Diquark Bound States at far beyond ladder truncation

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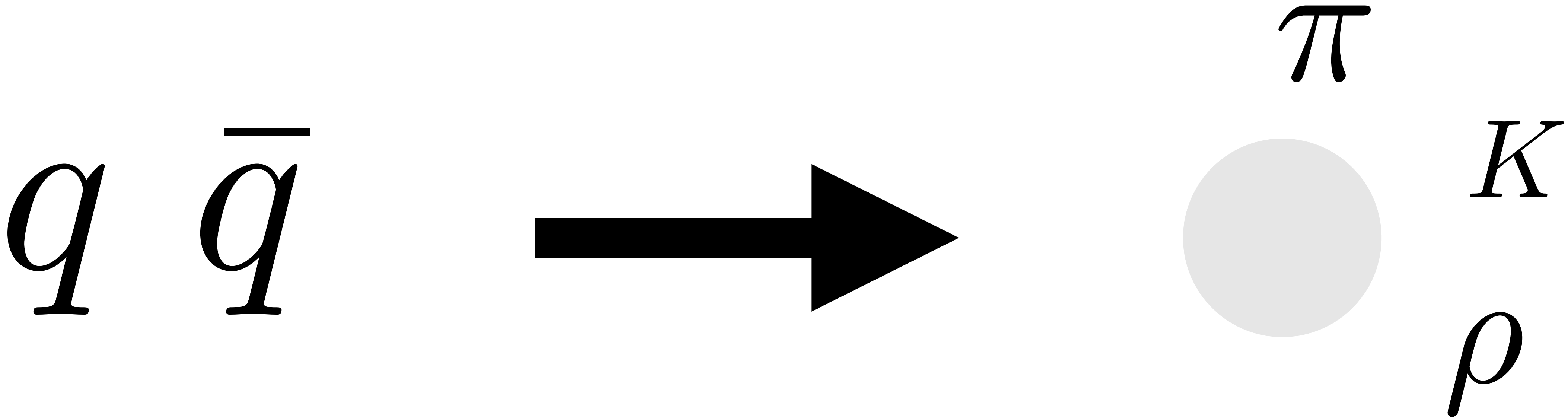
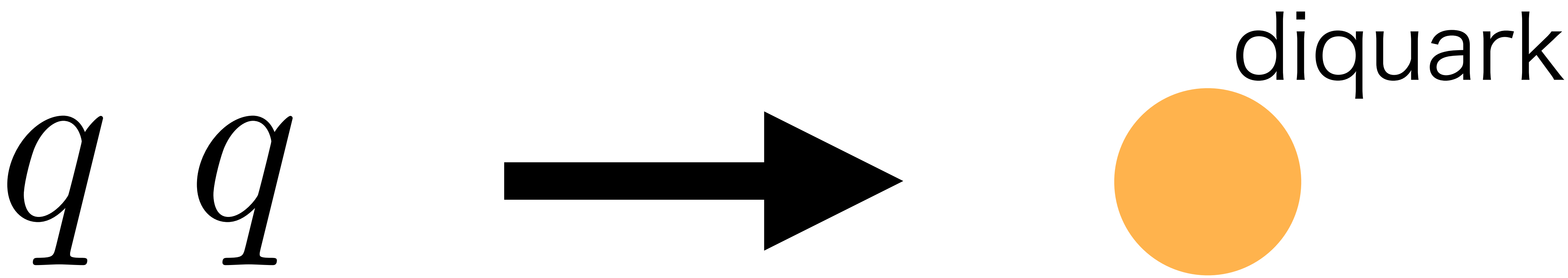
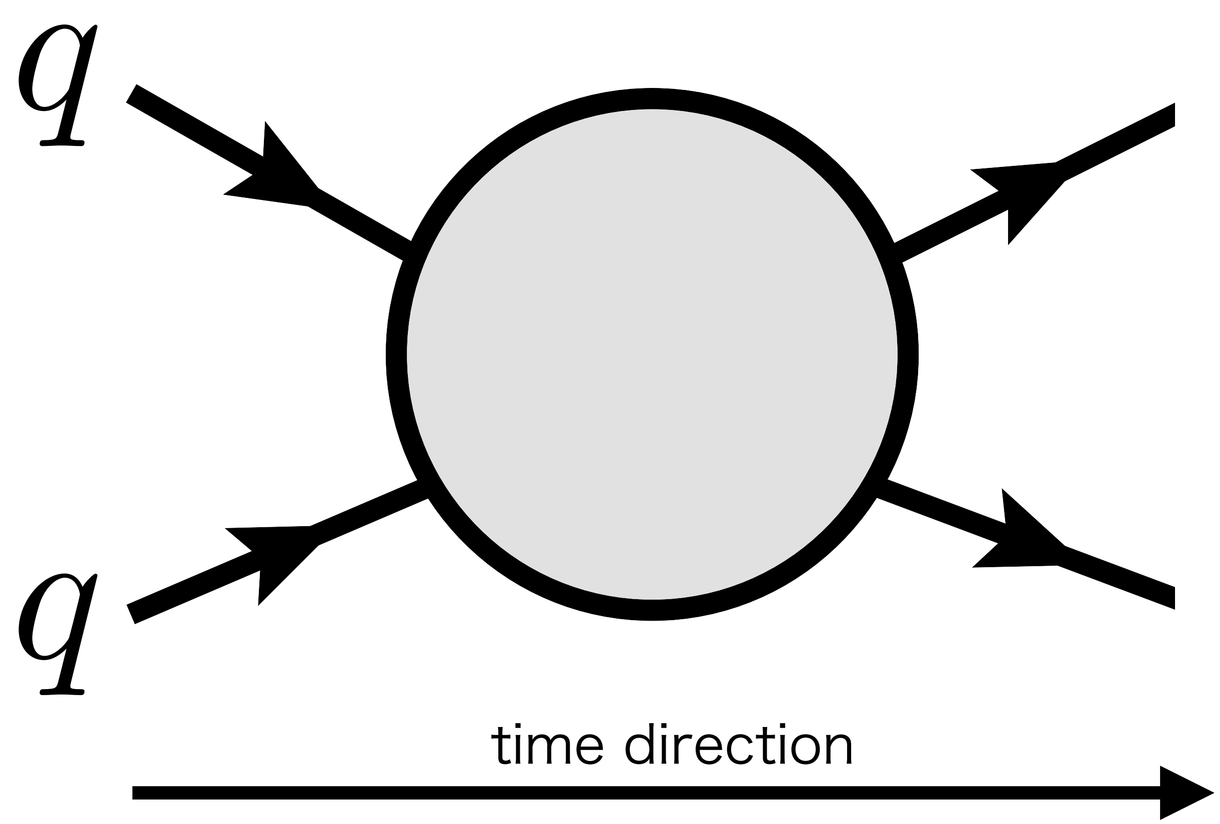


$$3 \otimes \bar{3} \to 1$$



$$3 \otimes 3 \to \bar{3} \oplus 6$$

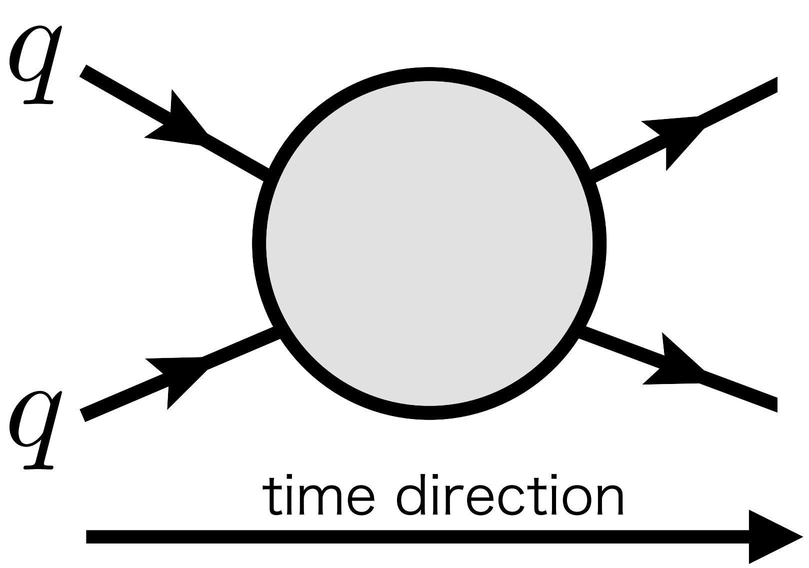
Our question



Does this amplitude have some singularities corresponding to bound states?

