

# Pushing the Frontiers of Perturbative QCD

Jesse Diaz Thaler

(Contrary to Indico,  
Kempner → Diaz in 2007)



Pheno 2015, University of Pittsburgh — May 5, 2015

# Pushing the Frontiers of Perturbative QCD

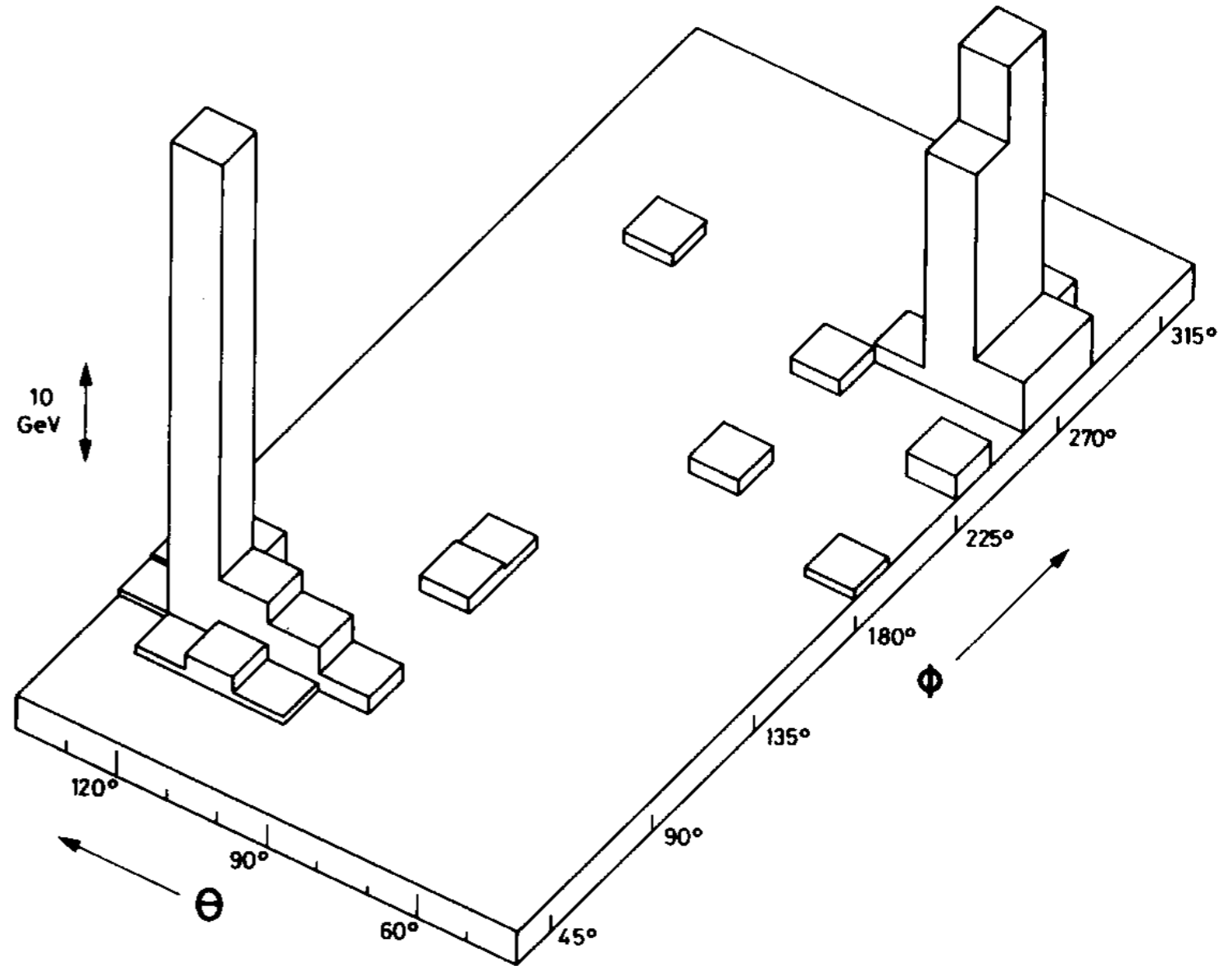
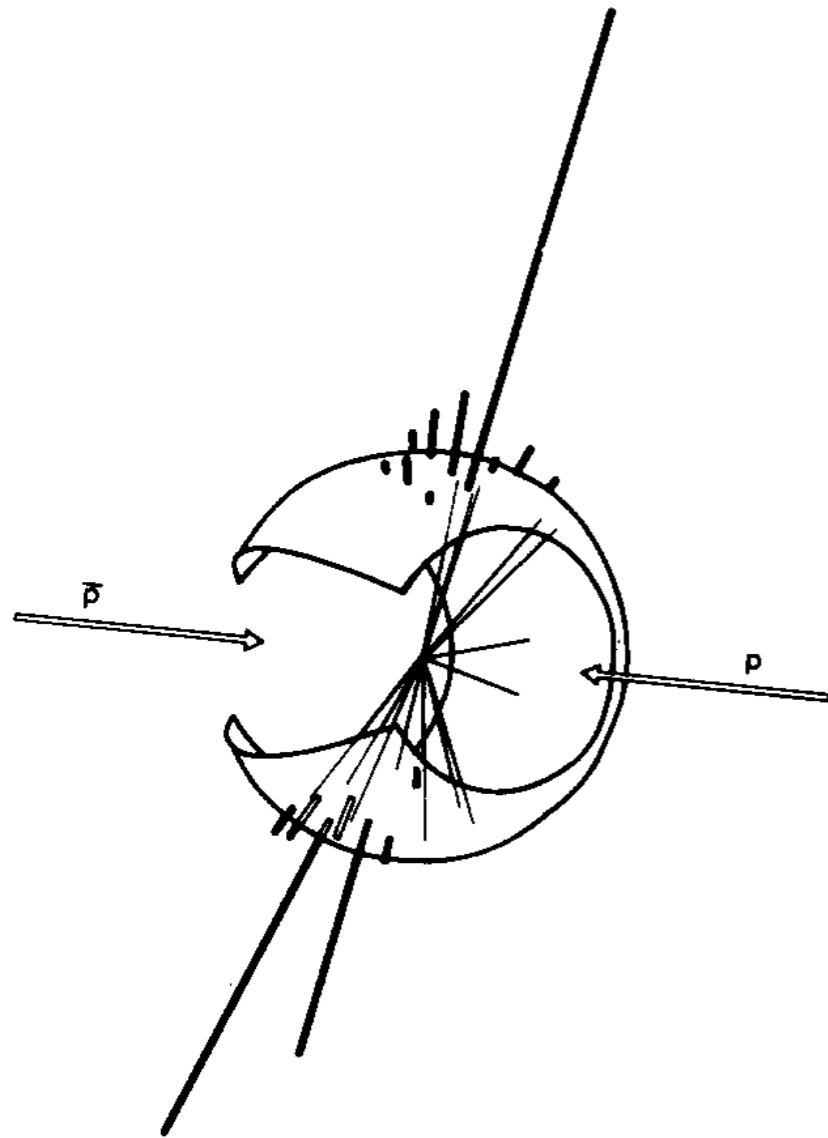
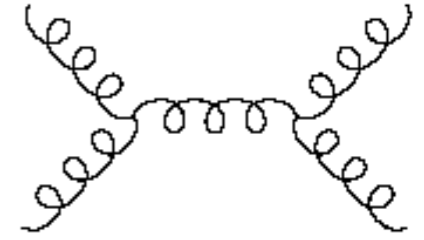
Has realized: “Jets!”



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# Four Decades of Jets and pQCD

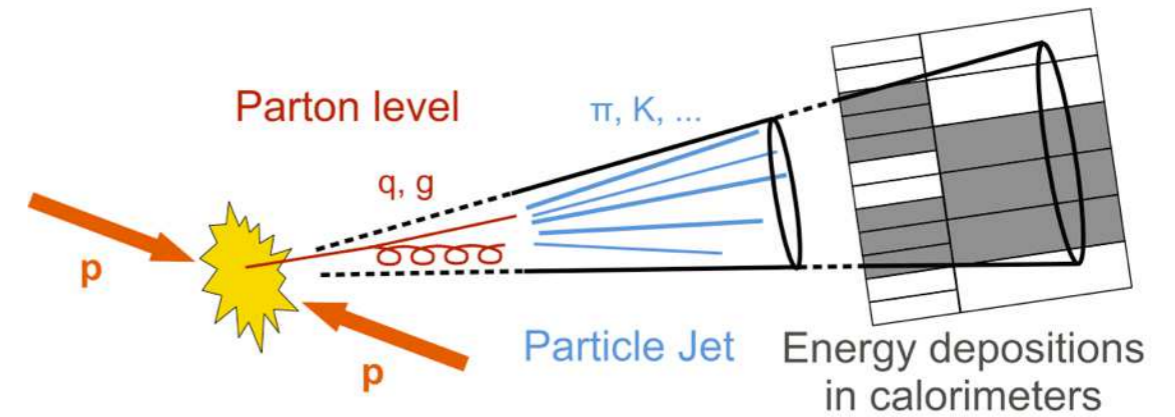
UA2, 1982



[see also SPEAR, 1975; PETRA, 1979]

# A QCD Renaissance

c. 2008–present

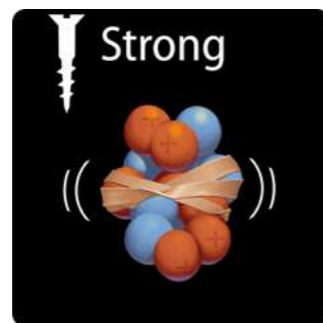


## LHC (vs. Tevatron)

Higher Energy ( $\approx \times 3.5-7$ )

Higher Luminosity ( $\approx \times 10-20$ )

Finer Segmentation ( $\approx \times 5$ )



## Theoretical Progress

New Jet Algorithms (esp. anti- $k_T$ )

Loop/Leg/Log Explosion

Jet Substructure

[Anti- $k_T$ : Cacciari, Salam, Soyez, 2008]

[BDRS: Butterworth, Davison, Rubin, Salam, 2008; see also Seymour, 1991, 1994]

# Challenging the Standard Lore

*c. 2012–present*

		IRC Safe	IRC Unsafe
Old Lore:	Calculable in pQCD?	✓	✗
	Controlled $\Lambda_{\text{QCD}}$ Effects?	✓	✗

# Challenging the Standard Lore

*c. 2012–present*

New Lore:

Calculable in pQCD?  
Controlled  $\Lambda_{\text{QCD}}$  Effects?

