Expanding the Axion Field Range via Mixings

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Based on the work with
Gary Shiu and Wieland Staessens
1503.01015, 1503.02965 [hep-th]

Introduction-Axions

- Axions: **CP-odd real scalars with continuous shift symmetry**; first introduced to solve strong CP problem

$$a(x) \to a(x) + \delta(x)$$

- Rich applications in particle physics and cosmology

Introduction-Axions

$$a(x) \to a(x) + \delta(x)$$

- **Continuous shift symmetry** ⟶ Derivative couplings ⟶ Possible light dark matter (DM) and inflaton? $\boxed{\frac{1}{f_a} da \wedge \star_4 J}$

- **Continuous shift symmetry is exact at most perturbatively.**

- Break continuous shift symmetry **down to discrete shift symmetry by non-perturbative** instantons

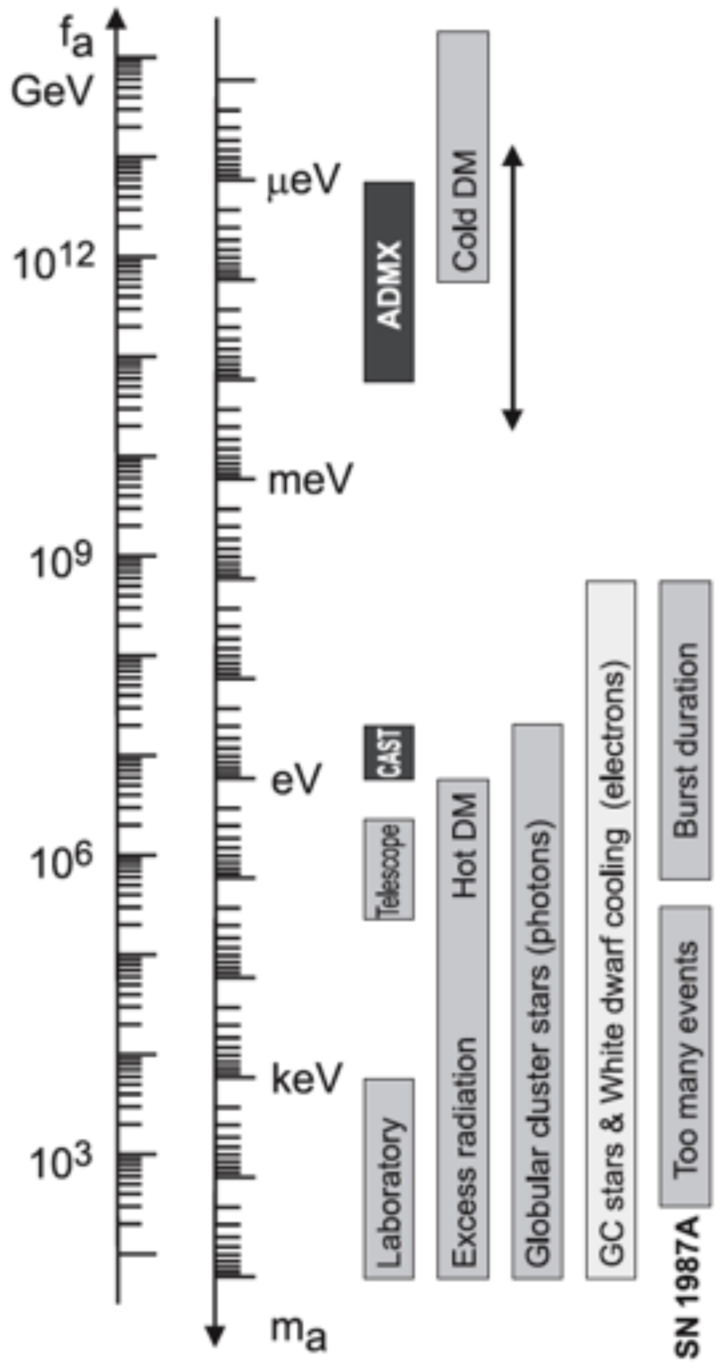
$$a(x) \to a(x) + 2\pi f_a$$

- Axion mass given by NP effects

$$m_a^2 = \frac{\partial^2 V_{eff}(a)}{\partial a^2} = \frac{\Lambda^4}{f_a^2}, \quad \Lambda = \text{NP scale}$$

