MSTW Parameterization and Uncertainties

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MSTW parameterization – in $\overline{MS}$ scheme.

At input scale $Q_0^2 = 1\ \text{GeV}^2$:

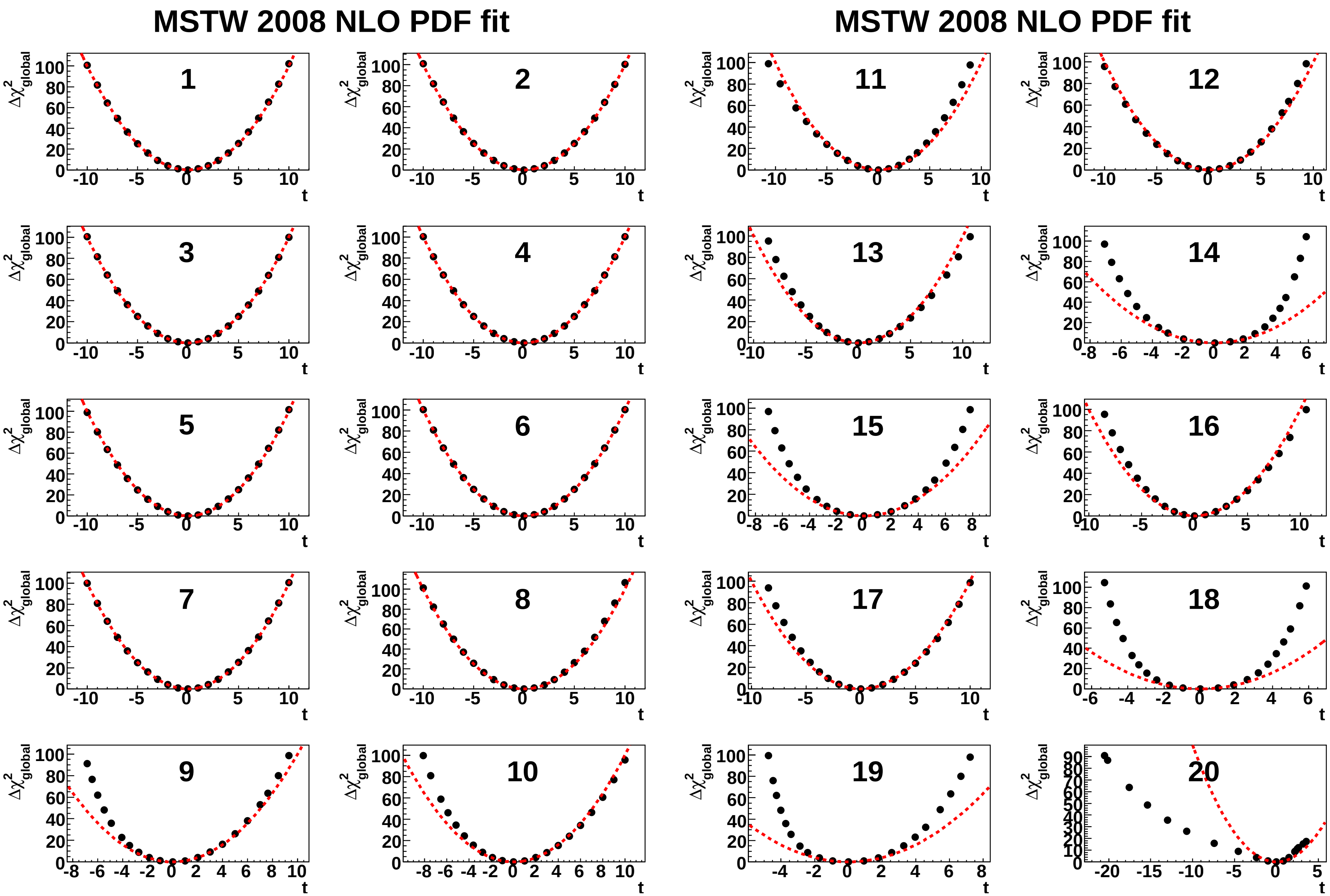
$$
\begin{aligned}
xu_v &= A_u\, x^{\eta_1}(1-x)^{\eta_2}(1+\epsilon_u\,\sqrt{x}+\gamma_u\, x)\\
xd_v &= A_d\, x^{\eta_3}(1-x)^{\eta_4}(1+\epsilon_d\,\sqrt{x}+\gamma_d\, x)\\
xS &= A_S\, x^{\delta_S}(1-x)^{\eta_S}(1+\epsilon_S\,\sqrt{x}+\gamma_S\, x)\\
x\bar{d}-x\bar{u} &= A_\Delta\, x^{\eta_\Delta}(1-x)^{\eta_S+2}(1+\gamma_\Delta\, x+\delta_\Delta\, x^2)\\
xg &= A_g\, x^{\delta_g}(1-x)^{\eta_g}(1+\epsilon_g\,\sqrt{x}+\gamma_g\, x)+A_{g'}\, x^{\delta_{g'}}(1-x)^{\eta_{g'}}\\
xs+x\bar{s} &= A_+\, x^{\delta_S}\,(1-x)^{\eta_+}(1+\epsilon_S\,\sqrt{x}+\gamma_S\, x)\\
xs-x\bar{s} &= A_-\, x^{\delta_-}(1-x)^{\eta_-}(1-x/x_0)
\end{aligned}
$$

Overall 28 parameters used to obtain best fit (δ_- actually fixed).

4 new in strange quarks (2008), 3 new in second gluon term (2001) and 2 new in $\bar{d}-\bar{u}$ in (1998).

However, $\rightarrow$ some very flat directions in eigenvector space. Some parameters very highly (anti)-correlated, e.g. ϵ_g, γ_g trade off nearly precisely.

Vary 20 highlighted in red. Other 8 fixed at best value. CTEQ similar procedure.



Beyond 20 we find Hessian approach breaks down totally due to correlations. (Behaviour of no. 20 special case – η_- has physical limit.)

Use parameterization inspired by simple spectator counting rules at high x, i.e. expect

