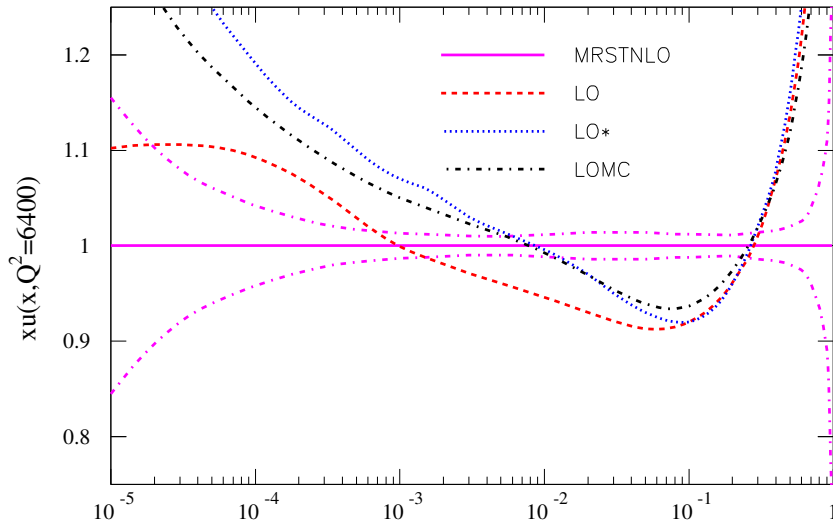
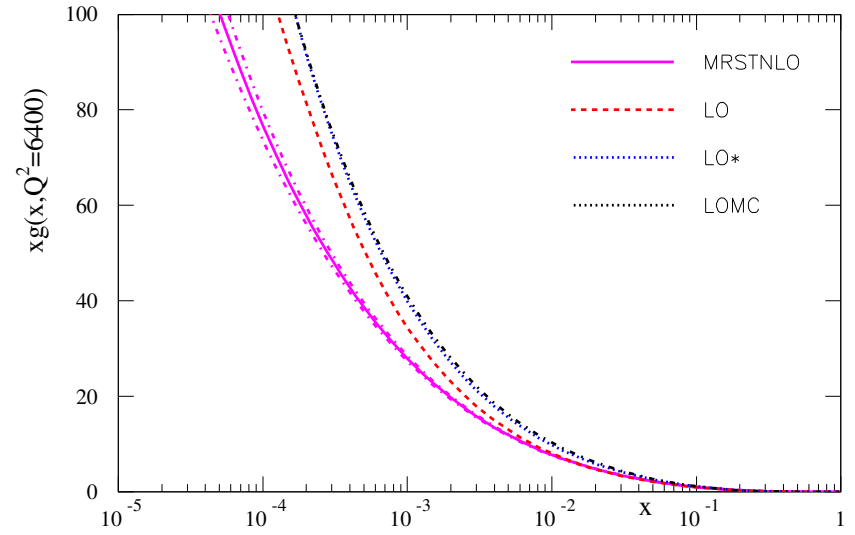
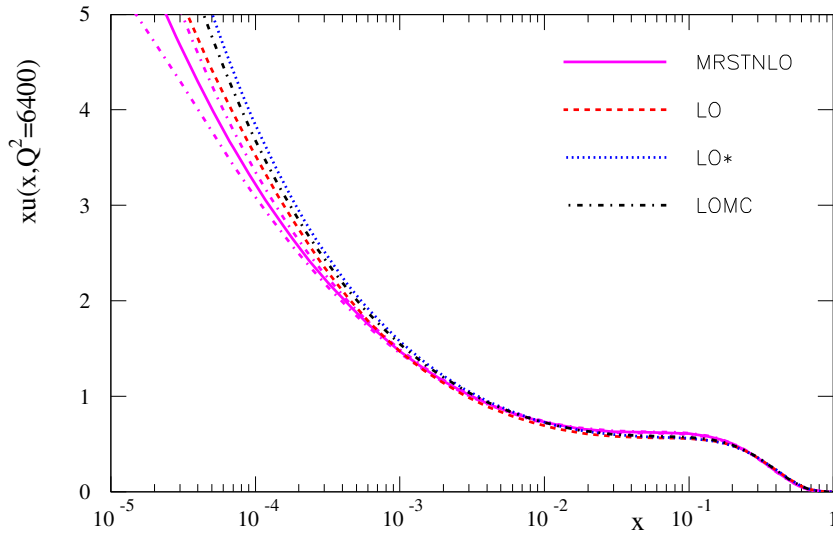
Quality of fit improves by  $\sim 50$  units, mainly for high- $x$  structure functions where resummation speeds evolution.

Allows  $\alpha_S(M_Z^2)$  to lower to 0.115.