Previous works

[A.Bender, C.D.Roberts, L.V.Smekal, 1996]
[G.Hellstern, R.Alkofer, H.Reinhardt, 1997]
[A.Bender, W.Detmold, C.D.Roberts, A.W.Thomas, 2002]



Diquark singularity disappears in the author's model with their truncation.

- We think their argument is not conclusive.

(and show that the singularity appears in the same model with more reasonable truncation)

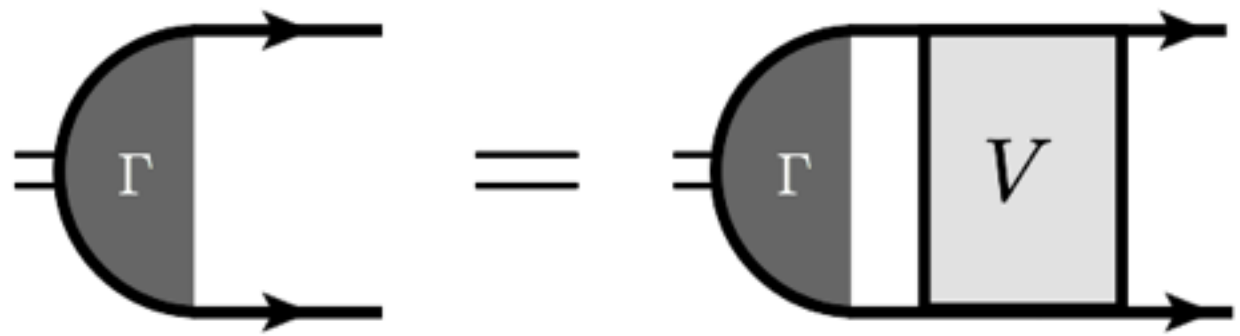
The method

(Bethe-Salpeter eq.)

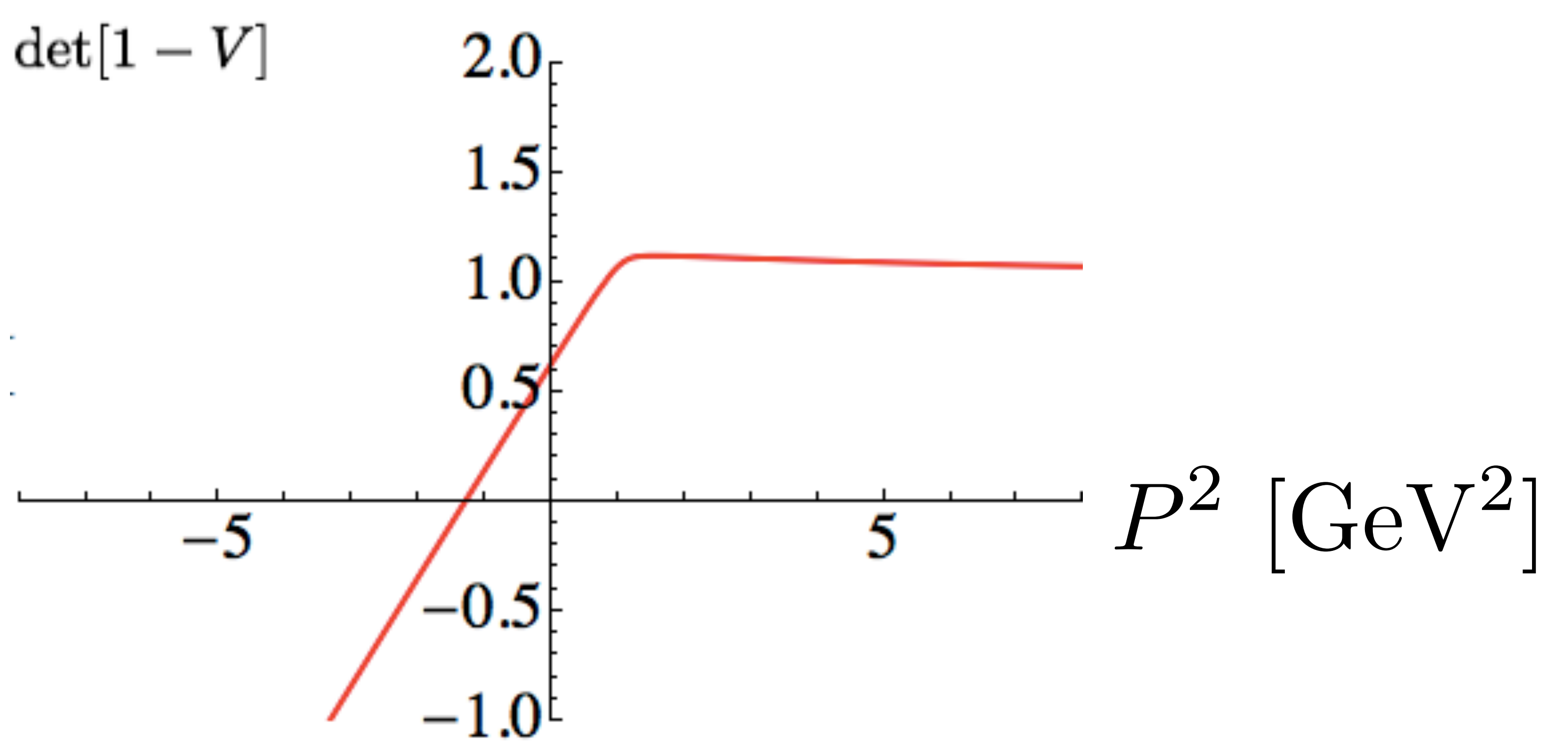
contents

- The method
- Previous works
- Our study

Bethe-Salpeter equation



eigen value condition $\det\left[\mathbb{1} - V(P^2)\right]\Big|_{P^2=-M^2} = 0$



an example of the case where a bound state exists

Previous works

[A.Bender, C.D.Roberts, L.V.Smekal, 1996]

contents

- The method
- Previous works
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Model

[H.J.Munczek, A.M.Nemirovsky, 1983]

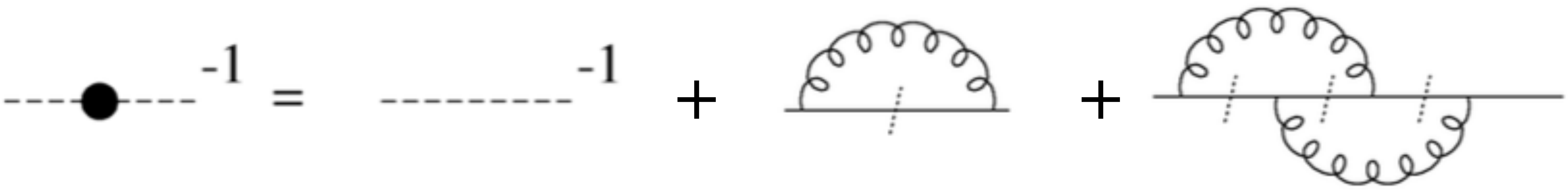
- gluon propagator $$\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)(2\pi)^4\delta^4(q)G$$
- quark-gluon vertex $$\mathrm{i}\gamma^\mu t^a$$
- quark propagator $$-\mathrm{i}\not{p}\mathcal{A}(p^2) + \mathcal{B}(p^2)$$

determined by self-consistency eq.