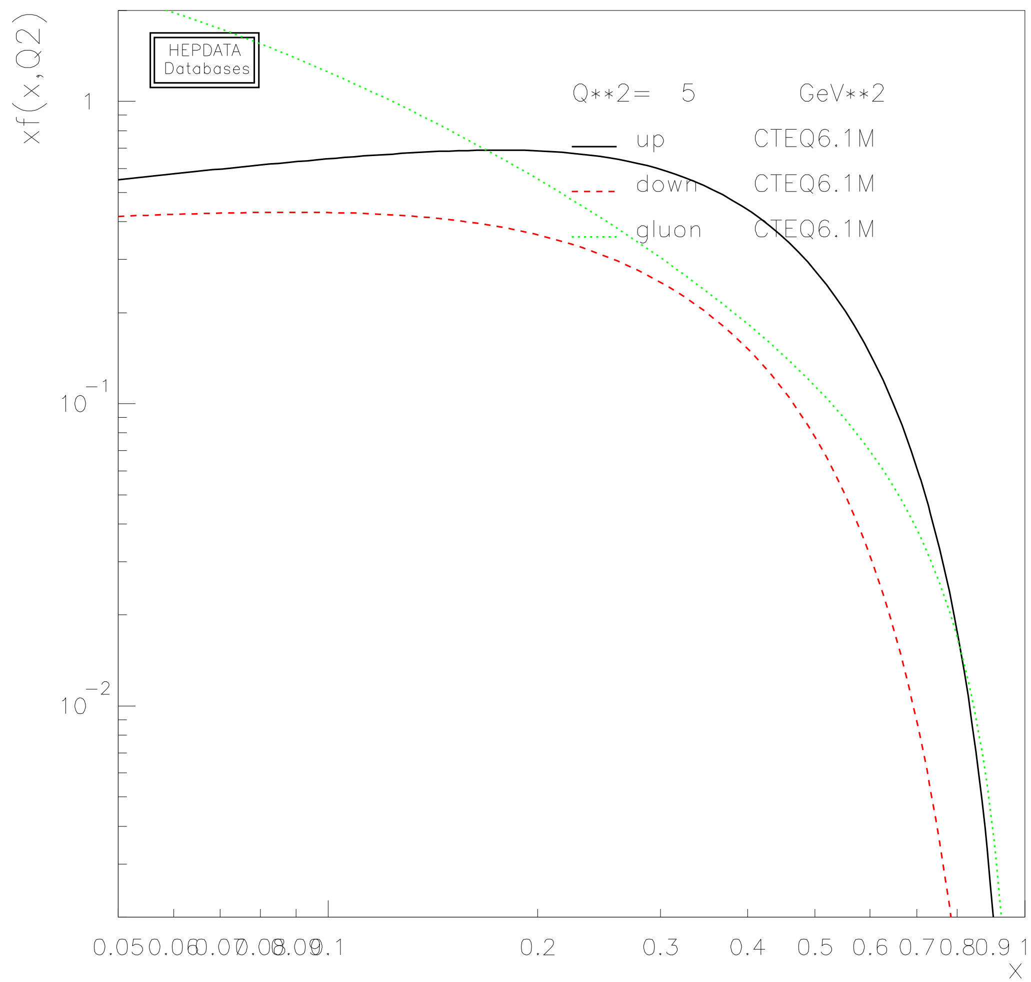
$$q_V(x) \sim (1-x)^3, \; g(x) \sim (1-x)^5$$

Not incredibly flexible. η determined by data at $x \sim 0.5$ prescribes shape at $x \to 1$ fairly strongly.

However, this seems likely. Avoids extreme behaviour for x very near 1

Illustrated by CTEQ6 partons which gave good jet fit.

Gluon is hardest as $x \to 1$.



Valence Quarks – Uncertainties

In each case

$$xf_V(x,Q_0^2) = A_V(1-x)^{\eta_V}(1+\epsilon_V x^{0.5}+\gamma_V x)x^{\delta_V}.$$

3 free parameters contribute to eigenvectors. A_V fixed by sum rule.

High-x –

$$f_V \pm \Delta f_V \sim A_V(1-x)^{\eta_V \pm \Delta\eta_V}$$

$$f_V \pm \Delta f_V \sim f_V(1-x)^{\pm \Delta\eta_V}$$

$$f_V \pm \Delta f_V \sim f_V[1 \pm \Delta\eta_V \ln(1-x)]$$

$\rightarrow$ very large possible uncertainty as $x \rightarrow 1$.

Small-x –

$$f_V \pm \Delta f_V \sim A_V x^{\delta_V \pm \Delta\delta_V}$$

$$f_V \pm \Delta f_V \sim f_V x^{\pm \Delta\delta_V}$$

$$f_V \pm \Delta f_V \sim f_V[1 \pm \Delta\delta_V \ln(1/x)]$$

$\rightarrow$ linear in $\ln(1/x)$ growth as $x \rightarrow 0$.

ϵ_V gives further uncertainty at intermediate x.

Up Valence

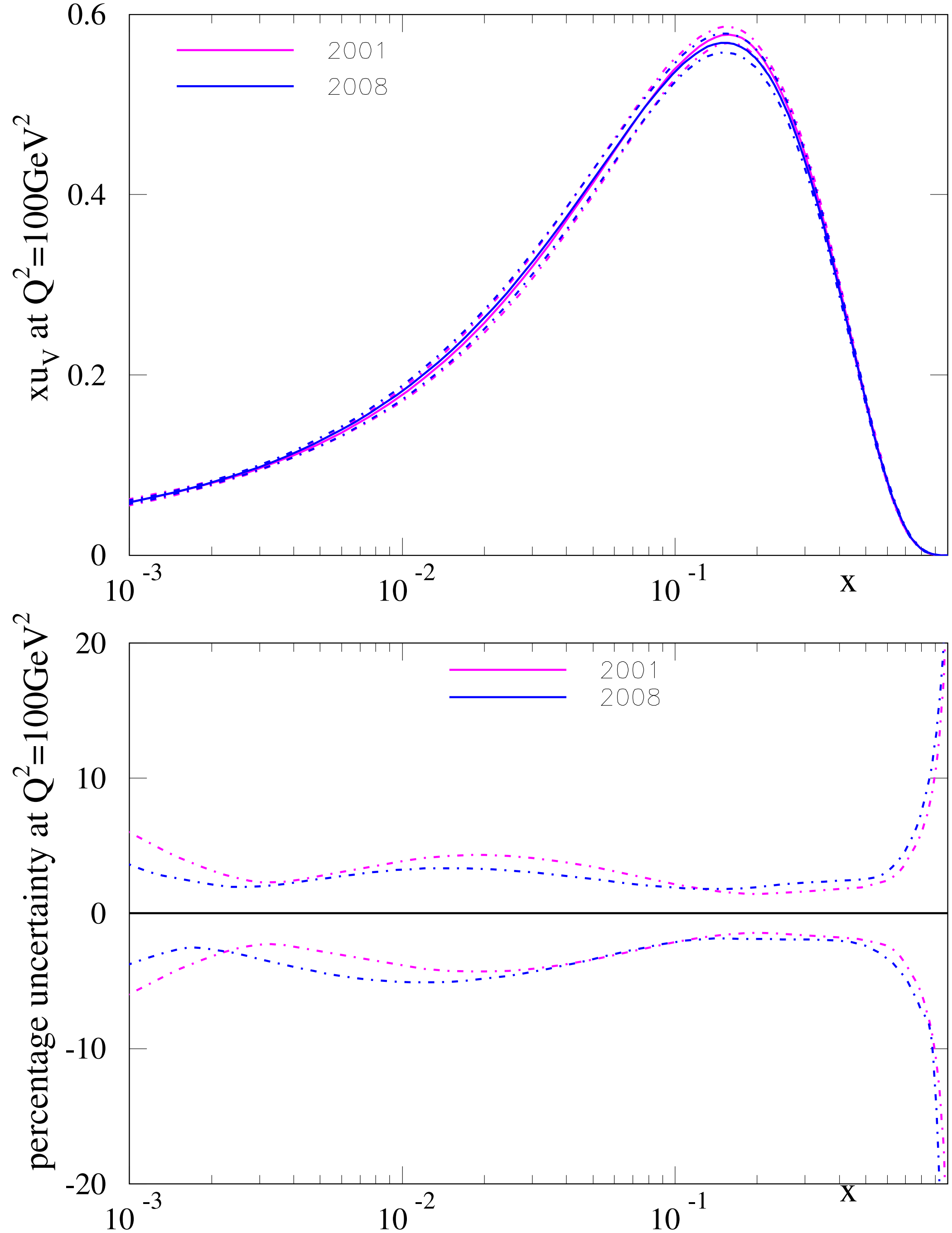
For $u_V(x, Q_0^2)$ same free parameters in eigenvectors as for 2001.

