

Quick reference guide

Introduction

This is a model experiment for electron spin resonance, for clear demonstration of interaction between the magnetic moment of the electron spin with a superimposed direct or alternating magnetic field.

A ball with a central rod magnet, which rotates with low friction on an air cushion, acts as model electron. Two pairs of coils generate a constant magnetic field B_0 and an alternating magnetic field B_1 . The axes of both fields intersect perpendicularly at the centre of the ball.



The table is slightly inclined to start the electron gyroscope with an air draught (Magnus effect). If the direct magnetic field B_0 acts on the ball, a precession of the magnet axis is observed. Precession frequency increases with the intensity of field B_0 . With a second pair of coils and a pole changeover switch (commutator), a supplementary alternating field B_1 is generated. If the change of poles occurs at the right phase, the angle between the gyroscope axis and the direction of the direct field is continuously increased, until the magnetic axis of the ball is opposed to the field direction (spin flip).

Experimental procedure

1. Tilt the table by turning the knob on one of the table's leg as shown in Fig. 3.

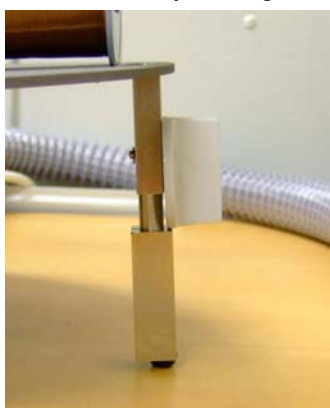


Fig. 3: Turning knob to tilt gyroscopic table

2. Place the gyroscopic ball on the blowhole such that the north pole of the central magnet (mark with a red ring) is almost parallel to the B_0 field.
3. Start the air blower by turning the knob to a setting of between 3.5 to 4.
4. When the spin of the ball is steady, switch on the B_0 field using the switch with a setting of between 15 to 20 Vdc on the power supply.



5. Once the field is switch on, the central magnet axis will start to precess¹.
6. Choose two reference points on the precession cycle that are on opposite sides, flip the commutator switch² from one side to another when the north pole of the magnet reaches these reference points.
7. If your timing is good, the pole of the magnet will switch direction³ in about two minutes (less with practice).

Observations and drawing parallels to quantum ideas

Precession of central magnet axis

When the main magnetic field B_0 is applied, the magnetic axis of the ball will undergo precession (refer to the theory section on Larmor precession). Since the precession frequency is proportional to the applied field, one may want to change the voltage from the power supply to observe this effect.

Spin resonance

From quantum theory, a photon of frequency (i.e., energy) matching the energy difference of an electron in a uniform field with spin up or down can cause its spin to flip. This experiment provides learners the opportunity to reconcile the quantum effect of electron (proton) spin resonance with classical mechanics. When the frequency of the second magnetic field (analogous to the electromagnetic radiation or photon) is close to the precession frequency (analogous to the spin of the electron in a magnetic field), the magnetic axis of the ball will flip to oppose the main magnetic field (resonance).

Some guidance to teachers

- High school students should have the pre-requisite knowledge of resonance. Hence it may be useful to recap on the concept of resonance before the experiment or demonstration.
- You may want to discuss about gyroscopic precession (if it has not been done in classical mechanics) in general with the usual bicycle wheel and turntable demonstration.
- When the magnetic field B_0 is switch on, the torque produced by the magnetic dipole of the ball and the field causes the magnetic axis to precess. This is similar to the classical explanation of Larmor precession of an electron in a magnetic field.
- The concept of spin in quantum theory has no equivalent in the classical world. In order to provide the link between quantum spin precession and classical spin, we need a spinning ball to go with the central magnet the ball otherwise precession is not possible. It is good to remind students that although there are similarity between quantum spin and this demonstration, electron spin is an intrinsic property which got its name due to historical reason.

¹ If the magnetic axis of the ball is parallel to the magnetic field B_0 of the coils, precession is small. You may want to start by having the magnetic axis of the ball slightly at an angle to B_0 .