IRC Safe



IRC Unsafe



# Challenging the Standard Lore

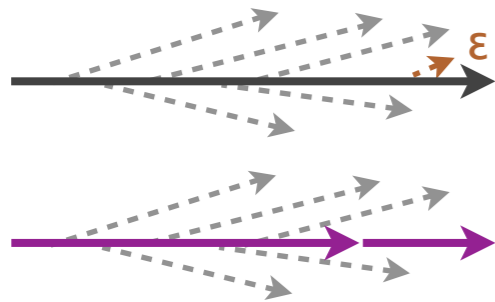
*c. 2012–present*

		IRC Safe	IRC Unsafe
New Lore:	Calculable in pQCD?	✓	✓*
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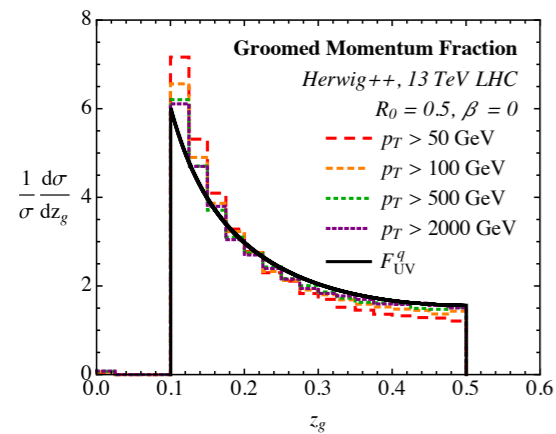
*All observables are calculable, but some observables are more calculable than others.*

*≈ George Orwell, Animal Farm*

# Outline



## Review of Safe Observables



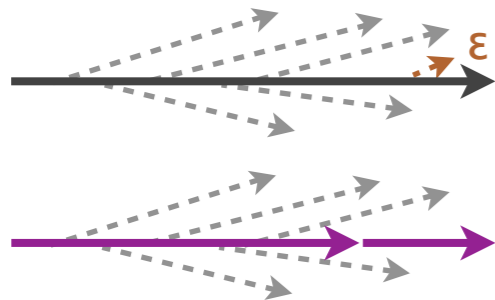
## A Standard Candle for Jets

$$\frac{d\sigma}{d\mathcal{O}_{\text{unsafe}}}$$

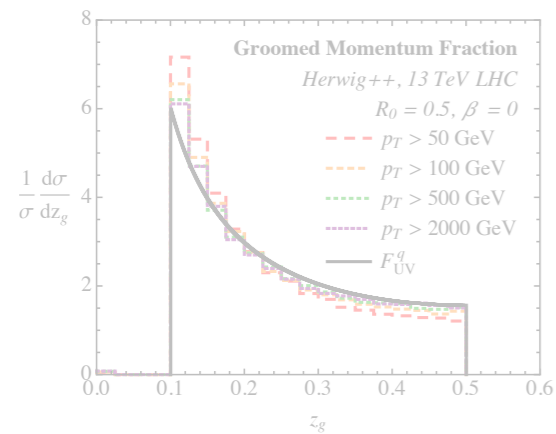
## Two New Tools for pQCD



# Review of Safe Observables



## A Standard Candle for Jets

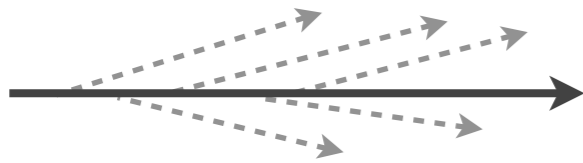


## Two New Tools for pQCD

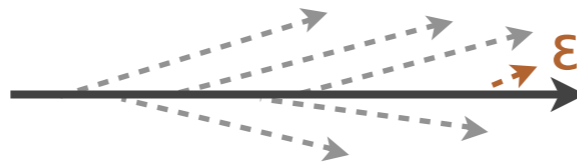
$$\frac{d\sigma}{d\mathcal{O}_{\text{unsafe}}}$$

# Infrared/Collinear Safety

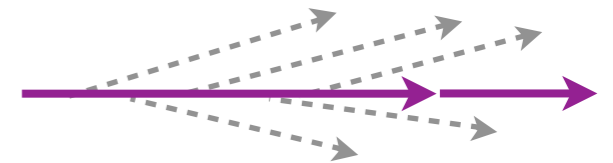
Original Jet



Infrared



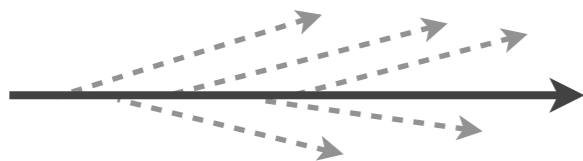
Collinear



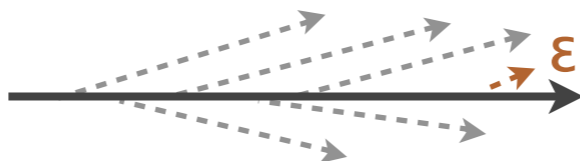
**IRC Safe Observable:** Insensitive to **IR** or **C** emissions

# Infrared/Collinear Safety

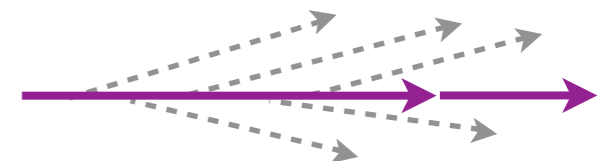
Original Jet



Infrared



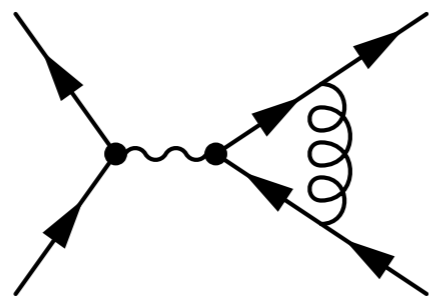
Collinear



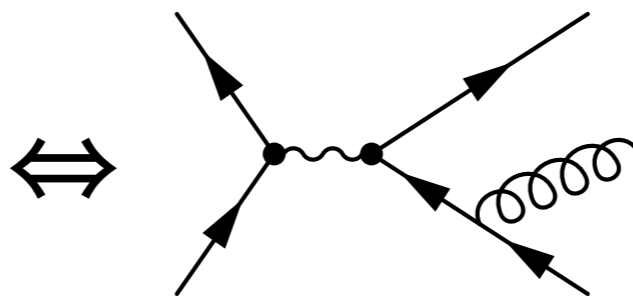
IRC Safe Observable: Insensitive to **IR** or **C** emissions

Formally:

Virtual Diagrams



$\Sigma$  Real Emissions



IRC divergences cancel order-by-order in  $\alpha_s$

# Examples from Jet Substructure

Jet  $p_T$ :  $\sum_{i \in \text{jet}} p_{T,i}$  **IRC Safe**

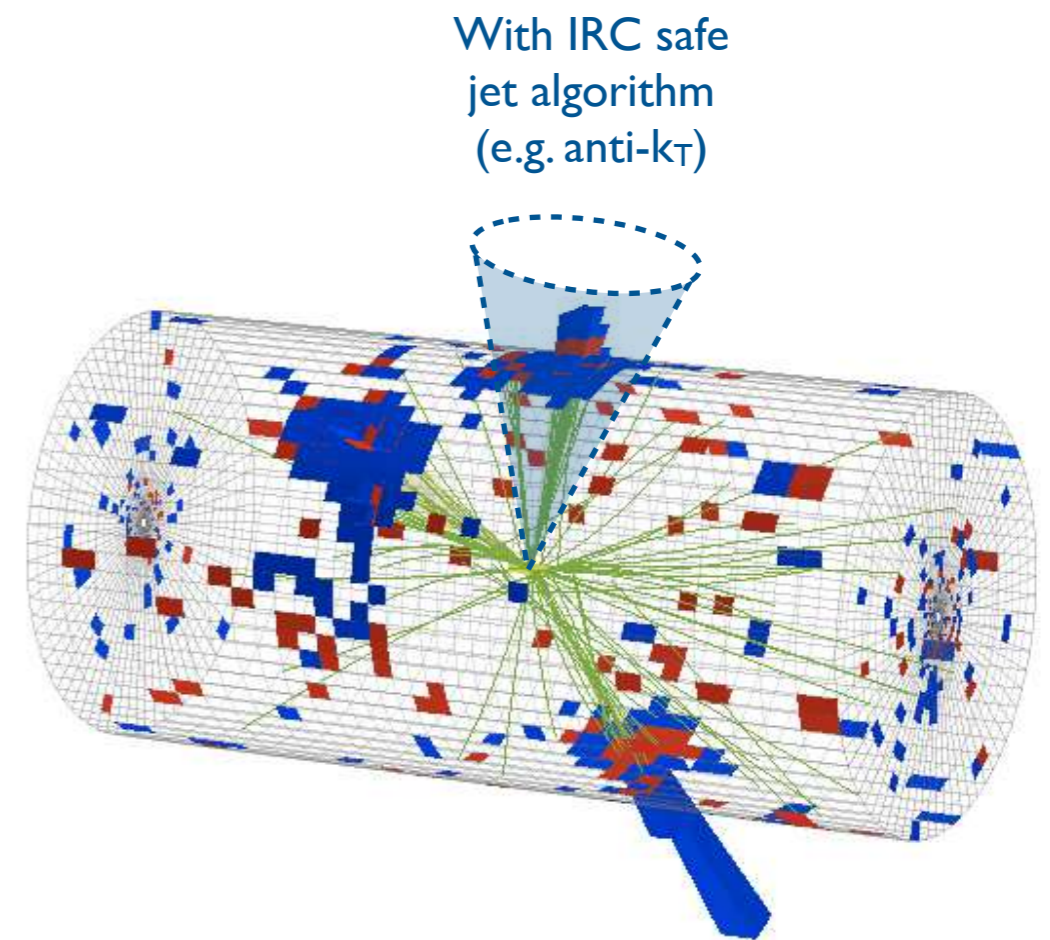
$p_T^D$ :  $\sum_{i \in \text{jet}} \frac{p_{T,i}^2}{p_{T,\text{jet}}^2}$  **IR Safe**  
**C Unsafe**  
 [CMS HIG-11-027]

Multiplicity:  $\sum_{i \in \text{jet}} 1$  **IRC Unsafe**

Jet Mass:  $\sum_{i,j \in \text{jet}} p_i \cdot p_j$  **IRC Safe**

N-subjettiness:  $\sum_{i \in \text{jet}} p_{T,i} \min \{ \Delta R_{i,1}, \Delta R_{i,2}, \dots, \Delta R_{i,N} \}^\beta$  **IRC Safe**

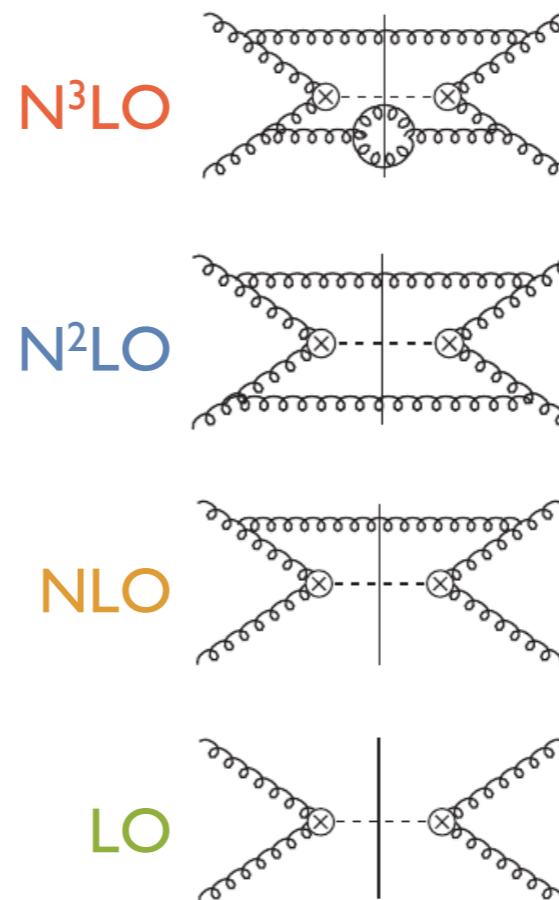
[JDT, Van Tilburg, 1011.2268, 1108.2701]



