Model Independent Approach to Inelastic Dark Matter Scattering

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PRD**90** with G. Barello and Spencer Chang.

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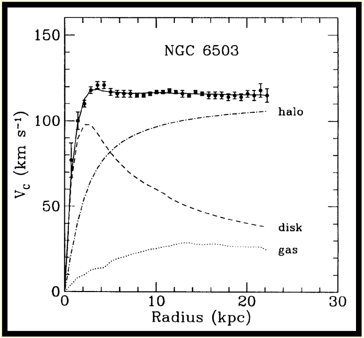
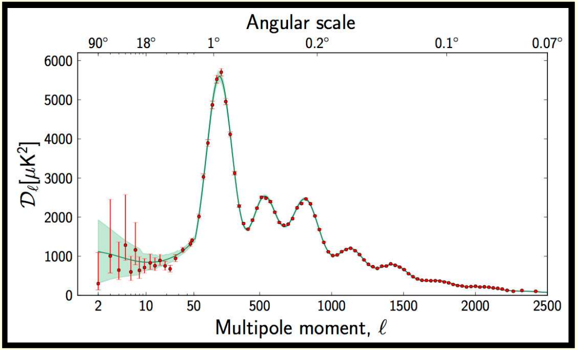
Detection of Dark Matter

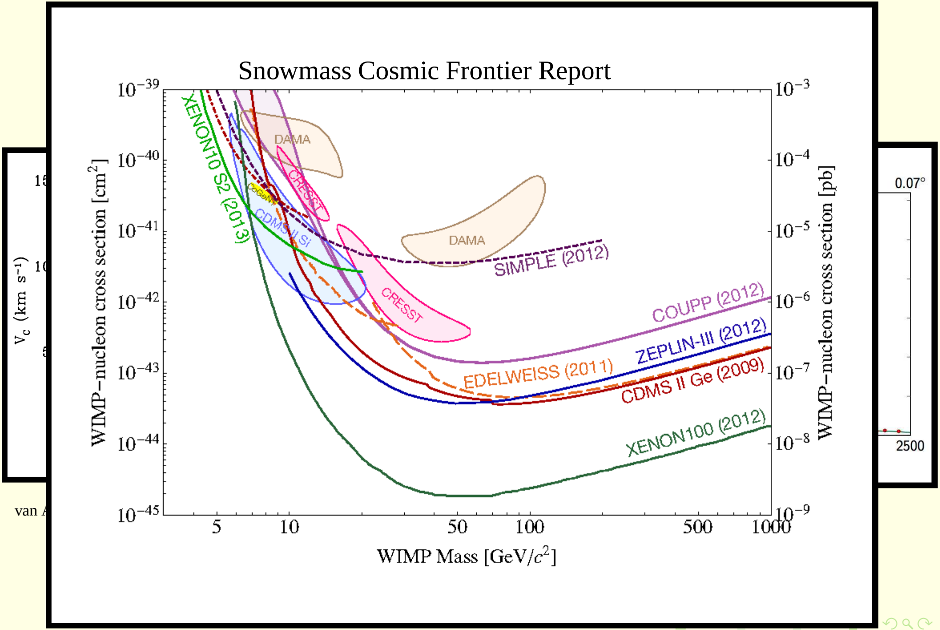


van Albada *et. al.* Astrophys.J. **295**. 305-313 (1985)



PLANCK

Detection of Dark Matter



van A

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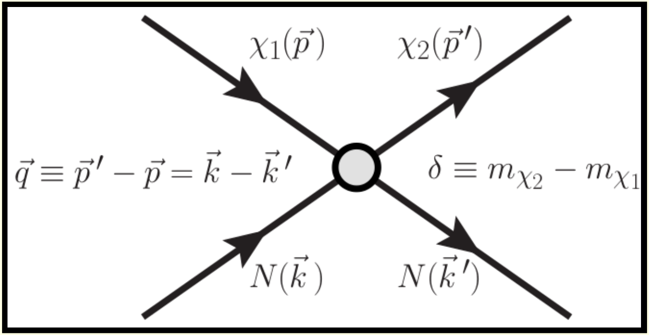
- Highlighted new operators beyond spin-independent and -dependent interactions
- Anand *et. al.* have provided some *Mathematica* code to calculate the nuclear form factors (arXiv:1308.6288)
- We generated the Effective Theory for Inelastic Dark Matter
  - And updated the *Mathematica* code



$$\delta \sim 100\,\mathrm{keV}$$

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For the Majorana mass eigenstates ($\chi_1 \simeq i(\eta - \xi),\ \chi_2 \simeq \eta + \xi$) vector current becomes