Similar uncertainties.

Perhaps underestimate for very small x.

Valence sum rule for $x = 0.01 - 0.75$ (region of data fit) contributes $\sim 75\%$.

$x < 0.01$ makes contribution – not much freedom.



Down Valence

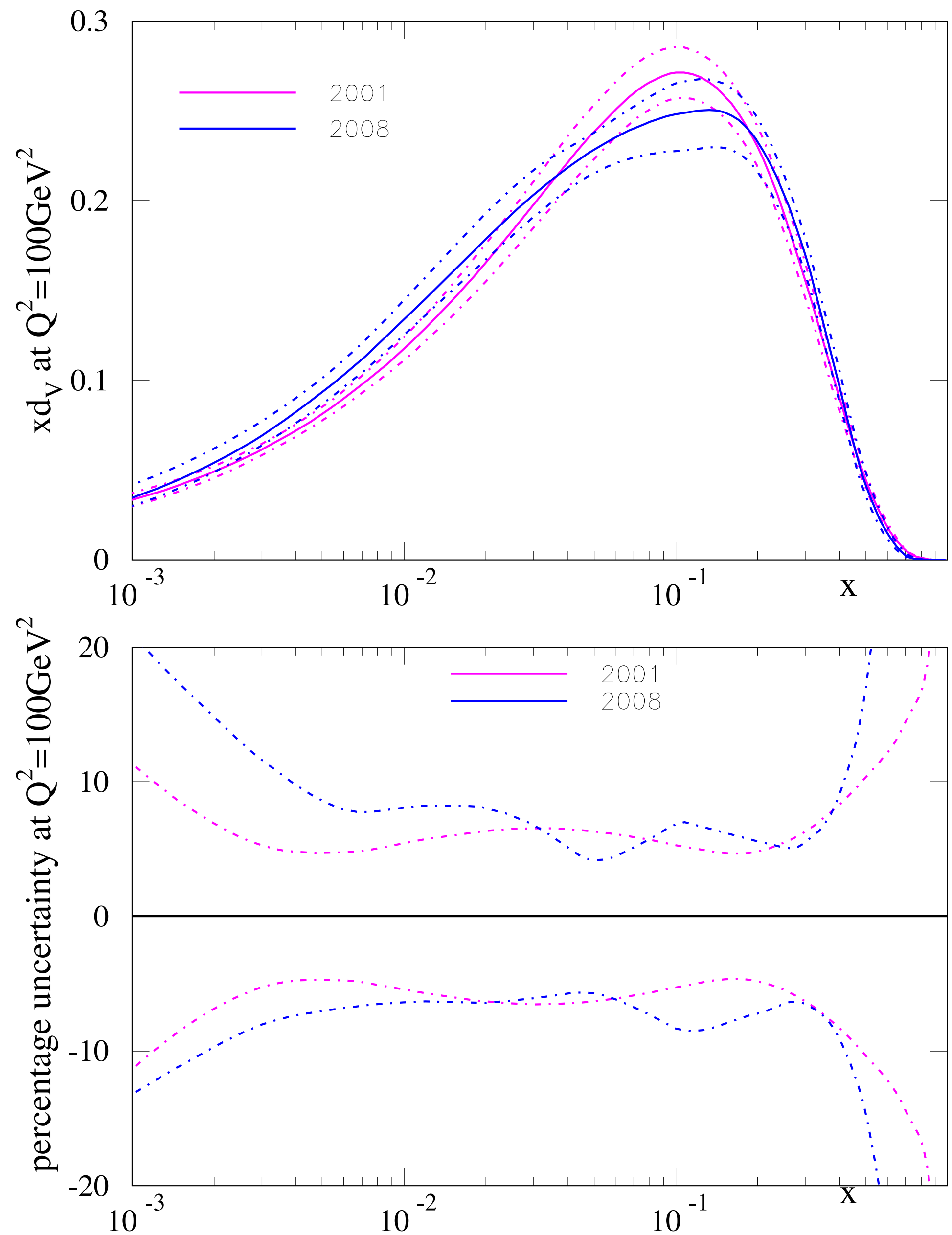
New data constraints affect $d_V(x,Q_0^2)$.

Overall $d_V(x,Q^2)$ now chooses a different type of shape.

Additionally – in 2001, $\eta_d, \epsilon_d, \gamma_d$ contributed to eigenvectors. Now same as for $u_V(x,Q^2)$.

Better balance between small and large x.

Uncertainty growing more quickly as $x \to 0$ and $x \to 1$ than before due to better parameterisation in determining uncertainty eigenvectors.



Gluon Distribution

For all other parameterisations at small x – $xg(x,Q_0^2) \sim x^{\delta_g}$.

This means $g \pm \Delta g \sim g[1 \pm \Delta\delta_g \ln(1/x)]$, i.e. uncertainty grows linearly with $\ln(1/x)$.

No scope for rapidly expanding uncertainty as data constraints run out.

Moreover $\Delta g(x,Q_0^2) \sim g(x,Q_0^2)\Delta\delta_g \ln(1/x)$, and smaller $g(x,Q_0^2)$ the smaller $\Delta g(x,Q_0^2)$.

If $g(x,Q_0^2)$ very small absolute input uncertainty very small, at higher Q^2 determined by evolution from higher-x better determined gluon.

Most determinations find at low Q^2 that $xg(x,Q^2)$ is small and, if extrapolated backwards, complicated at small x.

At small x MRST/MSTW gluon

$$\sim xg(x,Q_0^2) = xg_1(x,Q_0^2) + xg_2(x,Q_0^2) \sim A_1 x^{\delta_{g1}} + A_2 x^{\delta_{g2}}.$$

More flexible than single power. Allows possibility to turn negative at very small x.

Particularly important for uncertainty.

$$\Delta g(x,Q_0^2) \sim \pm g_1(x,Q_0^2)\Delta\delta_{g_1} \pm g_2(x,Q_0^2)\Delta\delta_{g_2}.$$

Interplay between two terms allows for large uncertainty at $x < 0.0001$ where data constraint (from $F_2(x,Q^2)$ evolution) diminishes rapidly.