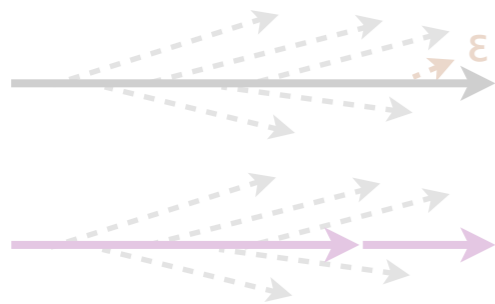
*All IRC safe observables are alike;  
each IRC unsafe observable is unsafe in its own way.*

≈ Leo Tolstoy, Anna Karenina

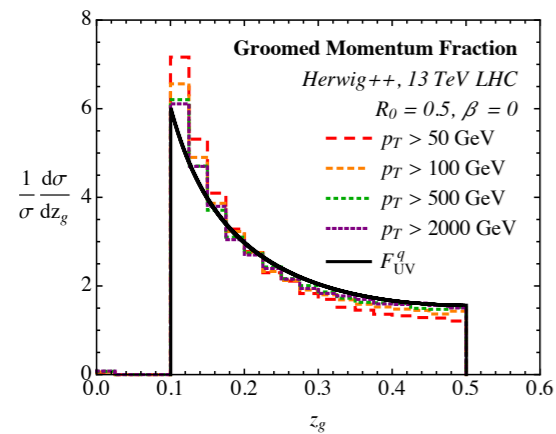
(see backup for  
recent precision  
fixed-order calculations)



# Review of Safe Observables



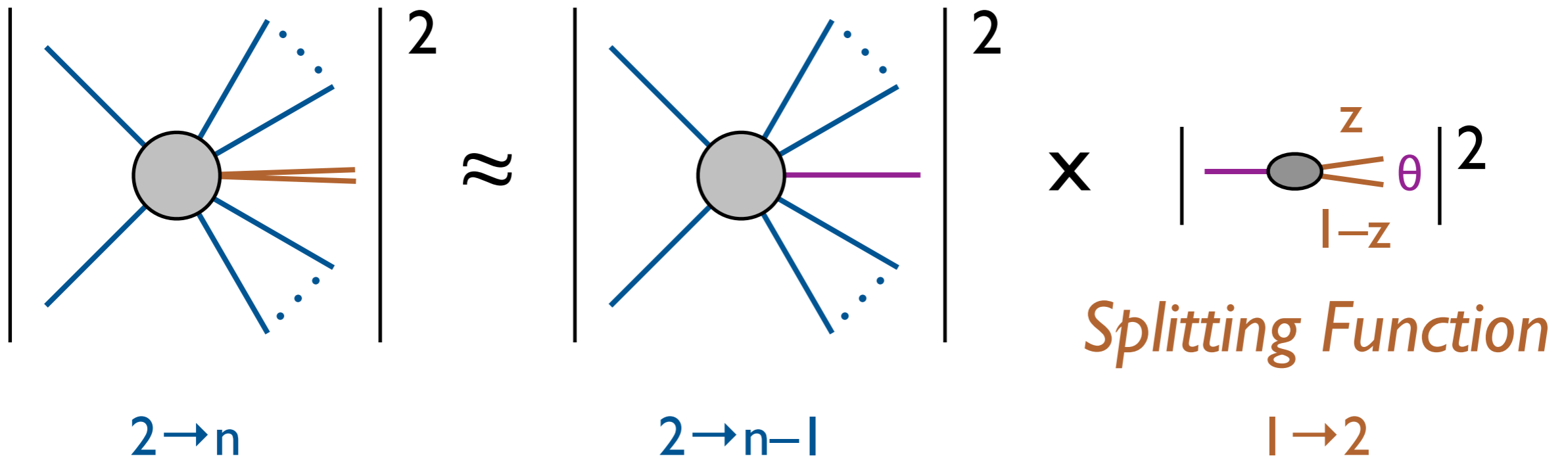
## A Standard Candle for Jets



## Two New Tools for pQCD

$$\frac{d\sigma}{d\mathcal{O}_{\text{unsafe}}}$$

# Textbook QCD



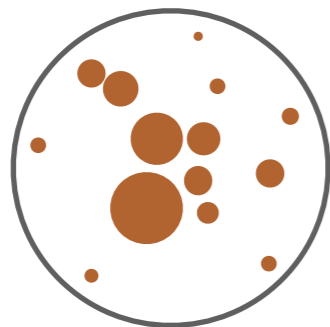
Basis of parton shower MC generators,  
PDF evolution, NLO subtractions, ...

**Measurable?**

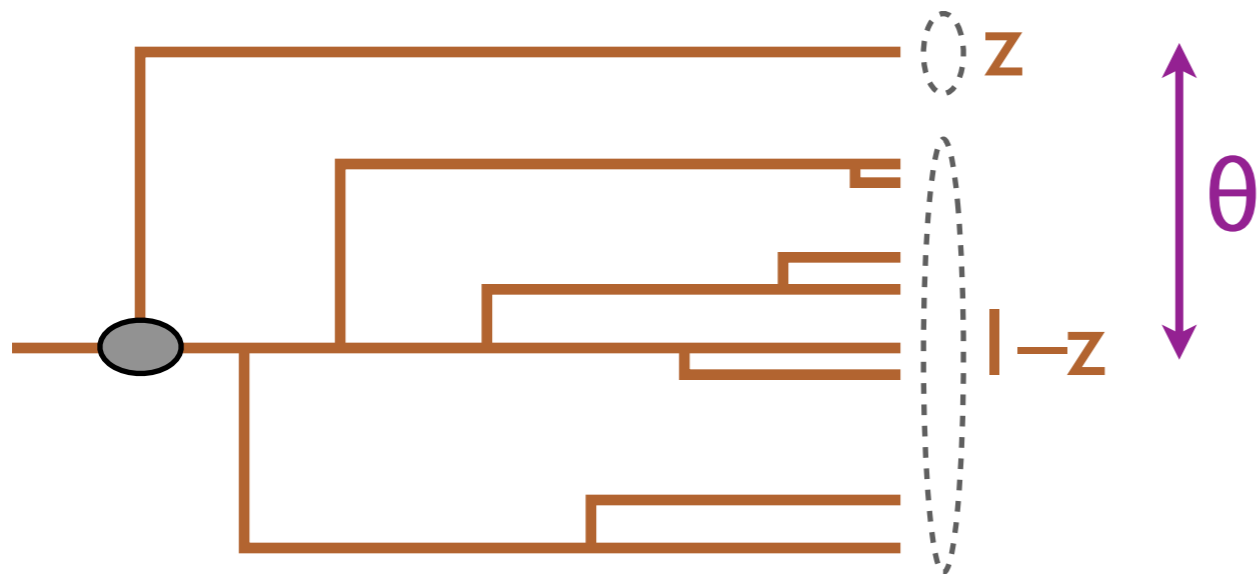
$$\int \underbrace{\frac{d\theta}{\theta}}_{\text{Collinear singularity}} \underbrace{dz P(z)}_{\text{Soft singularity}}$$

$$P(z) \simeq \frac{1}{z}$$

# Measure Universal Singularity?

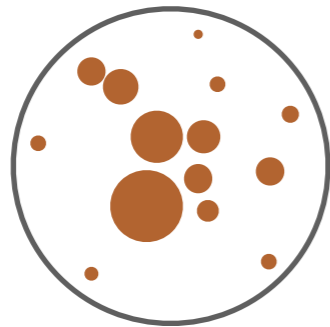


Angular-ordered tree...

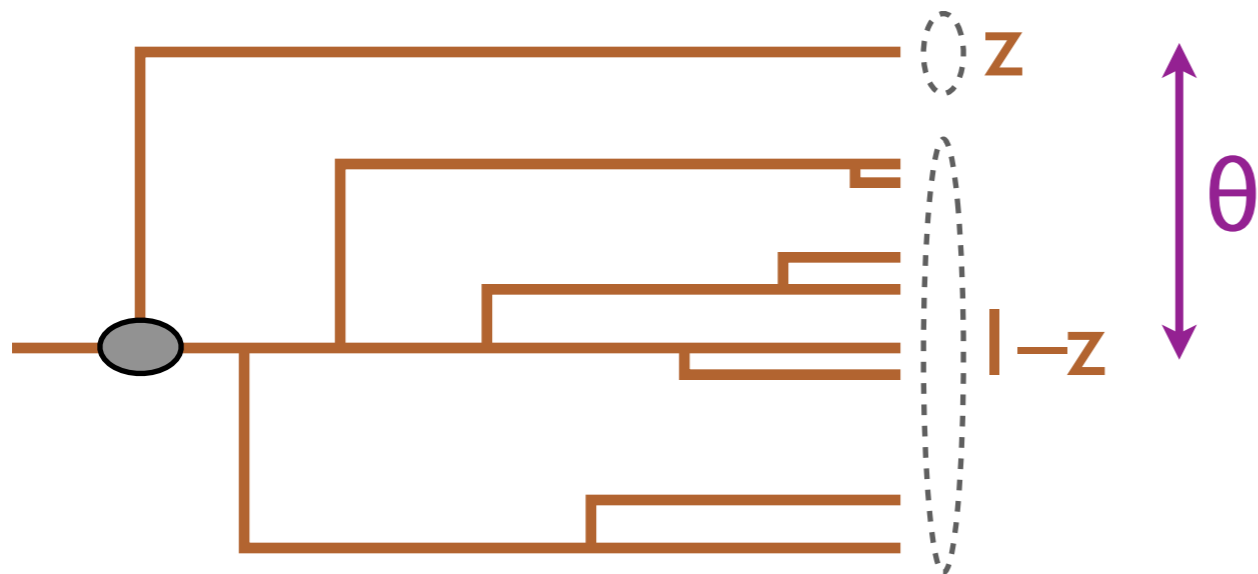




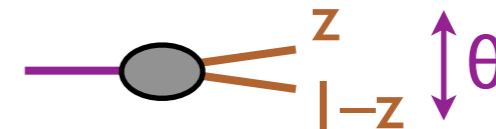
# Measure Universal Singularity?



Angular-ordered tree...



