Holographic Models with a Small Cosmological Constant at Finite T

Based on
Work in progress with Jay Hubisz, Don Bunk [arXiv:1505.xxxxx]

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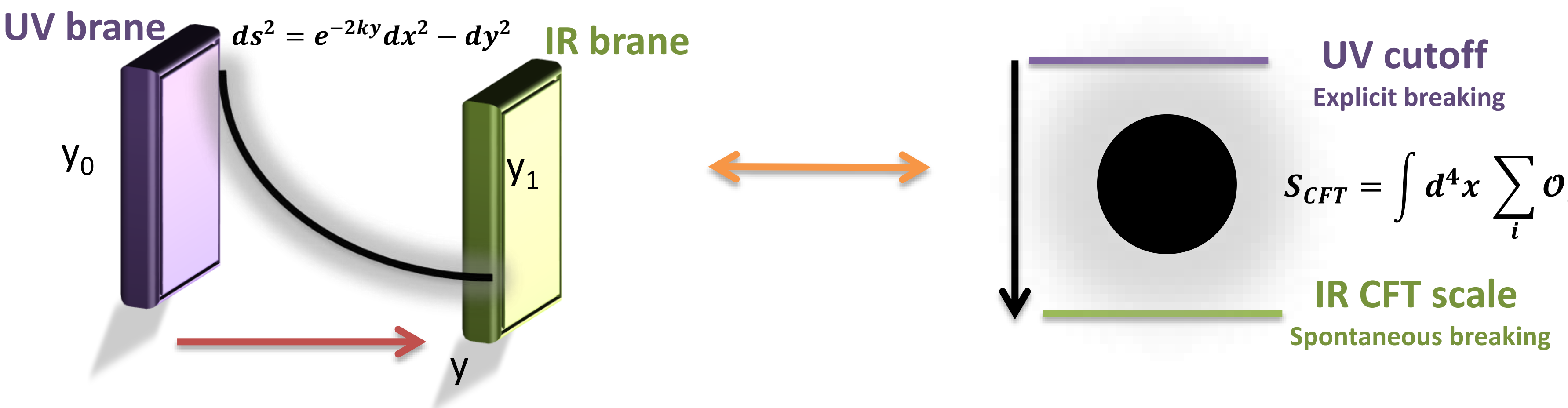
AdS/CFT phenomenological correspondence

Randall Sundrum I Setup



AdS_5	⟷	**CFT_4**
e^{-ky}	⟷	μ
UV brane, e^{-ky_0}	⟷	μ_0
IR brane, e^{-ky_1}	⟷	χ
radion	⟷	**dilaton**

The brane separation-hierarchy of scales, is fixed by $\langle\chi\rangle = f$

AdS_5		CFT_4
$\Lambda_0 = \Lambda_{5D}$	⟷	$\Lambda_{bare} \to 0$, bare tuning
$\Lambda_1 = -\Lambda_{5D}$	⟷	$Af^4 \to 0$ quartic tuning
Goldberger Wise mechanism	⟷	explicit conformal invariant breaking
Small back-reaction in GW Scalar	⟷	partial solution to quartic tuning
$\Downarrow$		
Small mistunes in Brane tension		

Some RS models plagued by **eternal inflation** [1]

[1] Creminelli et. al (2001)

Soft-wall Model

- Bulk scalar potential with "soft" dependence on ϕ ⇒ Spontaneous breaking of scale invariance (SBSI) [2]

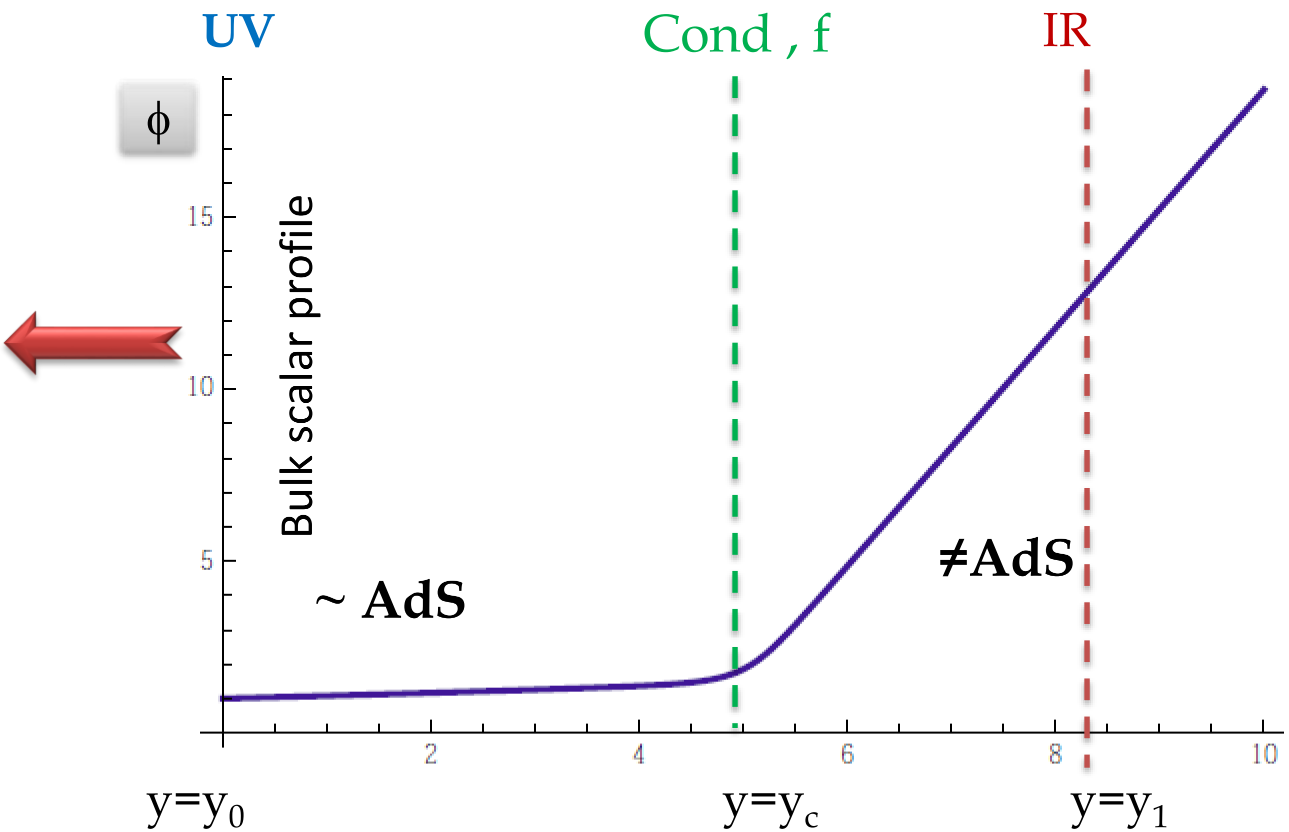
[2] Bellazzini et. al (2013)

$$S = \int d^5x\sqrt{g}\left[\frac{1}{2}(\partial_M\phi)^2 - V(\phi) - \frac{1}{2\kappa^2}\mathcal{R}\right] - \int d^5x\sqrt{g_0}\,V_0(\phi) - \int d^5x\sqrt{g_1}\,V_1(\phi)$$

Soft wall *geometric* model of SBSI

Soft wall generic when IR brane tension >0

IR brane plays **subdominant** role: just cuts off growth of scalar field and curvature