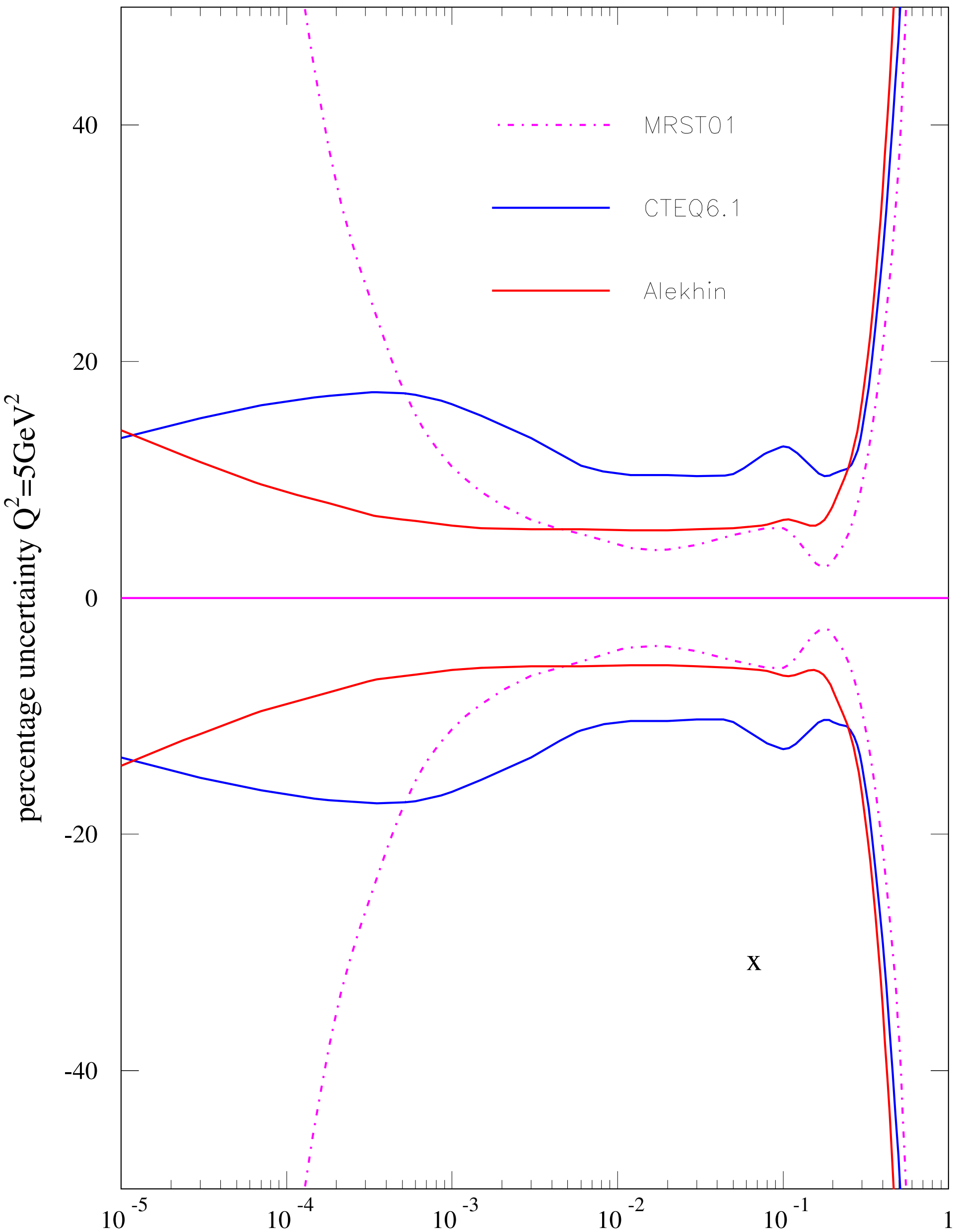
At high x – do not use high-x gluon enhancement inspired by transformation from DIS scheme to $\overline{MS}$ scheme present in 2004 and 2006 sets.

Run II jet data happier with simple $(1-x)^\eta$ prescription.

Gluon Comparisons

MRST uncertainty blows up for very small x, whereas Alekhin (and ZEUS and H1) gets slowly bigger, and CTEQ saturates (or even decreases).

Related to input forms and scales.



MRST (MSTW) parameterise at $Q_0^2 = 1\mathrm{GeV}^2$ but allow negative and positive small x contributions. Very flexible. Represent true uncertainty at low x?

Alekhin and ZEUS gluons input at higher scale – behave like $x^{-\lambda}$ at small x. Uncertainty due to uncertainty in one parameter.

CTEQ gluons input at $Q_0^2 = 1.69\mathrm{GeV}^2$. Behave like x^{λ} at small x where λ large and positive. Input gluon valence-like.

Requires fine tuning. Evolving backwards from steep gluon at higher scale valence-like gluon only exists for very narrow range of Q^2 (if at all).

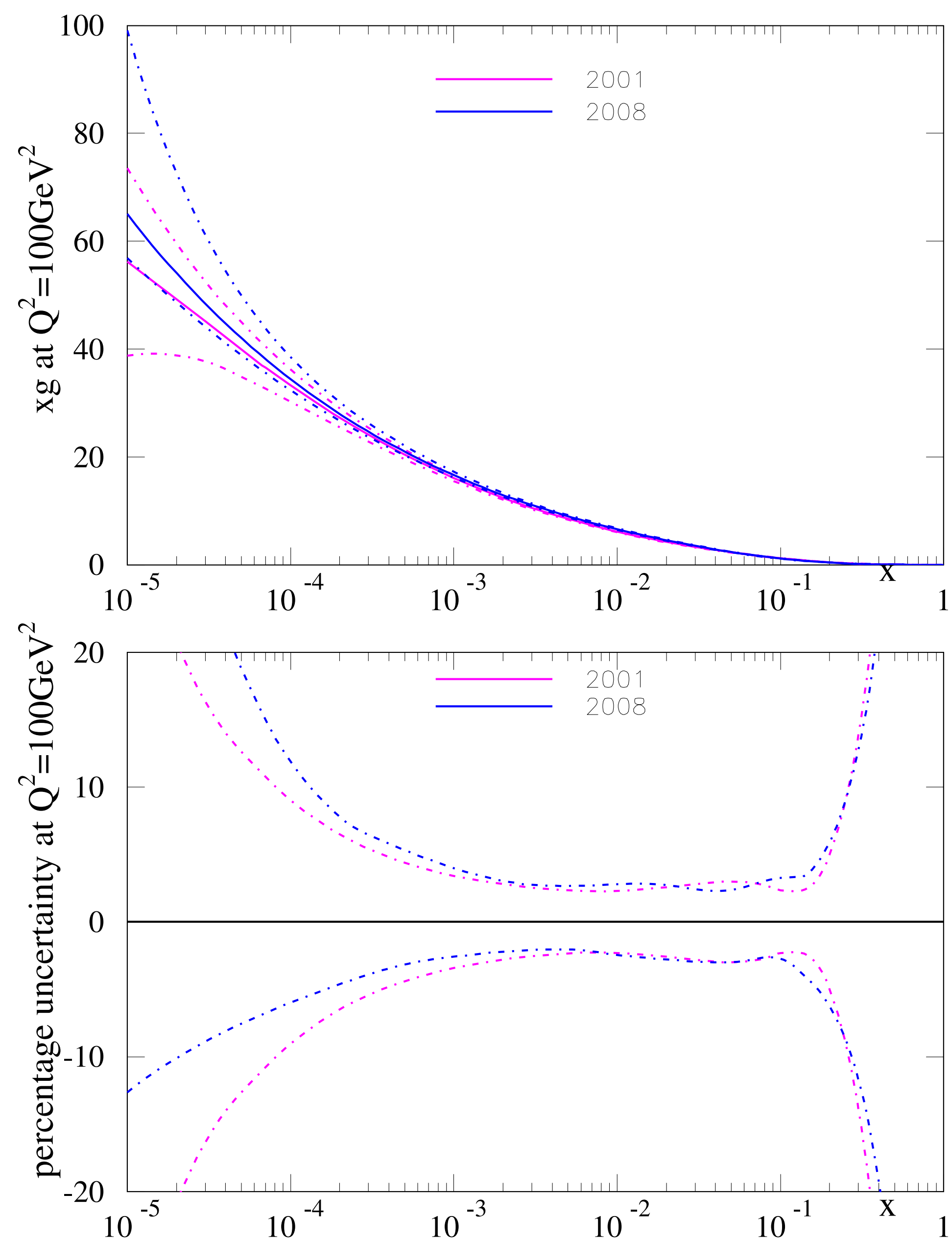
Small x input gluon tiny – very small absolute error. At higher Q^2 all uncertainty due to evolution driven by higher x, well-determined gluon. Very small x gluon no more uncertain than at $x = 0.01 - 0.001$.

