W tagging using jet functions

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Work in progress with Yang Bai and Zhenyu Han

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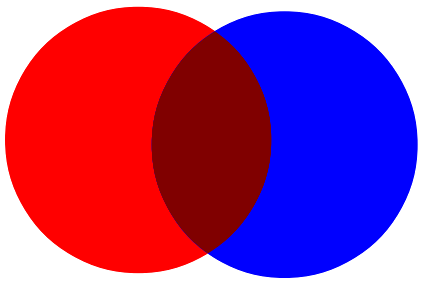
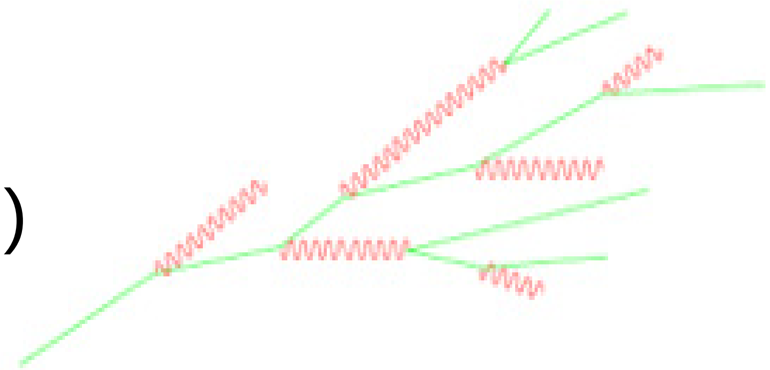
Jet Definitions

- Sterman-Weinberg (1977)
- Cone Algorithm
  - Stable Cone
  - Split and Merge



- Clustering Algorithm
  - Pair-Wise Merging
  - Undo Shower (k_t algorithm)
  - Nearest Neighbor



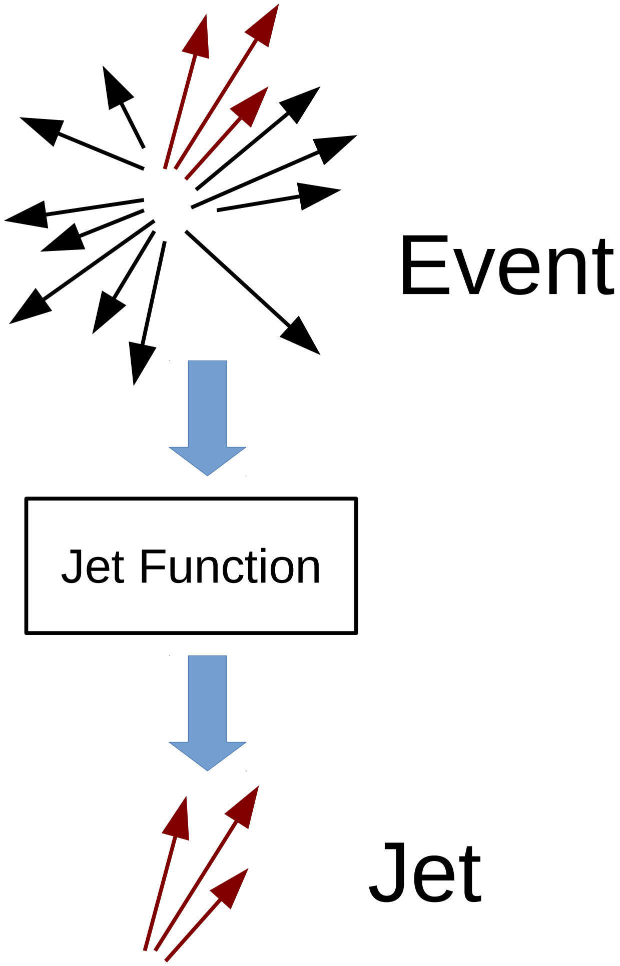
- No time to review all of them...

Jet Function

- Jets Maximize the Jet Function
- Function of Jet 4-momentum $J(P)$
  - Increases with increasing energy
  - Decreases with increasing mass
- Concrete Example:

$$J = E - \beta \frac{m^2}{E}, \beta \geq 0$$

- Cone Jet: $\sin\theta \leq \sqrt{\frac{1}{\beta}}$



H. Georgi, arXiv:1408.1161

Generalization (for hadron collider)

Y.Bai, Z. Han, RL, JHEP 3 (2015) 102

- New parameter $\alpha, (0 < \alpha < 2)$

$$J_{E_T^\alpha} = E_T^\alpha \left(1 - \beta \frac{m^2}{E_T^2} \right) \qquad E_T^2 = p_T^2 + m^2$$