Zero temperature

$$V_{\text{bulk}} = \Lambda_5 + \epsilon B(\phi)$$

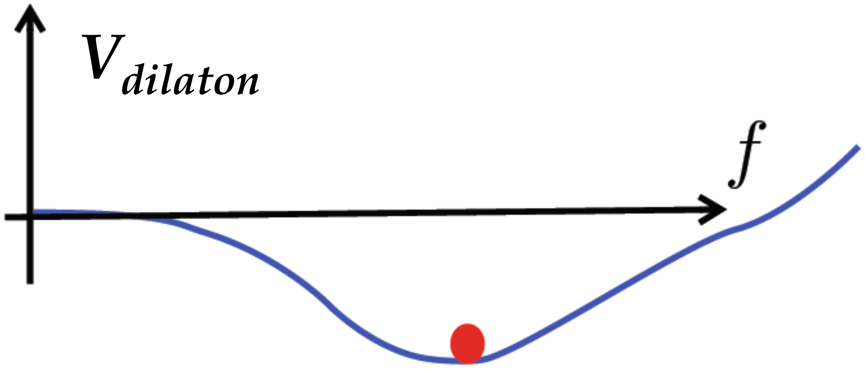
Weak scalar field dependence

Using holography to compute the effective dilaton (f) potential

$$V_{\text{dilaton}} = -\epsilon \lambda_\epsilon \mu_0^\epsilon f^{4-\epsilon} + \lambda_4 f^4$$

$$V_{\text{min}} \approx -\frac{\epsilon \lambda_4 f^4}{4}$$

CC small



$$m^2_{\text{dilaton}} = \epsilon f^2$$

Light dilaton

Soft wall realisation of small condensate contribution to CC in models with non-linearly realised SBSI

Different from GW mechanism

Finite temperature

Geometry includes a black hole horizon at a finite point, y_h in extra dimension, y.

Hawking radiation from this BH allows BH to reach in **equilibrium with the thermal bath.**

Partition function associated with these Classical solutions

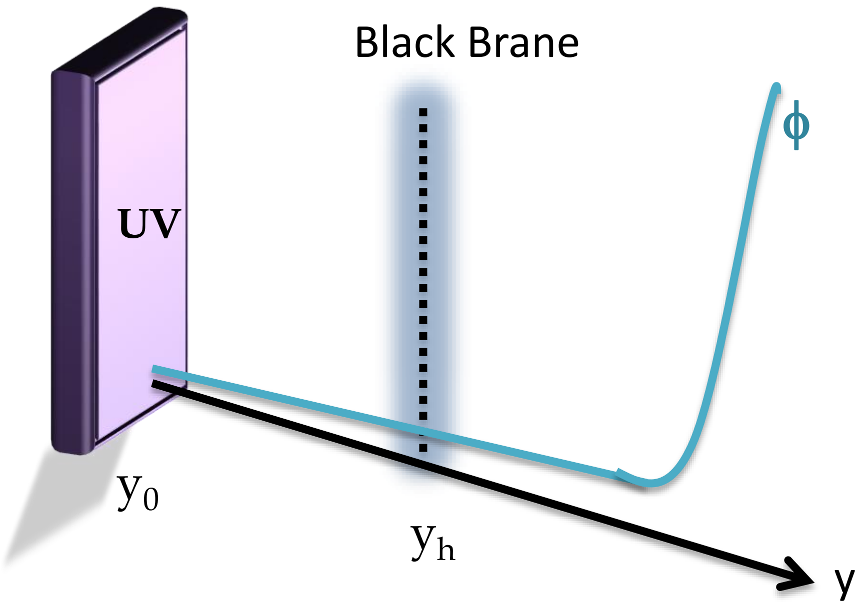
$$\Downarrow$$

Free energy of system

Free energy, $V = V_{UV} + V_{horizon}$

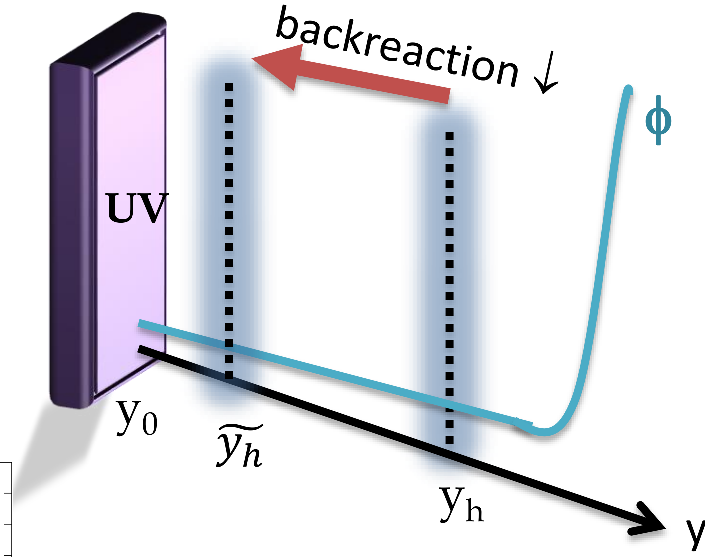
$= U - T\ S$

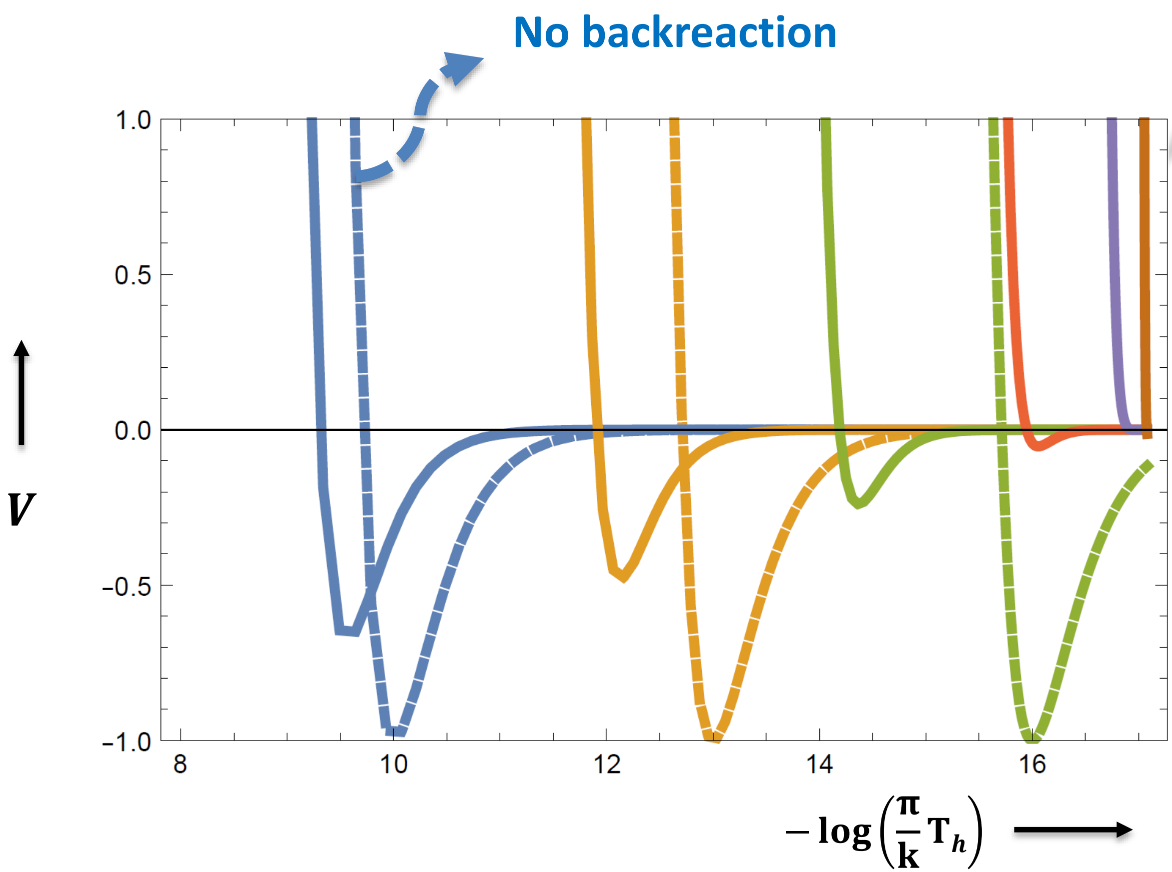
IR brane has been replaced with a BH horizon at y_h



$$T_{eq} = T_h = \frac{k}{\pi} e^{-y_h} e^{4(y_h - \tilde{y}_h)} \quad (at \quad V_{min})$$

