

W tagging using jet functions

Ran Lu

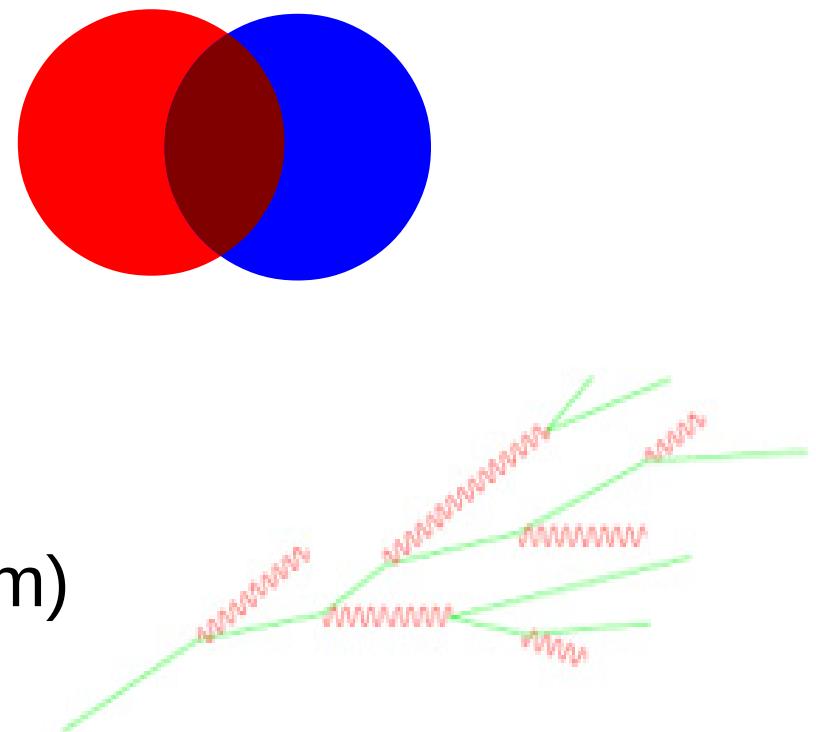
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Work in progress with Yang Bai and Zhenyu Han

2015-05-04

Jet Definitions

- Sterman-Weinberg (1977)
- Cone Algorithm
 - Stable Cone
 - Split and Merge
- Clustering Algorithm
 - Pair-Wise Merging
 - Undo Shower (k_t algorithm)
 - Nearest Neighbor
- No time to review all of them...

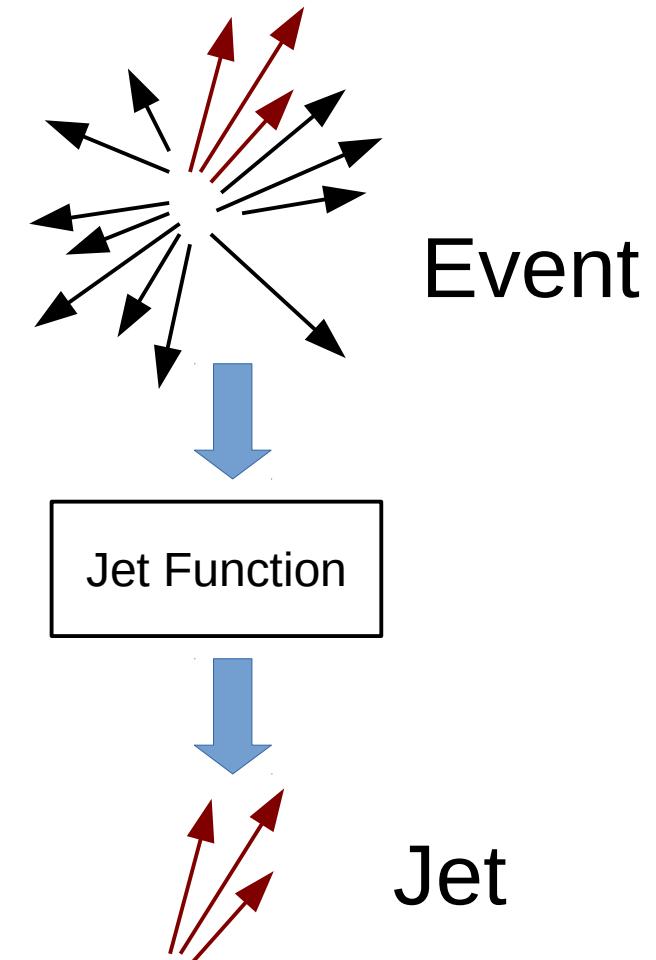


Jet Function

- Jets Maximize the Jet Function
- Function of Jet 4-momentum $J(P)$
 - Increases with increasing energy
 - Decreases with increasing mass
- Concrete Example:

$$J = E - \beta \frac{m^2}{E}, \beta \geq 0$$

- Cone Jet: $\sin \theta \leq \sqrt{\frac{1}{\beta}}$



Generalization (for hadron collider)

