Anarchy In Unified Theories

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Outline of the talk

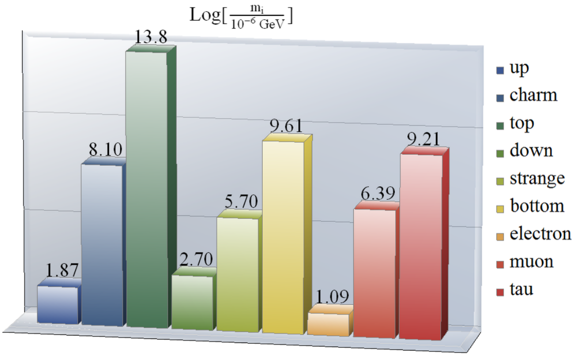
- Hierarchy in Fermion masses and mixings
- Strong Hierarchy & Small Mixing Angles in Charged Fermion Sector and Large Mixing Angles & Mild Hierarchy in the Neutrino Sector in Unified Theories
- Random Matrices in Unified Theories

Charged Fermion Mass Spectrum and Hierarchies [in the unit of m_t]

- **up-type quarks**
- $m_u \sim 6.5 \times 10^{-6}$
- $m_c \sim 3.3 \times 10^{-3}$
- $m_t \sim 1$

- **down-type quarks**
- $m_d \sim 1.5 \times 10^{-5}$
- $m_s \sim 3 \times 10^{-4}$
- $m_b \sim 1.5 \times 10^{-2}$

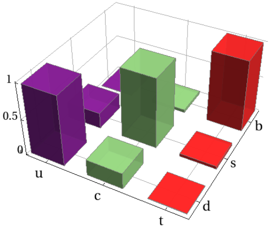
- **charged leptons**
- $m_e \sim 3 \times 10^{-6}$
- $m_\mu \sim 6 \times 10^{-4}$
- $m_\tau \sim 1 \times 10^{-2}$



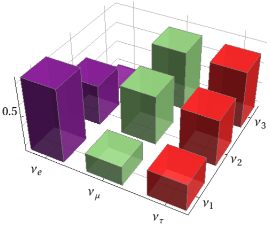
charged fermion mass spectrum strongly hierarchical

CKM and PMNS matrices and Neutrino mass differences

$$V_{CKM} \sim \begin{bmatrix} 0.976 & 0.22 & 0.004 \\ -0.22 & 0.98 & 0.04 \\ 0.007 & -0.04 & 1 \end{bmatrix}$$



$$U_{PMNS} \sim \begin{bmatrix} 0.85 & -0.54 & 0.16 \\ 0.33 & 0.62 & -0.72 \\ -0.40 & -0.59 & -0.70 \end{bmatrix}$$



Quark mixing angles are small and Leptonic mixing angles are large

- **neutrinos** (assuming normal hierarchy)
- $\Delta m^2_{sol} \sim 7.5 \times 10^{-5} eV^2$; $m_2 \sim 8.5 \times 10^{-12}$ GeV
- $\Delta m^2_{atm} \sim 2.5 \times 10^{-3} eV^2$; $m_3 \sim 5 \times 10^{-11}$ GeV

Neutrino mass spectrum shows mild hierarchy

Mixing between Ordinary-fermions and Vectorlike-fermions

Hierarchy among generations can arise as a consequence of mixing between the ordinary three families and vectorlike-fermions which have masses of order of the GUT scale.

$$U = H^T U_0 H$$
$$D = D_0 H$$
$$L = H^T L_0$$
$$N = N_0$$

with each of the matrices U_0, D_0, L_0, and N_0 has no hierarchy among elements.

The matrix, H comes due to the exogenous mixing and gives rise to the intergenerational hierarchy of masses.

Such structure of mass matrices can arise if there is exogenous mixing present for all the fermion types contained in 10-plet of $SU(5)$.