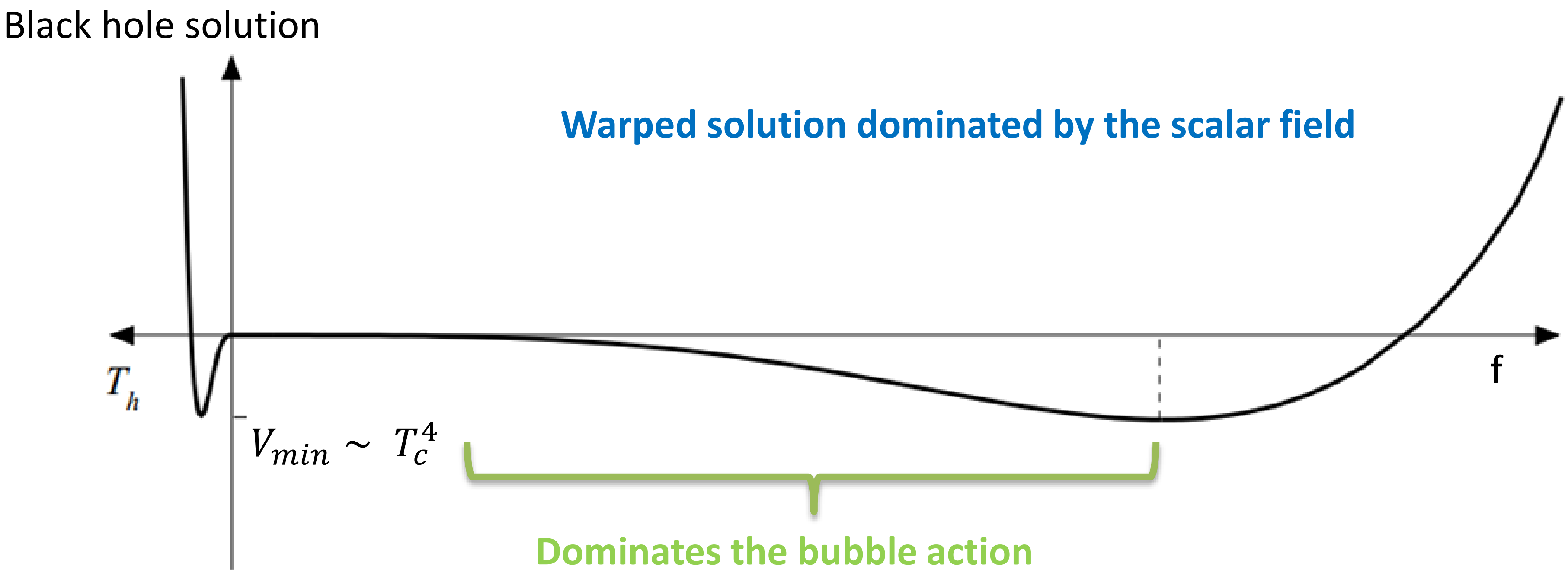
where $\tilde{y}_h \leq y_h$





With back-reaction included Phase transition would occur at higher temperatures

Phase transitions



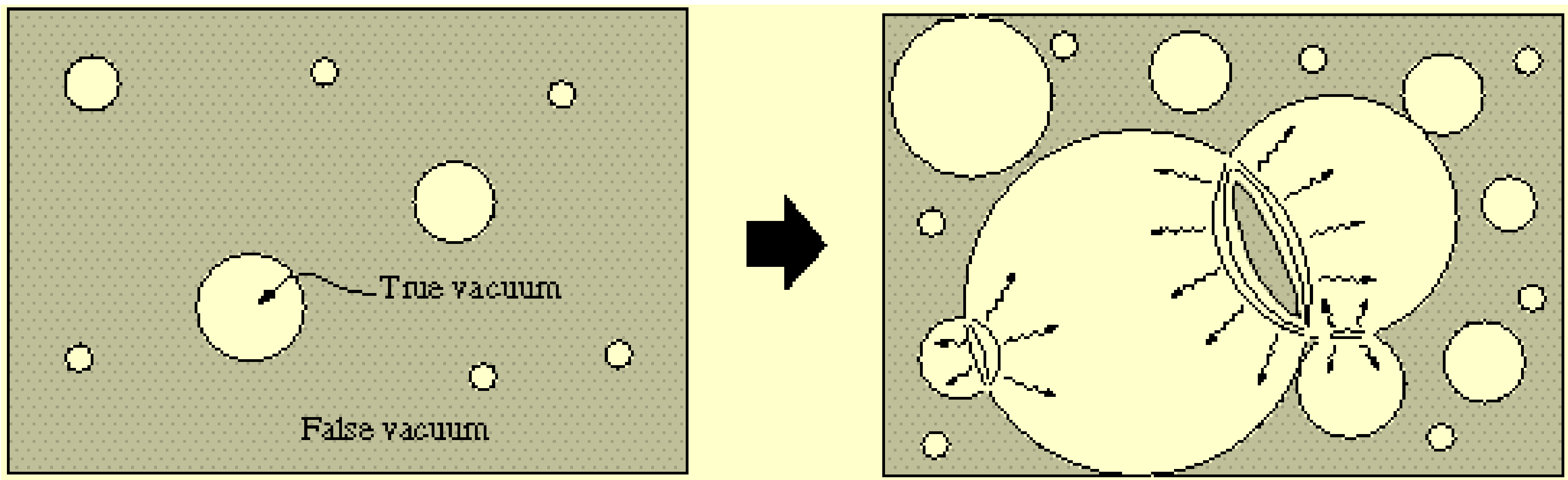
At $T=T_c$, first order phase transition occurs

At $T<T_c$ system is in warped scalar field dependent phase

Bubble nucleation

System at $T = T_c$
In 4D , True vacuum bubble forms

Expands until false vacuum bubble disappears .



In 5D, spherical brane patches form on the horizon

They expand and coalesce to form a complete 3-brane

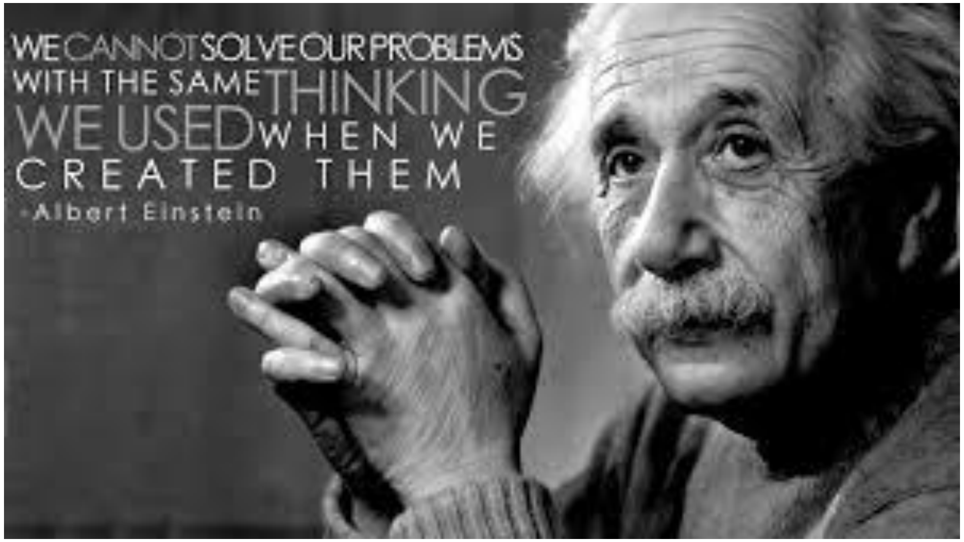
- For a successful phase transition:
- Bubble nucleation rate > Rate at which the universe expands
- $\Rightarrow$ Bubble action, $S_E > 140$
- Exact calculation for S_E are **not possible**
- **Approximate methods** are used
- ΔV > height of the barrier $\Rightarrow$ thickness of bubble wall ~ size of the bubble, R (Thick- wall Approximation) [3]
- Cosmological constraint as a upper bound on N