Y.Bai, Z. Han, RL, JHEP 3 (2015) 102

- New parameter α , ($0 < \alpha < 2$)

$$J_{E_T^\alpha} = E_T^\alpha \left(1 - \beta \frac{m^2}{E_T^2} \right) \quad E_T^2 = p_T^2 + m^2$$

- Boundary

$$\frac{1}{|p||P|} (P_x p_x + P_y p_y + \kappa P_z p_z) = \frac{\kappa}{\nu}$$

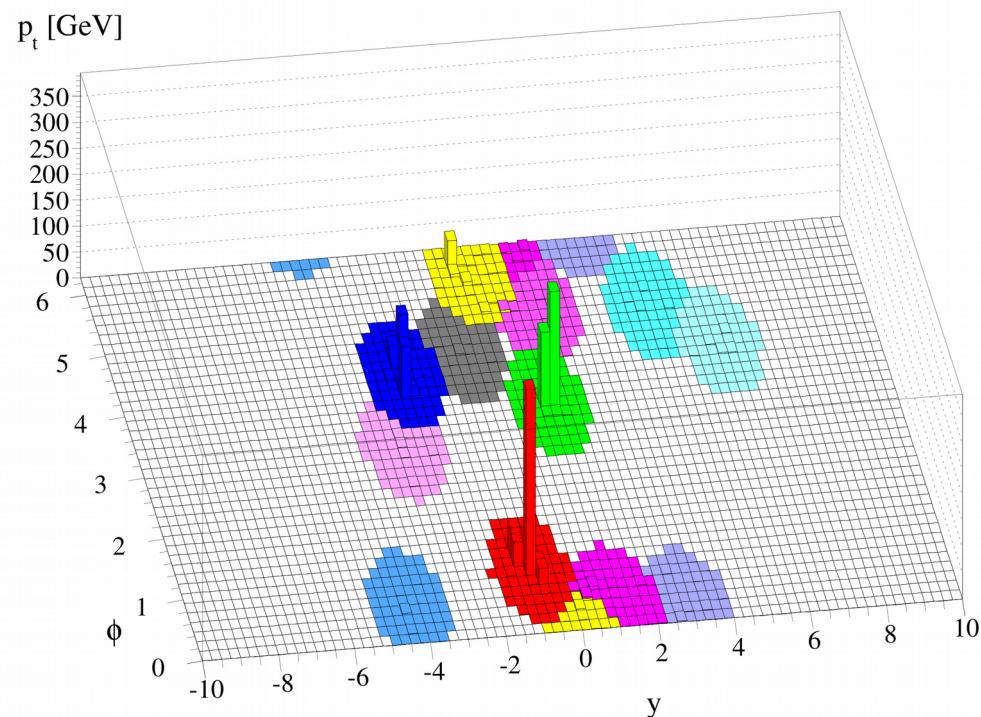
$$\kappa = 1 - \frac{\alpha}{2\beta} + \frac{\alpha-2}{2} \frac{m^2}{E_T^2}$$

- Center: $(P_x, P_y, \kappa P_z)$ $\cos \theta' = \frac{\kappa}{\nu \sqrt{1 - (1 - \kappa^2) \cos^2 \theta_z}}$