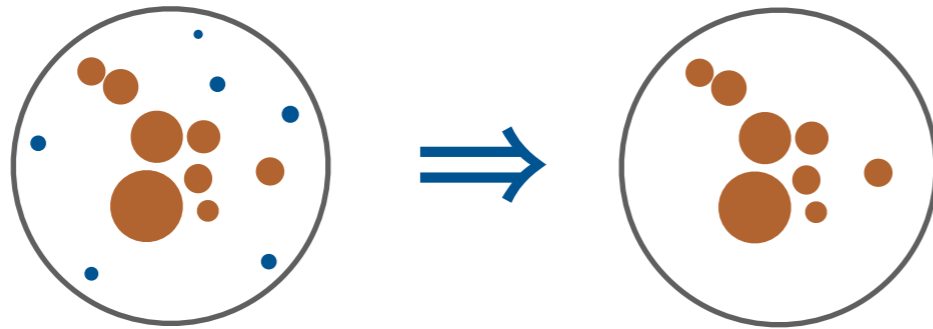
...gives splitting function?



$$\int \frac{d\theta}{\theta} dz P(z)$$

**Z** IRC Unsafe

# Measure Universal Singularity?

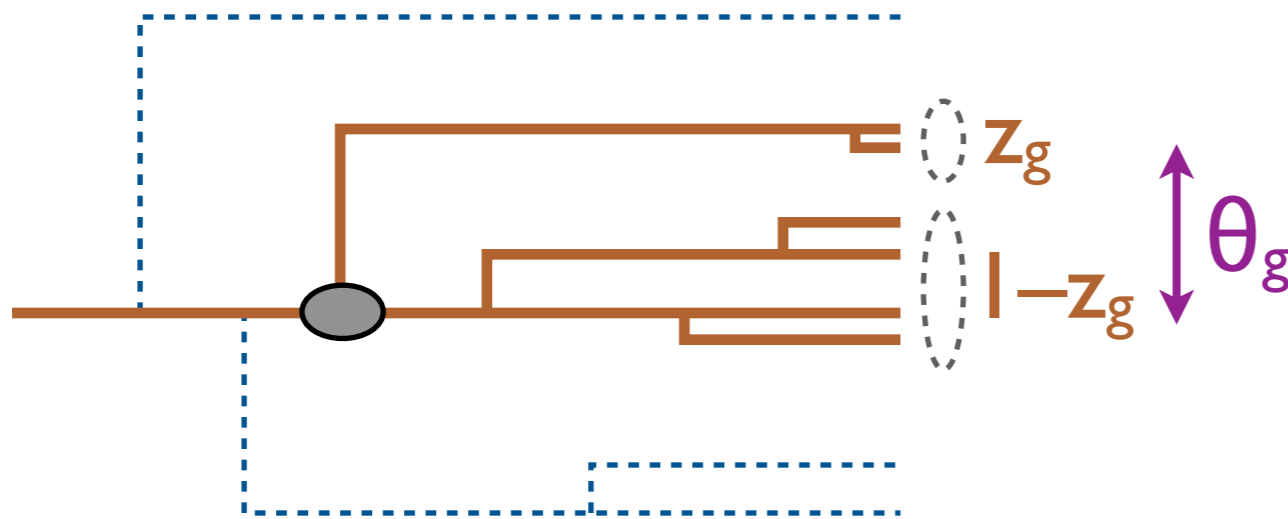


Soft Drop ( $\beta = 0$ )

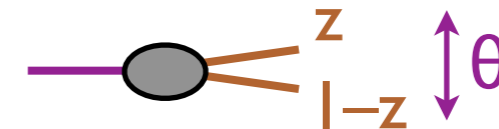
$$z > z_{\text{cut}}$$

↑  
energy  
threshold

Groomed angular-ordered tree...



...gives splitting function?



$$\int \frac{d\theta}{\theta} dz P(z)$$

$z_g$  IR Safe  
C Unsafe

[Larkoski, Marzani, Soyez, JDT, 1402.2657]

[see also Butterworth, Davison, Rubin, Salam, 0802.2470; Dasgupta, Fregoso, Marzani, Salam, 1307.0007]

# Measure Universal Singularity?



Soft Drop ( $\beta = 0$ )

$$z > z_{\text{cut}}$$

↑  
energy  
threshold

One prong jet...



$$\theta_g = 0$$

vs.

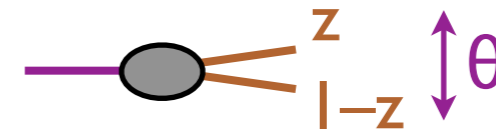


$z_g$

$1-z_g$

$$\theta_g = 0$$

...gives splitting function?

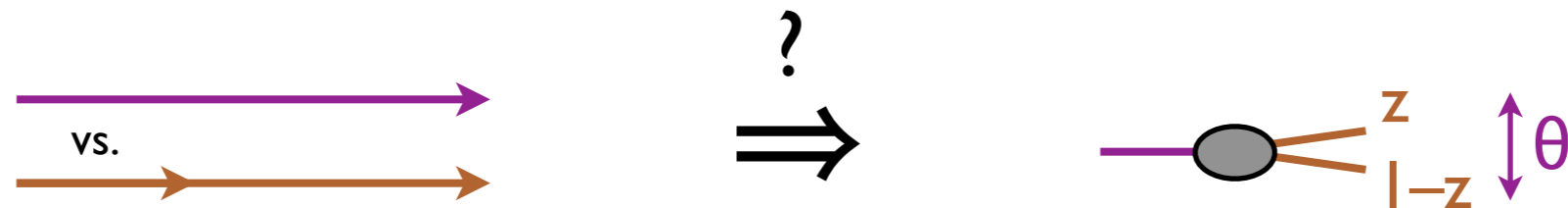


$$\int \frac{d\theta}{\theta} dz P(z)$$

$z_g$  IR Safe  
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[Larkoski, Marzani, Soyez, JDT, 1402.2657]

[see also Butterworth, Davison, Rubin, Salam, 0802.2470; Dasgupta, Fregoso, Marzani, Salam, 1307.0007]



$Z_g$  IR Safe  
C Unsafe

## *How to calculate from first principles?*

(see backup for how our elders addressed this in 1978)

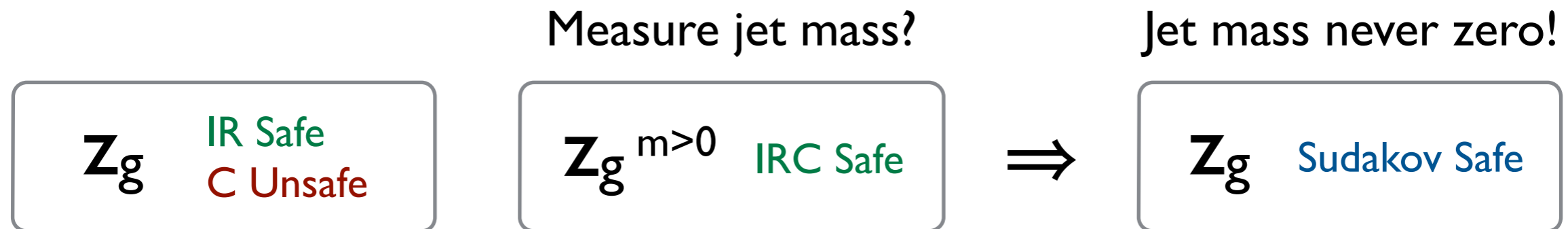
# I. Use Sudakov Form Factors

Measure jet mass?