[3] Randall, Servant (2007)

Summary

- We have explored "soft-wall" realization of the Randall Sundrum geometry
- SBSI manifests as continuous geometry (soft wall SBSI) with IR brane playing lesser role of a cutoff
- As the universe cools, a first order phase transition between two phases, AdS-S like (high T) to warped geometry (low T) occurs
- Back reaction is a **crucial element** in assisting phase transition at high T
- Punchline: no eternal inflation ⇒ cosmology is safe
  (bubble nucleation rates faster than RS)

THANK YOU!


WE CANNOT SOLVE OUR PROBLEMS
WITH THE SAME THINKING
WE USED WHEN WE
CREATED THEM
-Albert Einstein


Backup

Cosmological Constant Problem

Cosmological Constant(CC)

- Conventionally denoted by Λ and Dimensionful → units of length^{-2}
- **Parameter describing the energy density of vaccum** - perspective from particle physics
- When we start considering scales of various contributions to vacuum energy we find that

"Theoretical expectations for the cosmological constant exceed observational limits by some 120 orders of magnitude." – Weinberg (Rev. Mod. Phys. **61**, 1)

- Miraculously !!

Vacuum zero point energy+ fluctuations

\+

QCD gluon and quark condensates

\+

Higgs field

\+

GUTs? String theory? All unknown contribution!!

= net CC ~ $\mathbf{10^{-48}}$ $\boldsymbol{GeV^4}$ **(obs)**

- **No known special symmetry enforcing this vanishing vacuum energy & being consistent with the laws of nature** - Cosmological constant problem

One can have **different meanings** to the notion of the CC problem

1. **A physics problem: QFT ↔ Λ .** Vacuum energy densities are estimated from QFTs describing particles and forces. Certain assumptions are made wrt general relativity and QFT for cosmological implications of this vacuum energy.

2. **An "expected" scale problem for Λ.** Dimensional considerations of the theory of quantum gravity would have M_{PL} as a fundamental scale , $\Lambda \sim 1$ in Planck units – **Big hierarchy problem!**

Why masses in SM≪ scales of a more fundamental theory (Planck)?

Different approaches to CC Problem

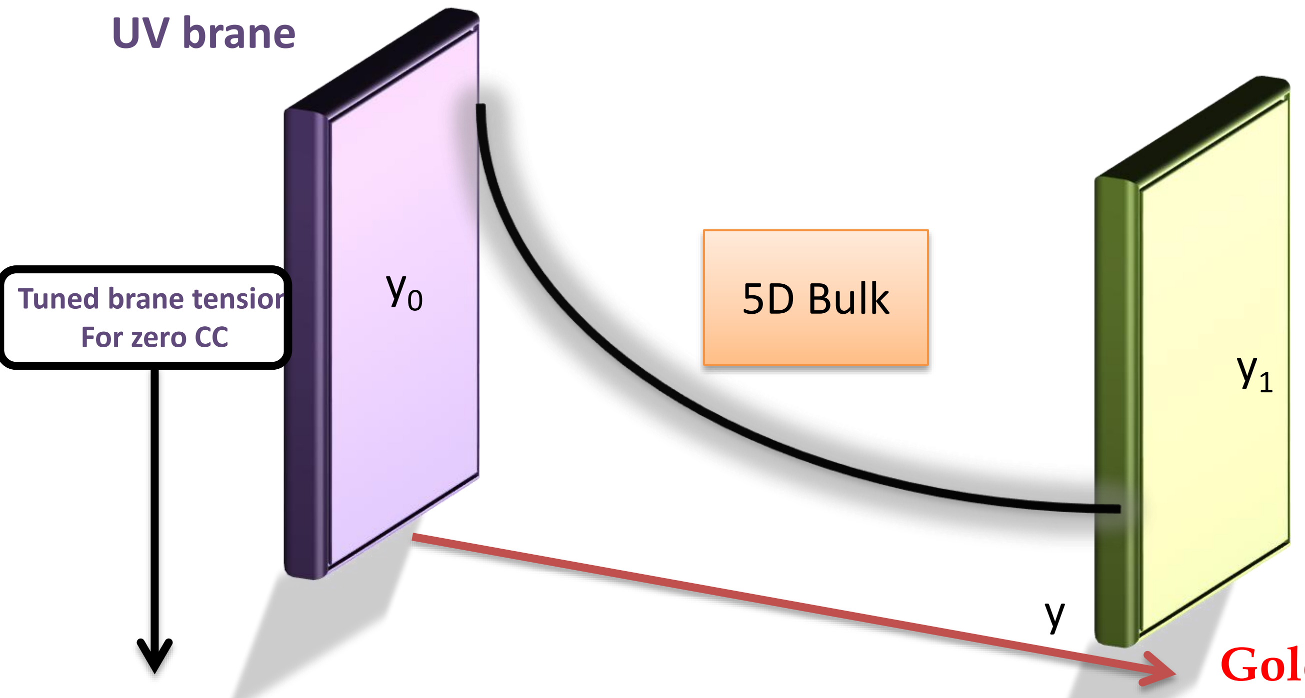
- **Deep symmetries**
  - SUSY in flat space claims to solve the CC problem halfway (on a log scale)
  - In curved space time- Supergravity has to be considered – fine tuning can fantastically make the Kahler derivative vanish and give small CC(?)
- **Miscellaneous Adjustment Mechanisms**
  - Conformal invariance canceling gravity effects to give small CC
  - S_{EH}+ scalar , which evolves to make CC vanish …..
- **Anthropic principle**
  - ``observers will only observe conditions which allow for observers''

Any solution to the CC problem is likely to have a wider impact on other areas of physics/astronomy

$$S = \int d^4x\, dy \sqrt{g} \left(-\frac{1}{2\kappa^2} R - \Lambda \right) - \int d^4x\, dy \sqrt{g}\; V_1\, \delta(y - y_1) - \int d^4x\, dy \sqrt{g}\; V_0 \delta(y - y_0)$$

Bulk — IR brane tension — UV brane Tension

$$ds^2 = e^{-2ky} dx^2 - dy^2$$



$$V_0 = -V_1 = 12\, k\, M^3$$

$$\Lambda_{IR} = M_{PL} e^{-kL}$$

IR CFT scale

Spontaneous breaking

Tuned brane tension for
Zero radion potential

Goldberger Wise Mechanism:
Prevents branes from colliding or flying away
V_1 and V_0 depend on scalar field ϕ

Scaling and Dilaton Basics

- Under Dilatations : $x \to x' = e^{-\alpha} x$
- Operators transform: $\mathcal{O}(x) \to \mathcal{O}'(x) = e^{\alpha\Delta}\mathcal{O}(e^{\alpha} x)$
- Δ has dimensions of the operators
- Now, the linearized transformation of the action sourced with $\mathcal{O}$ is