- Boundary

$$\frac{1}{|p||P|} \left(P_x p_x + P_y p_y + \kappa P_z p_z \right) = \frac{\kappa}{v}$$

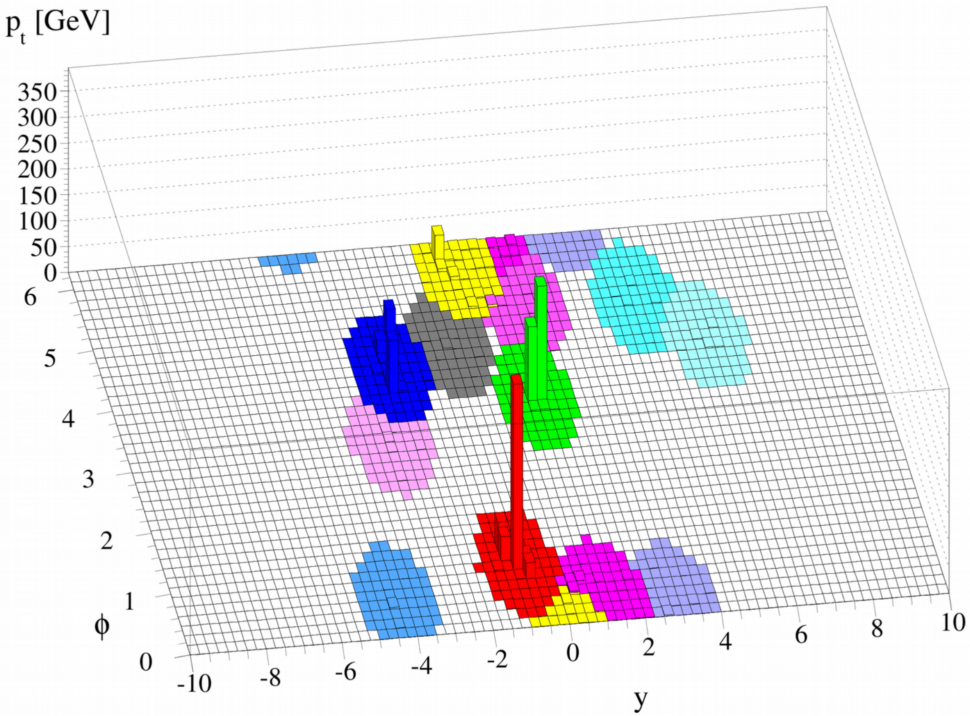
$$\kappa = 1 - \frac{\alpha}{2\beta} + \frac{\alpha - 2}{2} \frac{m^2}{E_T^2}$$

Still a cone

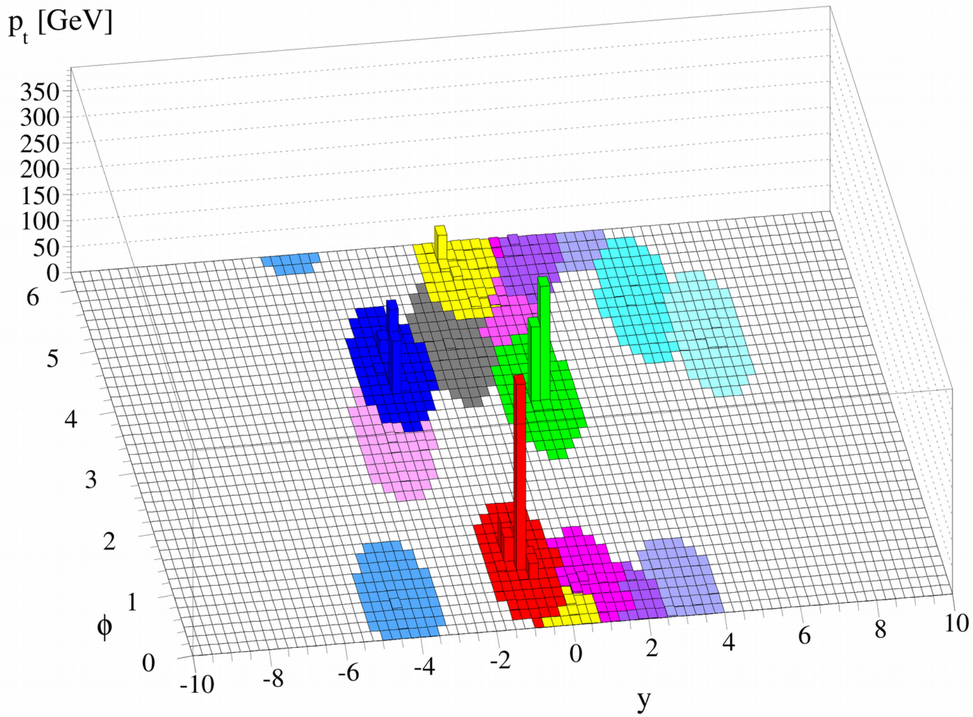
- Center: $\left(P_x, P_y, \kappa P_z \right)$ $\qquad \cos\theta' = \frac{\kappa}{v\sqrt{1 - (1 - \kappa^2)\cos^2\theta_Z}}$