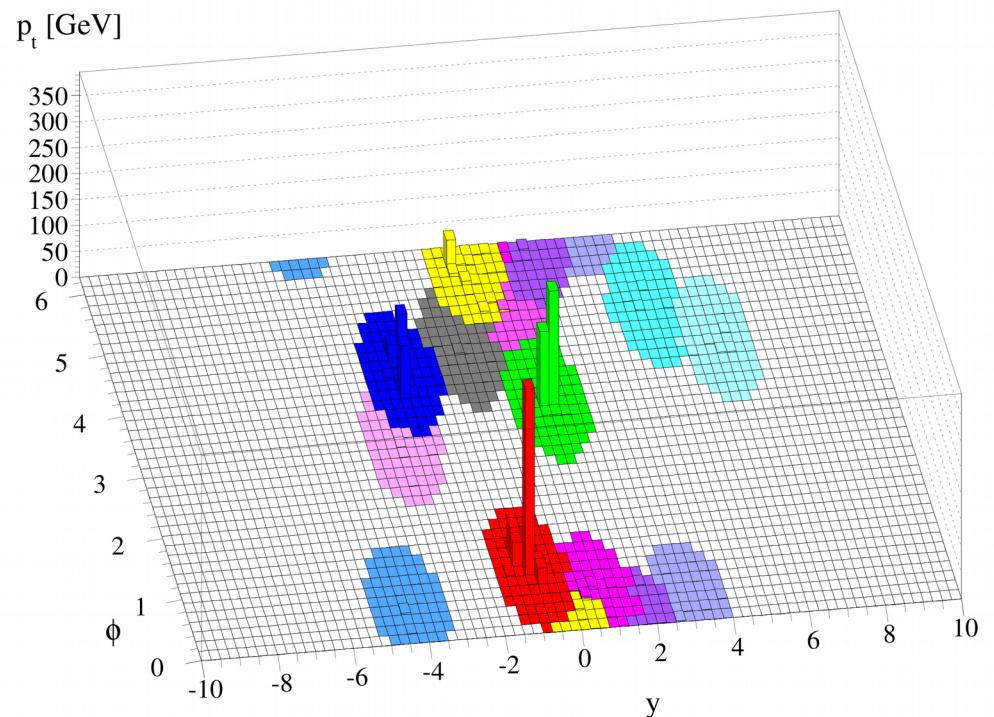
Still a cone

Comparison: Shape

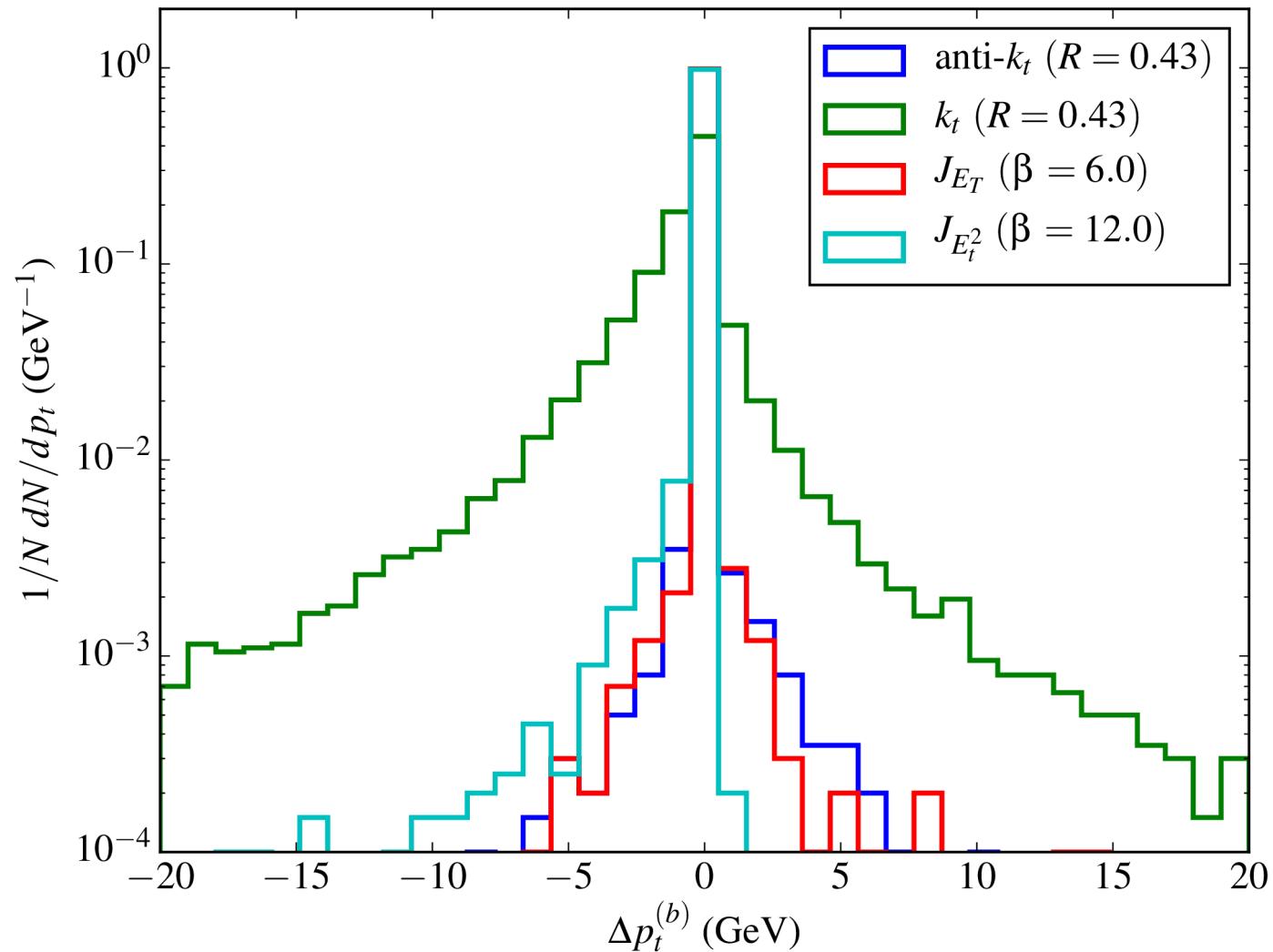
$J_{E_T} : \beta = 1.3$



Anti- k_T : $R = 1.0$



Comparison: Backreaction



Implementation

<https://github.com/LHCJet/JET>

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LHCJet / JET Watch 2 Star 0 Fork 0

109 commits 1 branch 3 releases 1 contributor

branch: master JET / +

Fix memory leak in python binding ranlu authored on Mar 3 latest commit 777c3be730

File	Description	Time
cmake/modules	Summary.cmake adapted from Clementine	4 months ago
examples	Forgot to add it	4 months ago
fastjet	Separate examples and internal codes	4 months ago
python	Fix memory leak in python binding	2 months ago
sample	Some event sample to test with	7 months ago
src	Build the version number into the library	4 months ago
CMakeLists.txt	Bump the version number	4 months ago
README.md	Format	4 months ago
cmake_uninstall.cmake.in	Proper make uninstall target	5 months ago
config.h.cmake	Build the version number into the library	4 months ago

HTTPS clone URL <https://github.com/LHCJet/JET>
You can clone with [HTTPS](#) or [Subversion](#).