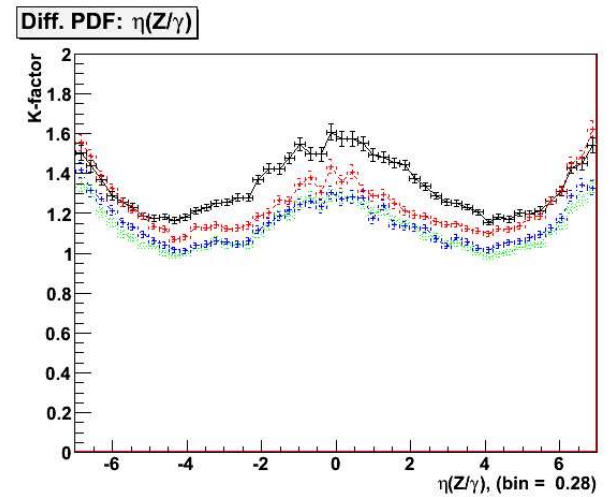
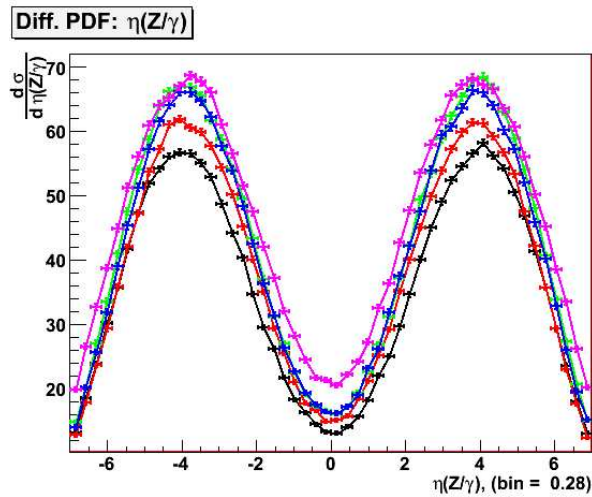
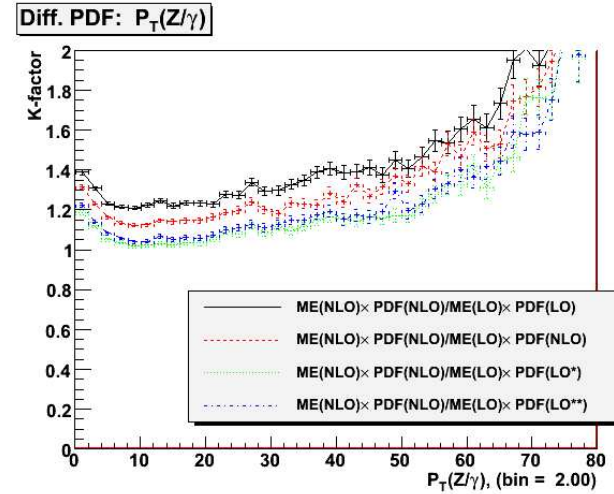
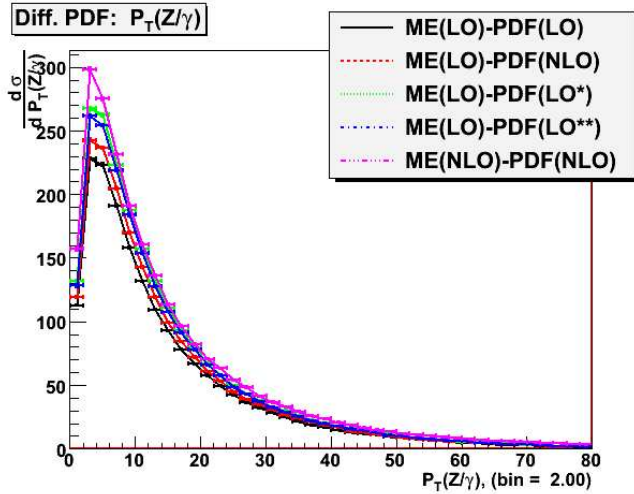
Input partons now carry 117% momentum, but this now falls with  $Q^2$  since modified coupling leads to increased branching of high- $x$  quarks.

Overall change in partons LOMC compared to LO\* very modest.

Partons rather insensitive to change. **LOMC** far more similar to **LO\*** than to **LO** and **NLO**.



Look at distributions.