Axion decay constant: periodicity, field range

Introduction-Scenario 1: Axions as DM



$$10^9\,\text{GeV} \leq f_a^{DM} \leq 10^{12}\,\text{GeV}$$

(From PDG 2013)

Introduction-Scenario 2: Axions as Inflaton

- Natural inflation: periodic inflaton potential

$$V(a) = \Lambda^4[1 - \cos\frac{a}{f_a}]$$

- Slow roll:

$$\boxed{\epsilon \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1,\ |\eta| \equiv M_P^2\left|\frac{V''}{V}\right| \ll 1 \Rightarrow f_a > M_P}$$

Large field inflation

Introduction-Stringy Axions

- Arise from dimensional reduction of higher forms (bulk fields)
- Shift symmetry as remnant of gauge symmetry of the forms in extra dimensions
- Constraint from string compactifications (no axion mixing):

P. Svrcek and E. Witten, JHEP **0606**, 051 (2006)
T. Banks, M. Dine, P. J. Fox and E. Gorbatov, JCAP **0306**, 001 (2003)

$$f_a \lesssim \frac{g^2}{8\pi^2} M_P$$

Sub-Planckian!

typically a little lower than string scale

Not in ADM or slow-roll inflation range

Introduction-Motivation

- Widen the axion window without violating the string axion bound
- Apply both to field and string theories

Axion Mixings

- General lagrangian

Fermion and/or GCS terms
to ensure the gauge invariance

Abelian ***Non-Abelian***

$$\mathcal{S}^{\rm eff} = -\int \Bigg[\sum_{\alpha,\,\beta=1}^{M} f_{\alpha\beta} F^{\alpha}\wedge\star_4 F^{\beta} + \sum_{A=1}^{P}\frac{1}{g_A^2}{\rm Tr}\,(G^A\wedge\star_4 G^A) - \frac{1}{2}\sum_{i,j=1}^{N}\mathcal{G}_{ij}(da^i - \sum_{\alpha=1}^{M} k^i_{\alpha}A^{\alpha})\wedge\star_4(da^j - \sum_{\beta=1}^{M} k^j_{\beta}A^{\beta}) - \frac{1}{8\pi^2}\sum_{A=1}^{P}\left(\sum_{i=1}^{N} r_{iA}a^i\right){\rm Tr}(G^A\wedge G^A) - \frac{1}{8\pi^2}\sum_{\alpha,\beta=1}^{M}\left(\sum_{i=1}^{N} s_{i\alpha\beta}a^i\right)F^{\alpha}\wedge F^{\beta} + \dots \Bigg].$$

Winding number, anomalous coefficient etc

"Axion charge"

- Basis and normalization $k^i_{\alpha},\; r_{iA},\; s_{i\alpha\beta} \in \mathcal{Z}$

Compactness of U(1)'s

$$U(1)^{\alpha}:\;\; A^{\alpha}\to A^{\alpha}+d\eta^{\alpha},\;\; a^i\to a^i + k^i_{\alpha}\eta^{\alpha}$$

Axion Mixings

determined by the vev of saxions

$$\mathcal{S}^{\text{eff}} = -\int \Bigg[\sum_{\alpha,\,\beta=1}^{M} f_{\alpha\beta} F^{\alpha} \wedge \star_4 F^{\beta} + \sum_{A=1}^{P} \frac{1}{g_A^2} \text{Tr}\left(G^A \wedge \star_4 G^A\right) - \frac{1}{2} \sum_{i,j=1}^{N} \mathcal{G}_{ij} (da^i - \sum_{\alpha=1}^{M} k_{\alpha}^{i} A^{\alpha}) \wedge \star_4 (da^j - \sum_{\beta=1}^{M} k_{\beta}^{j} A^{\beta})$$
$$- \frac{1}{8\pi^2} \sum_{A=1}^{P} \left(\sum_{i=1}^{N} r_{iA} a^i \right) \text{Tr}(G^A \wedge G^A) - \frac{1}{8\pi^2} \sum_{\alpha,\beta=1}^{M} \left(\sum_{i=1}^{N} s_{i\alpha\beta} a^i \right) F^{\alpha} \wedge F^{\beta} + \ldots \Bigg].$$