² The commutator switch controls the direction of the magnetic field that is perpendicular to the main field B_0 . By alternating the current in the coil, an alternating magnetic field is produced which the magnetic dipole of the ball will feel. This alternating field is the driving force on the spinning ball.

³ When the magnetic axis changes direction, the precession frequency is rather high and it is hard to synchronize the commutator switch to the precession. The magnetic axis will become unstable and very soon it will return to its original orientation.

Theoretical background

Torque on a Current Loop

The torque on a current-carrying coil, as in a DC motor, can be related to the characteristics of the coil by the “magnetic moment” or “magnetic dipole moment”. The torque exerted by the magnetic force (including both sides of the coil) is given by

$$\tau = BILW \sin \theta$$

where L and W are the length and the width of the coil respectively. The coil characteristics can be grouped as

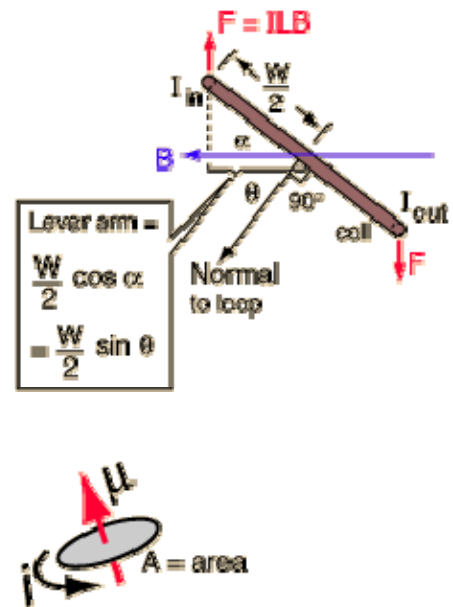
$$\mu = IA \quad (\text{or } \mu = NIA \text{ for } n \text{ loops})$$

called the magnetic moment of the loop ($A = L \cdot W$ is the area of the coil). The torque now can be written as

$$\tau = \mu B \sin \theta$$

The direction of the magnetic moment is perpendicular to the current loop in the right-hand-rule direction, the direction of the normal to the loop in the illustration. Considering torque as a [vector quantity](#), this can be written as the [vector product](#)

$$\tau = \mu \times B$$

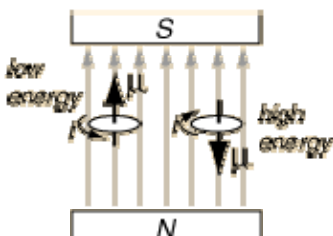


Since this torque acts perpendicular to the magnetic moment, then it can cause the magnetic moment to precess around the magnetic field at a characteristic frequency called the [Larmor frequency](#) (see below). If you exerted the necessary torque to overcome the magnetic torque and rotate the loop from angle 0° to 180° , you would do an amount of [rotational work](#) given by the [integral](#)

$$W = - \int_0^\pi \tau d\theta = - \int_0^\pi \mu B \sin \theta d\theta = -\mu B \cos \theta \Big|_0^\pi = 2\mu B$$

The position where the magnetic moment is opposite to the magnetic field is said to have a higher [magnetic potential energy](#).

As seen in the geometry of a current loop, the torque τ tends to line up the magnetic moment with the magnetic field B , so this represents its lowest energy configuration.



The potential energy associated with the magnetic moment is

$$U(\theta) = -\mu \cdot B$$

so that the difference in energy between aligned and anti-aligned is

$$\Delta U = 2\mu_B$$

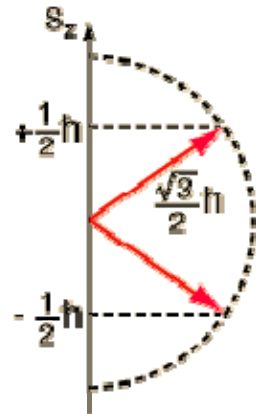
These relationships for a finite current loop extend to the magnetic dipoles of electron orbits and to the intrinsic magnetic moment associated with electron spin.