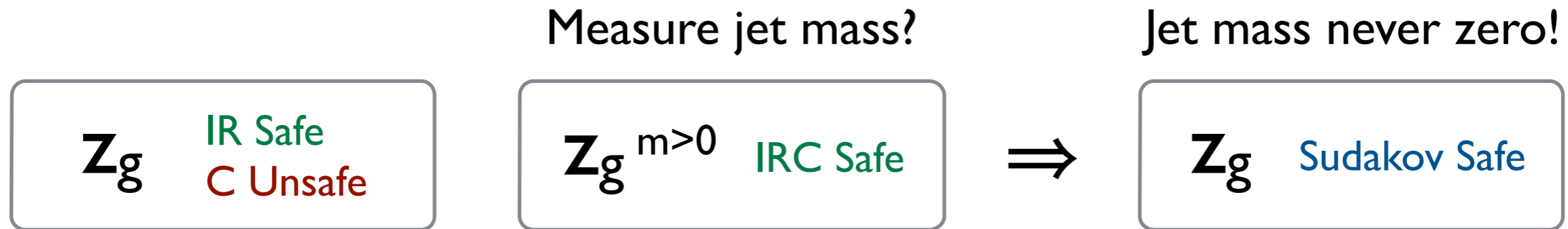
$Z_g$  IR Safe  
C Unsafe

$Z_g^{m>0}$  IRC Safe

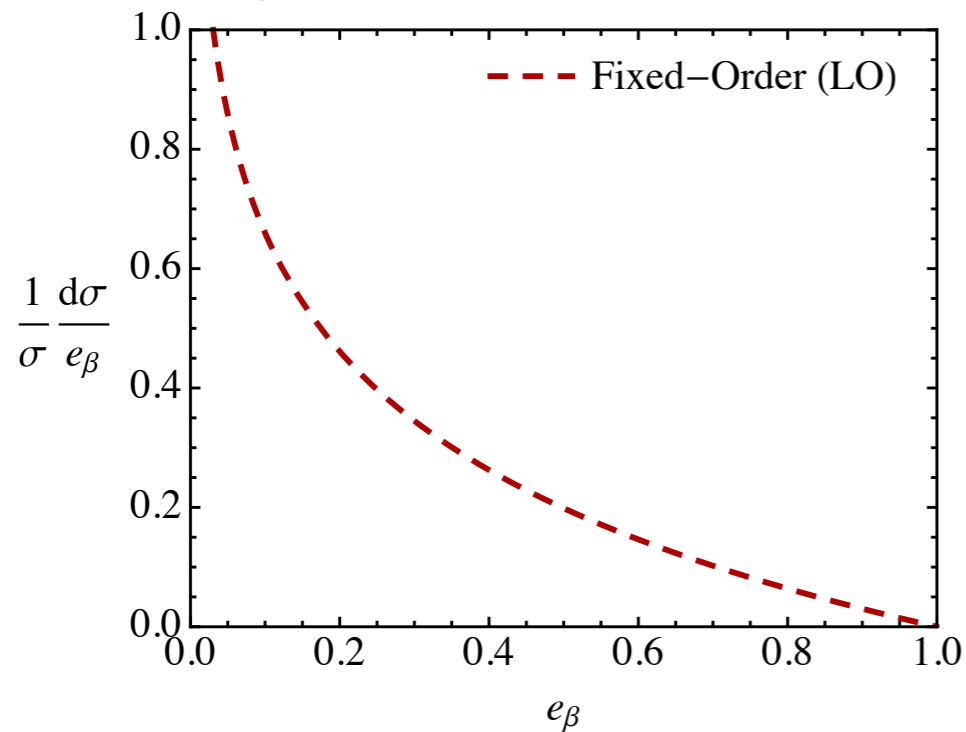
# I. Use Sudakov Form Factors



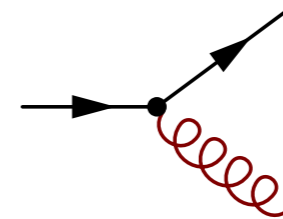
# I. Use Sudakov Form Factors



Jet mass distribution

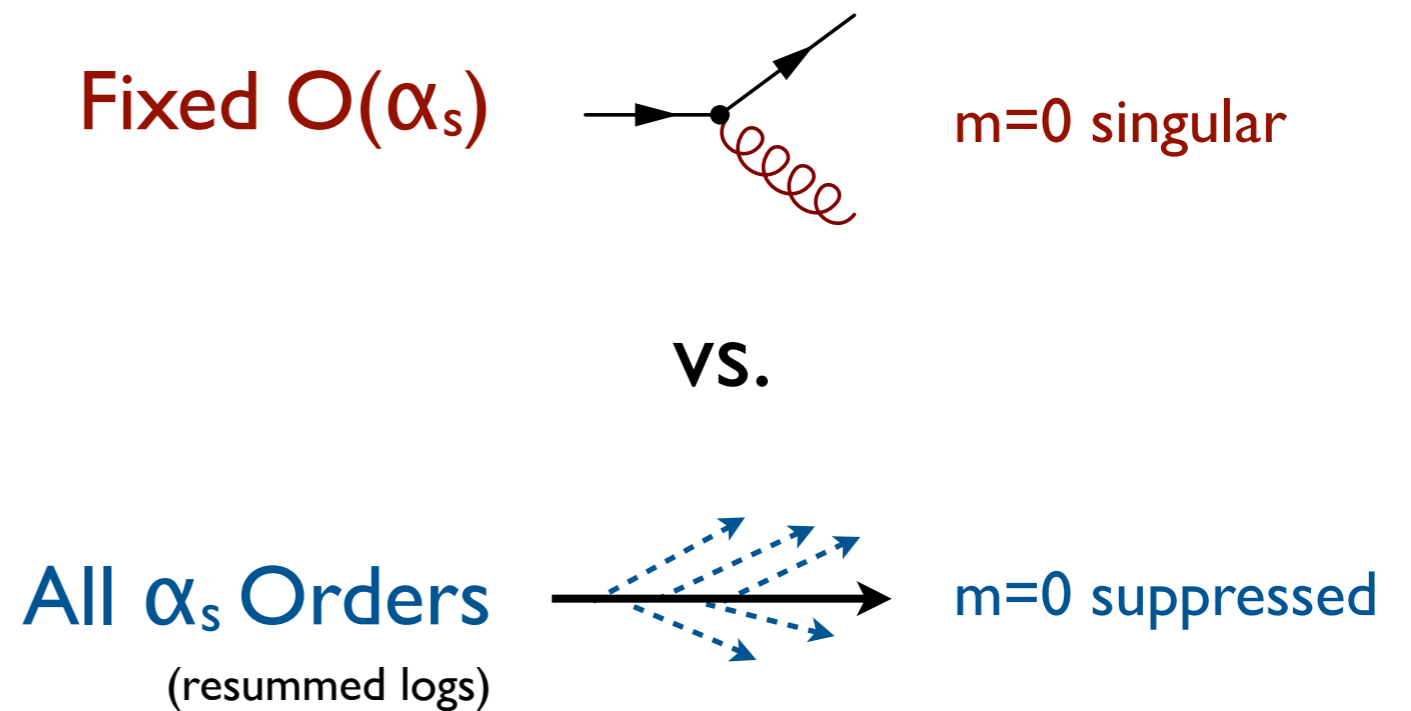
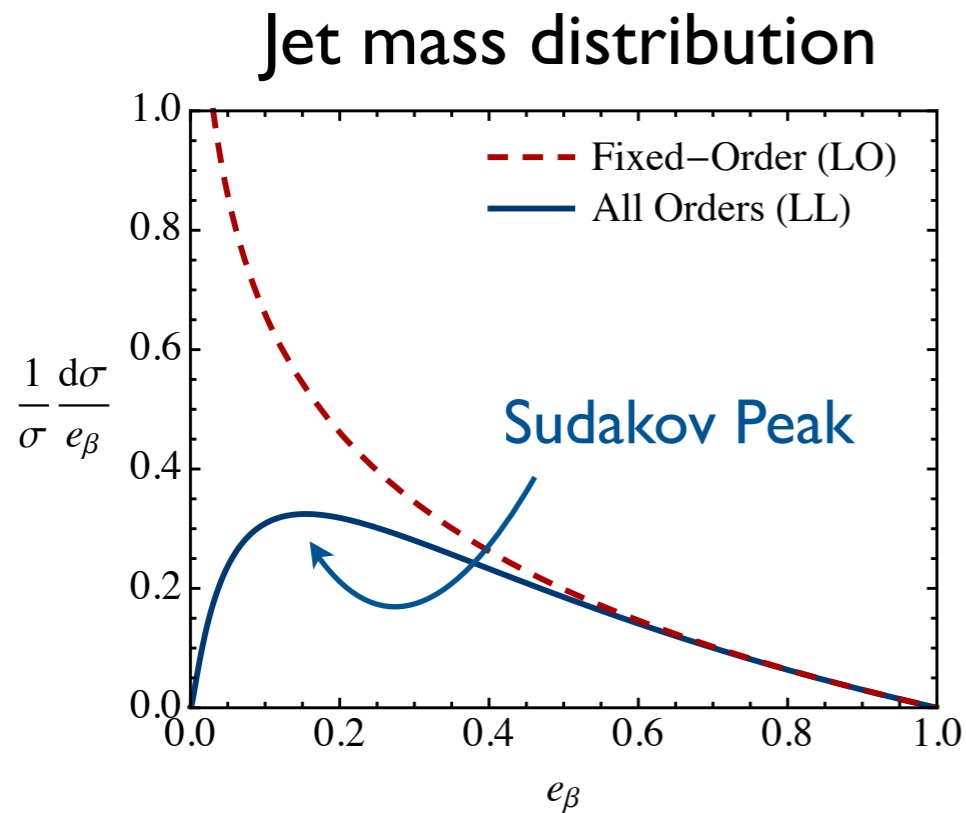
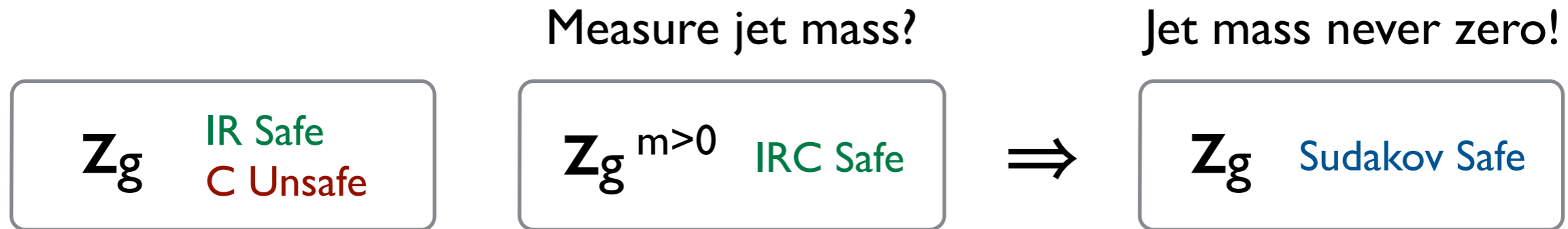


Fixed  $O(\alpha_s)$



$m=0$  singular

# I. Use Sudakov Form Factors



[Larkoski, JDT, 1307.1699]



## 2. Use Fragmentation Functions

$z_g$

IR Safe

C Unsafe

Absorb singularities  
into universal function  
(just like PDFs!)

$$\frac{d\sigma}{dz_g} \simeq F(z_g)$$



## 2. Use Fragmentation Functions

$z_g$  IR Safe  
C Unsafe

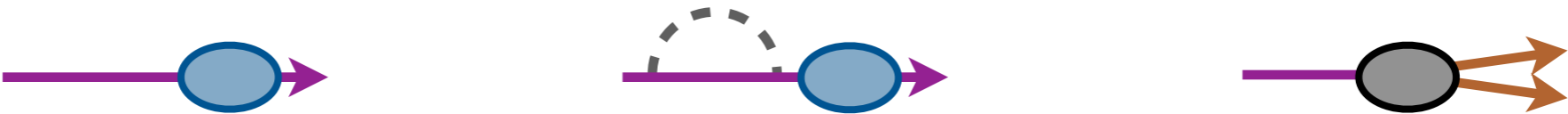
Absorb singularities  
into universal function  
(just like PDFs!)

$$\frac{d\sigma}{dz_g} \approx F(z_g) - \frac{1}{2\epsilon} \frac{\alpha_s C}{\pi} F(z_g) + \frac{\alpha_s C}{\pi} \int \frac{d\theta}{\theta} P(z_g)$$

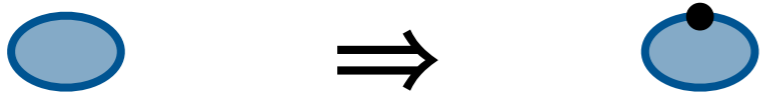
## 2. Use Fragmentation Functions

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$$\frac{d\sigma}{dz_g} \simeq F(z_g) - \frac{1}{2\epsilon} \frac{\alpha_s C}{\pi} F(z_g) + \frac{\alpha_s C}{\pi} \int \frac{d\theta}{\theta} P(z_g)$$


renormalize

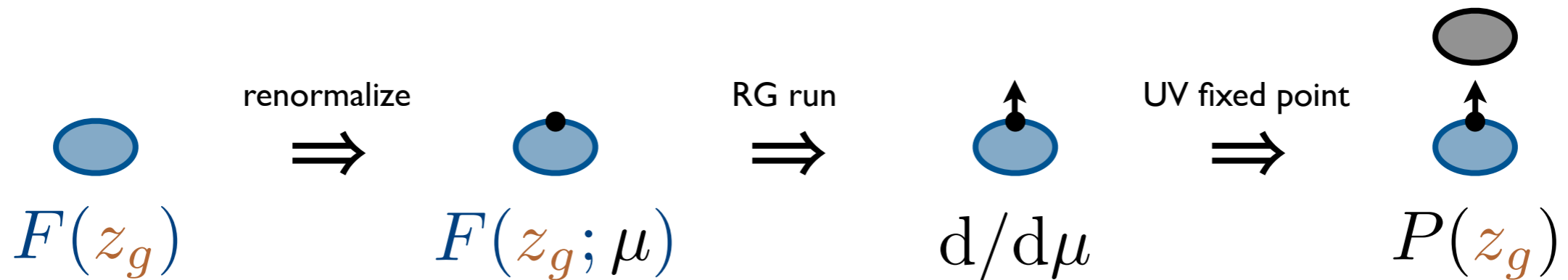
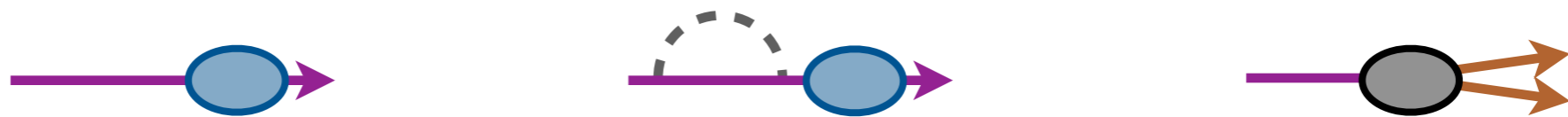
$$F(z_g) \Rightarrow F(z_g; \mu)$$


# 2. Use Fragmentation Functions

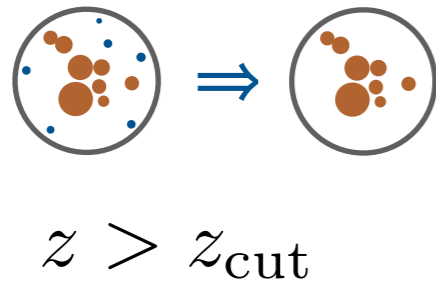
$z_g$  IR Safe  
C Unsafe

Absorb singularities into universal function (just like PDFs!)