Babu, Barr: Phys.Lett. B381 (1996) 202-208

[Composite Fermion Models: 10-plet of $SU(5)$ is composite.
Nelson, Strassler: Phys.Rev. D56 (1997) 4226-4237; Haba: Phys. Rev. D 59, 035011 (1999)

Fermions with Flavor Dependent Charge: 10-plet of $SU(5)$ GUT having $U(1)$-flavor charge.
Haba, Murayama: Phys.Rev. D63 (2001) 053010]

Random Matrices In Unified Theories

Starting with general class of such Unified theories, the Yukawa matrices can be written as,

$$Y^U = H^T Y_0^U H$$
$$Y^D = \epsilon_4 \; Y_0^D H$$
$$Y^L = \epsilon_4 \; H^T Y_0^L$$
$$Y^N = Y_0^N$$

where, ϵ_4 is a certain vev ratio.

Random matrices has $\sim O(1)$ elements and are in general complex

$$Y_0^F \sim \begin{bmatrix} O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \end{bmatrix}$$

F stands for U, D, L, N.

Hierarchy is introduced only by the matrix H, which has the form

$$H \simeq \begin{bmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_2 & 0 \\ 0 & 0 & \epsilon_3 \end{bmatrix}$$

with the assumption $\epsilon_1 << \epsilon_2 << \epsilon_3 \leq 1$.

Neutrino sector and Consequences of this class of Models

See-Saw Mechanism:

$$M_\nu = v\ {Y_0^N}^T Y_R^{-1} Y_0^N$$

$$Y_R \sim \begin{bmatrix} O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \end{bmatrix}$$

Y_R is complex symmetric matrix

Consequences (to the leading order):

- **generates hierarchies in the charged fermionic sector**

$$y_d : y_s : y_b \sim y_e : y_\mu : y_\tau \sim \epsilon_1 : \epsilon_2 : \epsilon_3$$
$$y_u : y_c : y_t \sim \epsilon_1^2 : \epsilon_2^2 : \epsilon_3^2$$

- **gives rise to small quark mixing angles but large neutrino mixing angles**

$$V_{ij}^{CKM} \sim \frac{\epsilon_i}{\epsilon_j}$$
$$V_{ij}^{lepton} \sim 1.$$

- **mild hierarchy in neutrino mass spectrum**

Gaussian Measure of the Random Matrices and Input Distributions

Basis independent random matrix, with all elements having no correlations among themselves, leads to Gaussian measure for the random matrix.

$$dM_{Dirac-type} = \prod_{ij} dM_{ij} \; e^{-|M_{ij}|^2}$$

$$dM_{Majorana-type} = \prod_{i} dM_{ii} \; e^{-|M_{ii}|^2} \; \prod_{i<j} dM_{ij} \; e^{-2|M_{ij}|^2}$$

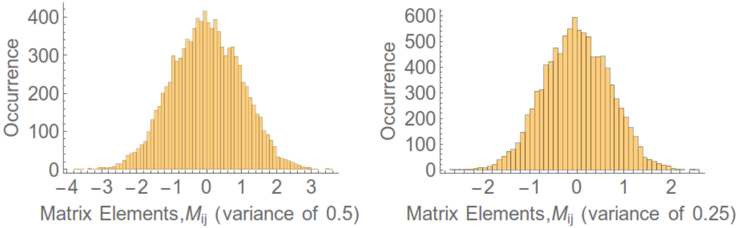


figure: Histogram distributions of the input random matrix elements. Left plot is for matrix elements with Gaussian distributions with variance 0.5 and the right plot with 0.25 (full ranges are shown in the plots). Sample size taken to be 10^4.