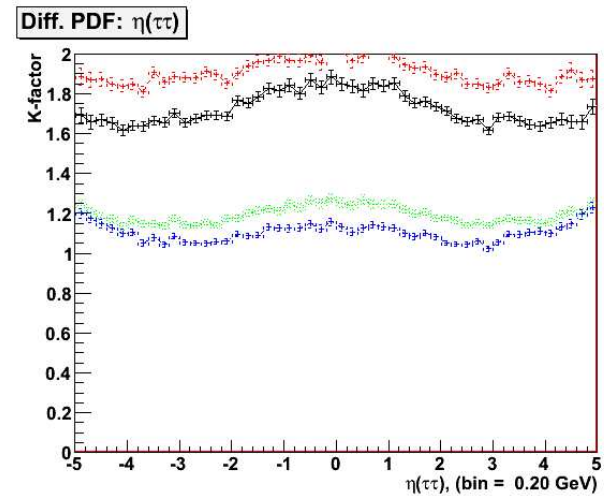
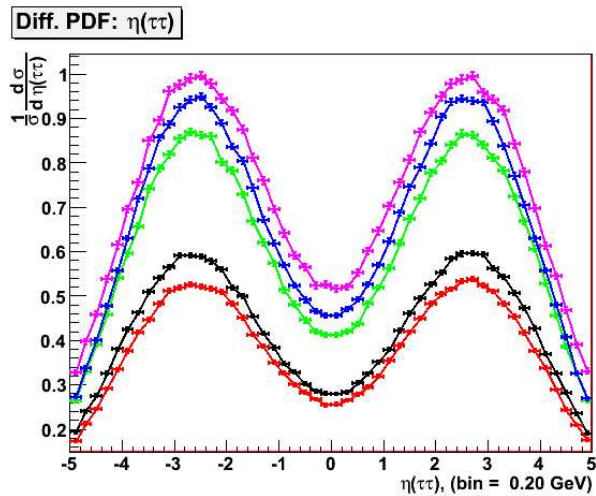
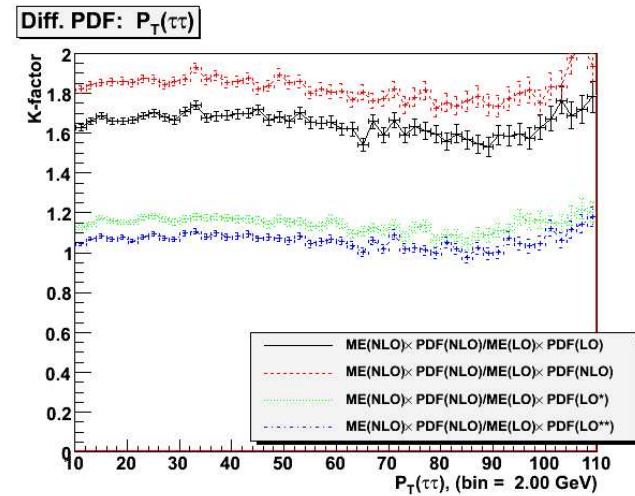
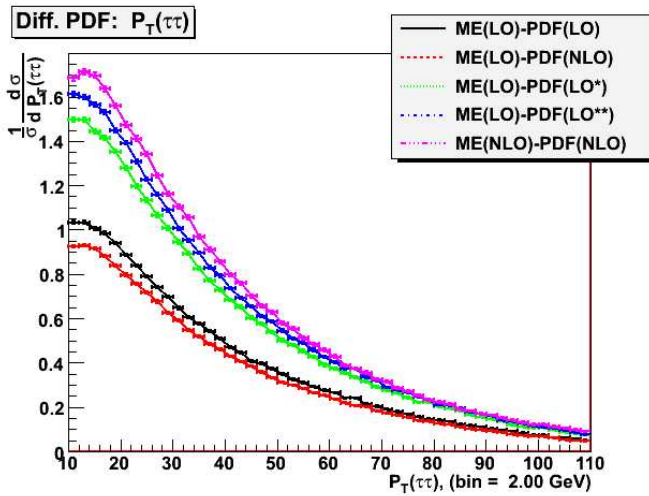


Results using **LO\*\*** partons extremely similar to those using **LO\*** partons for  $Z$  production.

Very similar for  $W$  production.



# Look at gluon dominated quantity



Results using **LO\*\*** partons extremely similar to those using **LO\*** partons for Higgs production from gluon-gluon fusion.

Very similar for  $b\bar{b}$  production.

Consider instead single top production with  $t \rightarrow \mu + \nu + b$  production at the LHC.  
Now a  $t$ -channel process.

$$\text{NLO}(\text{ME}) \otimes \text{NLO}(\text{pdf}) = 259 \text{pb}.$$

$$\text{LO}(\text{ME}) \otimes \text{LO}(\text{pdf}) = 238 \text{pb}.$$

$$\text{LO}(\text{ME}) \otimes \text{NLO}(\text{pdf}) = 270 \text{pb}.$$

$$\text{LO}(\text{ME}) \otimes \text{LO}^*(\text{pdf}) = 298 \text{pb}.$$

$$\text{LO}(\text{ME}) \otimes \text{LO}^*(\text{pdf}) = 288 \text{pb}.$$

$\text{LO}^{**}$  slightly better than  $\text{LO}^*$ . Similar for other  $t$ -channel process vector boson production of Higgs + two jets using NLO code VBFNLO (Zeppenfeld *et al*).

$$\text{NLO}(\text{ME}) \otimes \text{NLO}(\text{pdf}) = 4.52 \text{pb}.$$

$$\text{LO}(\text{ME}) \otimes \text{LO}(\text{pdf}) = 4.26 \text{pb}.$$

$$\text{LO}(\text{ME}) \otimes \text{NLO}(\text{pdf}) = 4.65 \text{pb}.$$

$$\text{LO}(\text{ME}) \otimes \text{LO}^*(\text{pdf}) = 4.95 \text{pb}.$$

$$\text{LO}(\text{ME}) \otimes \text{LO}^{**}(\text{pdf}) = 4.85 \text{pb}.$$

## Conclusions

Neither standard **LO** and **NLO** partons ideal for **LO** generators.