(Dyson-Schwinger eq.)

Dyson-Schwinger eq. for quark

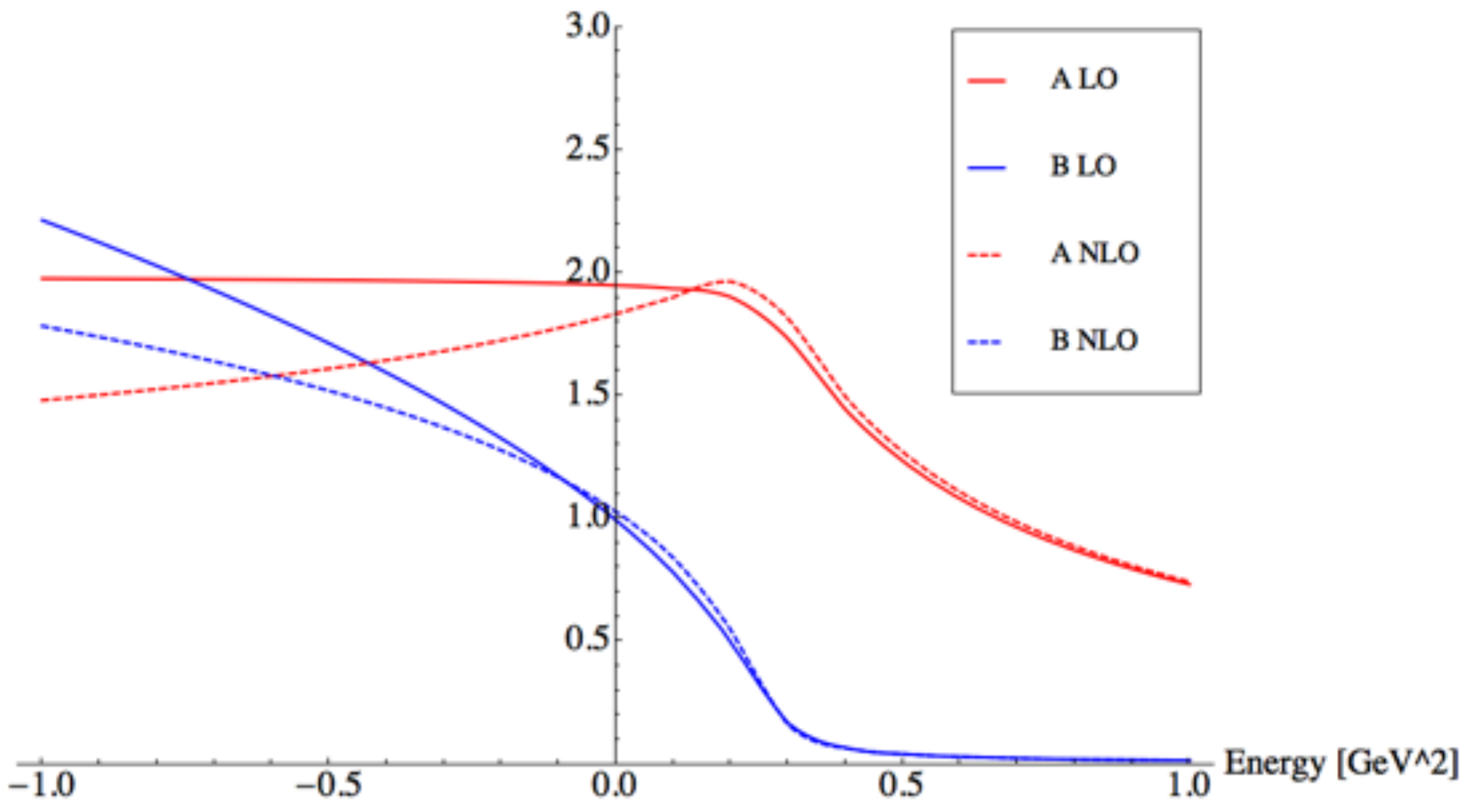
$$S^{-1}(p) = i\gamma \cdot p + m_q + G\,\gamma_\mu S(p)\gamma_\mu + \frac{1}{8}G^2\,\gamma_\mu S(p)\gamma_\nu S(p)\gamma_\mu S(p)\gamma_\nu$$



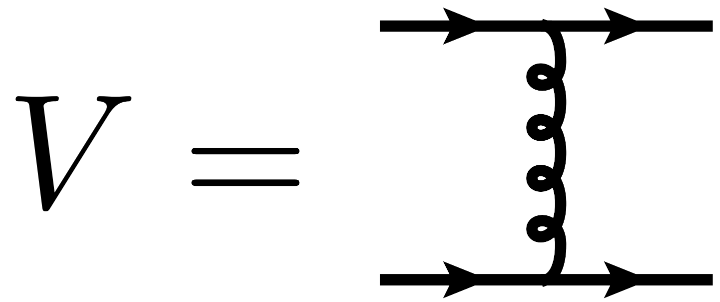
its solution

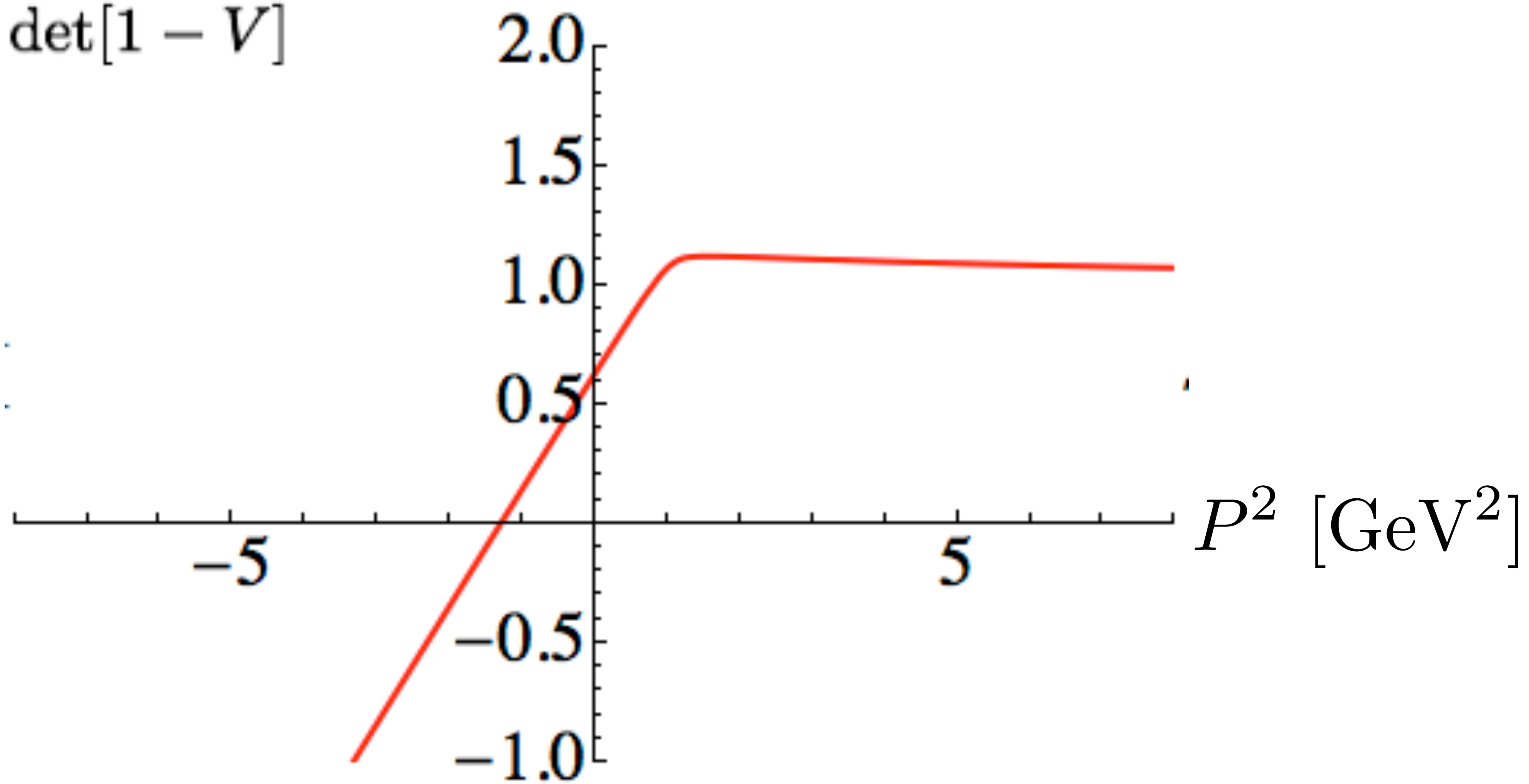
$$S(p) = -i\not{p}\mathcal{A}(p^2) + \mathcal{B}(p^2)$$