χ^2 minimization to fix the model parameters ϵ_i

- Model has only 4 parameters; Matrices are random inputs with Gaussian measure
- χ^2 minimization is done to fit the observables; 17 observables (n_{obs}) versus 4 parameters

χ^2 and pull are defined as

$$\chi^2_{dis} = \sum_j (pull_{dis\ j})^2 \quad ; with,$$

$$pull_{dis} = \frac{\text{theoretical mean value} - \text{experimental central value}}{\sqrt{\text{theoretical standard deviation}^2 + \text{experimental } 1\sigma \text{ uncertainty}^2}}$$

fitted model parameters: Best minimum $\frac{\chi^2_{dis}}{n_{obs}} \sim 1$ fixes the model parameters to be

$$\begin{aligned} \epsilon_1 &= 0.00211; \quad \epsilon_2 &= 0.02918 \\ \epsilon_3 &= 0.63287; \quad \epsilon_4 &= 0.00375 \end{aligned}$$

χ^2 best fit values of the observables

OBS	ECV	1σ exp	TMV	TSD	$\frac{TMV}{ECV}$	$Pull_{dis}$
y_u	2.51×10^{-6}	8.5×10^{-7}	9.47×10^{-6}	9.0×10^{-6}	3.77	0.764
y_c	1.37×10^{-3}	4.21×10^{-5}	1.42×10^{-3}	1.36×10^{-3}	1.03	0.03
y_t	0.543	4.7×10^{-3}	0.357	0.185	0.657	-1.00
y_d	2.89×10^{-6}	2.9×10^{-7}	7.02×10^{-6}	3.69×10^{-6}	2.43	1.11
y_s	5.68×10^{-5}	2.81×10^{-6}	1.456×10^{-4}	5.24×10^{-5}	2.56	1.68
y_b	3.45×10^{-3}	3.16×10^{-5}	3.94×10^{-3}	1.15×10^{-4}	1.14	0.42
y_e	1.18×10^{-6}	1.18×10^{-8}	6.93×10^{-6}	3.62×10^{-6}	5.87	1.58
y_μ	2.49×10^{-4}	2.49×10^{-6}	1.46×10^{-4}	5.25×10^{-4}	0.58	-1.97
y_τ	4.44×10^{-3}	4.44×10^{-5}	3.94×10^{-3}	1.17×10^{-3}	0.88	-0.42
$\lvert V_{us}\rvert$	0.22539	0.00072	0.1208	0.1030	0.53	-1.01
$\lvert V_{cb}\rvert$	0.04206	0.00064	0.0731	0.0715	1.73	0.43
$\lvert V_{ub}\rvert$	0.00363	0.00013	0.0071	0.0082	1.96	0.42
η_W	0.34	0.034	0.086	5.9	0.25	-0.07
$\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}$	0.031	0.001	0.134	0.184	4.33	0.56
$sin^2\theta_{12}$	0.308	0.017	0.501	0.288	1.62	0.66
$sin^2\theta_{23}$	0.3875	0.0225	0.500	0.292	1.29	0.38
$sin^2\theta_{13}$	0.0241	0.0025	0.3325	0.236	13.79	1.30
$\frac{\chi^2_{dis}}{n_{obs}}$						0.97

Histogram Distributions of up-type quark Yukawa couplings

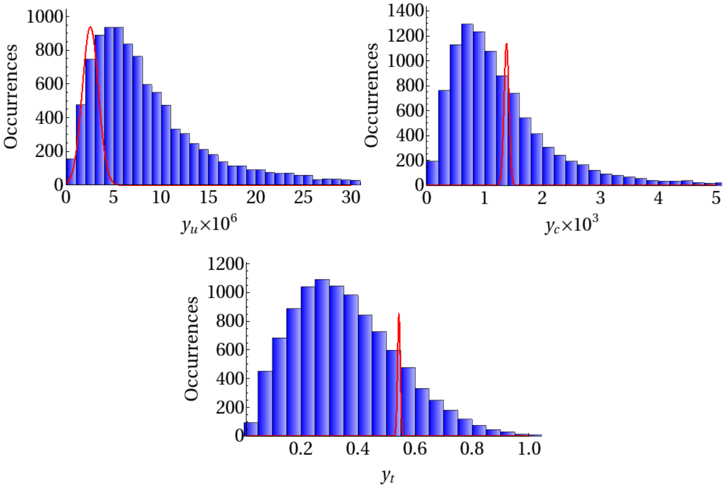


figure: Histograms showing the theoretical distributions of the up type quark Yukawa couplings picked according to anarchy hypothesis (blue) with fixed model parameters ϵ_i. Sample size taken to be 10^4. Red curves represent the experimental 1σ range.