This feature was present in 2001.

Now (perhaps due to more data) stability with 4 rather than 3 parameters contributing to eigenvectors.

Removes neck at $x \sim 0.015$.

Would be larger uncertainty at high-x, but better jet data.



Sea Quarks

$xS(x,Q_0^2) = A_S(1-x)^{\eta_S}(1+\epsilon_S x^{0.5}+\gamma_S x)x^{\delta_S}$.

In principle 5 free parameters. Only 3 can contribute to eigenvectors.

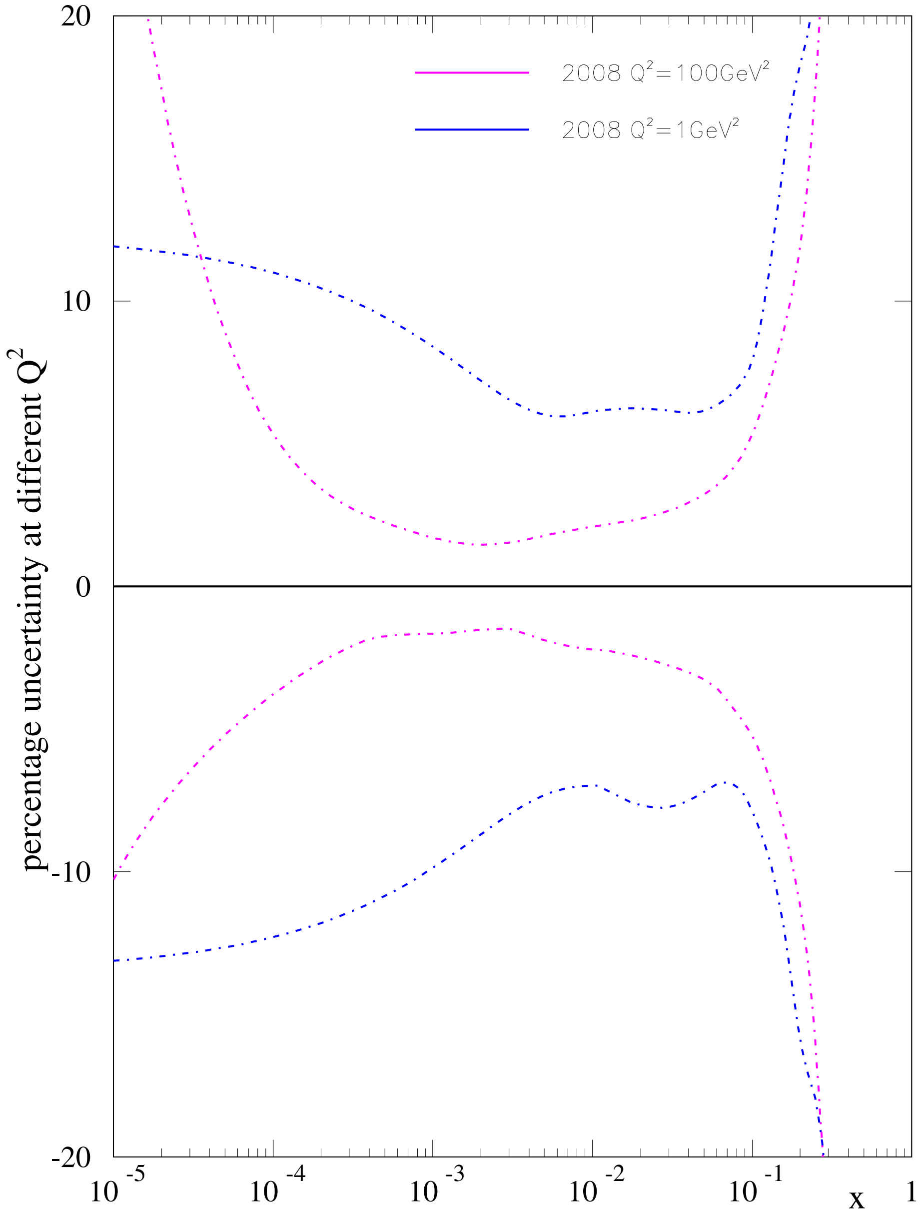
Would expect $\eta_S, \epsilon_s, \delta_S$, but δ_S has too large correlations.

At small x uncertainty input due to A_S

$\Delta S(x,Q_0^2) \propto S(x,Q_0^2)$.

But at higher Q^2 controlled by gluon evolution $\rightarrow$ larger uncertainty at very small x slightly.

Perhaps underestimate uncertainty at small Q^2 very small x.



But follows data constraint – $F_2(x,Q^2)$ constrains quarks down to $x=10^{-5}$ but only for small Q^2 range.

Strange Quarks

Now parameterise Strange quarks separately rather than assume

$$(s(x,Q_0^2)+\bar{s}(x,Q_0^2)) = \kappa\, 0.5 * (\bar{u}(x,Q_0^2)+\bar{d}(x,Q_0^2)), \qquad \kappa = 0.4 - 0.5$$

Use direct constraint from CCFR and NuTeV dimuon data – $\nu_\mu s(\bar{\nu}_\mu \bar{s}) \to c\mu^-(\bar{c}\mu^+)$.

Could have free parameterization, but data only for $x > 0.01 \to$ enormous uncertainty for $x \leq 0.01$. Realistic? Regge considerations – all flavours same power as $x \to 0$.

Strange has some non-insignificant mass, and this should qualitatively lead to difference/suppression compared to light sea quarks up and down.

When c and b turn on they evolve like massless quarks, but always lag behind. $\to$ some suppression at all x for finite Q^2. But at small x roughly just normalization suppression. Evolution makes small-x powers the same.