w/ $m_q = 13\ \mathrm{MeV}, G = 0.25\ \mathrm{GeV}^2$
fitted by meson masses



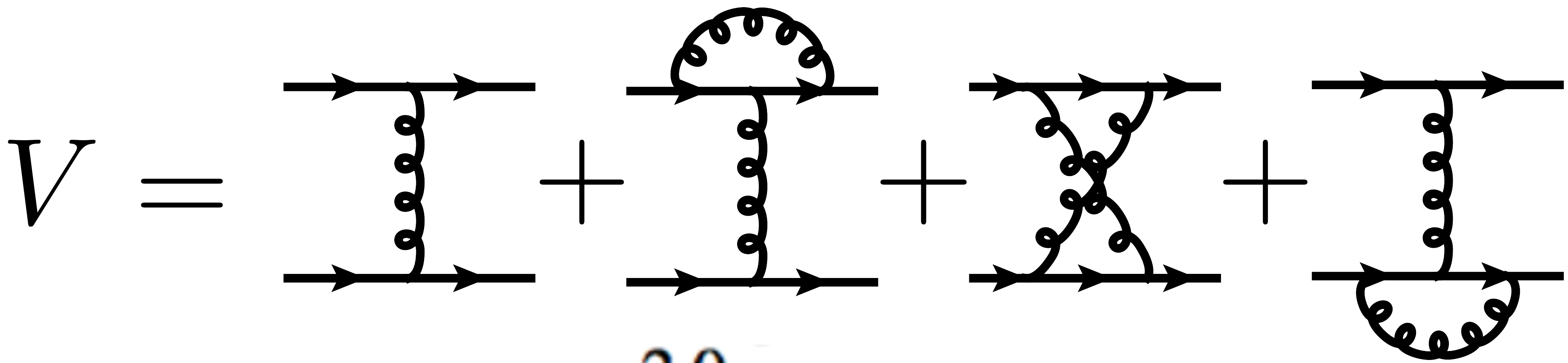
Diquark forms bound state at LO.

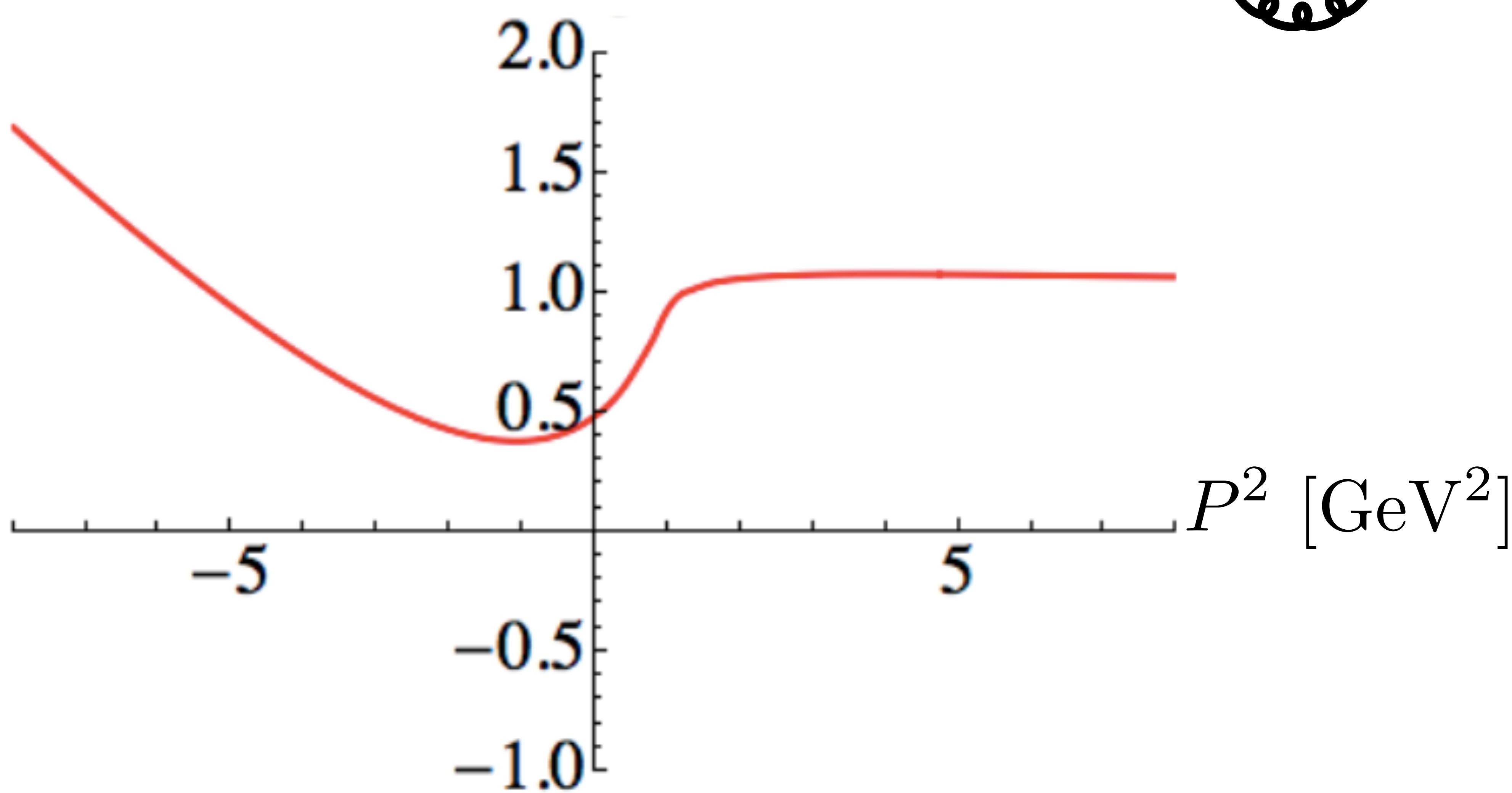




There is a intersection.

Diquark does not form bound state at NLO.