  $S \to S + \sum_i \int d^4x \, \alpha \, g_i \, (\Delta - 4)\mathcal{O}_i(x)$

Spontaneous breaking

- When CFT operator gets a VEV

$$\langle \mathcal{O} \rangle = f^n$$ → Classical dimension of $\mathcal{O}$

- Spontaneous breaking of scale invariance(SBSI) occurs ⇒ goldstone boson for scale transformations exist i.e dilaton

$$\sigma(x) \rightarrow \sigma(e^{\alpha} x) + \alpha f$$

- Non-linear realization in effective theory

$$f \rightarrow f\chi \equiv f e^{\sigma/f}$$

- Gives low-energy effective theory…

Dilaton Quartic

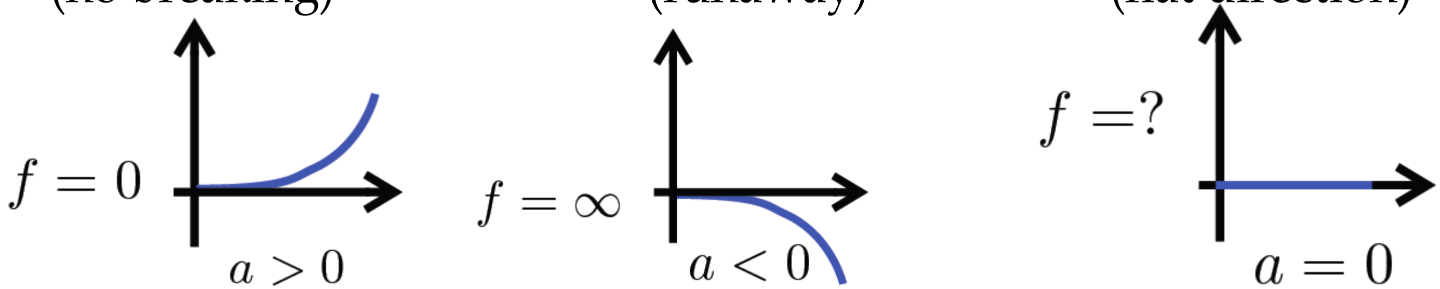
Requirement of scale invariance gives:

$$\mathcal{L}_{eff} = -af^4\chi^4 + \frac{f^2}{2}(\partial_\mu \chi)^2 + \textbf{higher derivative terms}$$

Dilaton quartic → not allowed in standard Goldstone Boson
Plays a crucial role in SBSI

Obstruction to SBSI (Fubini' 76), 3 possibilities

a > 0 → f = 0 (no breaking) | a < 0 → f =□ (runaway) | a=0 → f= anything (flat direction)



Near-marginal deformation

Add Explicit breaking term with an almost marginal operator

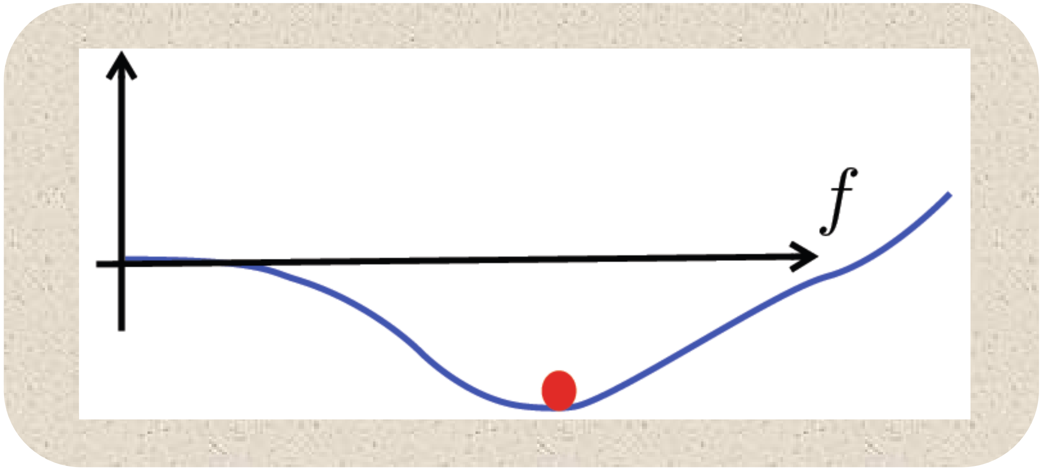
$$\delta S = \int d^4x \lambda(\mu) \mathcal{O}$$

Dilaton quartic has dependence on near marginal coupling:

$$V(\chi) = a\chi^4 \rightarrow V(\chi) = \chi^4 F(\lambda(\chi))$$

Near marginal

deformation can stabilize f away from origin



Dilaton mass

Stationarity Condition for V: $V' = f^3[4F(\lambda(f)) + \boldsymbol{\beta} F(\lambda(f))'] = 0$

Explicit breaking

F at minimum: $F(\lambda) = a + \sum_n a_n \lambda^n$

$$m_{dil}{}^2 \simeq 4f^2 \boldsymbol{\beta} F'\big(\lambda(f)\big) = -16 f^2 F\big(\lambda(f)\big)$$

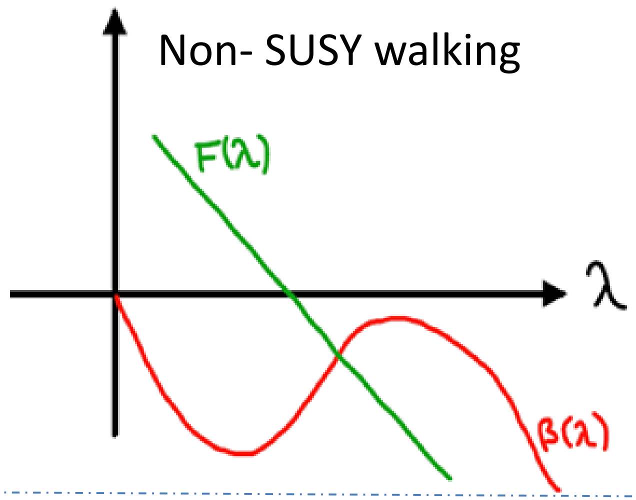
$$\beta' \ll 1$$

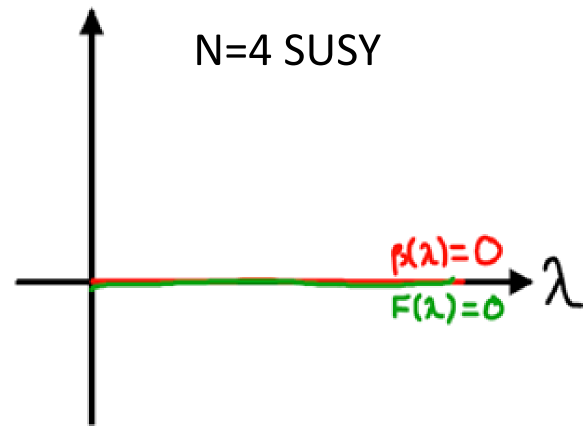
Reparametrize, dilaton as $\chi \equiv f e^{\sigma/f}$ with $<\boldsymbol{\sigma}>=0$