**NLO** gluon much smaller at small  $x \rightarrow$  qualitative changes. **LO** quarks usually too small.

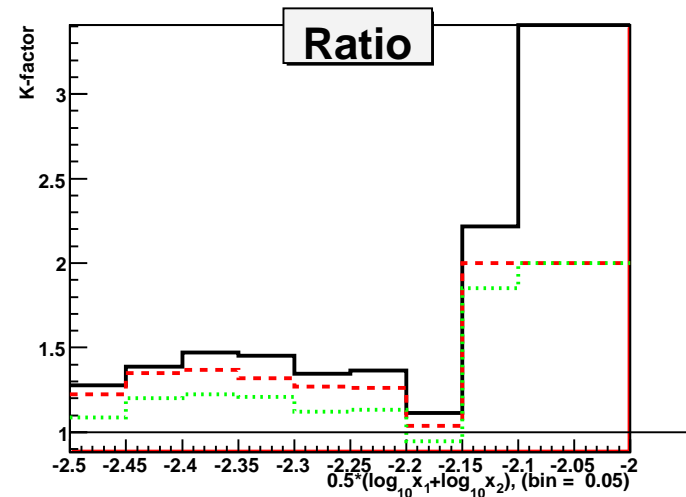
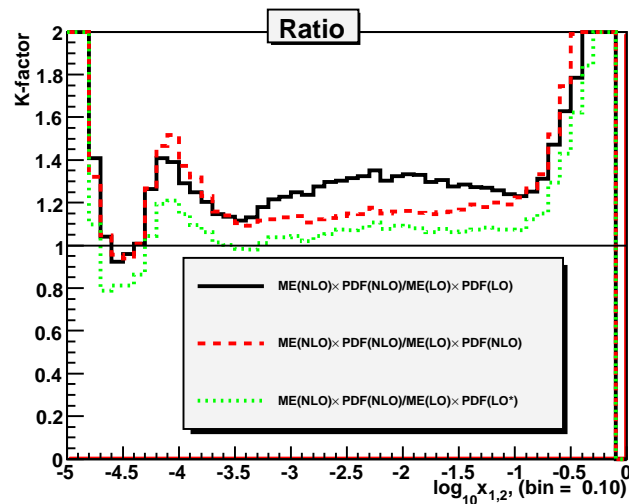
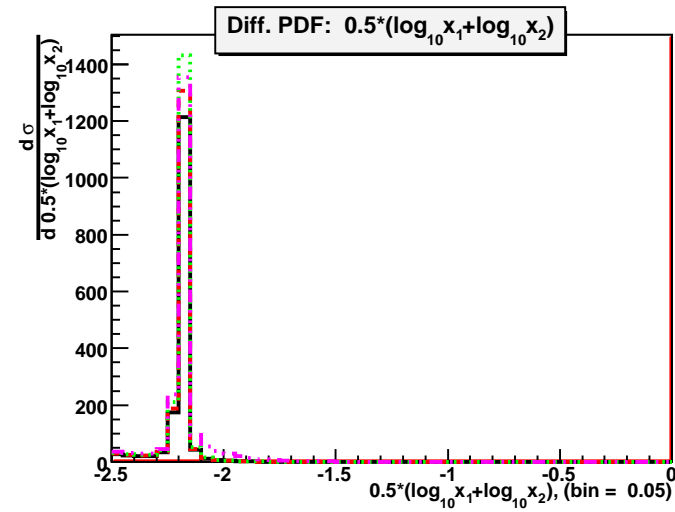
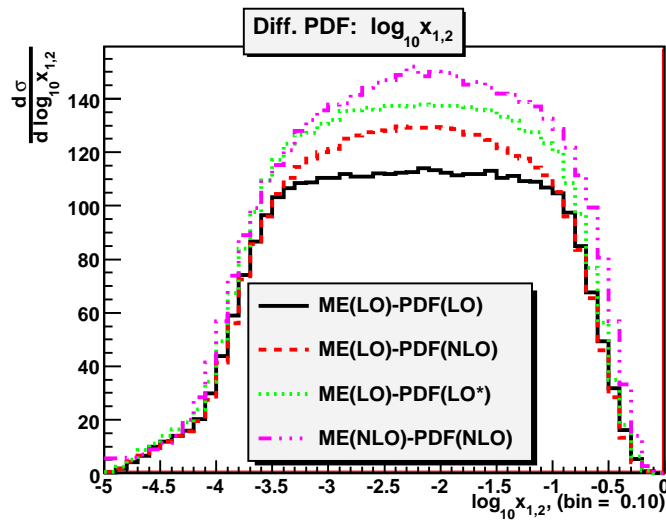
Introduce modified **LO\*** partons, i.e. momentum violation plus **NLO** coupling constant, and now Monte Carlo-inspired scale choice.