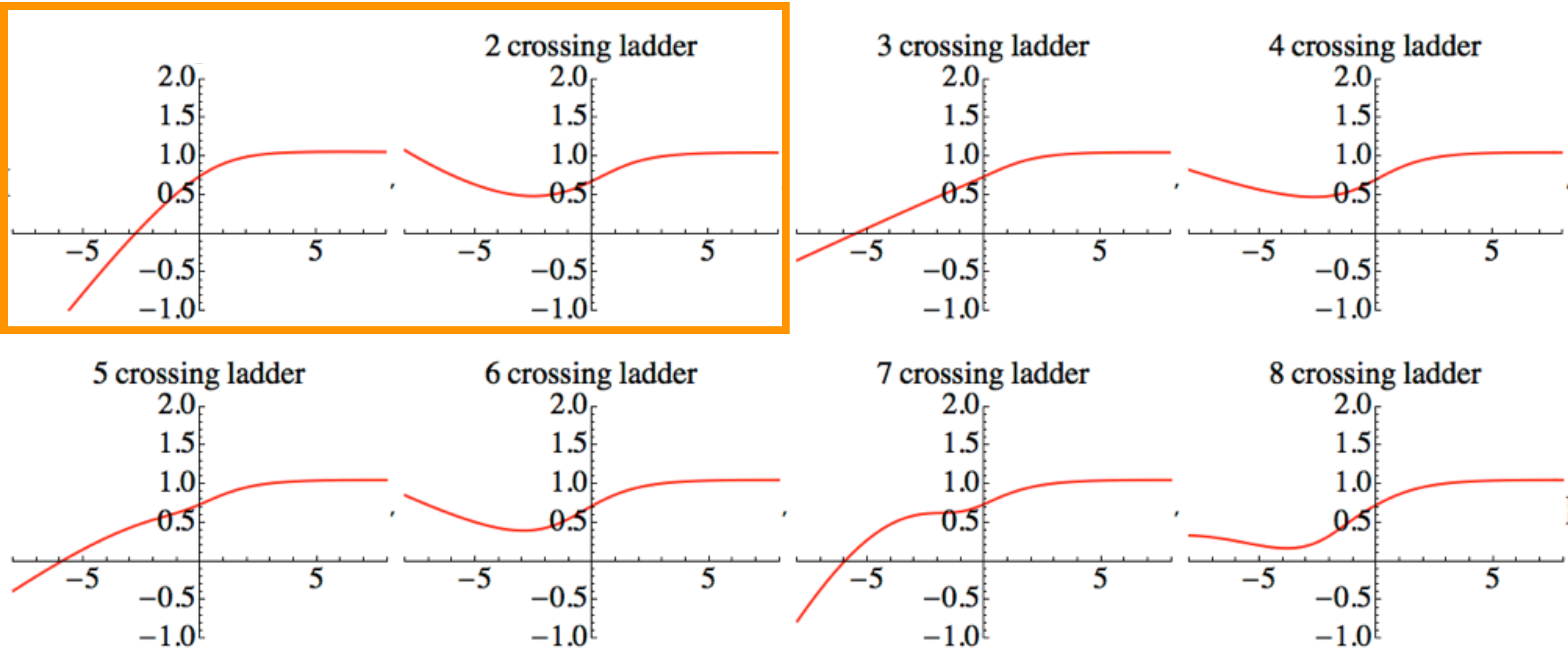
There is no intersection.

Our Study

contents

- The method
- Previous works
- Our study

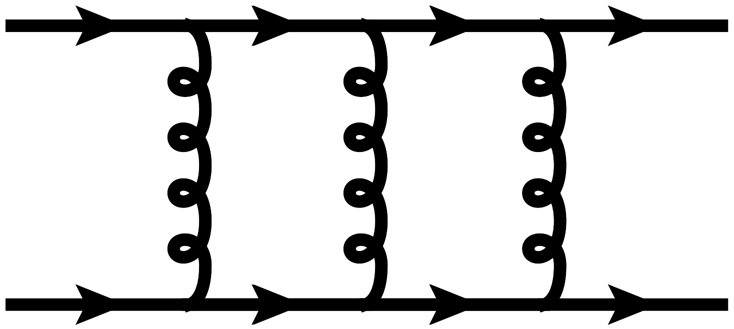
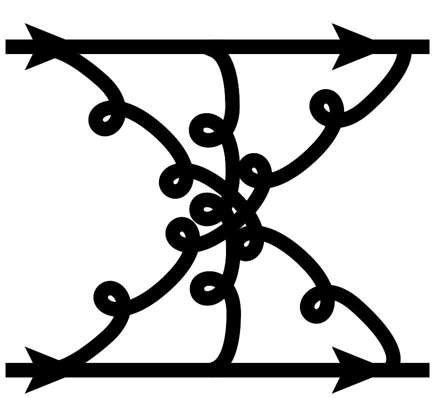
We observe the flipping repeatedly!



DSE-BSE matching is done as the previous works.

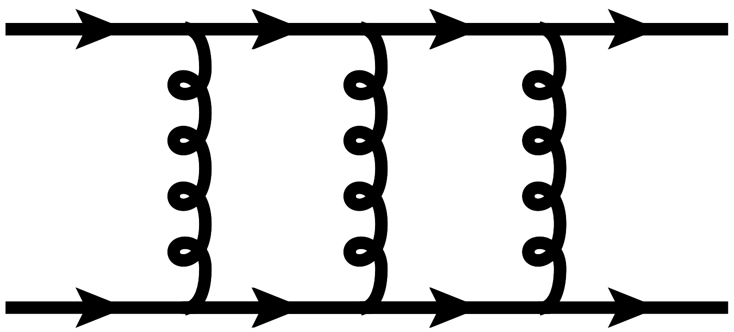
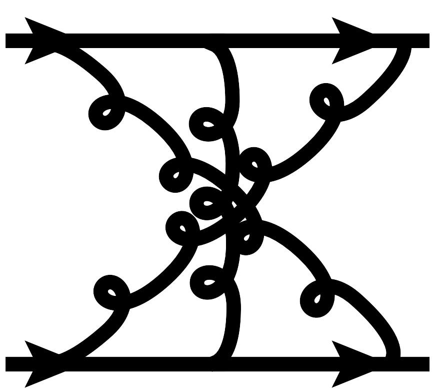
Coupling expansion seems to be not reasonable.

Color factor changes drastically.

$\mathrm{SU}(N)$	meson	diquark
ladder 	$t^{a_1} \cdots t^{a_n} t^{a_n} \cdots t^{a_1}$ $\sim N^n$	$t^{a_1} \cdots t^{a_n} \epsilon t^{\mathrm{T} a_n} \cdots t^{\mathrm{T} a_1}$ $\sim (-1)^n$
completely crossed ladder 	$t^{a_1} \cdots t^{a_n} t^{a_1} \cdots t^{a_n}$ $\sim N$ (for $n =$ odd) ~ -1 (for $n =$ even)	$t^{a_1} \cdots t^{a_n} \epsilon t^{\mathrm{T} a_1} \cdots t^{\mathrm{T} a_n}$ $\sim -N^{n-1}$

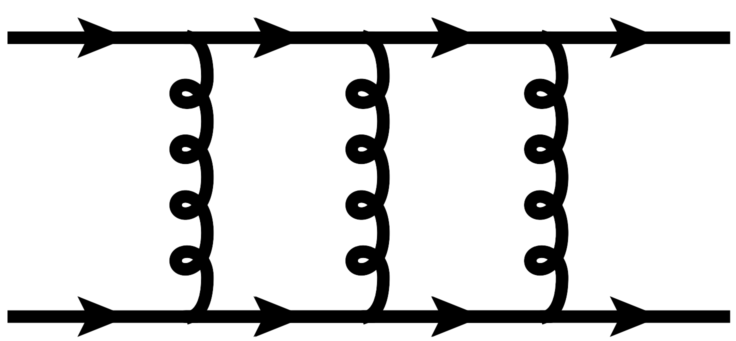
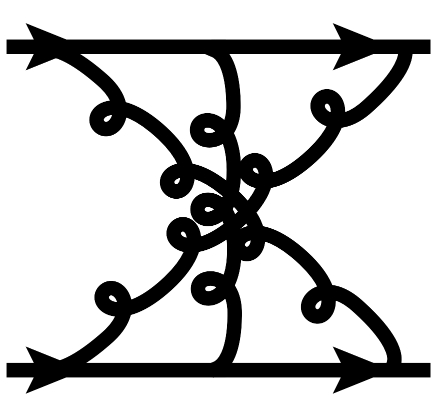
We obtain the exact formulae.