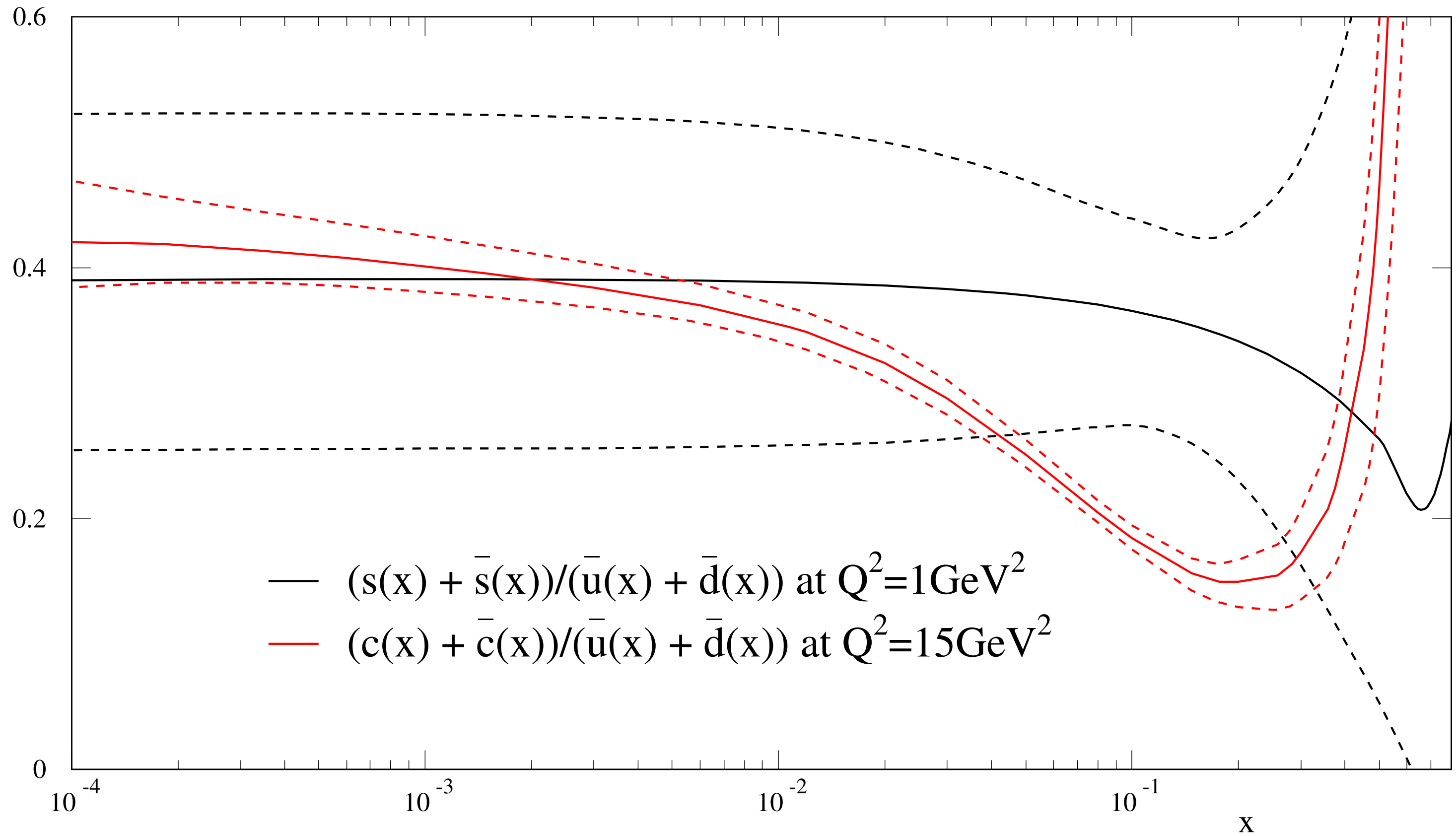
$\to$ only A_+, η_+ free parameters.

$$s^+(x,Q_0^2) \equiv s(x,Q_0^2)+\bar{s}(x,Q_0^2) = A_+(1-x)^{\eta_+} S(x,Q_0^2)$$

Small-x power fixed to δ_S. ϵ, γ not required in fit quality and $\to$ instability in eigenvectors.

$\sim 35\%$ normalization supression at Q_0^2 and some additional high-x suppression.

Suppression at *nonperturbative* $Q_0^2 = 1\mathrm{GeV}^2$ now ~ 0.3, i.e. value in hadronization models (probability to generate $\bar{s}s$ compared to $\bar{u}u, \bar{d}d$).



$c + \bar{c}$ evolved through $\sim 7 - 8$ times input scale similar to $s + \bar{s}$ at $Q^2 = 1\mathrm{GeV}^2$. Do not expect exact correspondence, but very good except $c + \bar{c}$ more suppressed at $x \sim 0.1$. (Implication for $s + \bar{s}$ from recent HERMES $K^{\pm}$ data).

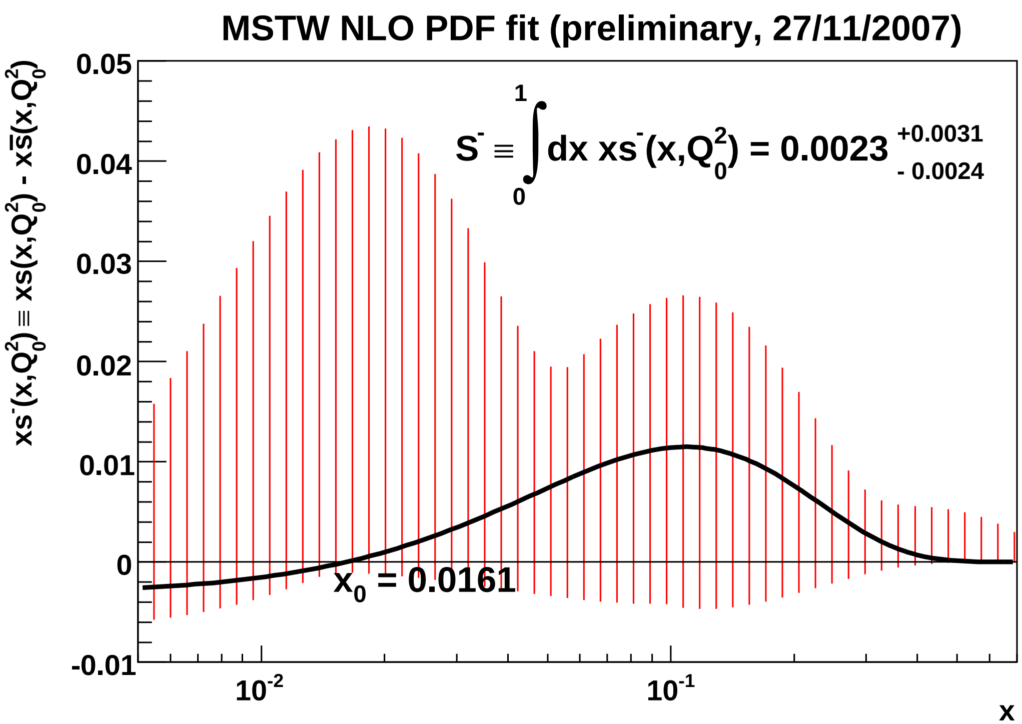
Dimuon data also constrain strangeness asymmetry. Hence, define

$$s^-(x,Q_0^2) \equiv s(x,Q_0^2) - \bar{s}(x,Q_0^2) = A_-(1-x)^{\eta_-}x^{-1+\delta_-}(1-x/x_0)$$

x_0 is a function of the other parameters and is determined by zero strangeness of proton, i.e.

$$\int_0^1 dx\, s^-(x,Q_0^2) = 0.$$

A_- and δ_- very highly correlated. $\delta_- = 0.2$ fixed, i.e. valence-like value similar to δ_{u_V,d_V}.



$\bar{d}(x,Q^2) - \bar{u}(x,Q^2)$

$= A_\Delta(1-x)^{\eta_S+2}(1+\gamma_\Delta x + \delta_\Delta x^2)x^{\eta_\Delta}$.

Constrained by data for $0.01 \leq x \leq 0.2$.

At high x choose the $(1-x)$ power to be $\eta_S + 2$ since $\bar{d}(x,Q^2) - \bar{u}(x,Q^2)$ becoming very small, and want to maintain $\bar{u}, \bar{d} \geq 0$.

Then need x^2 term for best fit.