Histogram Distributions of down-type quark Yukawa couplings

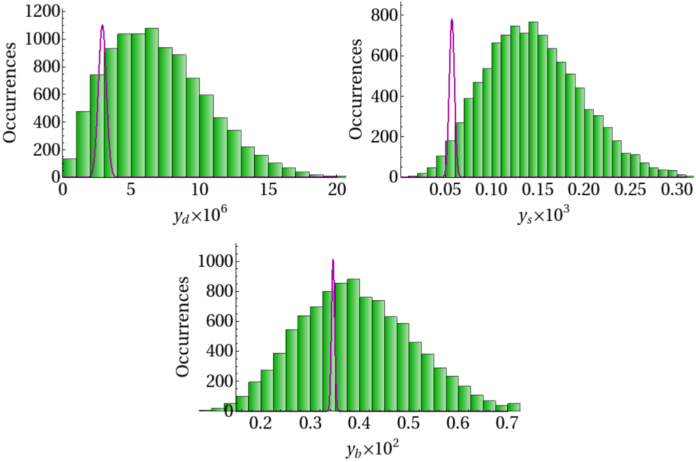


figure: Histograms showing the theoretical distributions of the down type quark Yukawa couplings according to anarchy hypothesis (green) with fixed model parameters ϵ_i. Sample size taken to be 10^4. Magenta curves represent the experimental 1σ range.

Histogram Distributions of charged lepton Yukawa couplings

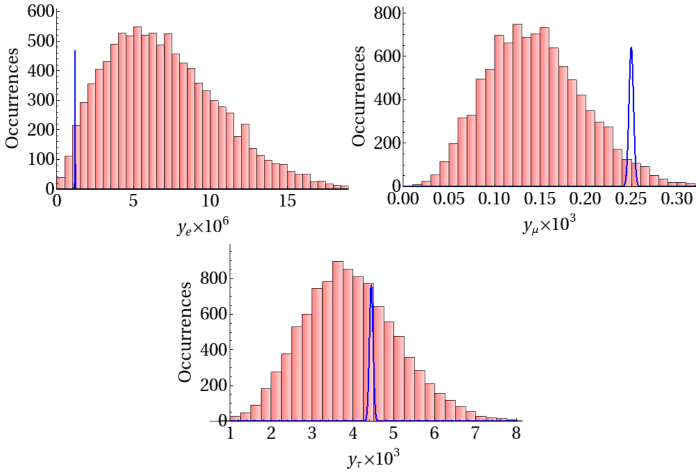


figure: Histograms showing the theoretical distributions of the charged leptons Yukawa couplings according to anarchy hypothesis (pink) with fixed model parameters ϵ_i. Sample size taken to be 10^4. Blue curves represent the experimental 1σ range.