Comparison with processes where **NLO** known suggests modified **LO\*** partons usually provides most reliable results – especially if sensitive to smallish  $x$ . Additional partons allowed by extra momentum compensate semi-universally for higher orders.

**LO\*\*** partons, with additional Monte Carlo-inspired scale choice for coupling work even better.

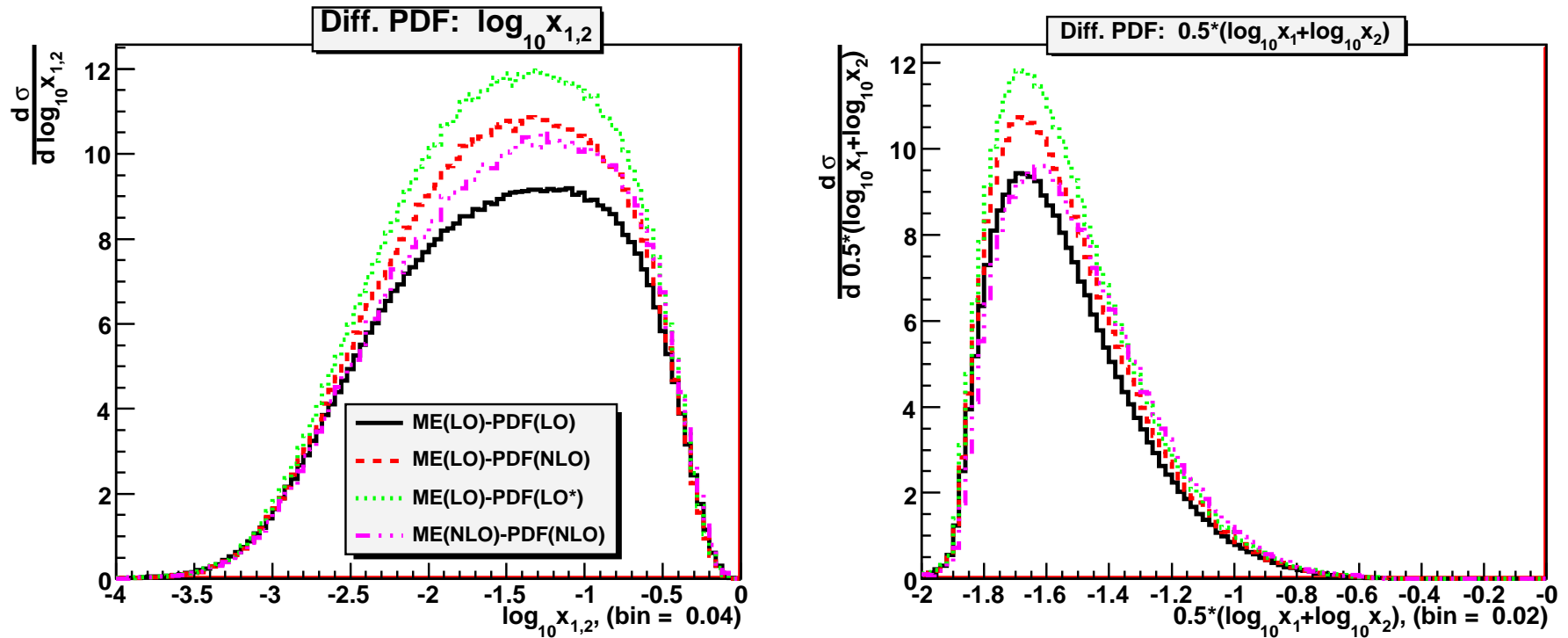
Not always most accurate way to predict full **NLO** cross-sections ( $t$ -channel processes). However, never badly wrong for any particular parton in any particular range, unlike standard fixed order.

Examination of values of  $x$  sampled in cross-section shows that deficit in **LO** rates due to lack of partons for  $x \sim 0.01$ .