$F = 0$ reminescent of a vanishing CC , $\Lambda_{eff} = F f^4$

Various dynamical possibilities

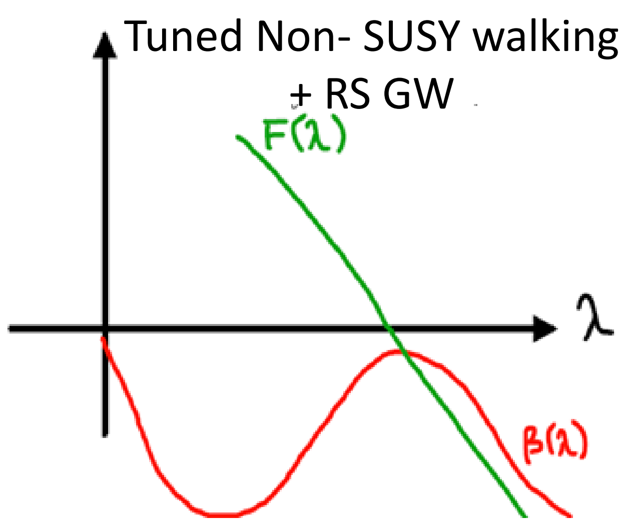
$F_0 \sim 16\,\pi^2$
$\beta \sim 4\pi$
$m_d^2 \sim 16\,\pi^2 f^2$

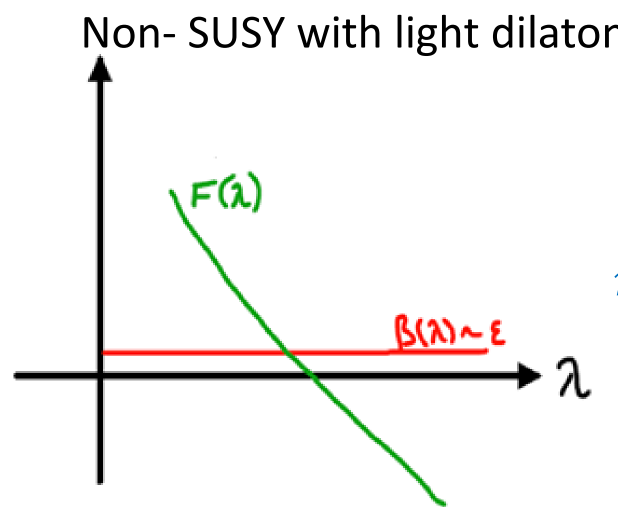




$F_0 \sim 0$
$\beta \sim 0$
$m_d^2 \sim \lambda^2 f^2$

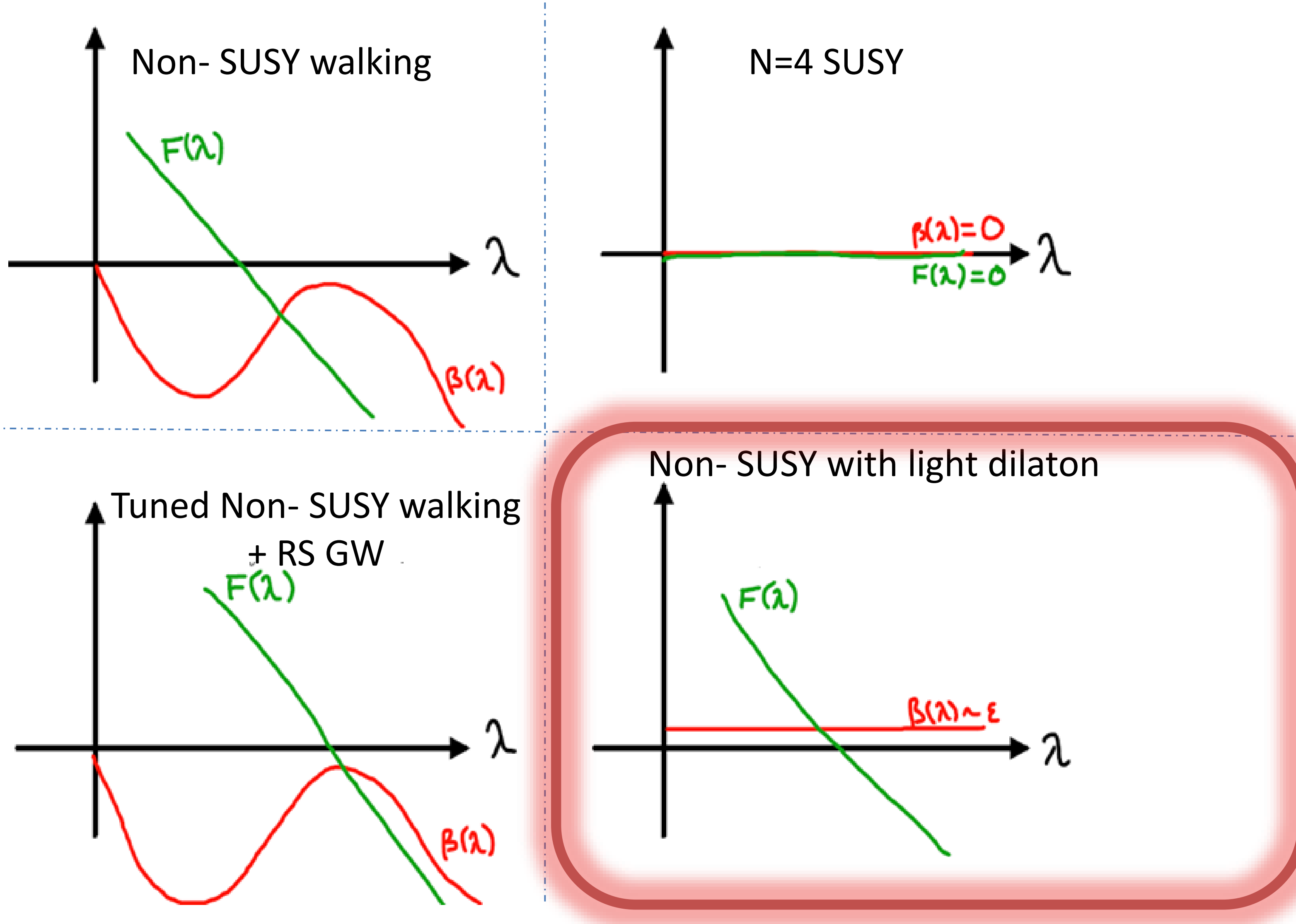
$F_0 \sim {(4\pi)^2}/{\Delta}$
$\beta \sim \epsilon\lambda$
$m_d \gtrsim \frac{2\,\Lambda_{IR}}{\Delta}$





$F_0 \sim (4\pi)^2$
$\beta \sim \epsilon\lambda$
$m_d \ll \Lambda_{IR}^2$

Various dynamical possibilities



CPR proposal*

- **"Presence of a flat direction, in the absence of perturbation is not required nor is it natural"**
- $F(\lambda)$ generically large, but if λ is near marginal for range of λ, theory will scan over F driven by the explicit breaking term

$$\frac{d\lambda}{d\log\mu} = \beta(\mu) \equiv \epsilon\, \mathrm{b}(\mu) \ll 1$$