Download ZIP

README.md

JETJet

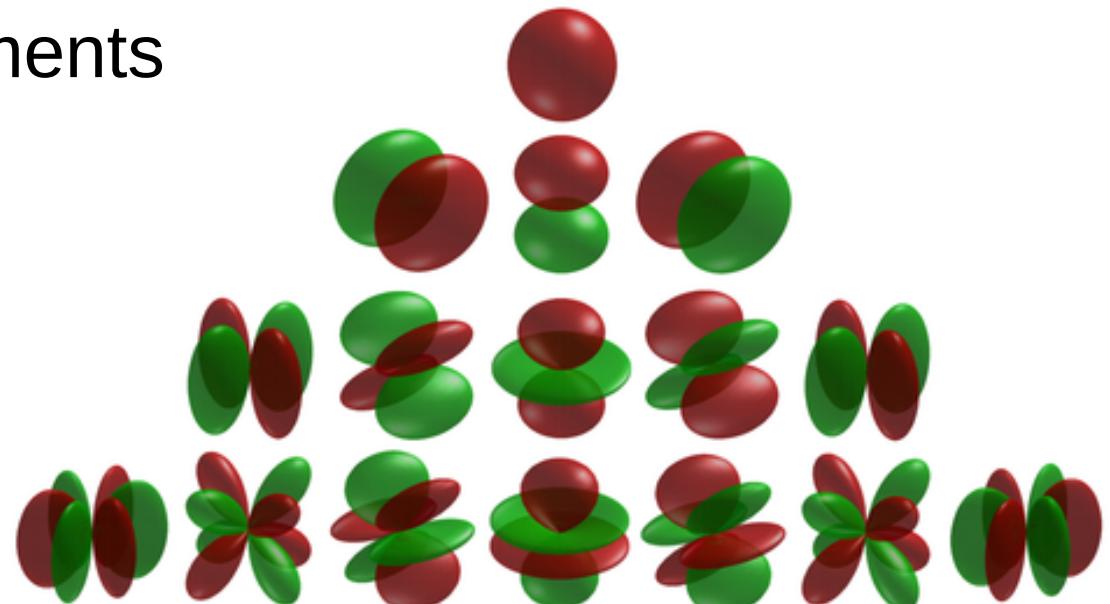
W tagging

- Many useful observables
 - (filtered/trimmed/pruned) Jet Mass
 - Mass Drop
 - N-subjettiness
 - Color Connection
 - Jet Charge
 - Energy Correlation Function
 - Planar Flow
 - Q-jet
- Multivariate Analysis (BDT)



Characteristic Function of W

- $J(P)$ is not enough
- 4-momentum: mass and velocity (point particle)
- “Shape” of the object
- Event/Jet shape variables
 - Fox-Wolfram Moments



Fox-Wolfram Moment

- General definition:

$$H_n = \sum_{i,j} \frac{|p_i||p_j|}{E_J^2} P_n(\cos\theta_{i,j})$$

- Examples:

$$H_0 = \sum_{i,j} \frac{|p_i||p_j|}{E_J^2} = 1$$

$$H_1 = \sum_{i,j} \frac{|p_i||p_j|}{E_J^2} \cos\theta_{i,j} = 1 - \frac{m^2}{E^2}$$

$$J = E((1-\beta)H_0 + \beta H_1)$$

Two-prong Object

- Second Moment

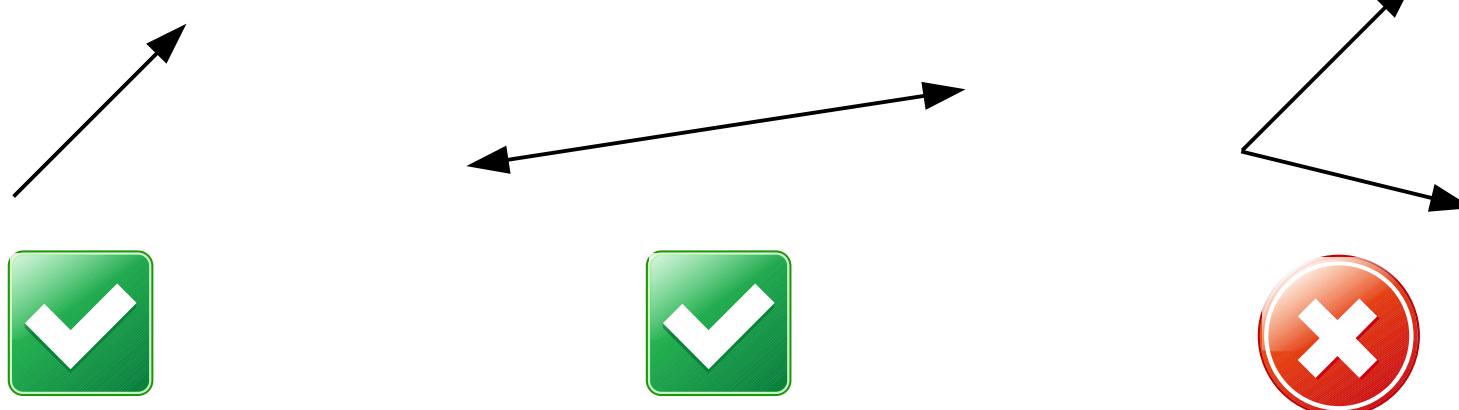
$$H_2 = \sum_{i,j} \frac{|p_i||p_j|}{E_J^2} \frac{(3\cos^2\theta_{i,j} - 1)}{2} \approx \sum_{i,j} \frac{|p_i||p_j|}{E_J^2} \cos^2\theta_{i,j}$$

- Only constrains $\cos^2\theta_{i,j}$
- Two-branch/prong boundary?

Back to back only?