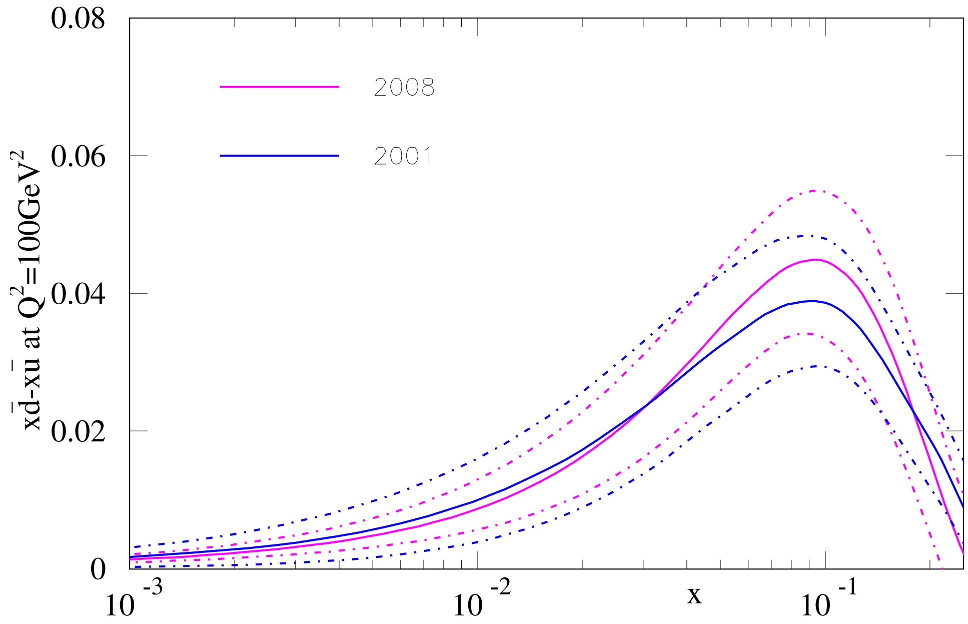
3 eigenvector parameters control normalization, high x and low x.

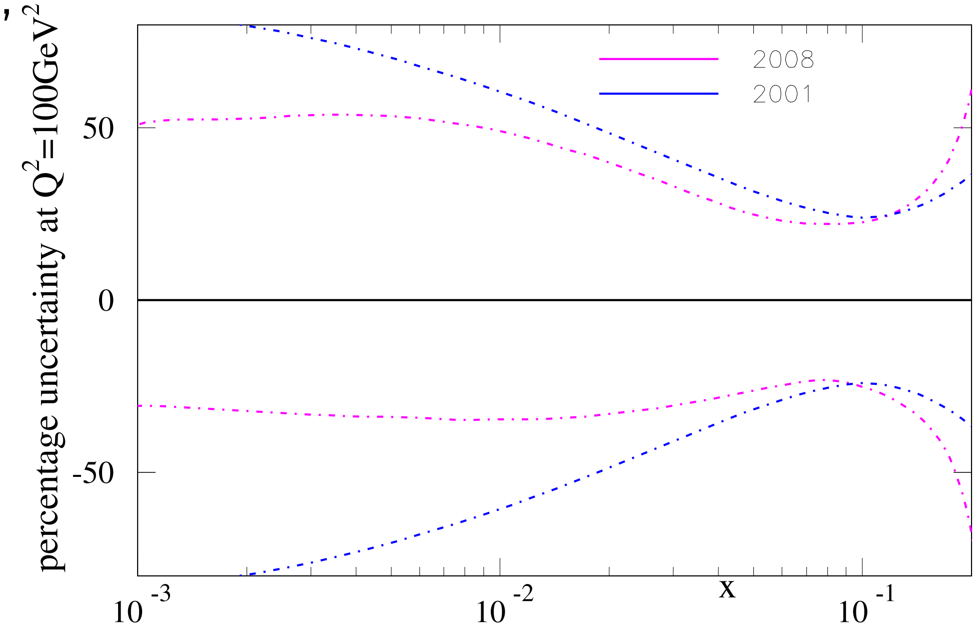
Differences to 2001 due to new data.

$\bar{d}(x,Q^2) - \bar{u}(x,Q^2)$ has become very small by $x = 0.001$ – $\eta_\Delta > 1$.

Does not allow for change of sign or turn back up in this region.

Change of sign within large high-x uncertainty.





Conclusions

Parameterise with simple forms. Doesn't allow bumps or shoulders in general.

Correlations between parameters determine stability reached at maximum of 20 free parameters, though 8 others not set zero.

Uncertainties can be very large at high x.

Parameterization allows flexible shape and very quickly growing uncertainty for small-x gluon.

Other small-x shapes restricted by sum rules (valence), data (sea), or by theory assumptions – i.e. stays extremely small ($\bar{d}-\bar{u}$) or behaves like mass-suppressed quark (strange).

Normalization a free parameter except where constraint from sum rule.

Believe we get a good span of possible uncertainty in ranges where constraint, since enormous amount of constraining data.

Possible underestimate in regions of extrapolation, e.g. small-x valence, $\bar{d}-\bar{u}$, and strange. However, for some of these uncertainty essentially unlimited without theory assumptions.

Change in Flavour Schemes

Check effect of change in flavour prescription for NLO.

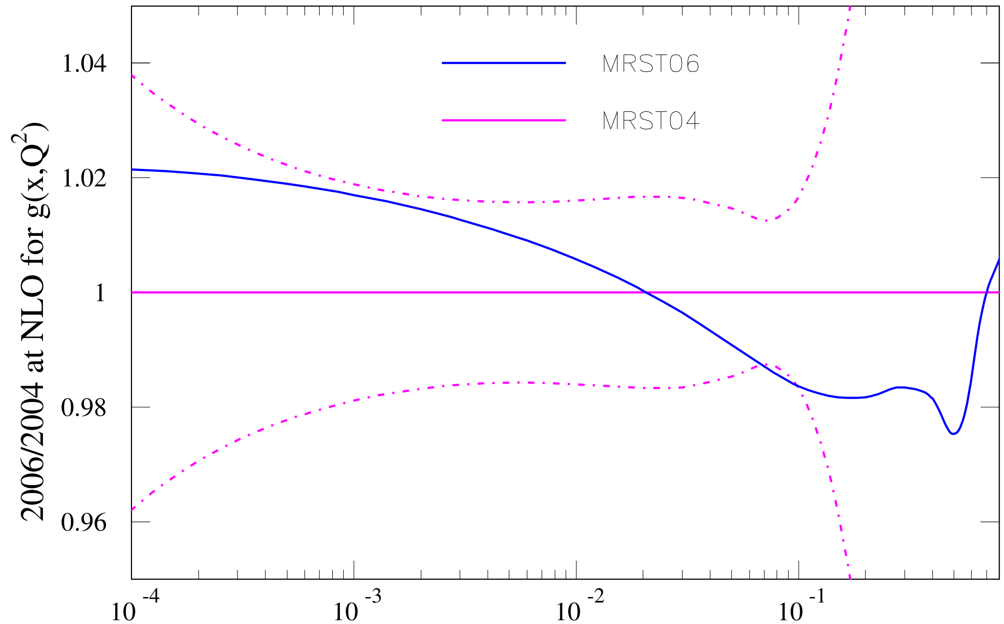
Compare MRST2004 (with 2001 uncertainties) to unofficial "MRST2006".