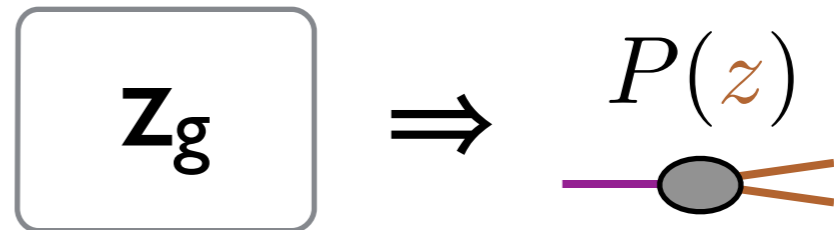
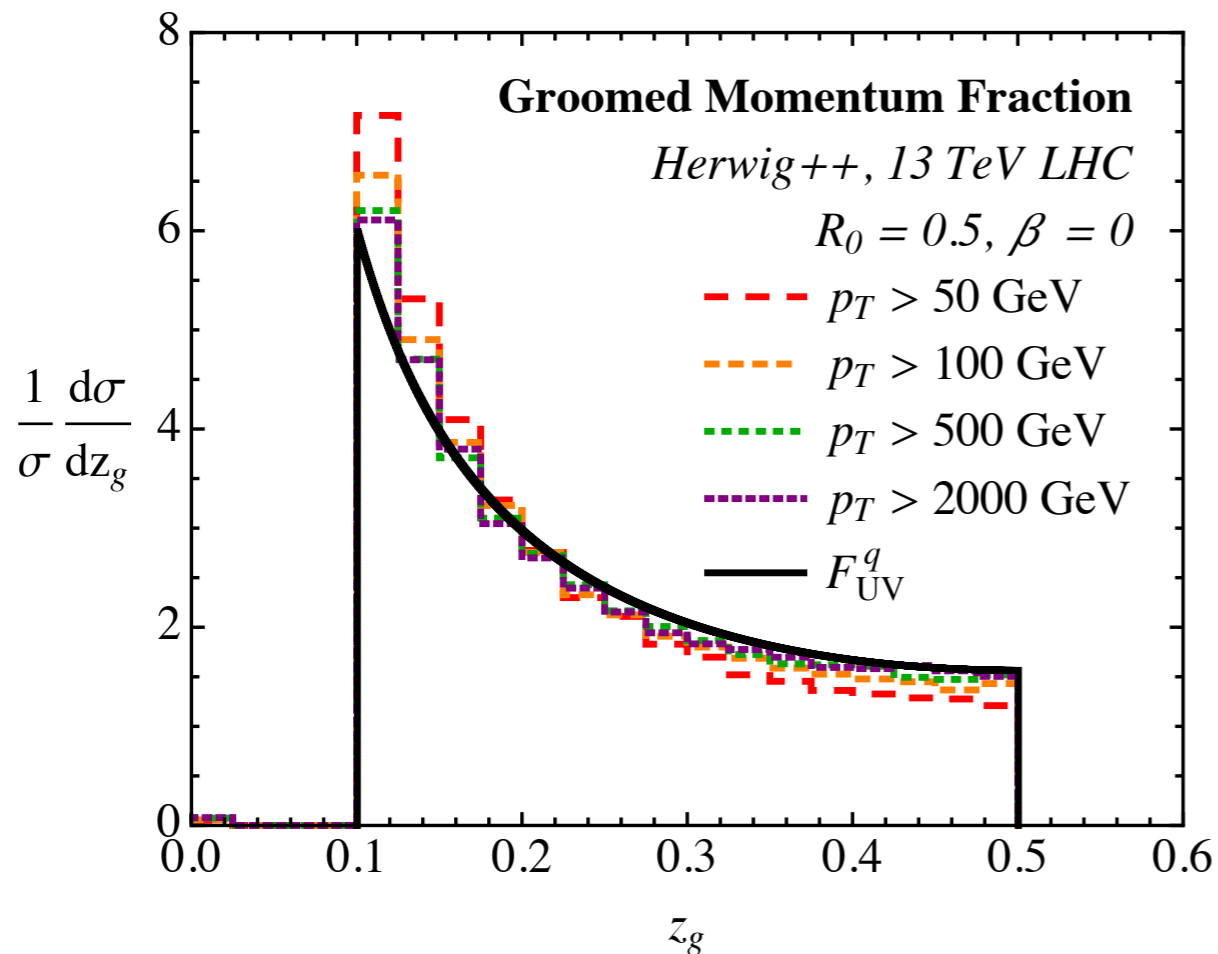
$$\frac{d\sigma}{dz_g} \simeq F(z_g) - \frac{1}{2\epsilon} \frac{\alpha_s C}{\pi} F(z_g) + \frac{\alpha_s C}{\pi} \int \frac{d\theta}{\theta} P(z_g)$$



# A Standard Candle for Jets



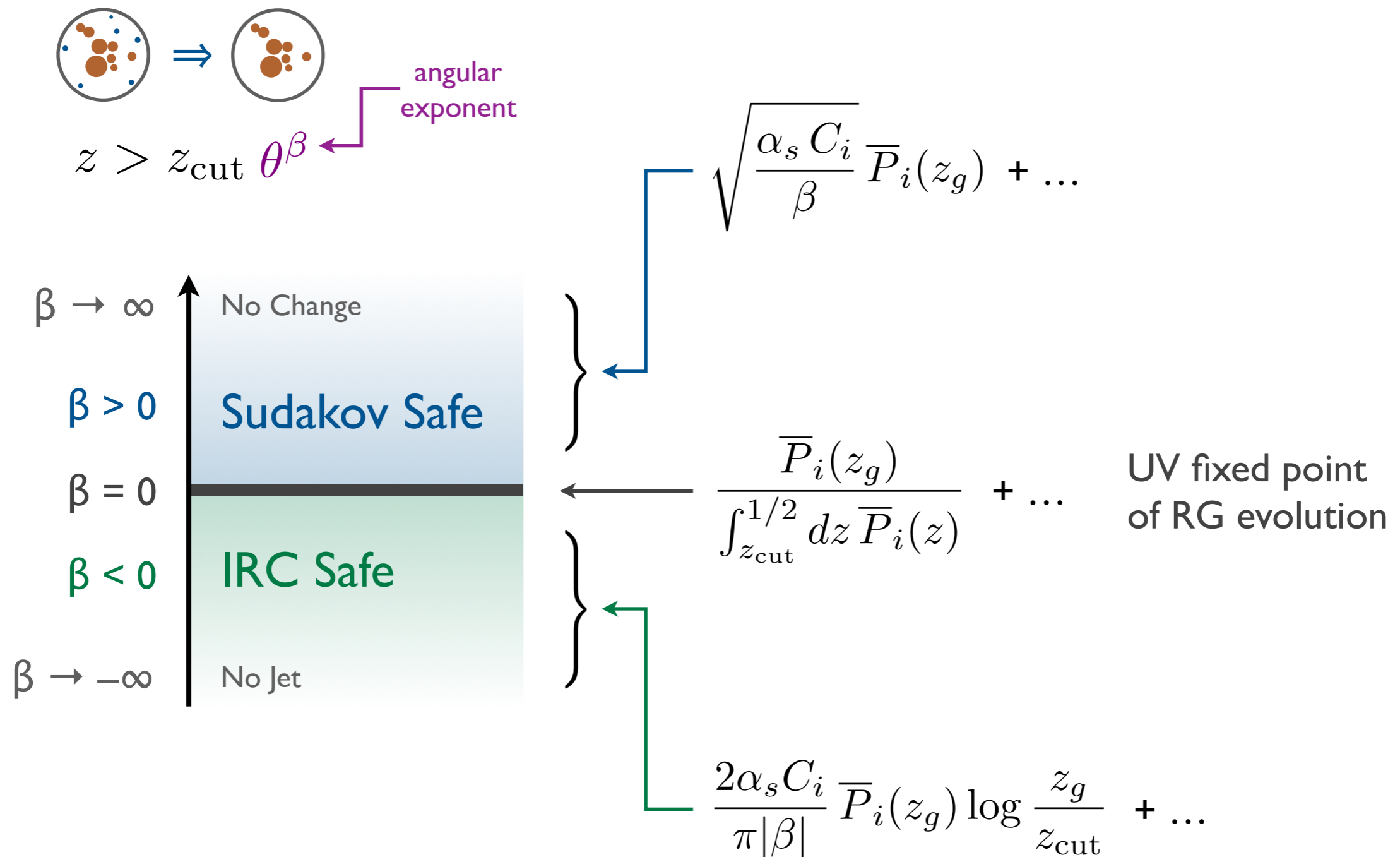
$$\frac{1}{\sigma} \frac{d\sigma}{dz_g} = \frac{\bar{P}_i(z_g)}{\int_{z_{\text{cut}}}^{1/2} dz \bar{P}_i(z)} + \dots$$

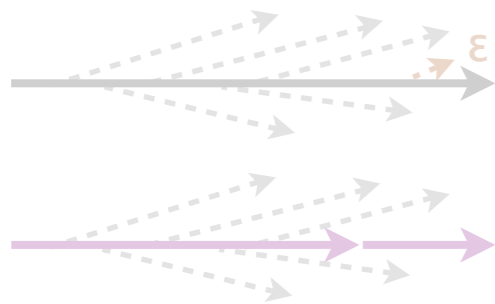


- $\approx$  independent of  $\alpha_s$  (!)
- $\approx$  independent of jet  $p_T$  and radius
- $\approx$  same for quarks and gluons

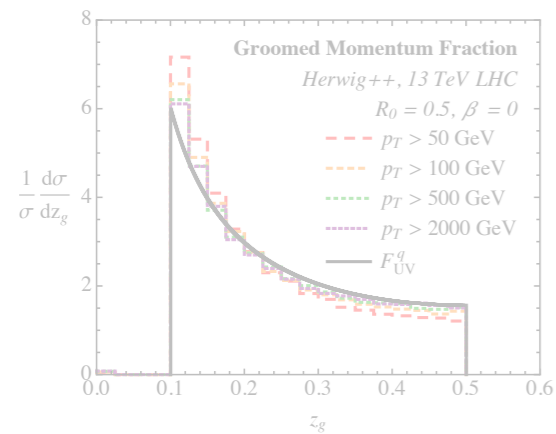
calculable deviations from universality

# “Phase Diagram” for Observables





## Review of Safe Observables



## A Standard Candle for Jets

$$\frac{d\sigma}{d\mathcal{O}_{\text{unsafe}}}$$

## Two New Tools for pQCD

# I. Isolated Singularity? *Try Sudakov Form Factor*

Unsafe  
↓  
Want:  $p(u) = \frac{1}{\sigma} \frac{d\sigma}{du}$



# I. Isolated Singularity? *Try Sudakov Form Factor*

Unsafe  
↓  
Want:  $p(u) = \frac{1}{\sigma} \frac{d\sigma}{du}$

Calculable...  
↓  
Need:  $p(u|s) = \frac{p(u, s)}{p(s)}$   
↑  
...with Safe companion