- Simplest option:

$$J_{\text{two-prong}} = E \left(1 - \beta \frac{m^2}{E^2} + \gamma H_2 \right)$$



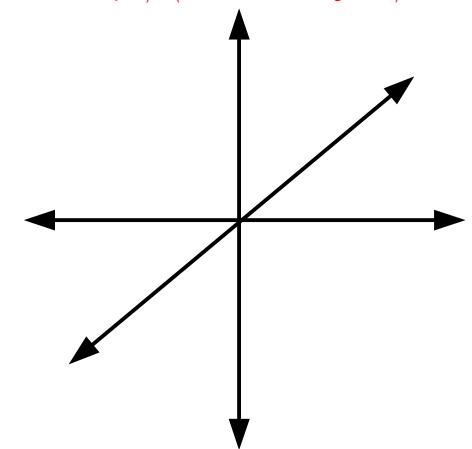
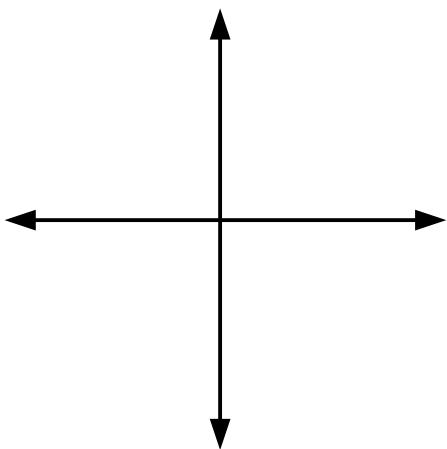
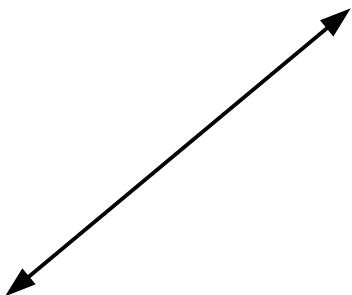
- Rest Frame?

$$\tilde{H}_2 = \left(\sum_{i,j} |p_i| |p_j| P_2(\cos(\theta_{i,j})) \right)_{\text{rest frame}}$$

Invariant Form

- Lorentz Invariance

$$\tilde{H}_2 = \left(\sum_{i,j} |p_i| |p_j| P_2(\cos(\theta_{i,j})) \right)_{\text{rest frame}} \equiv m^2 \sum_{i,j} \frac{(P_i^\mu p_{j\mu})^2}{(P^\mu p_{i\mu})(P^\nu p_{j\nu})} - m^2$$



$$\tilde{H}_2 = m^2$$

$$\tilde{H}_2 = \frac{1}{2} m^2$$

$$\tilde{H}_2 = \frac{1}{3} m^2$$

Invariant Form

- Lorentz Invariance

$$\widetilde{H}_2 = \left(\sum_{i,j} |p_i| |p_j| P_2(\cos(\theta_{i,j})) \right)_{\text{rest frame}} \equiv m^2 \sum_{i,j} \frac{(P_i^\mu p_{j\mu})^2}{(P^\mu p_{i\mu})(P^\nu p_{j\nu})} - m^2$$

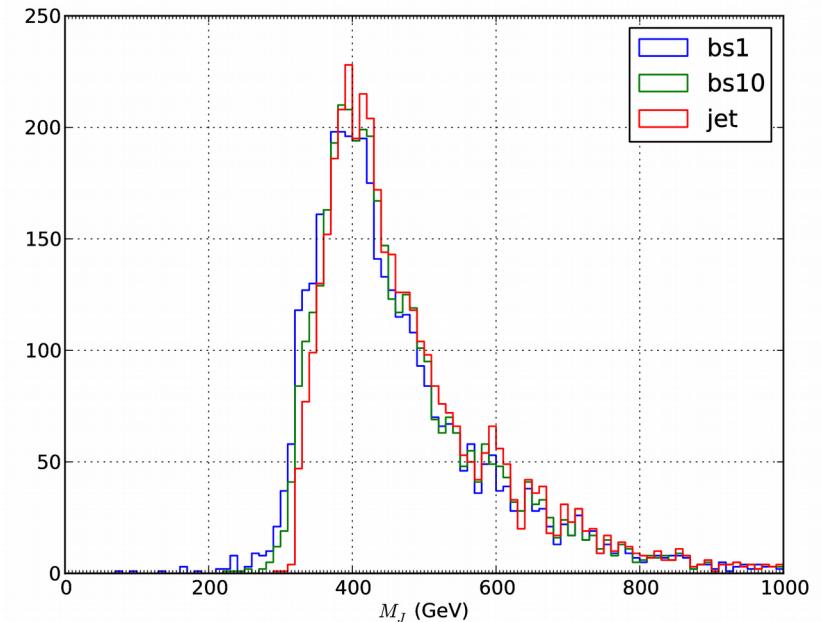
- Characteristic Function of two-prong objects

$$J_{\text{two-prong}} = E_T^2 \left(1 - \beta \frac{m^2}{E_T^2} + \gamma \frac{\widetilde{H}_2}{E_T^2} \right) \approx E_T^2 \left(1 - (\beta - \gamma) \frac{m^2}{E_T^2} \right)$$