Histogram Distributions of CKM mixing parameters

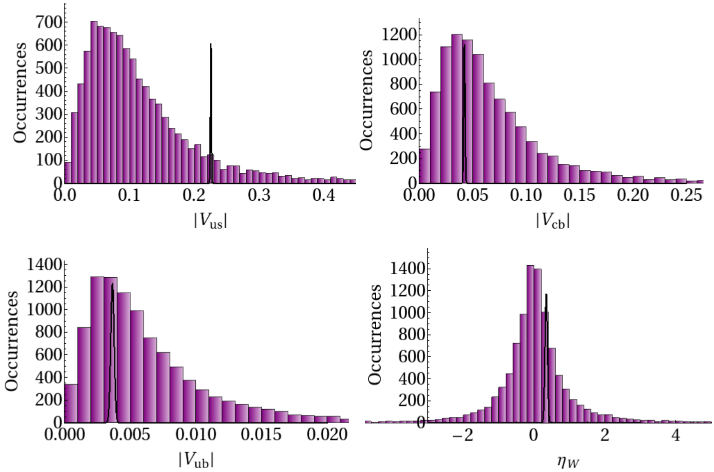


figure: Histograms showing the theoretical distributions of the CKM matrix observables according to anarchy hypothesis (purple) with fixed model parameters ϵ_i. Sample size taken to be 10^4. Black curves represent the experimental 1σ range.

Histogram Distributions in the Neutrino Sector

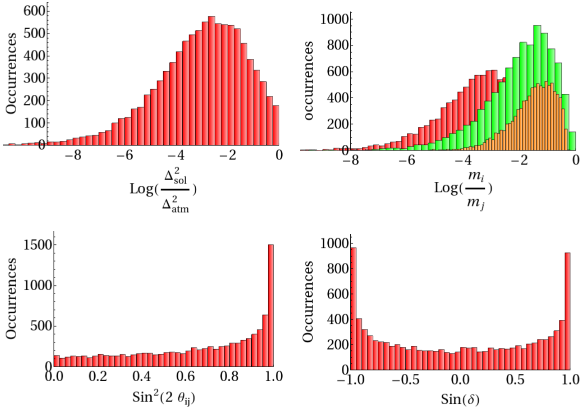


figure: Histogram distributions of $Log[\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}]$ (upper left); neutrino mass ratios $Log(m_{ij}) = Log(\frac{m_i}{m_j})$ [Red, green & orange histograms are for m_{13}, m_{12} & m_{23} respectively] (upper right); $sin(2\theta_{ij})^2$ (lower left) and $sin(\delta_{pmns})$ (lower right). Sample size taken to be 10^4.

Haba, Murayama: Phys.Rev. D63 (2001) 053010

Hall, Murayama, Weiner: Phys.Rev.Lett. 84 (2000) 2572-2575

Summary

- Random matrices do not have any hierarchy among the different entries rather of the order $\sim O(1)$.
- Hierarchy of Yukawa couplings among different generations are introduced only by the matrix H, which may have different origins.
- With only four model parameters, the charged fermion Yukawa coupling with strong hierarchy, the small quark mixing angles and large leptonic mixing angles can be reproduced.
- **Predictions**
- Assuming see-saw mechanism for neutrinos, model predicts mild mass hierarchy.
- Model prefers normal ordering of the neutrino mass spectrum.
- Model prefers order one CP violation in the leptonic sector.

Backup Slide-1: Explaining the Pattern of the Histogram Distributions

- Strong hierarchy in the up-quark sector nicely re-produced

$$y_u : y_c : y_t \sim \epsilon_1^2 : \epsilon_2^2 : \epsilon_3^2$$

- Model leads to

$$\frac{y_s}{y_b} \sim \frac{\epsilon_2}{\epsilon_3} \;\; \& \;\; \frac{y_\mu}{y_\tau} \sim \frac{\epsilon_2}{\epsilon_3}$$

  but at the GUT scale, $y_b \sim y_\tau$, which implies , $y_s \sim y_\mu$.
  So the histogram distributions of these two quantities are almost identical but observation dictates, $y_s \sim \frac{y_\mu}{4}$ (at the GUT scale).