Color factor changes drastically.

$\mathrm{SU}(N)$	meson
ladder 	$t^{a_1}\cdots t^{a_n}t^{a_n}\cdots t^{a_1}$ $\sim N^n$
completely crossed ladder 	$t^{a_1}\cdots t^{a_n}t^{a_1}\cdots t^{a_n}$ $\sim N$ (for $n=$ odd) ~ -1 (for $n=$ even)

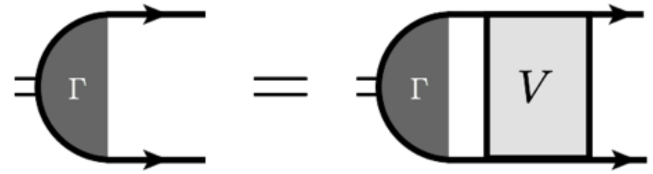
Resummation of ladder diagrams is reasonable for mason case, and crossed diagrams become less and less important in the sense of color factor magnitude.

Color factor changes drastically.

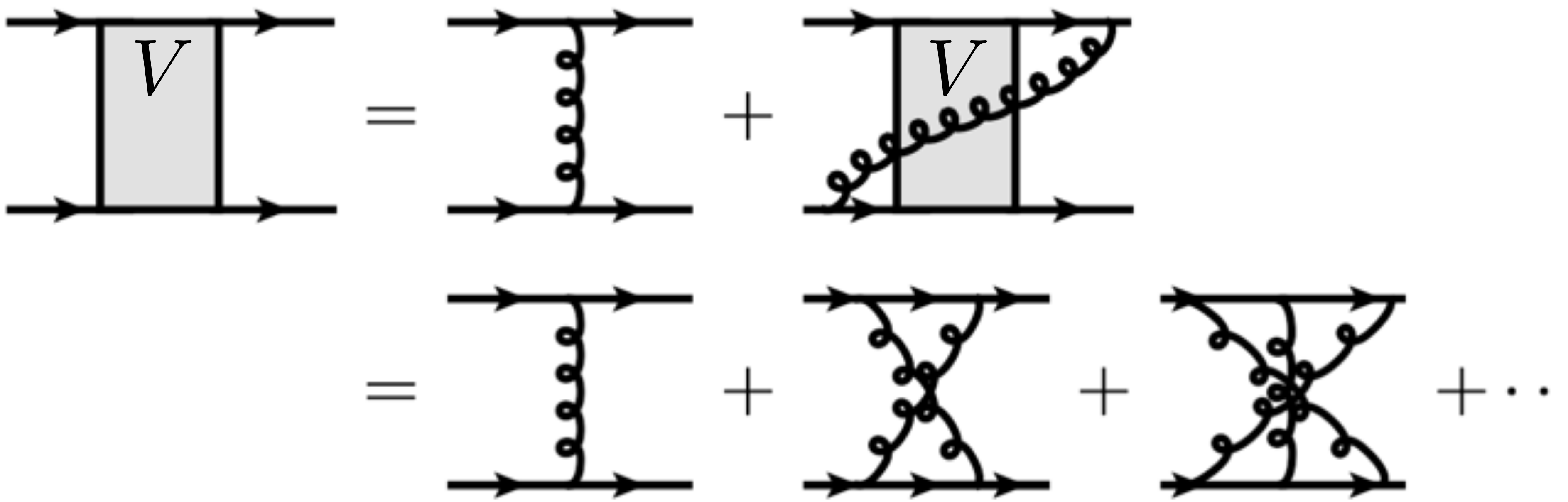
$\mathrm{SU}(N)$		diquark
ladder 		$t^{a_1}\cdots t^{a_n}\epsilon t^{\mathrm{T}a_n}\cdots t^{\mathrm{T}a_1}$ $\sim(-1)^n$
completely crossed ladder 	Crossed diagrams are more and more important.	$t^{a_1}\cdots t^{a_n}\epsilon t^{\mathrm{T}a_1}\cdots t^{\mathrm{T}a_n}$ $\sim -N^{n-1}$

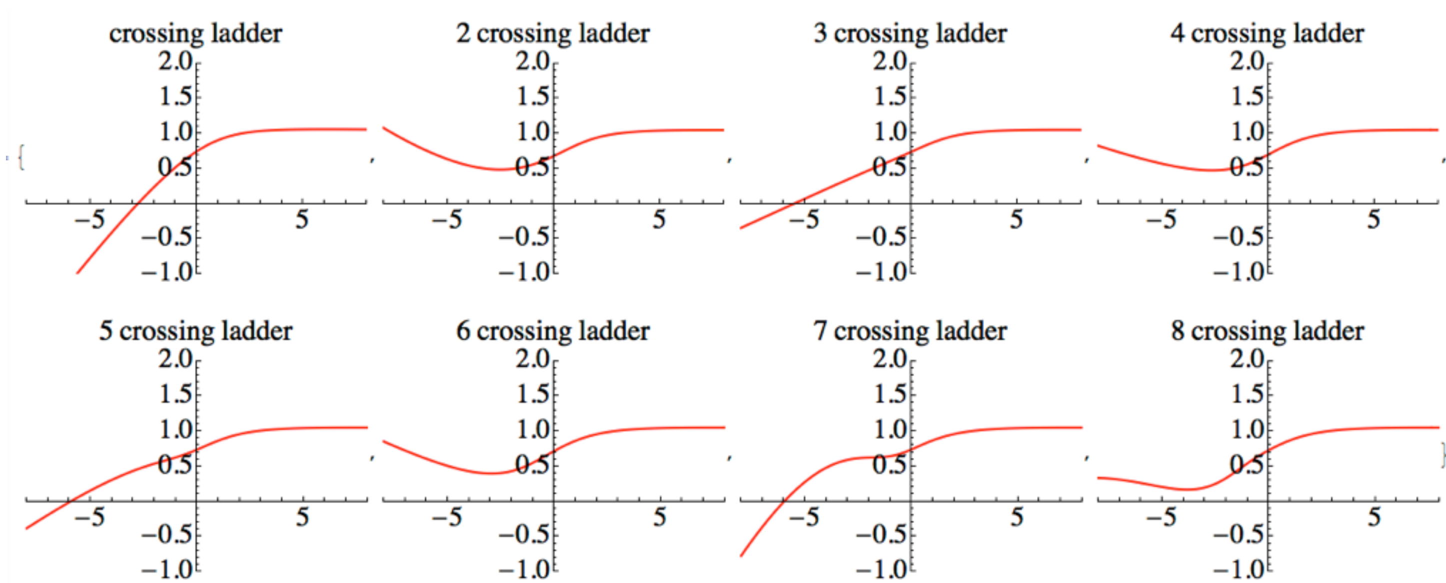
(Sign flipping is canceled with other “diquark factor”, so do not care here.)