Comparison: Shape

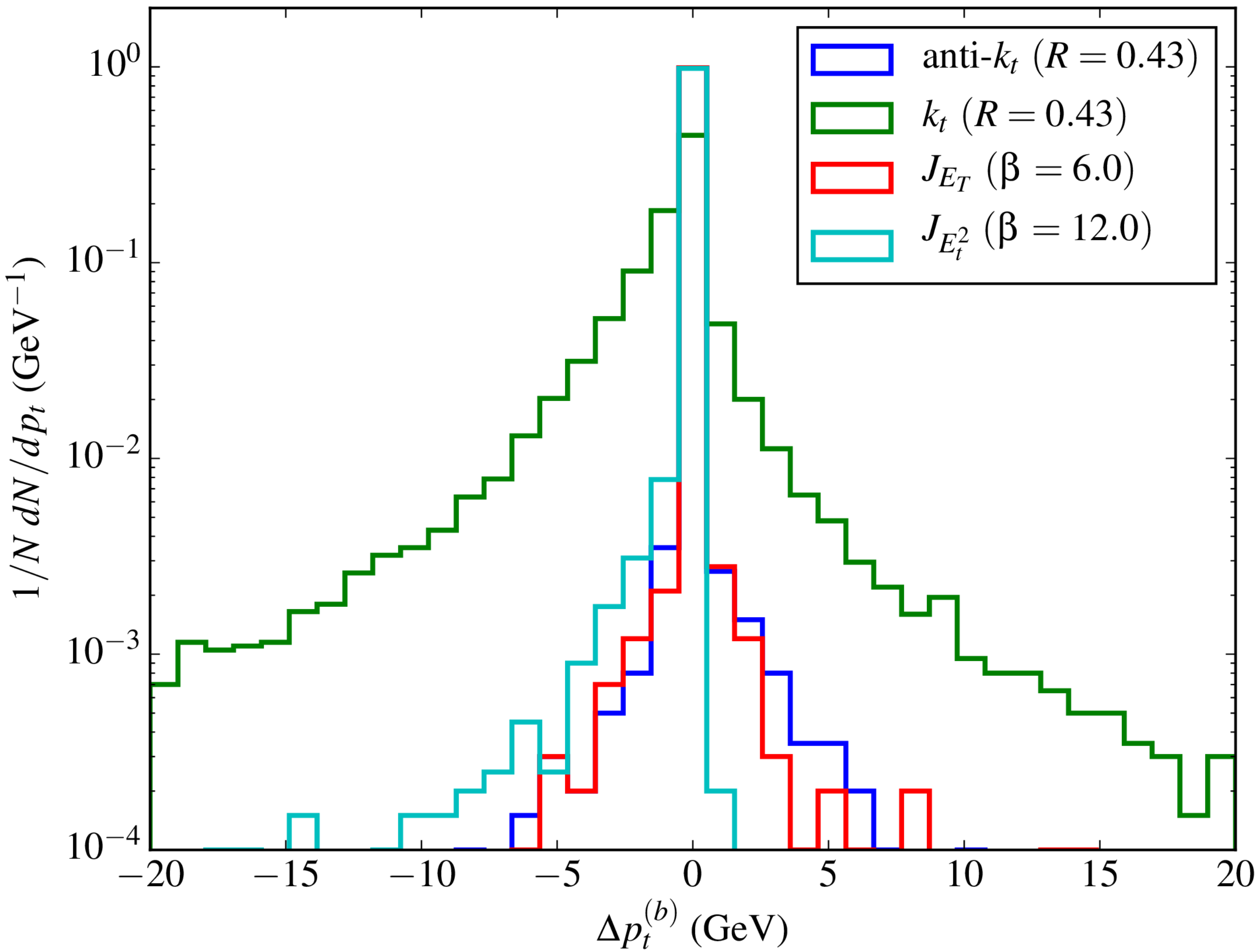
$J_{E_T} : \beta = 1.3$



$\text{Anti}-k_T : R = 1.0$

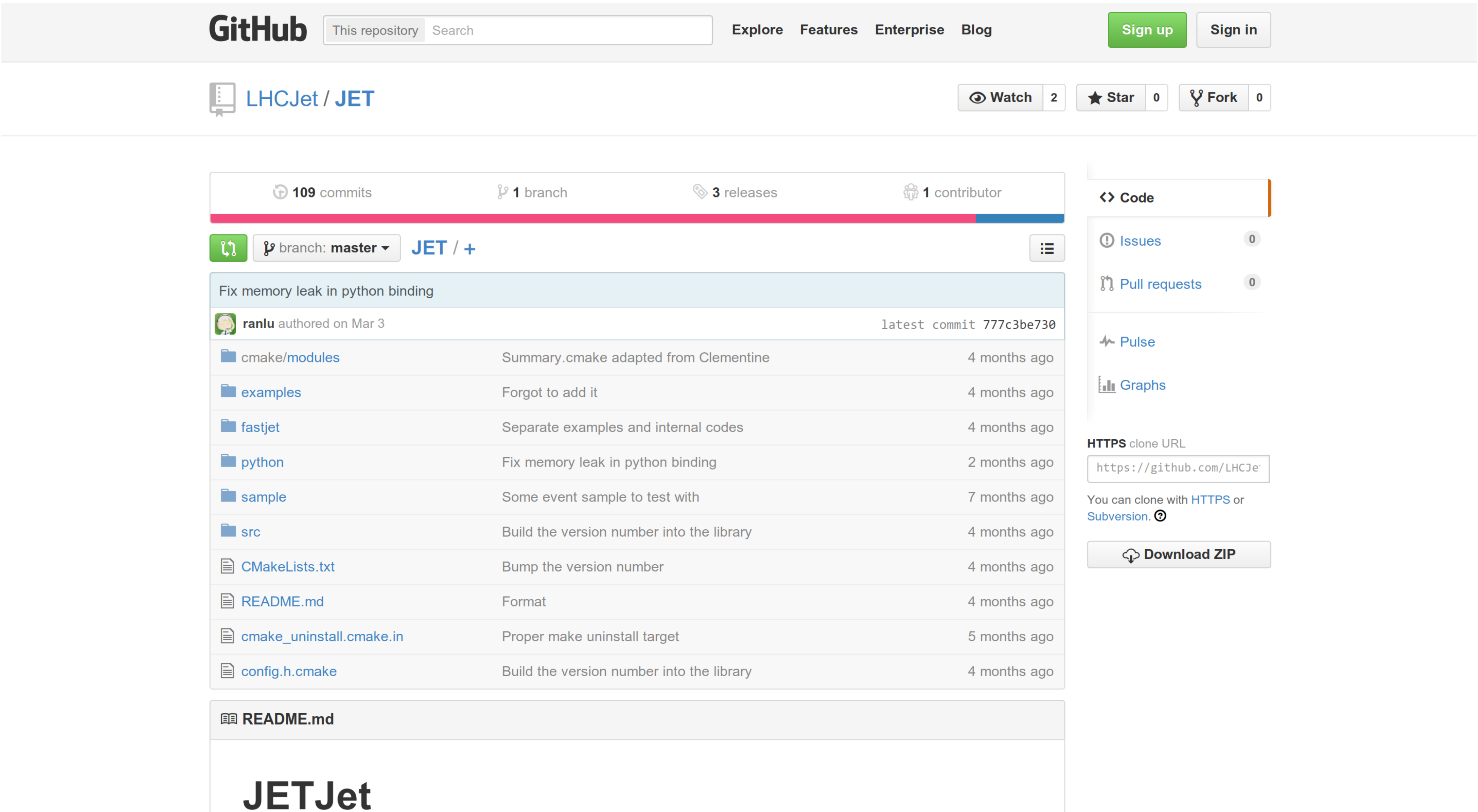


Comparison: Backreaction



Implementation

https://github.com/LHCJet/JET



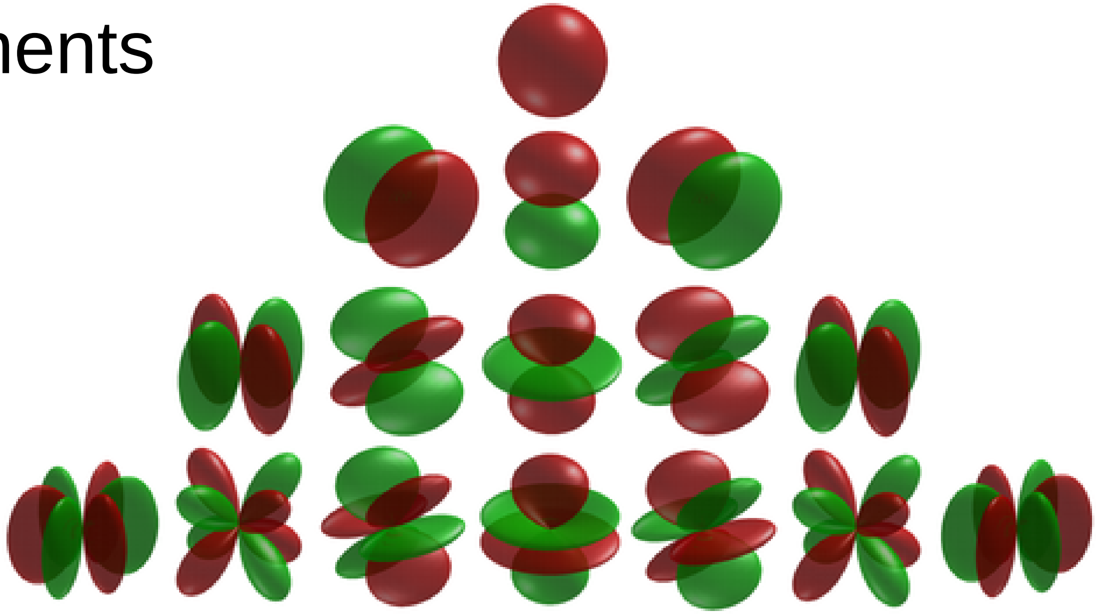
W tagging

- Many useful observables
  - (filtered/trimmed/pruned) Jet Mass
  - Mass Drop
  - N-subjettiness
  - Color Connection
  - Jet Charge
  - Energy Correlation Function
  - Planar Flow
  - Q-jet