$$\epsilon \ll 1,\ \ b(\lambda) = \mathcal{O}(1)$$

Dynamical (theory space) requirement

- Running will stop when $F \sim 0$ is reached

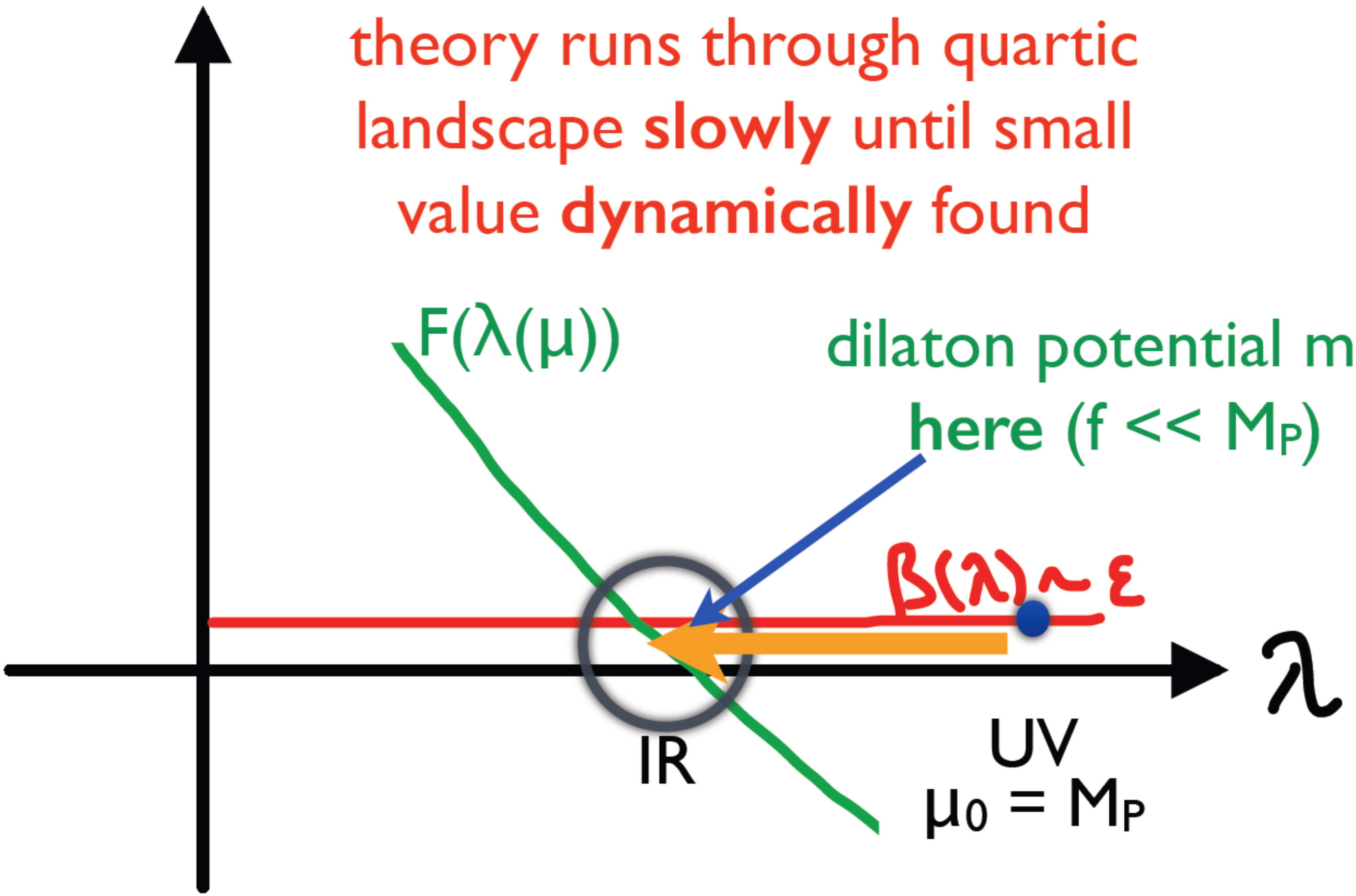
Explicit breaking at condensate scale

- **SBSI occurs** then
- We also have **a light dilaton→ $m_{dil} \propto \beta(\lambda(\mu)) \sim \epsilon$**
- **V at minimum = CC from phase transition from scale invariant to broken phase → will be suppressed too**

Contino, Pomarol, Rattazzi, ' talks in '10

Basic Principle

theory runs through quartic landscape **slowly** until small value **dynamically** found



Adding explicit breaking perturbation in AdS/CFT

AdS$_5$	⟷	**CFT$_4$**
ϕ	⟷	$\mathcal{O}$
$V'(\phi) = \frac{dV}{d\phi}$	⟷	$\beta(\lambda) = d\lambda / d\log\mu$
$V(\phi) = \Lambda_5$		exactly marginal
$\partial\phi\|_{y=y_0} = 0, \phi\|_{y=y_0}$	⟷	λ_{UV}

4D gravity is readily included

g_{MN}	⟷	dilaton+graviton

4D effective Lagrangian

$$\mathcal{L} = \sqrt{g}[\mathcal{L}_{CFT} + \lambda\,\mathcal{O} + M_P^2 R]$$

Generalized Randall-Sundrum

5D scalar minimally coupled to gravity

$$S = \int d^5x\sqrt{g}\left[\frac{1}{2}(\partial_M\phi)^2 - V(\phi) - \frac{1}{2\kappa^2}\mathcal{R}\right] - \int d^5x\sqrt{g_0}\,V_0(\phi) - \int d^5x\sqrt{g_1}\,V_1(\phi)$$

AdS/CFT : **Small □⇔ slowly changing $V(\phi)$** : $V(\phi) = \Lambda_5 + \epsilon f(\phi)$

Bellazzini, Csaki, Hubisz, Terning, Serra, '13
Coradeschi, Lodone, Pappadopulo, Rattazzi, Vitale, '13

Metric ansatz: flat 4d slices $\rightarrow$ $ds^2 = e^{-2A(\tilde{y})}\eta_{\mu\nu}\,dx^\mu dx^\nu - d\tilde{y}^2$

True scale coordinates for us : $\mu = ke^{-y}$

New notation !!

$A(\tilde{y}) = y\,,\qquad G(y) = A'\left(\tilde{y}(y)\right)^2$

$$ds^2 = e^{-2y}\,\eta_{\mu\nu}\,dx^\mu dx^\nu - \frac{dy^2}{G(y)}$$

Deviations from AdS encoded in G(y)

AdS/CFT: EOM captures running even when far from AdS

Scalar Einstein Equations

Equations of motion:

$$G = \frac{\frac{-\kappa^2}{6} V(\phi)}{1 - \frac{\kappa^2}{12}\dot{\phi}^2}$$

$$\frac{\dot{G}}{G} = \frac{2\kappa^2}{3}\dot{\phi}^2$$

$$\ddot{\phi} = \left(4 - \frac{1}{2}\frac{\dot{G}}{G}\right)\dot{\phi} + \frac{1}{G}\frac{\partial V}{\partial \phi}$$

Eliminate G(y) in the scalar field EOM to get the **Master Evolution equation** :

$$\ddot{\phi} = 4\left(\dot{\phi} - \frac{3}{2\kappa^2}\frac{\partial \log V(\phi)}{\partial \phi}\right)\left(1 - \frac{\kappa^2}{12}\dot{\phi}^2\right)$$

Backreaction term

AdS/CFT: $\phi \sim \log \lambda$

Captures running (and condensation) of sourced operators in ~CFT

Dilaton effective potential

Bulk action → total derivative (Bellazini et. al. 1305.3919)

Entire effective potential= boundary term with brane localized potentials and jump conditions

$$\mathbf{V}_{\text{eff}} = \mathbf{e}^{-4\mathbf{y}_0}\left[\mathbf{V}_0(\phi(\mathbf{y}_0)) - \frac{6}{\kappa^2}\sqrt{\mathbf{G}(\mathbf{y}_0)}\right] + \mathbf{e}^{-4\mathbf{y}_1}\left[\mathbf{V}_1(\phi(\mathbf{y}_1)) + \frac{6}{\kappa^2}\sqrt{\mathbf{G}(\mathbf{y}_1)}\right]$$

UV brane

$$\mu_0 = \mathbf{k}\mathbf{e}^{-\mathbf{y}_0}$$

IR brane

$$\mu_1 = \mathbf{k}\mathbf{e}^{-\mathbf{y}_1}$$

What is the behaviour of the dilaton effective potential for various bulk scalar potentials?