# I. Isolated Singularity? *Try Sudakov Form Factor*

**Unsafe**  
 ↓  
**Want:**  $p(u) = \frac{1}{\sigma} \frac{d\sigma}{du}$

**Calculable...**  
 ↓  
**Need:**  $p(u|s) = \frac{p(u, s)}{p(s)}$   
 ↑  
 ...with Safe companion

**Finite distribution**  
 ↓  
**Insight:**  $p(u) = \int ds p(s) p(u|s)$

↑  
**Sudakov form factor**  
 (all orders in  $\alpha_s$ )

↑  
**Perturbative**  
 (fixed order in  $\alpha_s$ )

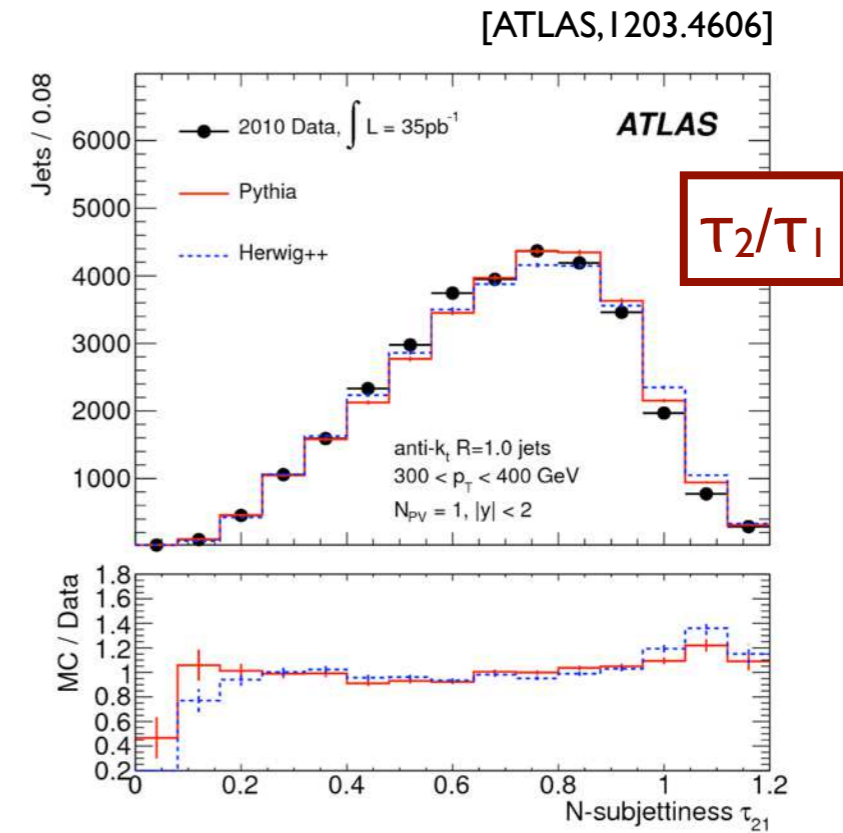
Suppresses isolated singularities...

...at each perturbative order

# Ratio Observables

Ubiquitous in jet substructure  
(e.g. N-subjettiness)

Fantastic data/MC agreement



IRC Safe Numerator

IRC Safe Denominator

singularity at zero...

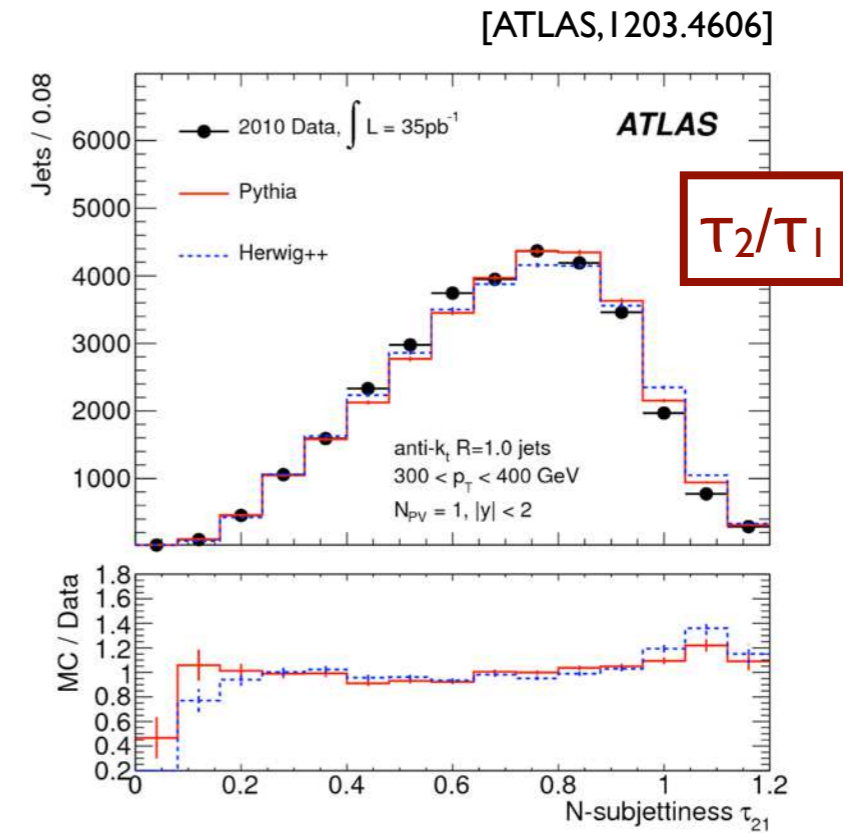
= IRC Unsafe Ratio

[Soyez, Salam, Kim, Dutta, Cacciari, I21 I.28 I ]

# Ratio Observables

Ubiquitous in jet substructure  
(e.g. N-subjettiness)

Fantastic data/MC agreement



IRC Safe Numerator

IRC Safe Denominator

singularity at zero...

...regulated by Sudakov

(in parton showers  
by construction)

= IRC Unsafe Ratio

[Soyez, Salam, Kim, Dutta, Cacciari, 1211.2811]

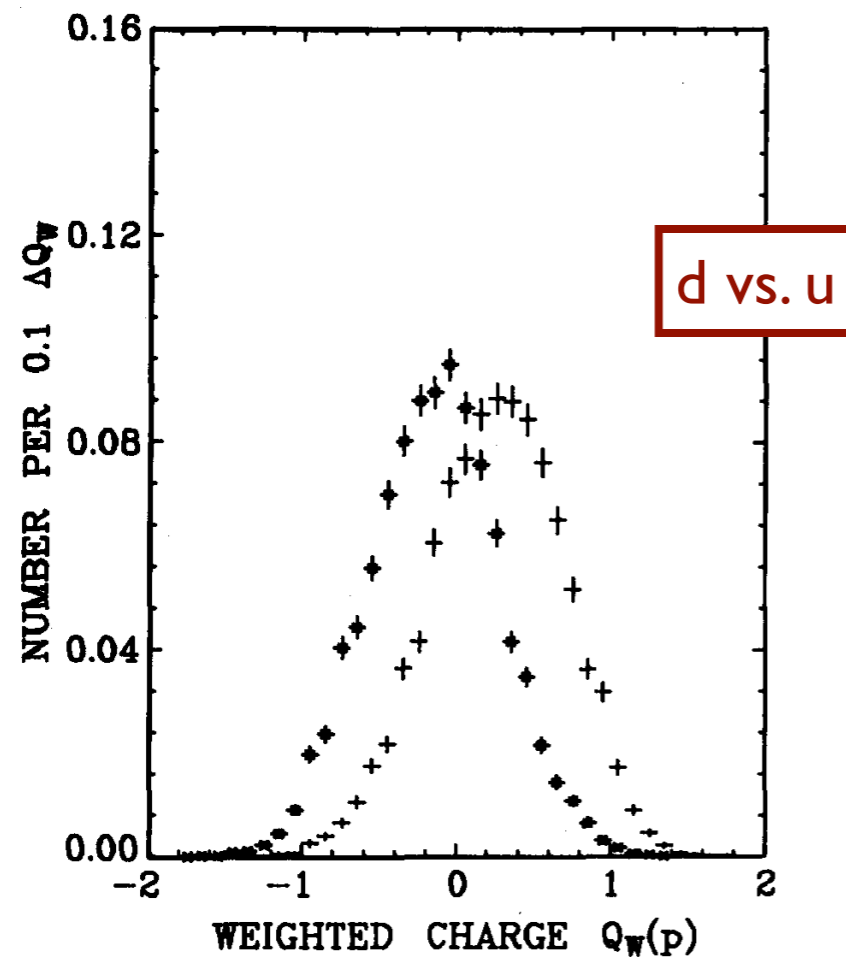
= Sudakov Safe Ratio

[Larkoski, JDT, 1307.1699;  
see also Larkoski, Mout, Neill, 1401.4458;  
Procura, Waalewijn, Zeune, 1410.6483]

## 2. Collinear Unsafe? *Try Fragmentation Function*

Weighted Jet Charge...

$$Q_W(p) = \sum_i z_i^p q_i$$

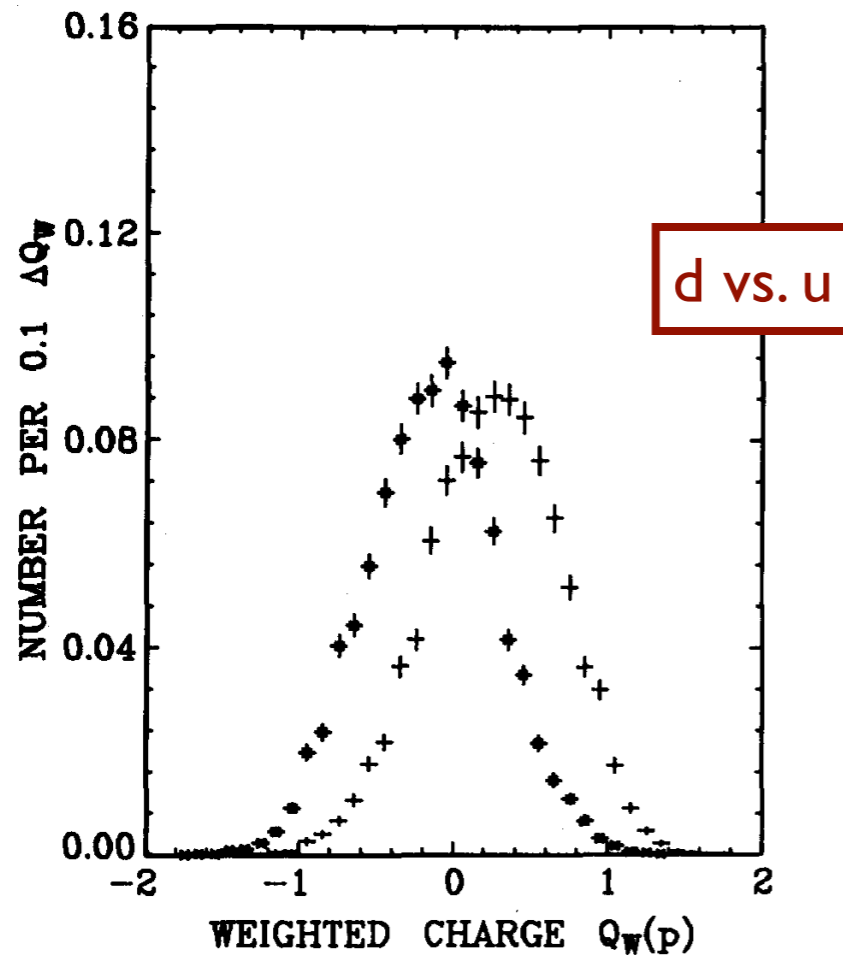


[Feynman, Field, 1978]