- V_{us} has the tendency to lean toward the smaller value, but still the theoretical mean is only a factor of 2 less than observed value.
  model predicts $V_{us} \sim \frac{\epsilon_1}{\epsilon_2}$, at the same time,

$$\frac{\epsilon_1}{\epsilon_2} \sim \sqrt{\frac{y_u}{y_c}} \sim \frac{1}{20} \sim 0.05$$

Backup Slide-2:Models of Composite Fermions

10 and 1 representations in $SU(5)$ GUT model are composite states, they produce mass hierarchy which induces large mixing angles in the lepton sector. Masses of quark and lepton are produced from irrelevant operators suppressed by the Planck scale. Yukawa interactions which include composite states (10 and 1) are suppressed by a dimension-less parameter ϵ.

$$U \sim v \begin{bmatrix} O(1)\epsilon^4 & O(1)\epsilon^3 & O(1)\epsilon^2 \\ O(1)\epsilon^3 & O(1)\epsilon^2 & O(1)\epsilon^1 \\ O(1)\epsilon^2 & O(1)\epsilon^1 & O(1) \end{bmatrix} \qquad D \sim \bar{v} \begin{bmatrix} O(1)\epsilon^2 & O(1)\epsilon^2 & O(1)\epsilon^2 \\ O(1)\epsilon^1 & O(1)\epsilon^1 & O(1)\epsilon^1 \\ O(1) & O(1) & O(1) \end{bmatrix}$$

$$N \sim v \begin{bmatrix} O(1)\epsilon^2 & O(1)\epsilon^1 & O(1) \\ O(1)\epsilon^2 & O(1)\epsilon^1 & O(1) \\ O(1)\epsilon^2 & O(1)\epsilon^1 & O(1) \end{bmatrix} \qquad L \sim \bar{v} \begin{bmatrix} O(1)\epsilon^2 & O(1)\epsilon^1 & O(1) \\ O(1)\epsilon^2 & O(1)\epsilon^1 & O(1) \\ O(1)\epsilon^2 & O(1)\epsilon^1 & O(1) \end{bmatrix}$$

$$M_R \sim M_p \begin{bmatrix} O(1)\epsilon^4 & O(1)\epsilon^3 & O(1)\epsilon^2 \\ O(1)\epsilon^3 & O(1)\epsilon^2 & O(1)\epsilon^1 \\ O(1)\epsilon^2 & O(1)\epsilon^1 & O(1) \end{bmatrix} \qquad M_\nu \sim \frac{v^2}{M_p} \begin{bmatrix} O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \end{bmatrix}$$

Nelson, Strassler: Phys.Rev. D56 (1997) 4226-4237
Haba: Phys. Rev. D 59, 035011 (1999)

Backup Slide-3:

Flavor charge assignment is:

$$\begin{array}{ccc} 10_1(+2) & 10_2(+1) & 10_3(0) \\ 5^*_1(0) & 5^*_2(0) & 5^*_3(0) \\ 1_1(0) & 1_2(0) & 1_3(0) \end{array} \quad || \; 10 = (Q, u^c, e^c),\ 5^* = (L, d^c)\ \&\ 1 = N^c$$

Hierarchy in the fermion masses and mixings are completely due to 10-plets. $U(1)$ flavor symmetry is broken by $\epsilon(-1) \sim \lambda^2$ leads to following Yukawa couplings:

$$D \sim \begin{bmatrix} O(1)\epsilon^2 & O(1)\epsilon^2 & O(1)\epsilon^2 \\ O(1)\epsilon^1 & O(1)\epsilon^1 & O(1)\epsilon^1 \\ O(1) & O(1) & O(1) \end{bmatrix}$$

$$L \sim \begin{bmatrix} O(1)\epsilon^2 & O(1)\epsilon^1 & O(1) \\ O(1)\epsilon^2 & O(1)\epsilon^1 & O(1) \\ O(1)\epsilon^2 & O(1)\epsilon^1 & O(1) \end{bmatrix}$$

$$U \sim \begin{bmatrix} O(1)\epsilon^4 & O(1)\epsilon^3 & O(1)\epsilon^2 \\ O(1)\epsilon^3 & O(1)\epsilon^2 & O(1)\epsilon^1 \\ O(1)\epsilon^2 & O(1)\epsilon^1 & O(1) \end{bmatrix}$$

$$N \sim \begin{bmatrix} O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \end{bmatrix}$$