- Multivariate Analysis (BDT)

Y. Cui, Z. Han and M. Schwartz Phys.Rev. D83, 074023
CMS Collaboration, JHEP 12 (2014), 017

Characteristic Function of W

- $J(P)$ is not enough
- 4-momentum: mass and velocity (point particle)
- “Shape” of the object
- Event/Jet shape variables
  - Fox-Wolfram Moments



Fox-Wolfram Moment

- General definition:

$$H_n = \sum_{i,j} \frac{|p_i||p_j|}{E_J^2} P_n(\cos\theta_{i,j})$$

- Examples:

$$H_0 = \sum_{i,j} \frac{|p_i||p_j|}{E_J^2} = 1$$

$$H_1 = \sum_{i,j} \frac{|p_i||p_j|}{E_J^2} \cos\theta_{i,j} = 1 - \frac{m^2}{E^2}$$

$$J = E\left(\left(1-\beta\right) H_0 + \beta H_1\right)$$

G. Fox and S. Wolfram, Phys.Rev.Lett. 41 (1978) 1581

Two-prong Object

- Second Moment

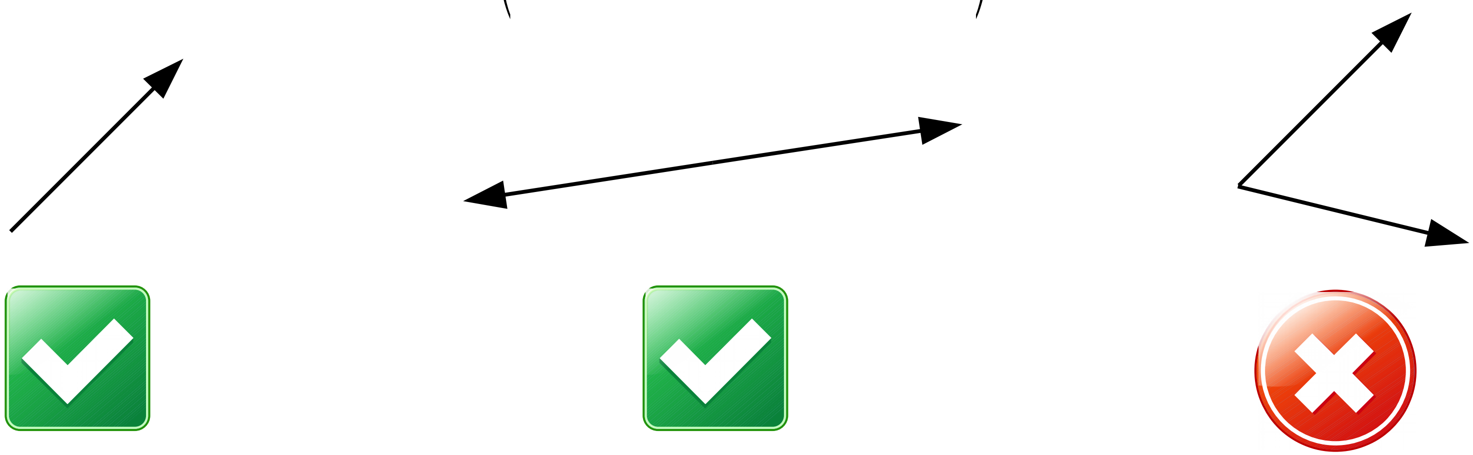
$$H_2 = \sum_{i,j} \frac{|p_i||p_j|}{E_J^2} \frac{\left(3\cos^2\theta_{i,j} - 1\right)}{2} \approx \sum_{i,j} \frac{|p_i||p_j|}{E_J^2} \cos^2\theta_{i,j}$$

- Only constrains $\cos^2\theta_{i,j}$
- Two-branch/prong boundary?