(various deformations of CFT)

Task = work out UV and IR asymptotics

How is spontaneously broken scale invariance manifested?

General Bulk Potentials (UV)

Small Bulk mass Term

$$\mathbf{V} = -\frac{6\mathbf{k}^2}{\kappa^2}\left(1 + \frac{\kappa^2}{3}\epsilon\phi^2\right)$$

Approximate (small backreaction) solution:

$$\phi = \phi_0 e^{\epsilon y_0} + \tilde{\lambda} e^{(4-\epsilon)(y_0 - y_c)}$$ Slow running

$$G(y) = k\left(1 + \mathcal{O}(e^{-8(y_c - y_0)})\right)$$ Almost AdS

UV contribution to dilaton effective potential

$$V_{UV} = \boxed{\Lambda_{bare}} + \boxed{a_{UV} f^{4-\epsilon} \mu_0^{-\epsilon}}$$

Bare CC **marginal "almost quartic"**

General Bulk Potentials (IR)

$$\ddot{\phi} = 4\left(\dot{\phi} - \frac{3}{2\kappa^2}\frac{\partial \log V(\phi)}{\partial \phi}\right)\left(1 - \frac{\kappa^2}{12}\dot{\phi}^2\right)$$

Solutions independent of V(ϕ)

Φ in the deep IR:

$$\dot{\phi} = \sqrt{\frac{12}{\kappa^2}}(1 - \delta v_{\mathrm{IR}})$$

Now, $G(y) = \frac{-\frac{\kappa^2}{6}V(\phi)}{1-\frac{\kappa^2}{12}\dot{\phi}^2} \approx -\frac{\kappa^2}{12}\frac{V(\phi)}{\delta_{IR}}$ **In deep IR**

Expand scalar EOM in small δv

Integrable in asymptotic limit! $G_{\mathrm{IR}}(y_1) = -\frac{\kappa^2}{12\tilde{\lambda}}V(\phi(y_c))e^{8(y_1-y_c)}$

Generates a pure quartic $V_{IR} \to a_{IR}\, f^4$

Dilaton Potential

$$V_{\text{dilaton}} = -\mu^{\epsilon} f^{4-\epsilon} + \lambda f^4$$

Bare CC set to zero

At min

$$\mathrm{f} \approx \mu \left(\frac{1}{\lambda}\right)^{1/\epsilon} \qquad V_{\text{dilaton}} = \frac{\epsilon}{4}\lambda f^4$$

CC is suppressed by ϵ!

$$\mathbf{m}^2_{\text{dilaton}} \sim \epsilon \mathbf{f}^2 \,/\, \mathrm{N}_{\text{eff}}$$

Soft wall realisation of small condensate contribution to CC in models with non-linearly realised SBSI

Black hole in AdS

Same action-euclidean signature, compactified time, t⊂ {0,□=1/T}

$$ds^2 = e^{-2y}\left[h(y)dt^2 + d\vec{x}^2\right] + \frac{1}{h(y)}\frac{dy^2}{G(y)}$$

EOMs

$$\frac{d}{dy}\log\frac{\dot h}{h} = 4\left(1 - \frac{1}{4}\frac{\dot h}{h} - \frac{\kappa^2}{12}\dot\phi^2\right)$$

$$\frac{\dot G}{G} = \frac{2\kappa^2}{3}\dot\phi^2$$

$$G = \frac{\frac{2\kappa^2}{3}\frac{V(\phi)}{h}}{\frac{d}{dy}\log\frac{\dot h}{h}}$$

$$\ddot\phi = \left(\dot\phi - \frac{3}{2\kappa^2}\frac{\partial \log V}{\partial\phi}\right)\frac{d}{dy}\log\frac{\dot h}{h}$$

Master Evolution Equation

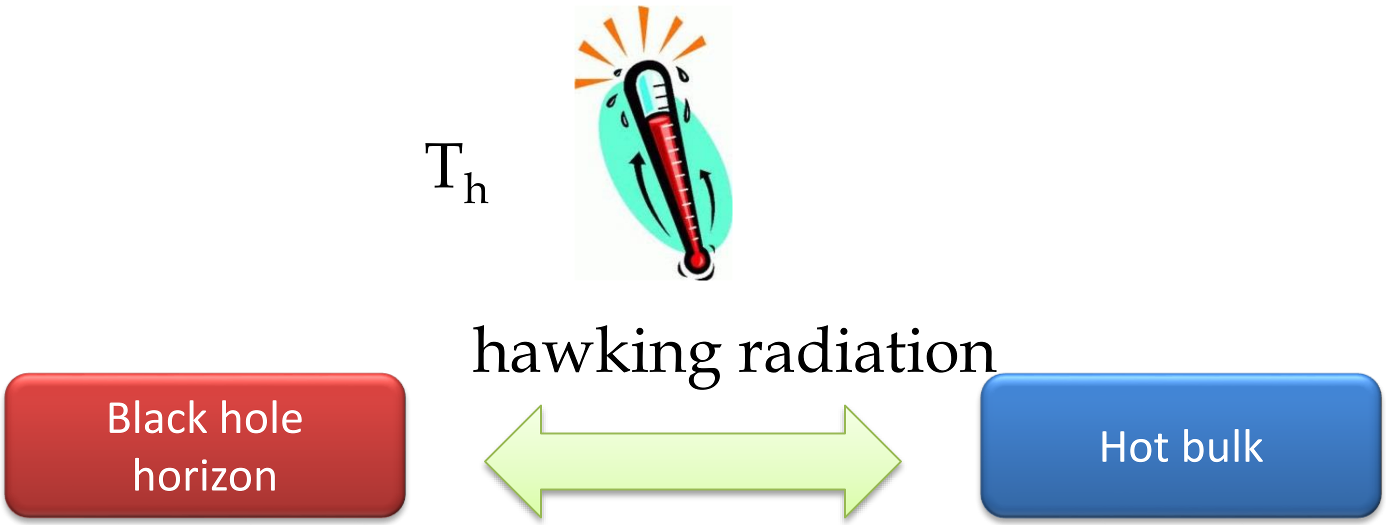
$$\ddot\phi = 4\left(\dot\phi - \frac{3}{2\kappa^2}\frac{\partial \log V}{\partial\phi}\right)\left(1 - \frac{1}{4}\frac{\dot h}{h} - \frac{\kappa^2}{12}\dot\phi^2\right)$$

+

Horizon BCs : $\dot\phi\Big|_{\mathbf{y_h}} = \frac{\mathbf{3}}{\mathbf{2\kappa^2}}\frac{\partial \log \mathbf{V}}{\partial\phi}\Big|_{\mathbf{y_h}}$

Holographic Dilaton Potential:

- At finite T, we have 2 different bulk spaces: warped and AdS-S describing states of thermodynamics equillibrium
- Canonical ensemble of AdS space = AdS- Schwarzschild (AdS- S) (Hawing, Page’83)



- Universe filled with Thermal CFT state with T_h
- Energy & Entropy of heat bath described by corresponding Black hole

Transition rate

The bubble nucleation rate per unit volume in a first order phase transition: $\Gamma = \Gamma_0 e^{-S}$ where $\Gamma_0 \sim TeV^4$

S: Euclidean action for the bubble solution interpolating between false and true minimum→ contains model dependent parameters

High T ⇒ $R_{bubble} > 1/T$ ⇒ instanton has O(3) symmetry ⇒ $S = S_3/T$

Low T ⇒ O(4) symmetric instanton ⇒ $S = S_4$

Focus on T close to T_c , two minima in the Free energy are degenerate

⇒ Thin Wall Approximation (Coleman ‘ 77)

Or $T > T_c$

Thick wall approximation