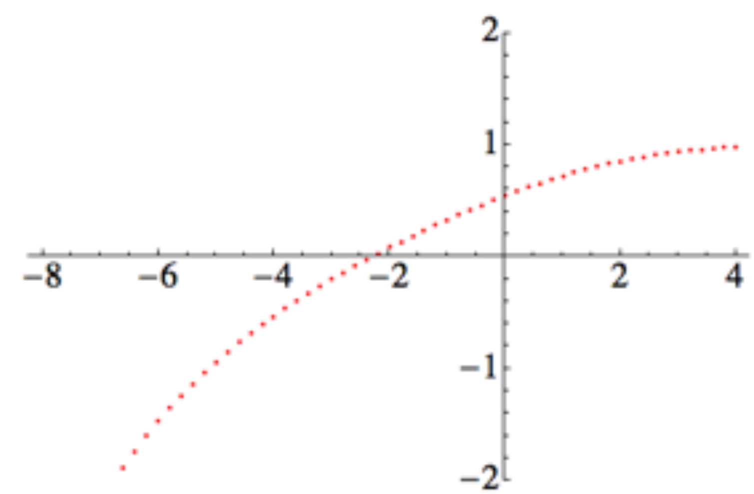
We have done “completely crossed diagram” resummation.

BSE 

together with another resummation







It is bound!

Summary

- We employ functional method with a model of “delta function gluon propagator”.
- We perform a resummation of the completely crossed ladder diagrams, which is consistent in light of 1/N counting.
- We found diquark bound states.
- However, the mass prediction is not reliable at this stage.

The basic formula is

$$(t^a)_{ij}(t^a)_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{kj} - \frac{1}{N}\delta_{ij}\delta_{kl}\right),$$

where $(t^a)_{ij}$ is the generator of SU(N) in fundamental representation. In the meson the ladder diagrams have a factor

$$t^{a_1}t^{a_2}\cdots t^{a_{n-1}}t^{a_n}t^{a_n}t^{a_{n-1}}\cdots t^{a_2}t^{a_1} = \frac{1}{2^n}\left(N - \frac{1}{N}\right)^n \equiv c_n^{(M)},$$

whereas the completely-crossed ladder diagrams have

$$t^{a_1}t^{a_2}\cdots t^{a_n}t^{a_1}t^{a_2}\cdots t^{a_n} = \frac{1}{2^{n+1}}\left(N - \frac{1}{N}\right)\left[\left(-1 - \frac{1}{N}\right)^{n-1} + \left(1 - \frac{1}{N}\right)^{n-1}\right] \equiv d_n^{(M)}.$$

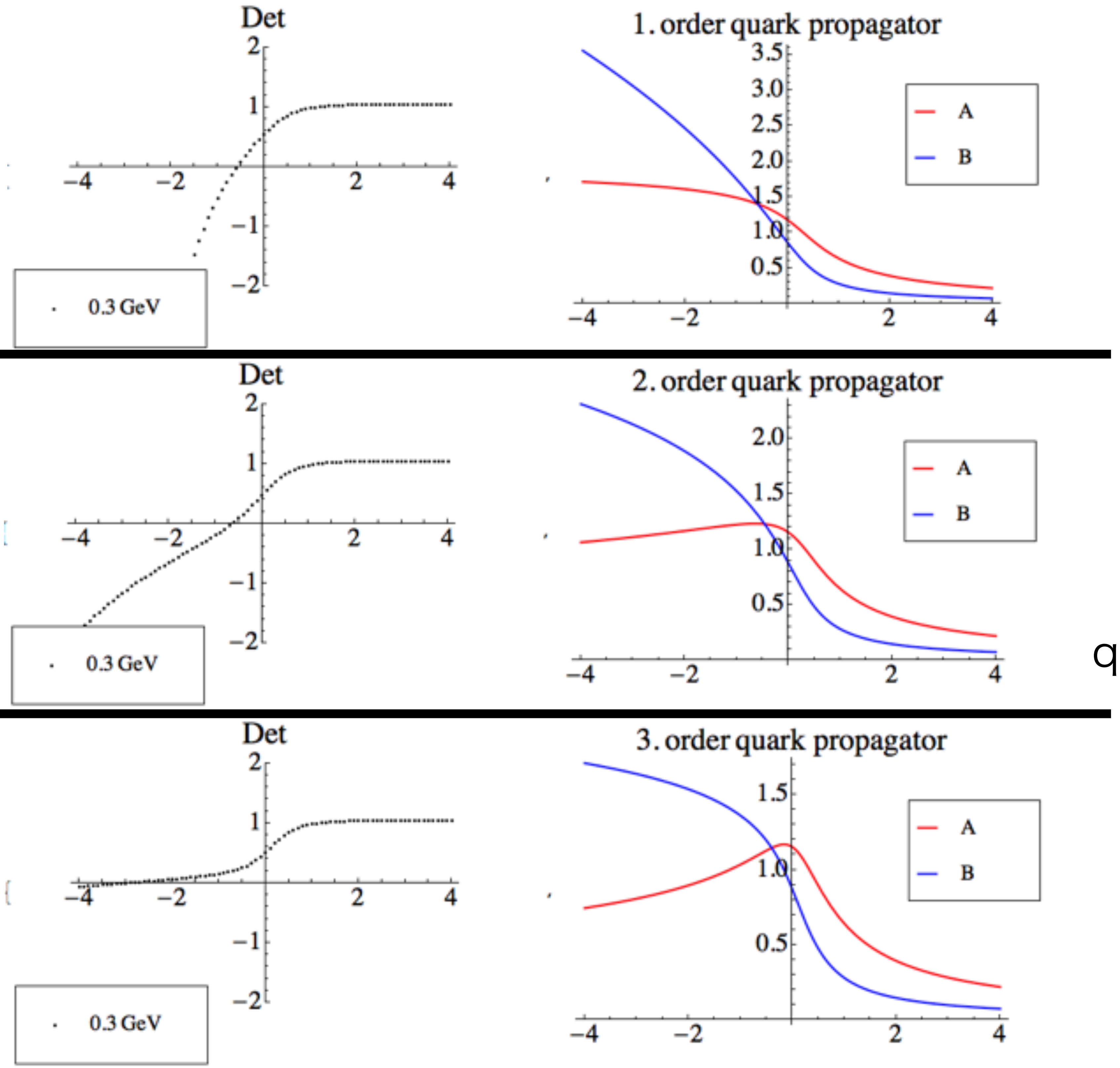
The ladder diagrams in the diquark channel have a factor

$$t^{a_1}t^{a_2}\cdots t^{a_{n-1}}t^{a_n}\epsilon t^{Ta_n}t^{Ta_{n-1}}\cdots t^{Ta_2}t^{Ta_1} = \frac{1}{2^n}\left(-1 - \frac{1}{N}\right)^n\epsilon \equiv c_n^{(D)}\epsilon,$$