- **3 types of axion mixings:**

- Mixing due to ***non-diagonal metric*** (referred as **kinetic mixing or metric mixing**)

non-vanishing charges

- Mixing due to ***Stueckelberg couplings*** (referred as **Stueckelberg mixing**)

- Mixing due to **mismatch between kinetic eigenstates and mass eigenstates**

Axion-Mixings (minimal)

$$S_{axion}^{N=2} = \int \left[-\frac{1}{2} \sum^{2}_{i,j=1} \mathcal{G}_{ij} \left(da^i - k^i A \right) \wedge \star_4 \left(da^j - k^j A \right) + \frac{1}{8\pi^2} \left(r_1 a^1 + r_2 a^2 \right) \mathrm{Tr}\, G \wedge G + ... \right]$$

- Minimal setup: **2 axions**, **1 Stueckelberg U(1)** and **1 non-Abelian gauge group**

Axion Mixings (minimal)

Kinetic eigenstates:

- **Axion eaten by U(1)**: part of the massive U(1) in appropriate gauge
- **Uneaten axion**: obtain **effective potential for inflation** by integrating out massive U(1) and non-Abelian gauge field

$$V_{\text{eff}}(\xi) = \Lambda^4 \left[1 - \cos\left(\frac{\xi}{f_\xi}\right)\right]$$

Single field potential for the remaining axion

$$f_\xi = \frac{\sqrt{\lambda_+ \lambda_-} M_{st}}{\cos\frac{\theta}{2}\left(\lambda_+ k^+ r_2 + \lambda_- k^- r_1\right) + \sin\frac{\theta}{2}\left(\lambda_- k^- r_2 - \lambda_+ k^+ r_1\right)}$$

mixing angle: amount of metric mixing

Axion Mixings (minimal)

Slow-roll inflation

$$\boxed{f_\xi = \frac{\sqrt{\lambda_+\lambda_-}M_{st}}{\cos\frac{\theta}{2}\left(\lambda_+k^+r_2+\lambda_-k^-r_1\right)+\sin\frac{\theta}{2}\left(\lambda_-k^-r_2-\lambda_+k^+r_1\right)}}$$