Back to back only?

- Simplest option:

$$J_{\mathrm{two-prong}} = E\left(1 - \beta\frac{m^2}{E^2} + \gamma H_2\right)$$



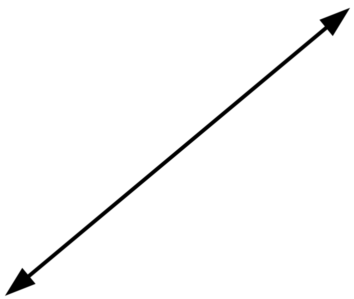
- Rest Frame?

$$\tilde{H}_2 = \left(\sum_{i,j} |p_i||p_j| P_2(\cos(\theta_{i,j}))\right)_{\text{rest frame}}$$

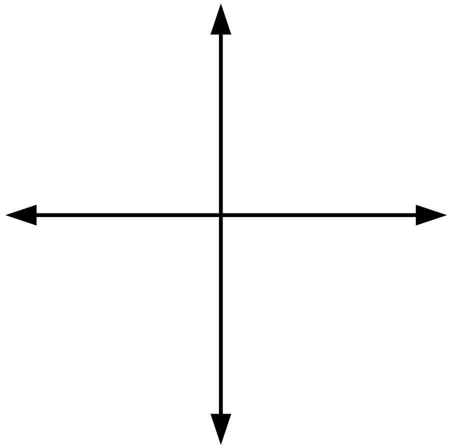
Invariant Form

- Lorentz Invariance

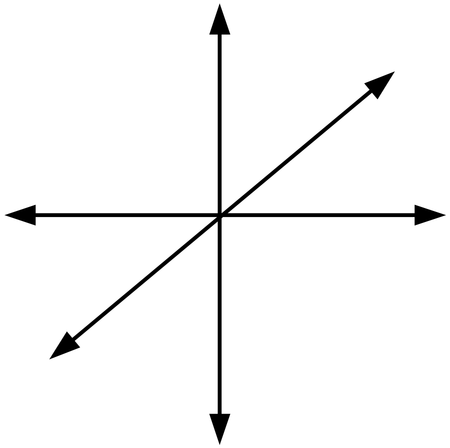
$$\tilde{H}_2 = \left(\sum_{i,j} |p_i||p_j| P_2\left(\cos\left(\theta_{i,j}\right)\right) \right)_{\text{rest frame}} \equiv m^2 \sum_{i,j} \frac{\left(p_i^{\mu} p_{j\mu}\right)^2}{\left(P^{\mu} p_{i\mu}\right)\left(P^{\nu} p_{j\nu}\right)} - m^2$$



$$\tilde{H}_2 = m^2$$



$$\tilde{H}_2 = \frac{1}{2} m^2$$



$$\tilde{H}_2 = \frac{1}{3} m^2$$

Invariant Form

- Lorentz Invariance

$$\tilde{H}_2 = \left(\sum_{i,j} |p_i||p_j| P_2(\cos(\theta_{i,j})) \right)_{\text{rest frame}} \equiv m^2 \sum_{i,j} \frac{(p_i^\mu p_{j\mu})^2}{(P^\mu p_{i\mu})(P^\nu p_{j\nu})} - m^2$$

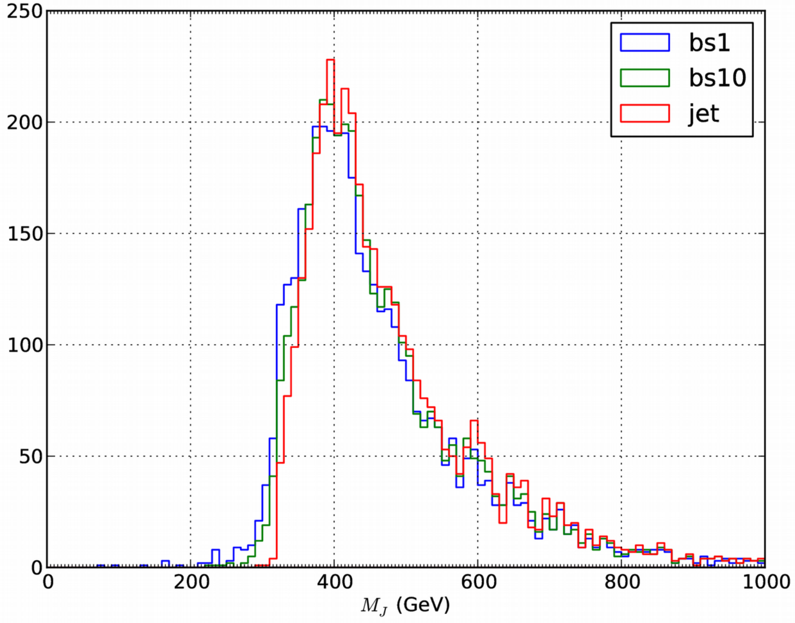
- Characteristic Function of two-prong objects