whereas the completely-crossed ladder diagrams have

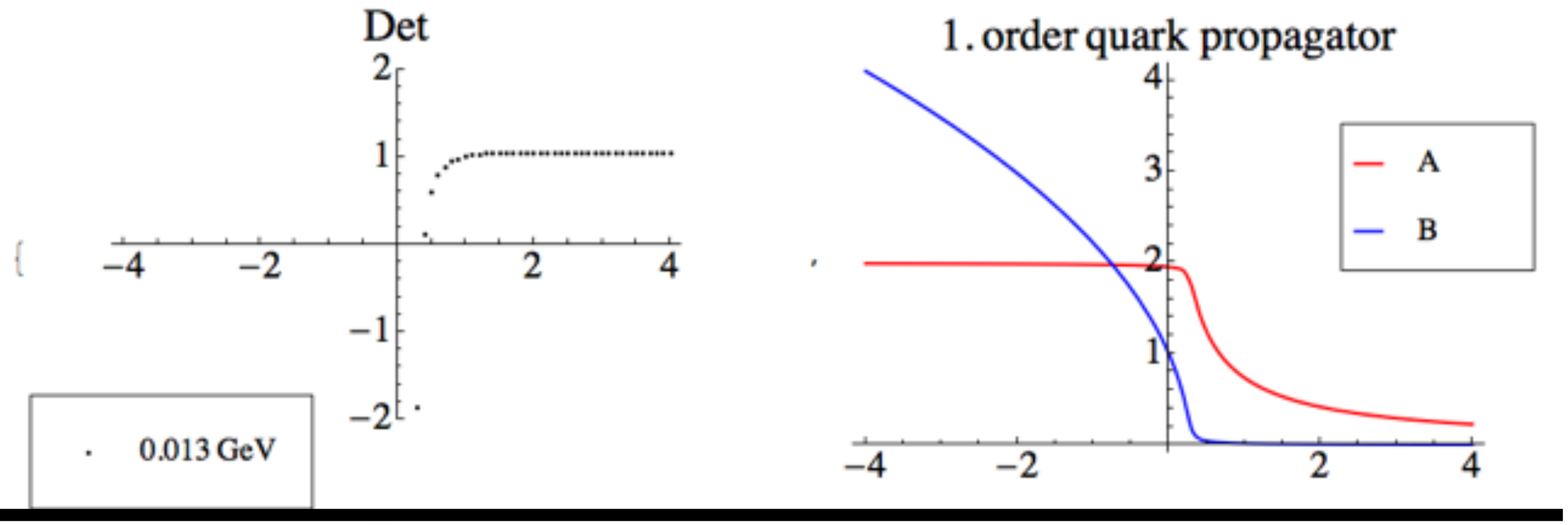
$$t^{a_1}t^{a_2}\cdots t^{a_n}\epsilon t^{Ta_1}t^{Ta_2}\cdots t^{Ta_n} = \frac{1}{2^n}\left[(-1)^n(N+1)\frac{1}{N^{n+1}} - \frac{1}{N}\left(N - \frac{1}{N}\right)^n\right]\epsilon \equiv d_n^{(D)}\epsilon.$$

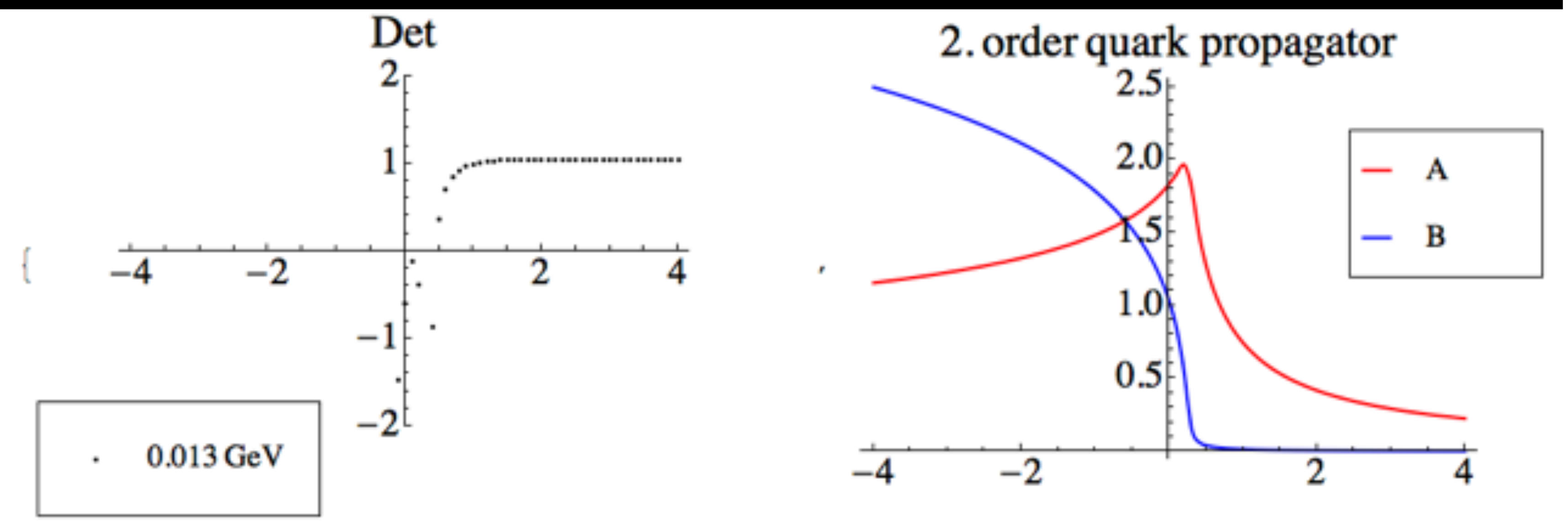
It is bound

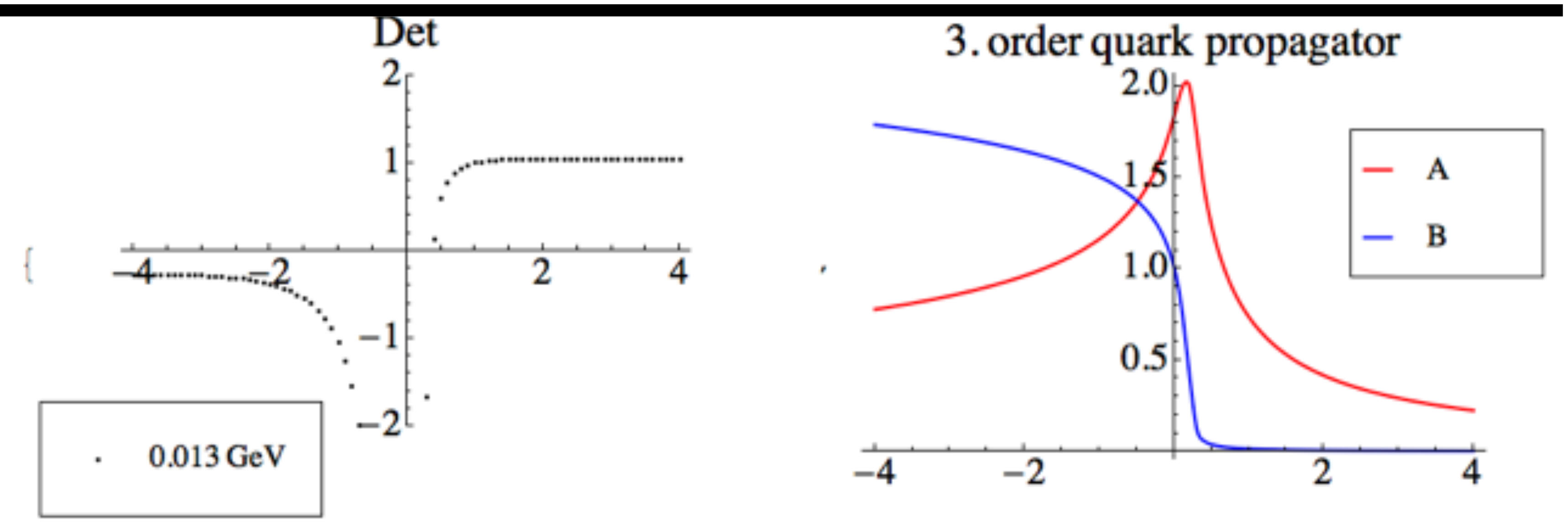


When we change the truncation scheme for the quark DSE, qualitative feature does not change although quantitative feature changes.

There are unpleasant space like poles.







There are some interpretations:

i) There is no light diquark.

ii) The gluon model is bad.

iii) The truncation scheme is bad.