- Explore axion field range - intermediate kinetic mixing

$$\varepsilon^2 \equiv \mathcal{G}_{22}/\mathcal{G}_{11}$$

White region $f_\xi > 10^2\sqrt{\mathcal{G}_{11}}$

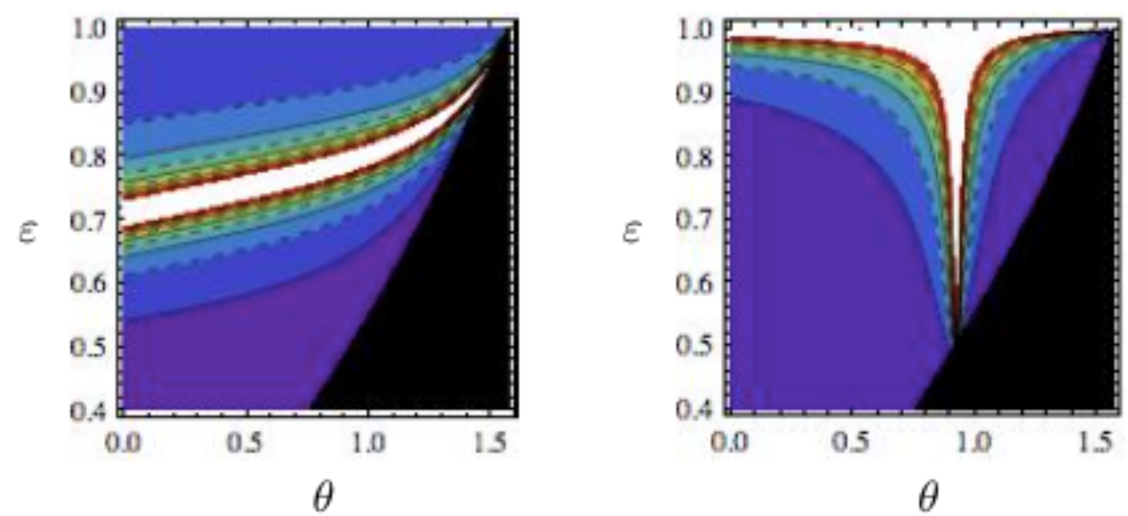


FIG. 1. Contour plots of decay constant $f_\xi(\theta,\varepsilon)$ for $2r_1 = 2r_2 = 2k^1 = k^2$ (left) and $r_1 = 2r_2 = k^1 = 2k^2$ (right). The f_ξ-values range from small (purple) to large (red) following the rainbow contour colors. Unphysical regions with complex f_ξ are located in the black band.

Axion Mixings (minimal)

Other axion inflation scenarios:

Both tied to number of DOF

- **N-flation**: S. Dimopoulos, S. Kachru, J. McGreevy and J. G. Wacker, JCAP **0808**, 003 (2008)

  $f_{eff} \propto \sqrt{N}$, $N =$ number of axions $=$ number of non-Abelian gauge instaontons

  N=2: some anomalous coefficients at order of 100

- **Aligned natural inflation**: J. E. Kim, H. P. Nilles and M. Peloso, JCAP **0501**, 005 (2005)

  $f_{eff} \propto \sqrt{N!}n^N$, N>2 version K. Choi, H. Kim and S. Yun, Phys. Rev. D **90**, no. 2, 023545 (2014)

  $N =$ number of axions $=$ number of non-Abelian gauge instantons,

  $n \in \mathcal{Z}$ anomalous coefficients

- Planck mass renormalization $\delta M_{Pl}^2 \sim N$

Spoil the slow roll condition?

Axion Mixings (minimal)

Our approach to get a super-Planckian inflation:

- Axion field range enhancement **not tied to the number of DOF**
- Relying on tuning **continuous parameters** in moduli space, not much on discrete parameters
- **Minimal (fewer DOF) setup works, which has less severe Planck mass renormalization issue.**

Axion Mixings (ADM)

Minimal setup

$$f_\xi = \frac{\sqrt{\lambda_+\lambda_-}M_{st}}{\cos\frac{\theta}{2}\left(\lambda_+k^+r_2+\lambda_-k^-r_1\right)+\sin\frac{\theta}{2}\left(\lambda_-k^-r_2-\lambda_+k^+r_1\right)}$$

- **To lower axion decay constants**: by tuning continuous parameters and choosing appropriate discrete parameters

$\varepsilon^2 \equiv \mathcal{G}_{22}/\mathcal{G}_{11}$

$\varrho^2 \equiv \mathcal{G}_{12}/\mathcal{G}_{11}$

$$\text{e.g. assuming } \varepsilon = 1,\ \theta = \pi,\ r_1 = -r_2,\ k^1 = k^2$$

$$f_\xi = \frac{\sqrt{\mathcal{G}_{11}^2-\mathcal{G}_{12}^2}}{\sqrt{2}r_2\sqrt{\mathcal{G}_{11}+\mathcal{G}_{12}}} = \frac{\sqrt{\mathcal{G}_{11}}\sqrt{1-\varrho^2}}{\sqrt{2}r_2} \quad r_2 \sim \mathcal{O}(1-10),\ \sqrt{\mathcal{G}_{11}} \sim \mathcal{O}(10^{15}-10^{17})\,\text{GeV},\ \text{large mixing } 1-\rho^2 \sim \mathcal{O}(10^{-4}-10^{-8})$$