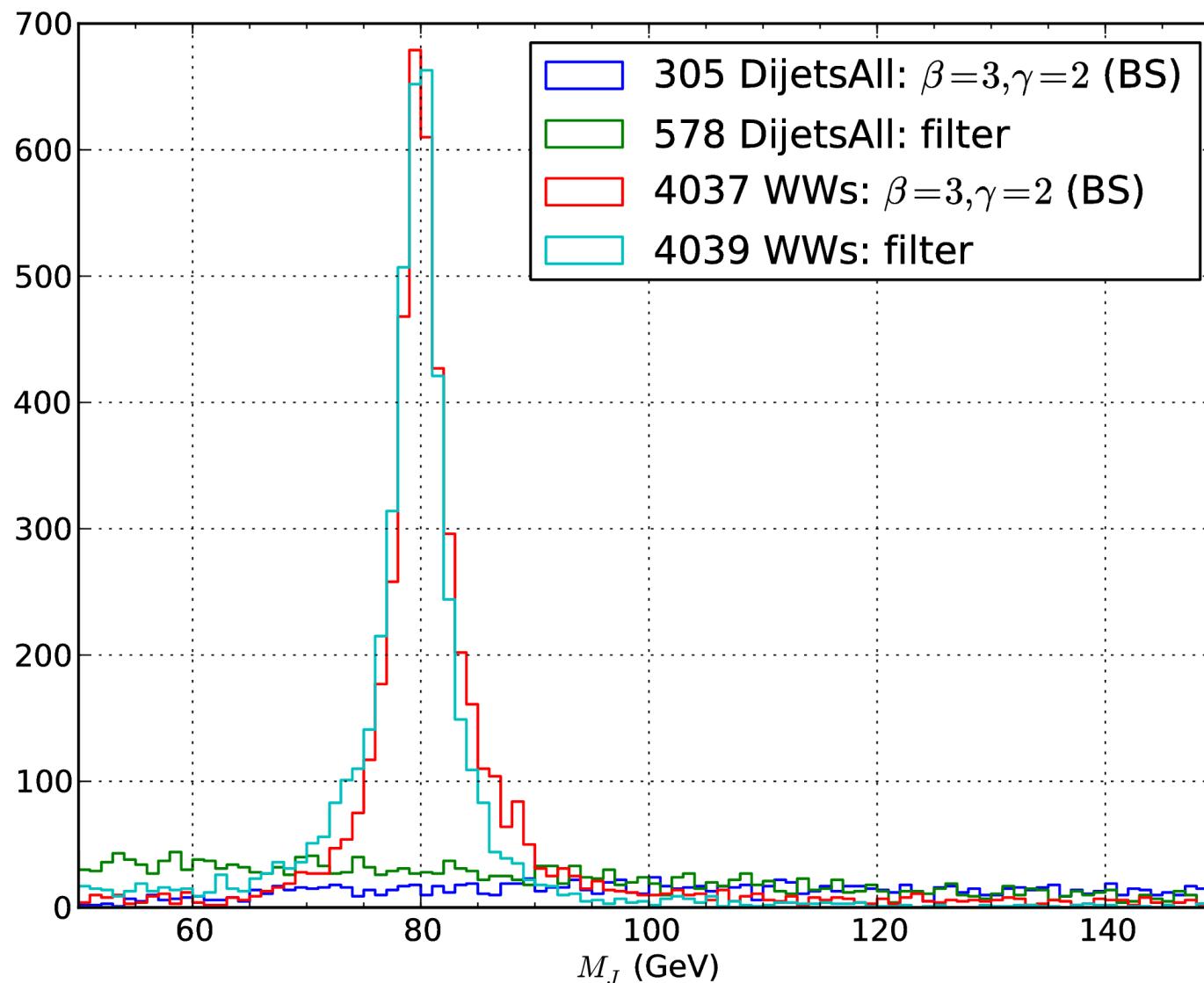
- $\beta - \gamma$: (maximal) size of the object