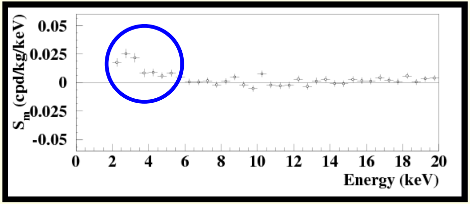
$$\bar{\psi}\gamma^{\mu}\psi \simeq i(\bar{\chi}_1\bar{\sigma}^{\mu}\chi_2 - \bar{\chi}_2\bar{\sigma}^{\mu}\chi_1) + \frac{\delta}{m}(\bar{\chi}_2\bar{\sigma}^{\mu}\chi_2 - \bar{\chi}_1\bar{\sigma}^{\mu}\chi_1)$$

And the masses are split by 2δ

Why IDM?

Originally used to explain DAMA's modulation signal.

- D. Tucker-Smith and N. Weiner, Phys. Rev. D **64**, 043502 (2001)



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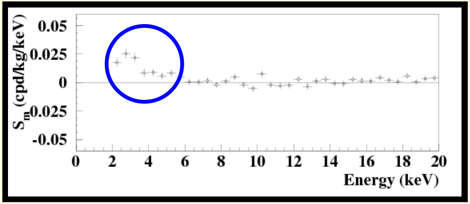

As Direct Detection limits got stronger, more models were introduced:

- A Theory of Dark Matter (N. Arkani-Hamed, D. P. Finkbeiner, T. R. Slatyer and N. Weiner, Phys. Rev. D **79**, 015014 (2009))
- Composite Inelastic Dark Matter (D. Alves, S. Behbahani, P. Schuster, J. Wacker, Phys. Lett. B **692** (2010) 323-326)
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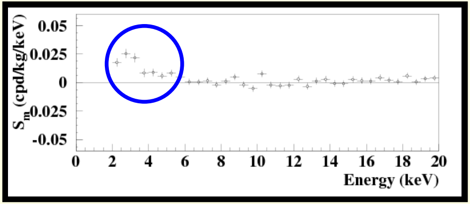

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One of the main motivations: Kinematics

IDM Kinematics

Raises minimum velocity to scatter:

$$v_{\min} = \frac{1}{\sqrt{2 m_N E_R}} \left(\frac{m_N E_R}{\mu_{N\chi}} + \boxed{\delta} \right)$$

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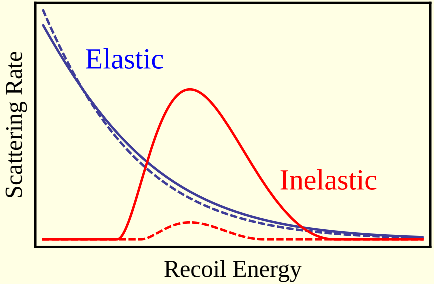
- Favors heavier targets

$$\begin{aligned} m_N \gg m_\chi :\ v_{\min} &\to \frac{1}{\sqrt{2m_N E_R}} \left(\frac{m_N}{m_\chi} E_R + \delta \right) \\ m_N \ll m_\chi :\ v_{\min} &\to \frac{1}{\sqrt{2m_N E_R}} (E_R + \delta) \end{aligned}$$

Raises minimum velocity to scatter:

$$v_{\min} = \frac{1}{\sqrt{2m_N E_R}} \left(\frac{m_N E_R}{\mu_{N\chi}} + \boxed{\delta} \right)$$

- Favors heavier targets
- Changes recoil spectrum
- Amplifies modulation



Proposed Theories have specific operators:

- Inelastic Dark Matter:
  - $\Phi_1 \Phi_2 \, \overline{q} q$
  - $\overline{\chi}_1 (g'_V \gamma^\mu + g'_A \gamma^\mu \gamma^5) \chi_2 \, \overline{q} (g_V \gamma^\mu + g_A \gamma^\mu \gamma^5) q$
  - $\Phi_1^\dagger \gamma^\mu \Phi_2 \, \overline{q} (g_V \gamma^\mu + g_A \gamma^\mu \gamma^5) q$
- A Theory of Dark Matter:
  - $\overline{\chi}_1 \gamma^\mu \chi_2 \, \overline{q} \gamma^\mu q$
- Composite Inelastic Dark Matter:
  - $\Phi_1^\dagger \gamma^\mu \Phi_2 \, \overline{q} \gamma^\mu q$
- Magnetic Inelastic Dark Matter:
  - $\overline{\chi}_1 (\sigma^{\mu\nu} \frac{q_\nu}{m_M}) \chi_2 \, \overline{q} \gamma_\mu q$

We would like to see if there are any others.

Galilean Invariants

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Invariant variables:

$$\vec{S}_\chi,\ \vec{S}_N,\ \vec{q},\ \vec{v}^\perp$$

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→These categorize nonrelativistic scattering

Invariant variables:

$$\vec{S}_\chi,\ \vec{S}_N,\ \vec{q},\ \vec{v}^\perp$$

Only $\vec{v}^\perp$ changes in inelastic case:

$$v^\perp_{\text{inel}} = \vec{v} + \frac{\vec{q}}{2\mu_N} + \boxed{\frac{\delta}{|\vec{q}^{\,2}|}\vec{q}}$$

This perpendicularness can be seen from conservation of energy:

$$E_f = E_i + \delta + \vec{q}\cdot\vec{v} + \frac{|\vec{q}^{\,2}|}{2\mu_N}$$

Nonrelativistic Operators

Only need to modify the $v_\perp$ of Fitzpatrick *et. al.*

$$\mathcal{O}_1 = \mathbf{1}_\chi \mathbf{1}_N, \quad \mathcal{O}_2 = (v_{\text{inel}}^\perp)^2, \quad \mathcal{O}_3 = i\vec{S}_N \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}_{\text{inel}}^\perp\right),$$

$$\mathcal{O}_4 = \vec{S}_\chi \cdot \vec{S}_N, \quad \mathcal{O}_5 = i\vec{S}_\chi \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}_{\text{inel}}^\perp\right),$$

$$\mathcal{O}_6 = \left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}\right)\left(\vec{S}_N \cdot \frac{\vec{q}}{m_N}\right), \quad \mathcal{O}_7 = \vec{S}_N \cdot \vec{v}_{\text{inel}}^\perp,$$

$$\mathcal{O}_8 = \vec{S}_\chi \cdot \vec{v}_{\text{inel}}^\perp, \quad \mathcal{O}_9 = i\vec{S}_\chi \cdot \left(\vec{S}_N \times \frac{\vec{q}}{m_N}\right),$$

$$\mathcal{O}_{10} = i\vec{S}_N \cdot \frac{\vec{q}}{m_N}, \quad \mathcal{O}_{11} = i\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}, \quad \mathcal{O}_{12} = \vec{S}_\chi \cdot \left(\vec{S}_N \times \vec{v}_{\text{inel}}^\perp\right),$$

$$\mathcal{O}_{13} = i\left(\vec{S}_\chi \cdot \vec{v}_{\text{inel}}^\perp\right)\left(\vec{S}_N \cdot \frac{\vec{q}}{m_N}\right),$$

$$\mathcal{O}_{14} = i\left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}\right)\left(\vec{S}_N \cdot \vec{v}_{\text{inel}}^\perp\right),$$

$$\mathcal{O}_{15} = -\left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}\right)\left((\vec{S}_N \times \vec{v}_{\text{inel}}^\perp) \cdot \frac{\vec{q}}{m_N}\right),$$

Relativistic Matching

$$v_{\text{inel}}^{\perp} = \vec{v} + \frac{\vec{q}}{2\mu_N} + \frac{\delta}{|\vec{q}^{\,2}|}\vec{q}$$