Electron Intrinsic Angular Momentum

Experimental evidence like the hydrogen fine structure and the Stern-Gerlach experiment suggest that an electron has an intrinsic angular momentum S , independent of its orbital angular momentum L . These experiments suggest just two possible states for this angular momentum, such that

$$S_z = \pm \frac{1}{2} \hbar$$

An angular momentum and a magnetic moment could indeed arise from a spinning sphere of charge, but this classical picture cannot fit the size or quantized nature of the electron spin. The property called electron spin must be considered to be a quantum concept without detailed classical analogy.



Electron Spin Magnetic Moment

Since the electron displays an intrinsic angular momentum, one might expect a magnetic moment which follows the form of that for an electron orbit. The z-component of magnetic moment associated with the electron spin would then be expected to be

~~$$\mu_z = \pm \frac{1}{2} \mu_B$$~~

(where $\mu_B = \frac{e\hbar}{2m_e}$ is the Bohr magneton) but the measured value turns out to be about twice that. The measured value is written

$$\mu_z = \pm \frac{1}{2} g \mu_B$$

where g is called “the electron spin g-factor” and is equal to 2.00232. The precise value of g was predicted by relativistic quantum mechanics and was measured in the Lamb shift experiment.

The electron spin magnetic moment is important in the spin-orbit interaction which splits atomic energy levels and gives rise to fine structure in the spectra of atoms. The electron spin magnetic moment is also a factor in the interaction of atoms with external magnetic fields (Zeeman effect).

2.4 Electron Spin Resonance

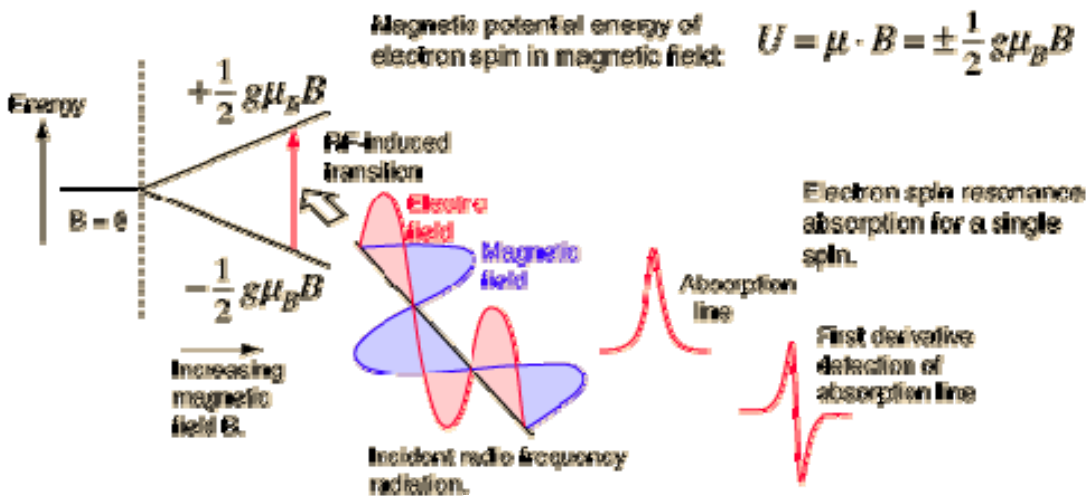
When the molecules of a solid exhibit paramagnetism as a result of unpaired electron spins, transitions can be induced between spin states by applying a magnetic field and then supplying electromagnetic energy, usually in the microwave range of frequencies. The resulting absorption spectra are described as electron spin resonance (ESR) or electron paramagnetic resonance (EPR).

ESR was first observed in [Kazan State University](#) by the [Soviet](#) physicist [Yevgeniy Zavoyskiy](#) in 1944, and was developed independently at the same time by [Brebis Bleaney](#) at [Oxford University](#).

ESR has been used as an investigative tool for the study of radicals formed in solid materials, since the radicals typically produce an unpaired spin on the molecule from which an electron is removed. Particularly fruitful has been the study of the ESR spectra of radicals produced as radiation damage from ionizing

radiation. Study of the radicals produced by such radiation gives information about the locations and mechanisms of radiation damage.