Not require the intrinsic axion field range to be too small

Non-minmal setup

For large N, eigenvalue repulsion

String Theory Embedding

- Approaching Planck scale: **physics is sensitive to Planck scale**

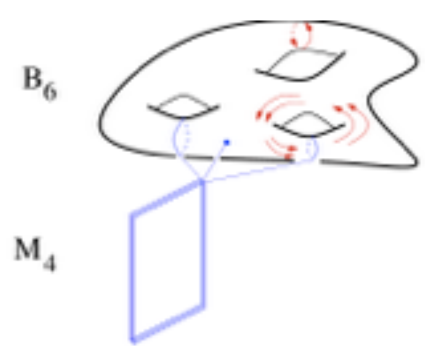
- Need a full theory description $\rightarrow$ String Theory

String Theory Embedding

- **Closed string axions (Ramond-Ramond axions)**: dimensional reduction of p-forms
- **Axion metric** $\mathcal{G}_{ij}$: moduli field dependent (stabilization of the saxion, volume of the internal cycle wrapped by p-forms)
- **U(1)**: Abelian factor of **world volume gauge group U(N)** for a stack of N D-branes
- **Gauge Dp-branes**: extending along Minkowski space and wrapping internal (p-3)-cycles
- **Wrapping numbers**$\rightarrow$Discrete parameters $k^i_\alpha \quad r_{i\alpha\beta}$

String Theory Embedding

- Instantons: **Gauge instantons** or **stringy instantons**
- **Gauge instanton**: on world volume of Dp-brane extending Minkowski space and wrapping internal (p-3)-cycles



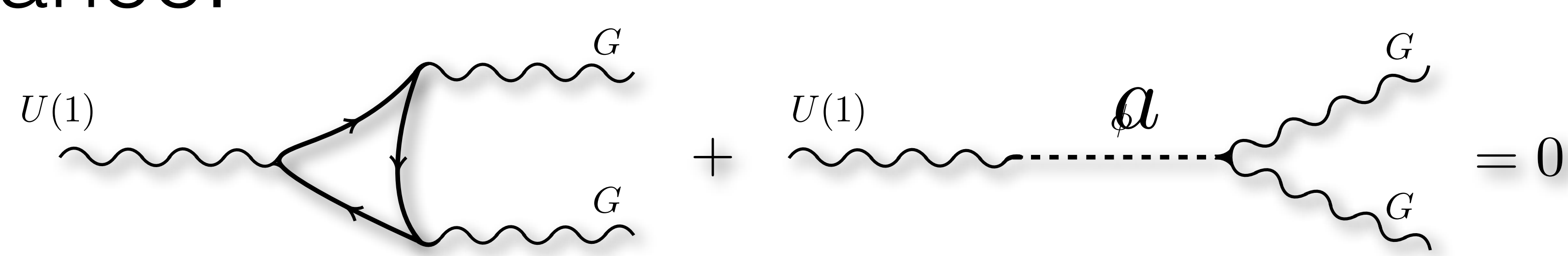
(Graph from 1003.4867)

- **Stringy instanton**: e.g. E(p-1)-instanton (D(p-1)-brane wrapping internal p-cycles, pointlike in 4d).
- **Instantons contribute only when fermonic zero modes are saturated**.

$$e^{-S_{E_{p-1}}} = e^{-\frac{2\pi}{g_s}\mathrm{Vol}(\gamma_i) - i\, a^i}$$

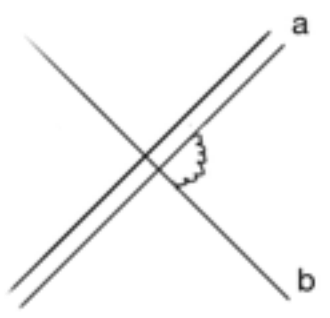
String Theory Embedding

- Introducing orientifold planes
- Consistency: tadpole cancellation
- Chiral spectrum: bi-fundamentals at intersections
- Green-Schwarz (GS) mechanism to ensure gauge invariance.