$$J_{\text{two-prong}} = E_T^2 \left(1 - \beta \frac{m^2}{E_T^2} + \gamma \frac{\tilde{H}_2}{E_T^2} \right) \approx E_T^2 \left(1 - (\beta - \gamma) \frac{m^2}{E_T^2} \right)$$

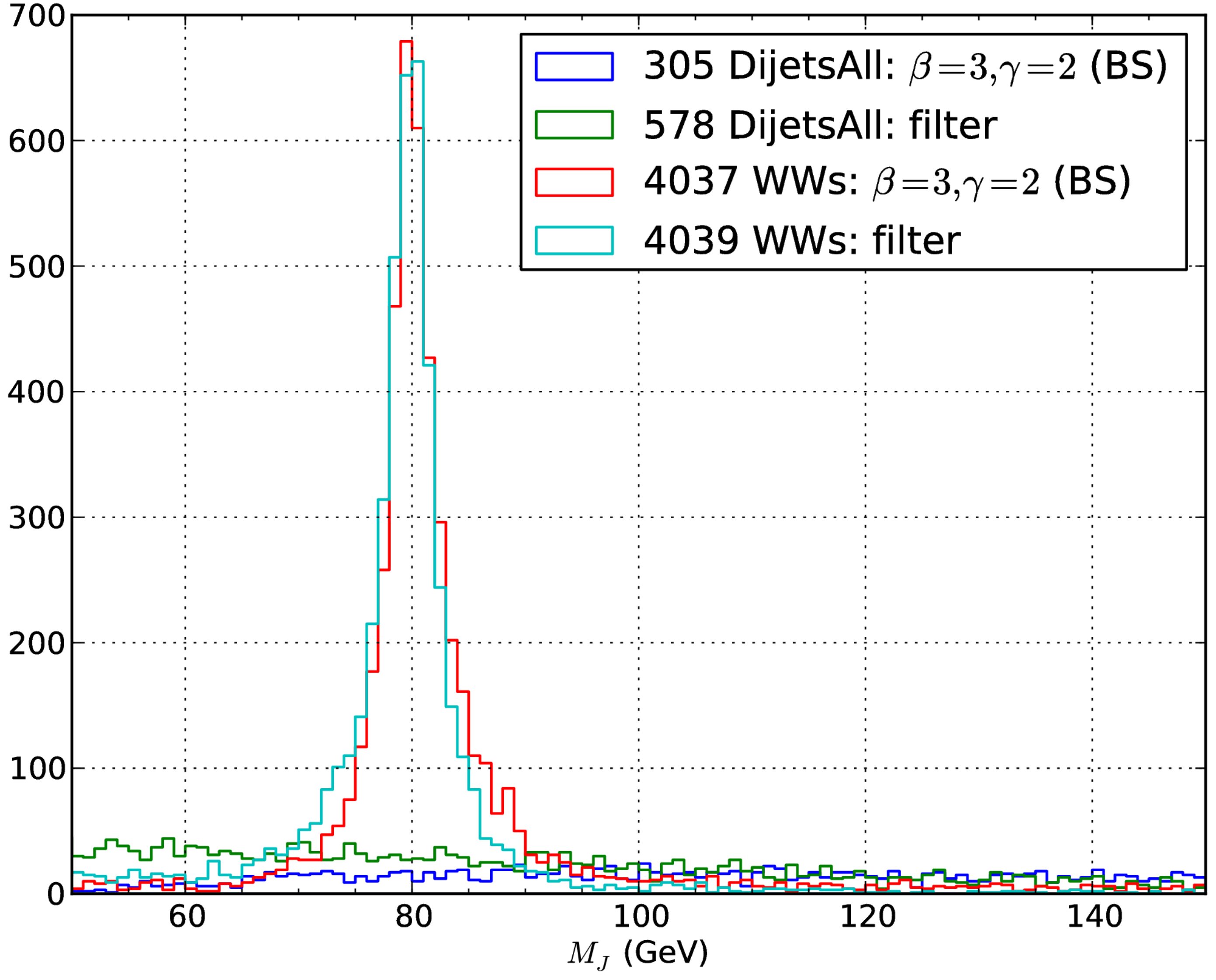
- $\beta - \gamma$: (maximal) size of the object