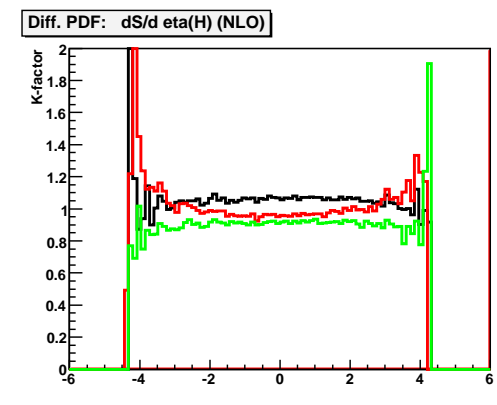
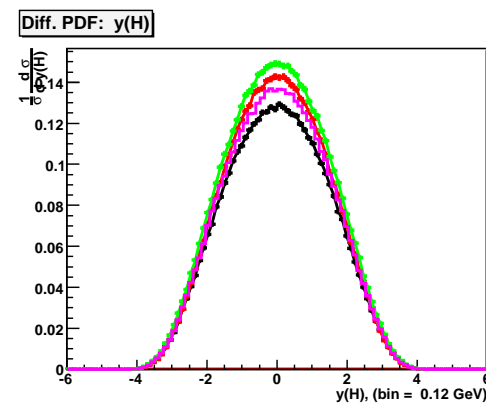
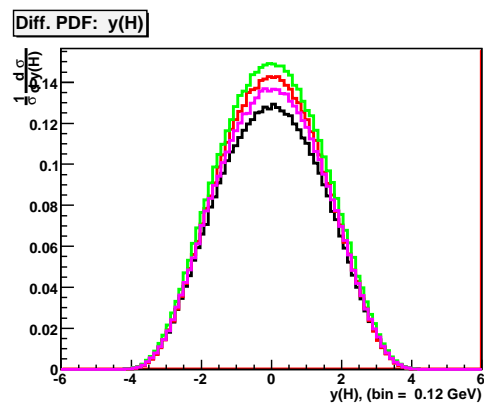
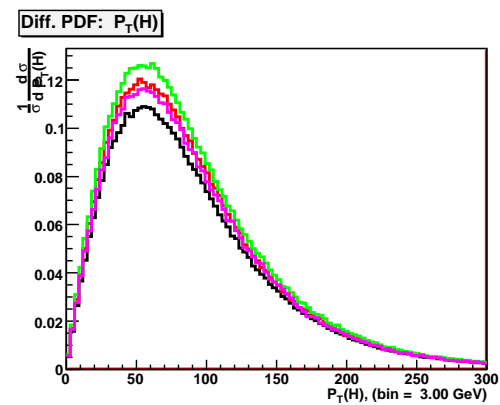
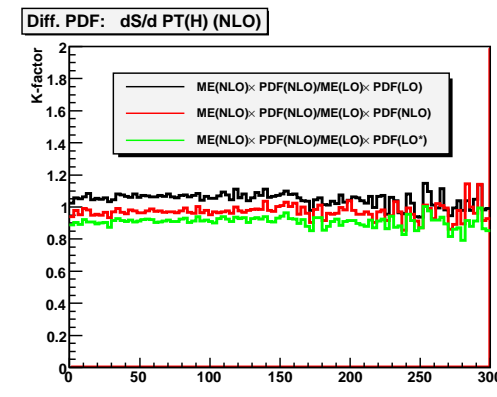
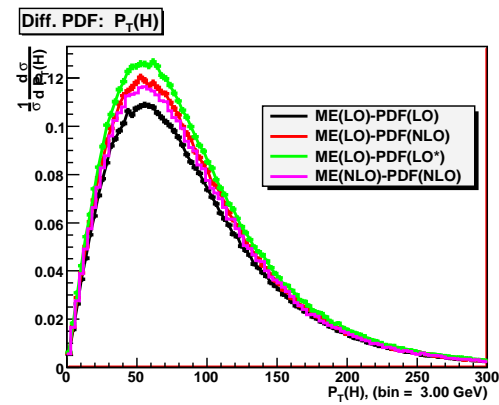
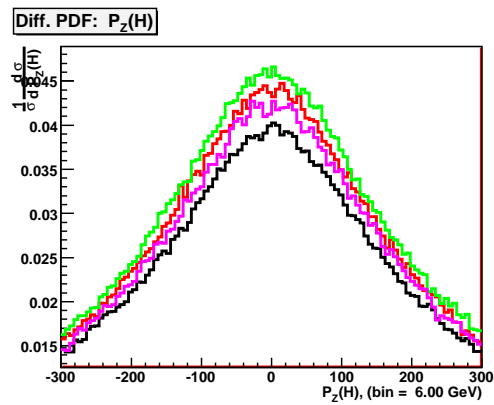
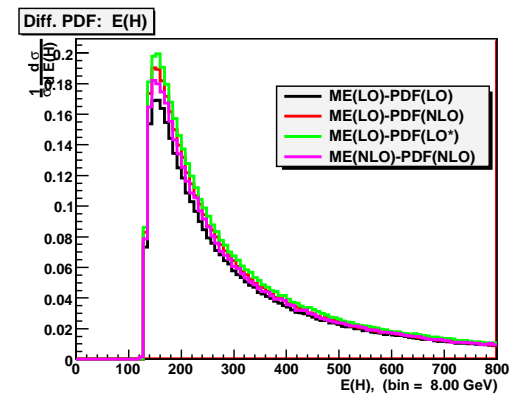


**NLO** partons have better distribution, but **LO\*** are good in normalization and shape.

These processes probe the fairly high- $x$  quarks, i.e.  $x \sim 0.01 - 0.1$ , but are much more smeared distributions than the  $s$ -channel processes.