String Theory Embedding

- Explicit D-brane example realizing super-Planckian excursion: “Large Field Inflation from Axion Mixing”, with G. Shiu and W. Staessens [hep-th/1503.02965]



Conclusion

- **Kinetic (Metric) and Stueckelberg mixings can enlarge axion window.**
- Applies to both field theory and string theory models with limited intrinsic axion field ranges.
- Axion field range enhancement is mainly through **tuning continuous parameters** (with discrete parameters properly chosen).
- **To lower the axion decay constant (for ADM)**: **no large compact cycles** needed (alleviate the requirement for intermediate string scale). **Allow high fundamental string scale** -> **new possibilities to detect string axions** through astrophysical, cosmological and laboratory ways
- **To increase the axion decay constant (for inflation)**: **not require large DOF** (-> minimal setup) and mainly depends on tuning continuous parameters.
- Model-dependent higher-order instanton corrections: quantum gravity at work to couple the multi-axion system to gravity. Deviation from a cosine potential -> measurable effect on the inflationary perturbation spectrum. Quantifying such deviation needs understanding of UV completion of inflation and moduli stabilization.

Thank you!

Backup

Gauge Invariance

$$\boxed{S^{N=2}_{axion} = \int \left[-\frac{1}{2}\sum^{2}_{i,j=1} \mathcal{G}_{ij} \left(da^i - k^i A\right) \wedge \star_4 \left(da^j - k^j A\right) + \frac{1}{8\pi^2}\left(r_1 a^1 + r_2 a^2\right) \mathrm{Tr}\, G \wedge G + \ldots \right]}$$

U(1) variant

$$U(1):\ A \to A + d\eta',\ a^i \to a^i + k^i \eta'$$

$$\text{non-Abelian: } B \to B + D\eta$$

- Solution 1: **Triangle anomalies + Variance in instanton coupling = 0 (GS mech.)**

- Solution 2: **Triangle anomalies + Variance in instanton coupling + Variance in GCS terms = 0**

$$\boxed{\mathcal{S}^{\mathrm{GCS}}_{sub} = -\int \frac{1}{8\pi^2}\mathcal{A}^{\mathrm{GCS}} A \wedge \Omega} \qquad \boxed{\mathrm{d}\Omega = \mathrm{Tr}\left(G \wedge G\right)}$$

The Gauge To Eat A Charged Axion

$$\mathcal{S}^{\text{full}}_{axion} = \int \Big[-\frac{f^2_{\tilde{a}^1}}{2} d\tilde{a}^1 \wedge \star_4 d\tilde{a}^1 - \frac{f^2_{\tilde{a}^2}}{2}\left(d\tilde{a}^2 - \tilde{k}^2 A\right) \wedge \star_4 \left(d\tilde{a}^2 - \tilde{k}^2 A\right) - \frac{1}{g_1^2} F \wedge \star_4 F$$
$$- \frac{1}{g_2^2}\mathrm{Tr}(G \wedge \star_4 G) + \frac{1}{8\pi^2}\left[\tilde{a}^1 + \tilde{a}^2\right] \mathrm{Tr}(G \wedge G) - \frac{1}{8\pi^2}\mathcal{A}^{\mathrm{GCS}} A \wedge \Omega + \dots \Big]$$

$$\boxed{A \longrightarrow A + \frac{1}{\tilde{k}^2} d\tilde{a}^2}$$

$$\mathcal{S}^{\text{full,unitary}}_{axion} = \int \Big[-\frac{f^2_{\tilde{a}^1}}{2} d\tilde{a}^1 \wedge \star_4 d\tilde{a}^1 - \frac{(f_{\tilde{a}^2}\tilde{k}^2)^2}{2} A \wedge \star_4 A - \frac{1}{g_1^2} F \wedge \star_4 F - \frac{1}{g_2^2}\mathrm{Tr}(G \wedge \star_4 G)$$
$$+ \frac{1}{8\pi^2}\tilde{a}^1 \mathrm{Tr}(G \wedge G) - \frac{1}{8\pi^2}\mathcal{A}^{\mathrm{GCS}} A \wedge \Omega + A \wedge \star_4 \mathcal{J}_\psi$$
$$+ \frac{1}{8\pi^2}\frac{\left(\tilde{k}^2 + \mathcal{A}^{\mathrm{GCS}} + \mathcal{A}^{\mathrm{mix}}\right)}{\tilde{k}^2}\tilde{a}^2 \mathrm{Tr}(G \wedge G) + \dots \Big] . \qquad (2.77)$$

Generating the Sinusoidal Potential

- There are different ways to generate the sinusoidal axion potential.