Approximation

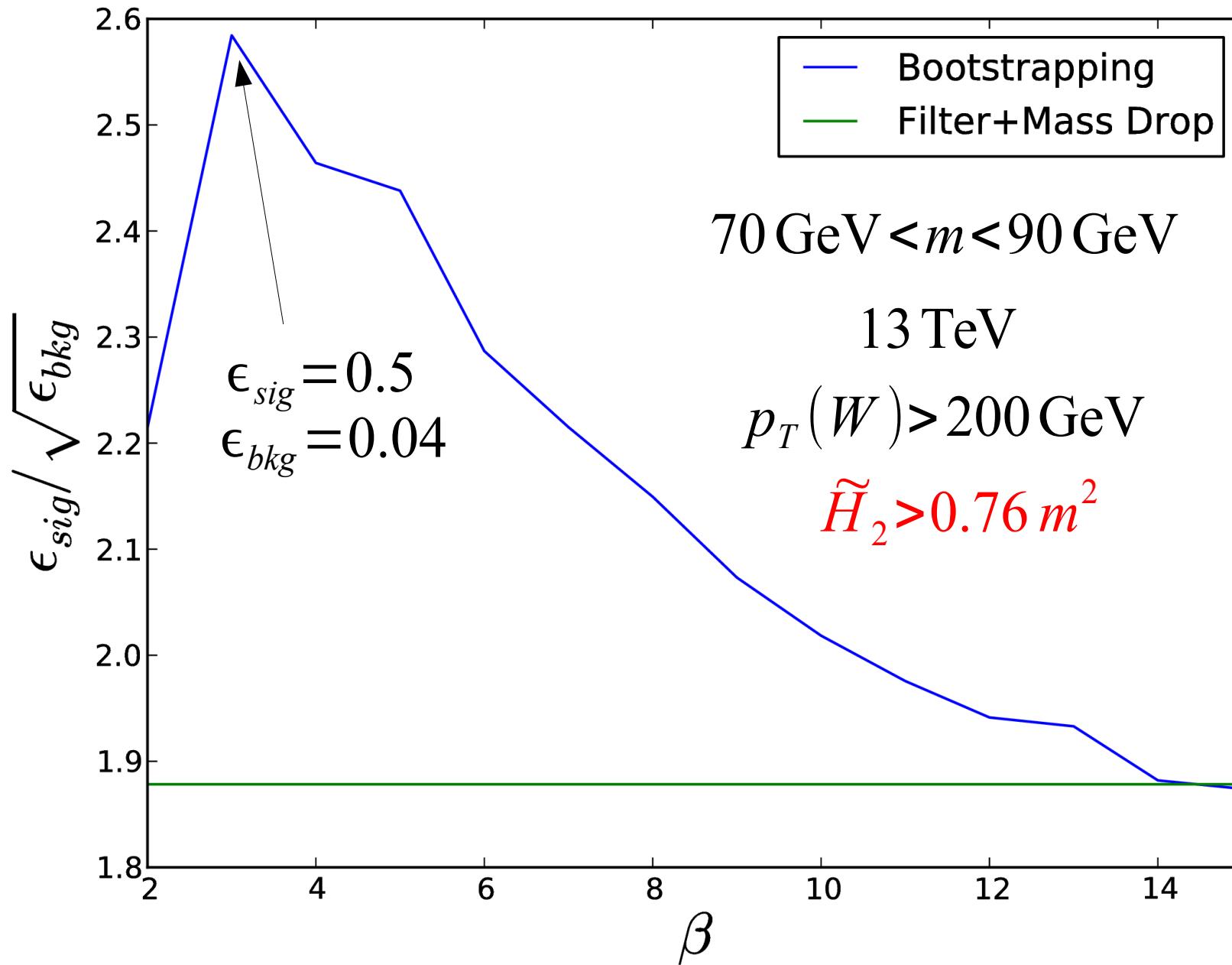
- Exact solution is still NP
- Iterative approximate solution:
 - (Bootstrapping) Exact solution for a small subset
 - Add/Remove one particle each time
 - Repeat until no improvement



Comparing with Filtering/Mass Drop



Comparing with Filtering/Mass Drop



Conclusion and Future Directions

- New approach to study objects
- Algorithm for QCD jet
- Potentially useful for W tagging
- Details about the efficiency
- Analytical understanding
- Even better characteristic function?