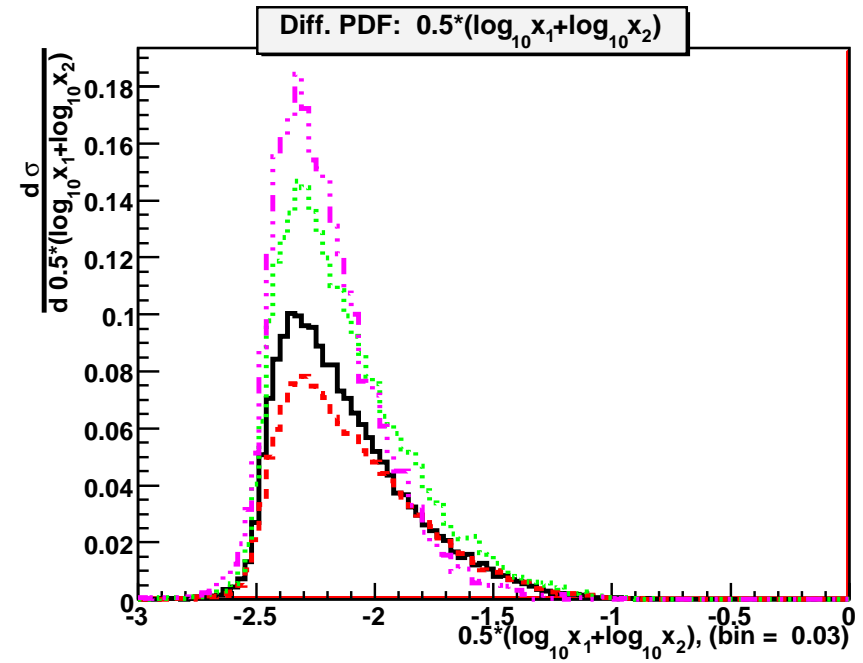
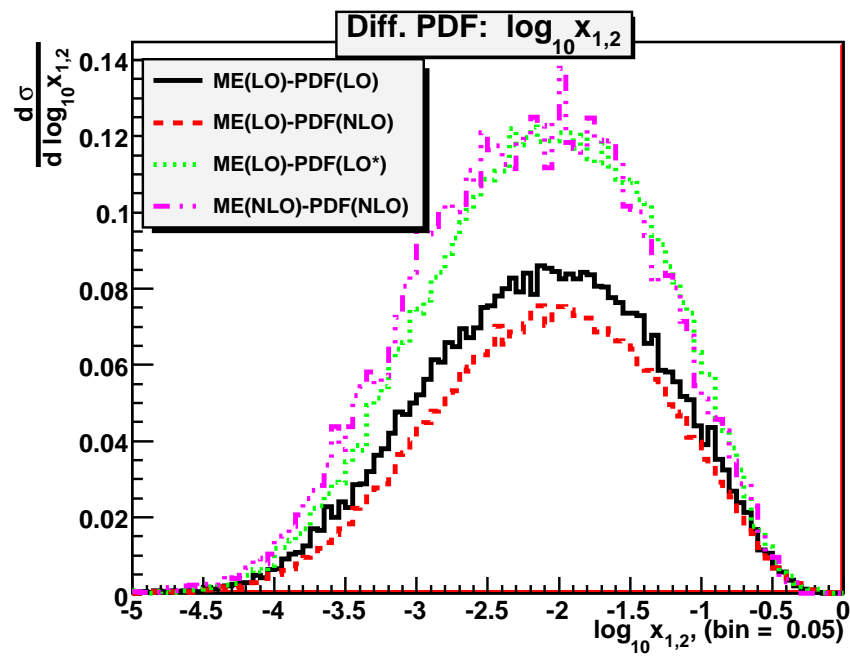


Also look at distributions for Higgs via [VBF](#).



Results using **LO\*** partons a bit high in normalization. **NLO** partons a little off in rapidity distribution.

$b\bar{b}$  probes the fairly small- $x$  quarks, i.e.  $x$  down to 0.0001.



Similar for di-jet production (using JETRAD). Cuts  $p_t > 20\text{GeV}$ ,  $|\eta(b)| < 5$ ,  $\Delta R(j, j) > 0.5$ .

$$\text{NLO(ME)} \otimes \text{NLO(pdf)} = 183\mu b.$$

$$\text{LO(ME)} \otimes \text{LO(pdf)} = 150\mu b.$$

$$\text{LO(ME)} \otimes \text{NLO(pdf)} = 116\mu b.$$

$$\text{LO(ME)} \otimes \text{LO}^*(\text{pdf}) = 178\mu b.$$

This process probes also the fairly small  $x$  gluon, so NLO partons are again worst.