The interaction of an external magnetic field with an electron spin depends upon the magnetic moment associated with the spin, and the nature of an isolated electron spin is such that two and only two orientations are possible. The application of the magnetic field then provides a magnetic potential energy which splits the spin states by an amount proportional to the magnetic field (Zeeman effect), and then radio frequency radiation of the appropriate frequency can cause a transition from one spin state to the other. The energy associated with the transition is expressed in terms of the applied magnetic field B , the electron spin g-factor g , and the constant μ_B .



If the radio frequency excitation was supplied by a klystron at 20 GHz, the magnetic field required for resonance would be 0.71 T, a sizable magnetic field typically supplied by a large laboratory magnet.

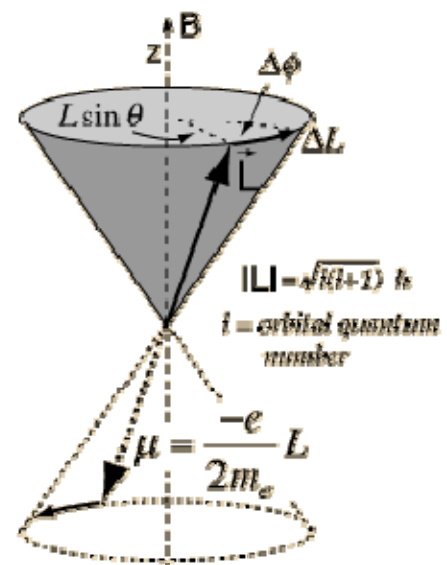
If you were always dealing with systems with a single spin like this example, then ESR would always consist of just one line, and would have little value as an investigative tool, but several factors influence the effective value of g in different settings. Much of the information obtainable from ESR comes from the splittings caused by interactions with nuclear spins in the vicinity of the unpaired spin, splittings called nuclear hyperfine structure.

Larmor Precession

When a magnetic moment μ is placed in a magnetic field B , it experiences a torque which can be expressed in the form of a vector product

$$\tau = \mu \times B$$

For a static magnetic moment or a classical current loop, this torque tends to line up the magnetic moment with the magnetic field B , so this represents its lowest energy configuration. But if the magnetic moment arises from the motion of an electron in orbit around a nucleus, the magnetic moment is proportional to the angular momentum of the electron. The torque exerted then produces a change in angular momentum which is perpendicular to that angular momentum, causing the magnetic moment to



Electron spin resonance



precess around the direction of the magnetic field rather than settle down in the direction of the magnetic field. This is called Larmor precession.

When a torque is exerted perpendicular to the angular momentum L , it produces a change in angular momentum ΔL which is perpendicular to L , causing it to precess about the z axis. Labeling the precession angle as ϕ , we can describe the effect of the torque as follows:

$$\tau = \frac{\Delta L}{\Delta t} = \frac{L \sin \theta \Delta \phi}{\Delta t} = |\mu B \sin \theta| = \frac{e}{2m_e} LB \sin \theta$$

The precession angular frequency (Larmor frequency) is

$$\omega_{\text{Larmor}} = \frac{d\phi}{dt} = \frac{e}{2m_e} B$$

These relationships for a finite current loop extend to the magnetic dipoles of electron orbits and to the intrinsic magnetic moment associated with electron spin. There is also a characteristic Larmor frequency for nuclear spins.

In the case of the electron spin precession, the angular frequency associated with the spin transition is usually written in the general form

$$\omega = \gamma \cdot B$$

$$\gamma = g \frac{e}{2m_e}$$

where $\frac{e}{2m_e}$ is called the gyromagnetic ratio (sometimes the magnetogyric ratio). This angular frequency is associated with the "spin flip" or spin transition, involving an energy change of $2 \cdot \mu \cdot B$.

The characteristic frequencies associated with electron spin are employed in electron spin resonance (ESR) experiments, and those associated with the nuclear spin in nuclear magnetic resonance (NMR) experiments.