Index	Relativistic Operator	Nonrelativistic Limit $\times \frac{1}{4m_N m_\chi}$	$\sum_i c_i \mathcal{O}_i$
1	$\bar{\chi}_2\chi_1\bar{N}N$	$\mathbf{1}_\chi \mathbf{1}_N$	$\mathcal{O}_1$
2	$i\bar{\chi}_2\chi_1\bar{N}\gamma^5 N$	$i\frac{\vec{q}}{m_N}\cdot\vec{S}_N$	$\mathcal{O}_{10}$
3	$i\bar{\chi}_2\gamma^5\chi_1\bar{N}N$	$-i\frac{\vec{q}}{m_\chi}\cdot\vec{S}_\chi$	$-\frac{m_N}{m_\chi}\mathcal{O}_{11}$
4	$\bar{\chi}_2\gamma^5\chi_1\bar{N}\gamma^5 N$	$-(\frac{\vec{q}}{m_\chi}\cdot\vec{S}_\chi)(\frac{\vec{q}}{m_N}\cdot\vec{S}_N)$	$-\frac{m_N}{m_\chi}\mathcal{O}_6$
5	$\bar{\chi}_2\gamma^\mu\chi_1\bar{N}\gamma_\mu N$	$\mathbf{1}_\chi \mathbf{1}_N$	$\mathcal{O}_1$
6	$\bar{\chi}_2\gamma^\mu\chi_1\bar{N}i\sigma_{\mu\nu}\frac{q^\nu}{m_M}N$	$\frac{\|\vec{q}\|^2}{2m_N m_M}\mathbf{1}_\chi\mathbf{1}_N$ $+2\left(\frac{\vec{q}}{m_\chi}\times\vec{S}_\chi + i\vec{v}_{\text{inel}}^{\perp}\right)\cdot\left(\frac{\vec{q}}{m_M}\times\vec{S}_N\right)$	$\frac{\|\vec{q}\|^2}{2m_N m_M}\left(\mathcal{O}_1 + \frac{4m_N}{m_\chi}\mathcal{O}_4\right)$ $-\frac{2m_N}{m_M}\left(\frac{m_N}{m_\chi}\mathcal{O}_6 + \mathcal{O}_3\right)$
7	$\bar{\chi}_2\gamma^\mu\chi_1\bar{N}\gamma_\mu\gamma^5 N$	$-2\vec{S}_N\cdot(\vec{v}_{\text{inel}}^{\perp} - \frac{\delta}{\|\vec{q}\|^2}\vec{q})$ $+2i\vec{S}_\chi\cdot(\vec{S}_N\times\frac{\vec{q}}{m_\chi})$	$-2\left(\mathcal{O}_7 + i\frac{m_N\delta}{\|\vec{q}\|^2}\mathcal{O}_{10} - \frac{m_N}{m_\chi}\mathcal{O}_9\right)$

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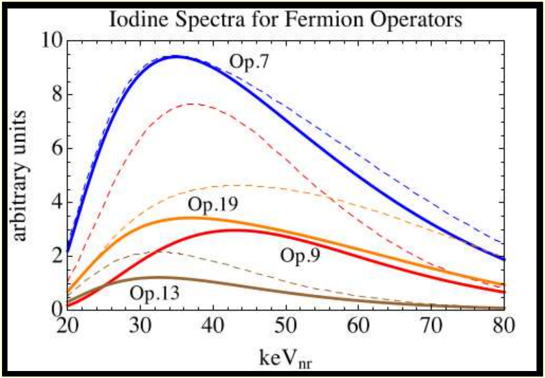
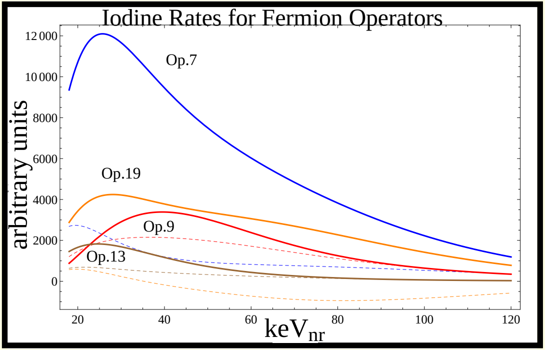


For $\delta > 0$

Solid $\rightarrow$ Correct
Dashed $\rightarrow$ Incorrect spectra with correct velocity threshold

Form Factor Dependence on $\vec{v}_{\text{inel}}^{\perp}$



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Conclusions

- Inelastic kinematics exhibit non-trivial effects
- Mapping relativistic operators onto nonrelativistic ones needs to be done carefully
- We updated the Anand *et. al. Mathematica* code
    - Check old experimental data
    - Useful for calculating constraints for new models
    - Experiments can use to check more types of operators

Any Questions?