- Way 1. Similar to calculations of QCD axion potential: integrating out the heavy mesons using non-linear sigma-models techniques

- Way 2. Gaugino condensates: break U(1) R symmetry. Supersymmetry obtains a NP correction

$$\mathcal{W} = \mathcal{W}_{per} + A\, e^{-\frac{2\pi}{N} T}$$ superfield T as $t + i\,a$

rank of the non-Abelian group

axion

$$K(T, \overline{T}) = -3\ln(T + \overline{T})$$

F-term potential for N=1 SUGRA

$$V_{\text{axion}}(a) = \frac{8\pi}{N} \frac{\langle t \rangle}{\mathcal{T}^2} |A|\, |\mathcal{W}_{per}|\, e^{-2\frac{\pi}{N} t} \cos\left(\frac{2\pi}{N} a + i\gamma\right)$$

Aligned Natural Inflation

N=2

$$\boxed{V(\Phi^1,\Phi^2) = \Lambda_1^4\left[1-\cos\left(\frac{n_1\Phi^1}{f_1}+\frac{n_2\Phi^2}{g_1}\right)\right] + \Lambda_2^4\left[1-\cos\left(\frac{m_1\Phi^1}{f_2}+\frac{m_2\Phi^2}{g_2}\right)\right], \qquad n_i,\, m_i \in \mathcal{Z}}$$

Perfect alignment $\dfrac{f_1/n_1}{g_1/n_2} = \dfrac{f_2/m_1}{g_2/m_2}$

Derivation from alignment $\alpha = g_2/m_2 - \dfrac{f_2/m_1}{f_1/n_1} g_1/n_2$

Along flat direction:

$f_{eff} \propto \dfrac{1}{\alpha}$ e.g. $f_1/n_1 = f_2/m_1 \ll g_1/n_2,\, g_2/m_2$

If want enhancement $\alpha \sim 10^{-2}\mathcal{O}(f_i,\, g_i)$, will require $\dfrac{n_2}{m_2} = \dfrac{99}{100}$

Too large integers!

Axion Bounds from String Compactifications

- Take weakly heterotic string for instance.

$$S_{bos} = M_s^8 \int d^{10}x\sqrt{-g}e^{-2\phi}(R - \frac{1}{2}|H_3|^2) - M_s^8 \int d^{10}x\sqrt{-g}e^{-2\phi}\frac{1}{2}|F_2|^2$$

$$H_3 = dB_2$$

Moduli from $B_{\mu\nu}$

dimensional reducing to 4d

$$M_s^8 \, l_s^6 = M_P^2$$

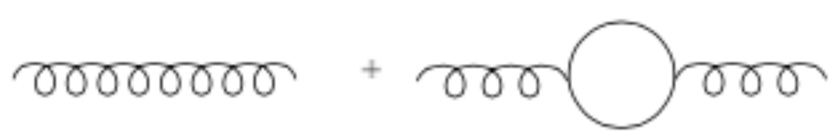
$$\frac{f_a^2}{M_p^2} = 1$$

Renormalization of the Planck Mass

- Running the Planck mass: $M_P \to M_P(\mu)$ for some scale μ

  $M_P(0) \sim 10^{19}\,\mathrm{GeV}$

- Graviton propagator:



- Integrating out (propagating) scalars, fermions and gauge bosons

$$M_P^2(\mu) \sim \frac{1}{G(\mu)} = \frac{1}{G(0)} - \frac{\mu}{12\pi}(n_0 + n_{1/2} - 4n_1)$$

n_i is the number of particles with spin-i

Moduli Stablization

- The metric in the axion moduli space is field dependent.
- In this work: assume the moduli have been stabilized at higher energy scale
- How stabilized? Model dependent and difficult.

  e.g. stabilized on orbifolds, Ruehle & Wieck, 1503.07183

- Rudelius 1409.5793, considered the moduli dependence of the metric, but assumed straight geodesics…