Approximation

- Exact solution is still NP
- Iterative approximate solution:
  - (Bootstrapping) Exact solution for a small subset
  - Add/Remove one particle each time
  - Repeat until no improvement



Comparing with Filtering/Mass Drop



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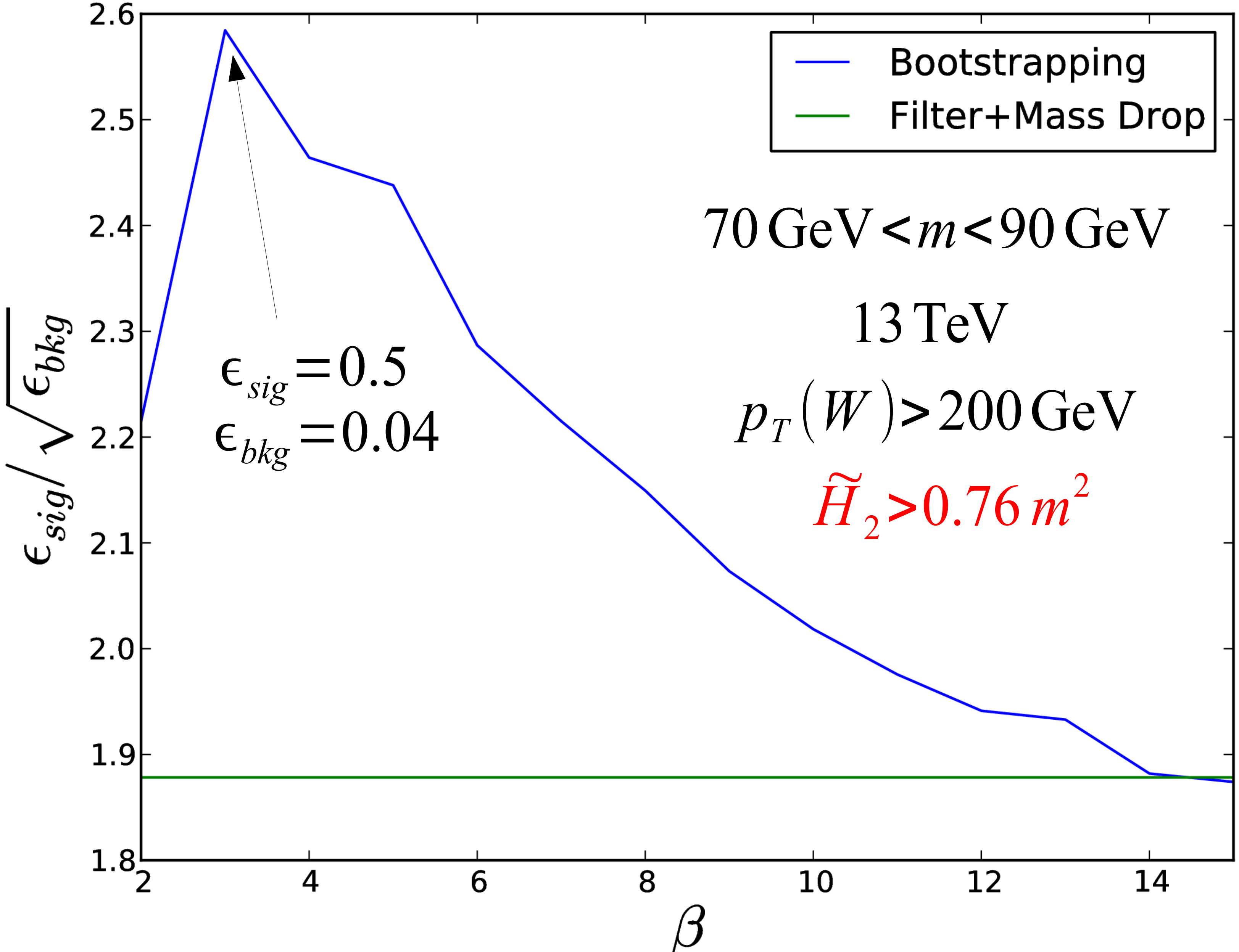

Conclusion and Future Directions

- New approach to study objects
- Algorithm for QCD jet
- Potentially useful for W tagging
- Details about the efficiency
- Analytical understanding
- Even better characteristic function?