Very similar for  $W + j$  production (using MCFM)

$$\text{NLO(ME)} \otimes \text{NLO(pdf)} = 21.1nb.$$

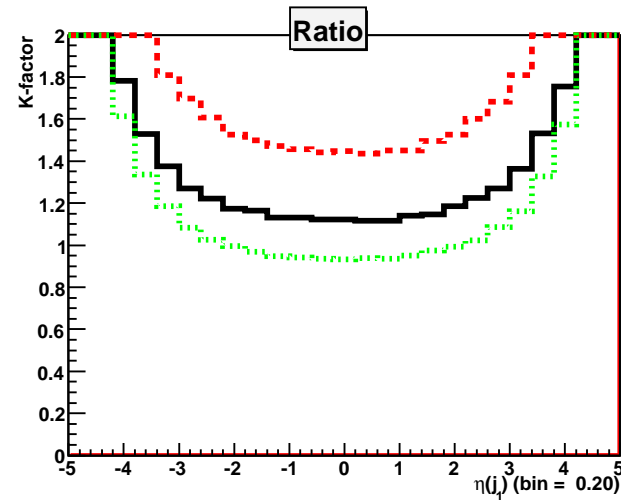
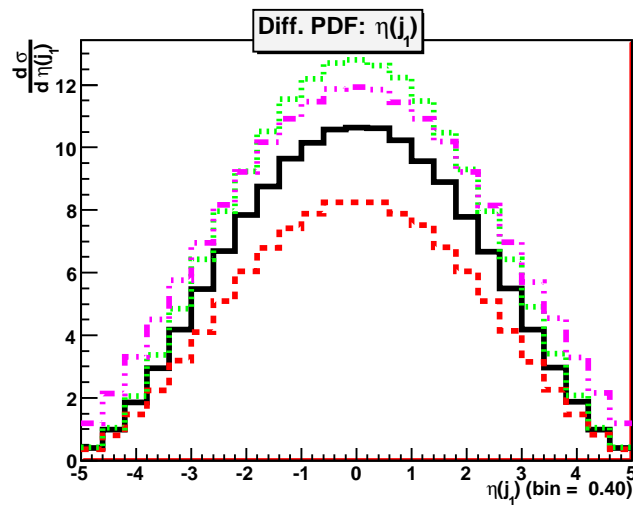
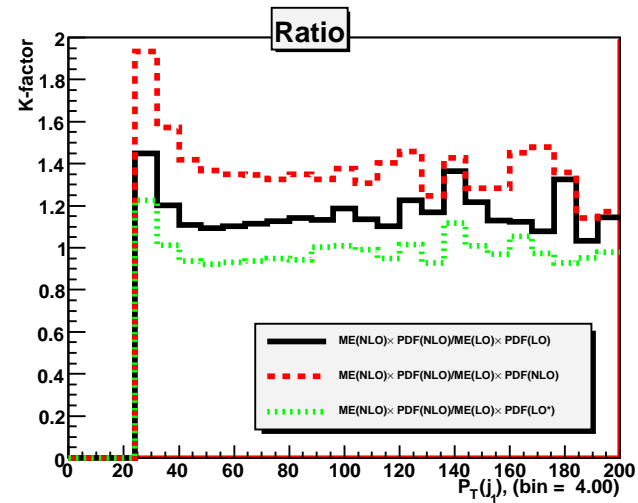
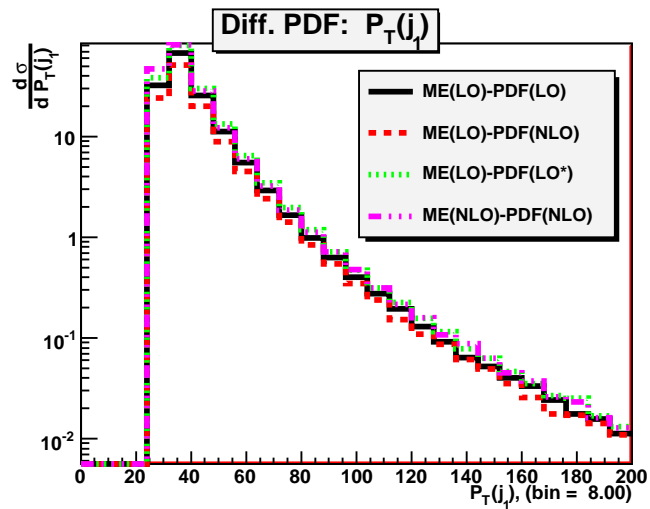
$$\text{LO(ME)} \otimes \text{LO(pdf)} = 17.5\mu b.$$

$$\text{LO(ME)} \otimes \text{NLO(pdf)} = 18.6\mu b.$$

$$\text{LO(ME)} \otimes \text{LO}^*(\text{pdf}) = 20.6\mu b.$$



Also look at distributions for di-jets



Results using **LO\*** partons clearly best in normalization. **NLO** again worst at low scales (i.e. small  $x$ ).