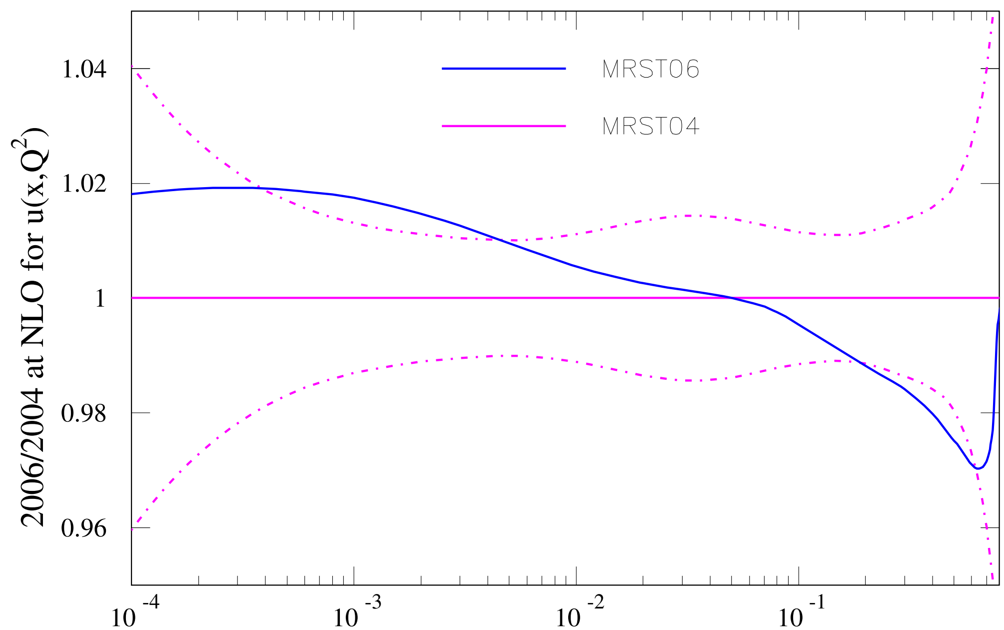
Fit to similar data.

Change between two perfectly acceptable NLO definitions of a GM-VFNS.

More recent $\rightarrow$ 2% increase in σ_W and σ_Z at the LHC.

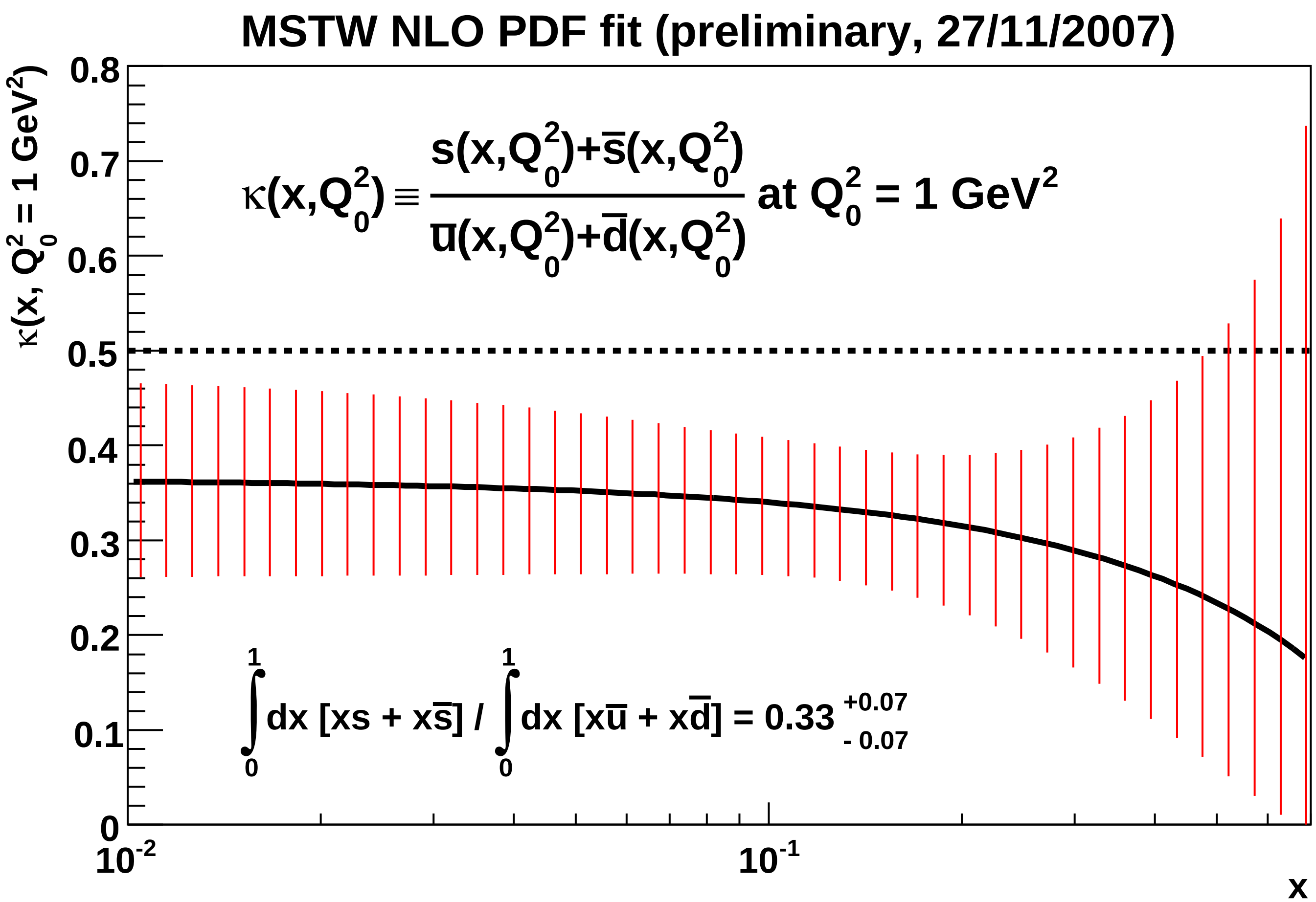
Can be same size as quoted uncertainties. This is a genuine theory uncertainty due to competing but equally valid choices. Ambiguity decreases at higher orders (to be demonstrated).





Find reduced ratio of strange to non-strange sea compared to previous default $\kappa = 0.5$.

Suppression at high x, i.e. low W^2. Effect of m_s?



Suppression at *nonperturbative* $Q_0^2 = 1\mathrm{GeV}^2$ now ~ 0.3, i.e. value in hadronization models (probability to generate $\bar{s}s$ compared to $\bar{u}u, \bar{d}d$).

Fitting to strange from NuTeV dimuon data affects uncertainties on partons other than strange.

Previously for us (and everyone else) strange a fixed proportion of total sea in global fit.

Genuine *larger* uncertainty on $s(x)$– feeds into that on $\bar{u}$ and $\bar{d}$ quarks.

Low x data on $F_2(x,Q^2)$ constrains sum $4/9(u+\bar{u})+1/9(d+\bar{d}+s+\bar{s})$.

Changes in fraction of $s+\bar{s}$ affects size of $\bar{u}$ and $\bar{d}$ at input.

The size of the uncertainty on the small x anti-quarks increases – $\sim 1.5\% \rightarrow \sim 2-2.5\%$, despite additional constraints on quarks in new fit.