# 2. Collinear Unsafe? *Try Fragmentation Function*

Weighted Jet Charge...

$$Q_W(p) = \sum_i z_i^p q_i$$

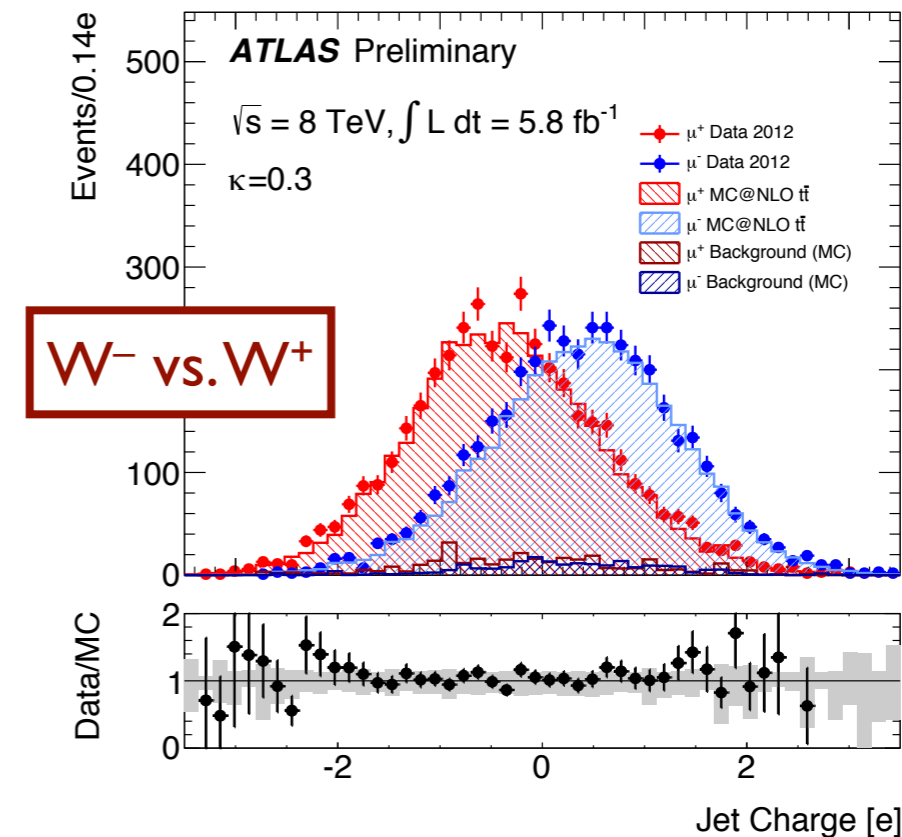


[Feynman, Field, 1978]

...on Firm Theoretical Ground

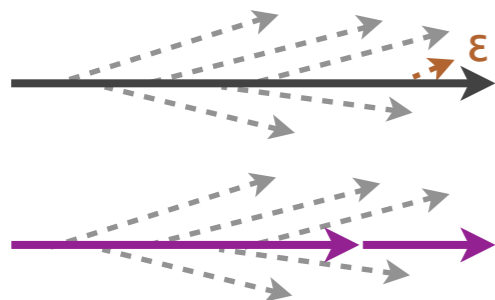
$$\mu \frac{d}{d\mu} D_i(Q, \kappa, \mu) = \frac{1}{2} \sum_j \int dQ_1 dQ_2 dz \gamma_{ij}^D(z, \mu) \times D_j(Q_1, \kappa, \mu) D_{a(ij)}(Q_2, \kappa, \mu) \times \delta[Q - z^\kappa Q_1 - (1-z)^\kappa Q_2]$$

(see backup for track function)



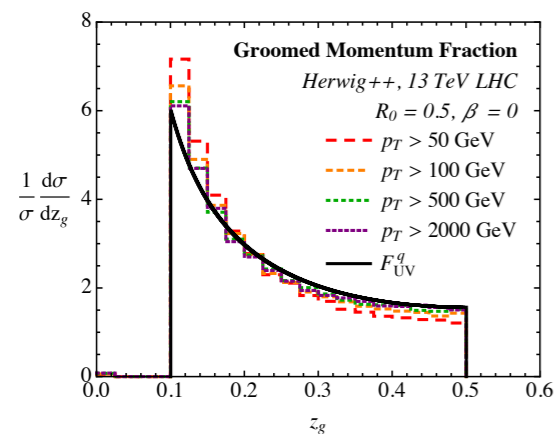
[Krohn, Schwartz, Lin, Waalewijn, 1209.2421; Waalewijn, 1209.3019] [see also Larkoski, JDT, Waalewijn, 1408.3122] [ATLAS-CONF-2013-086]

# Summary



## Review of Safe Observables

*Infrared and collinear safe observables defined order-by-order in  $\alpha_s$*   
*Successful (and growing) program of higher-order calculations*



## A Standard Candle for Jets

*New way to measure the universal singularity structure of QCD*  
*All orders in  $\alpha_s$  yields new insights into QFT*

$$\frac{d\sigma}{d\mathcal{O}_{\text{unsafe}}}$$

## Two New Tools for pQCD

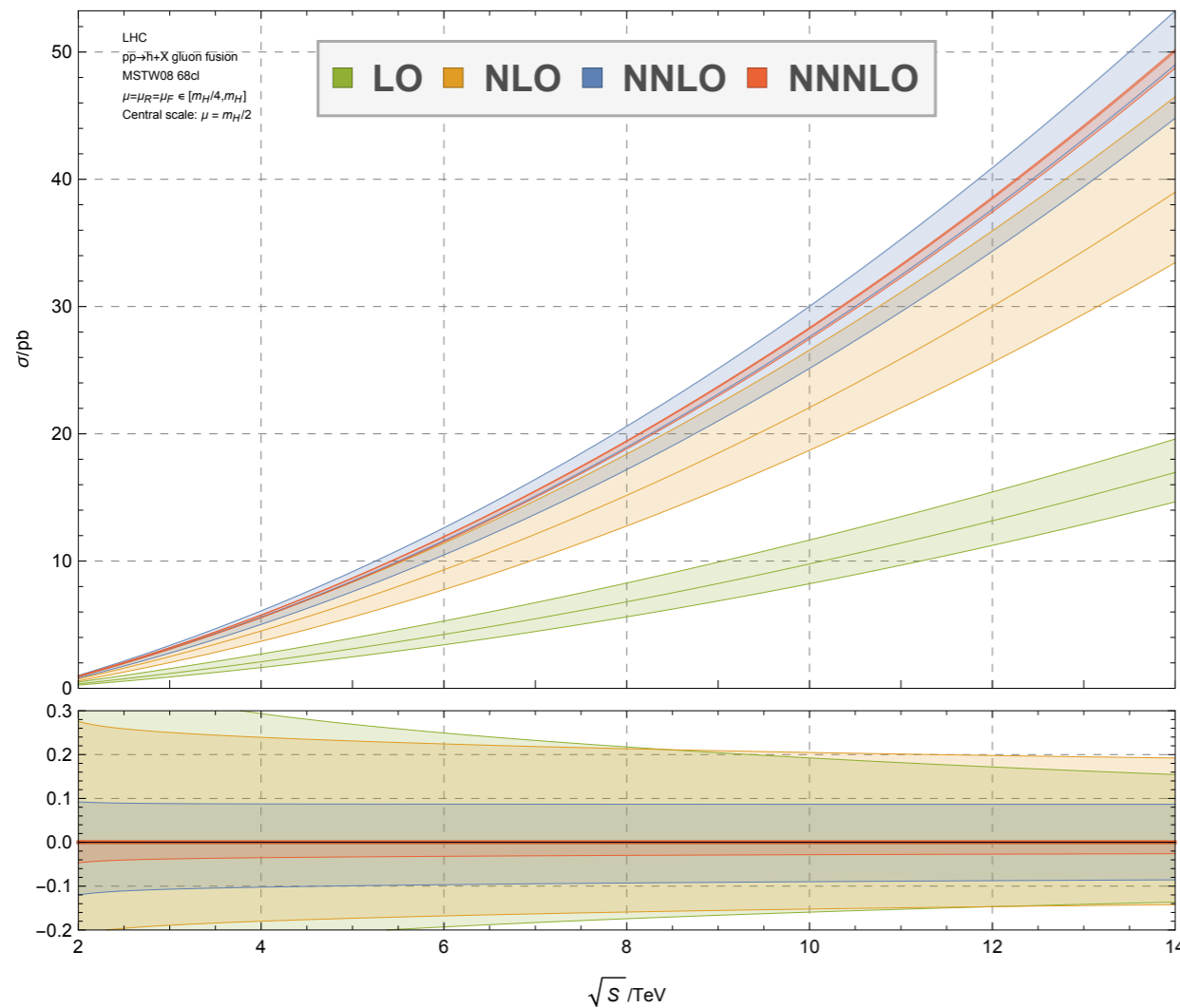
*Isolated singularity? Sudakov form factor! (e.g. Sudakov safety of ratios)*  
*Collinear unsafe? Fragmentation function! (e.g. jet charge, tracks,  $p_T^D$ )*

# *Backup Slides*

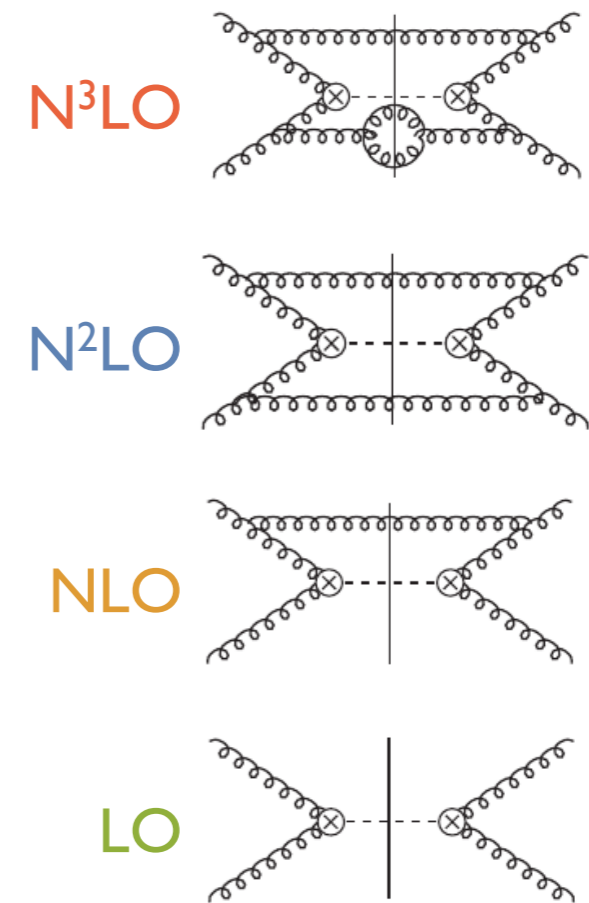


# Total Cross Sections

e.g. *Gluon Fusion to Higgs*



gg → H inclusive  
IRC Safe (with PDFs)



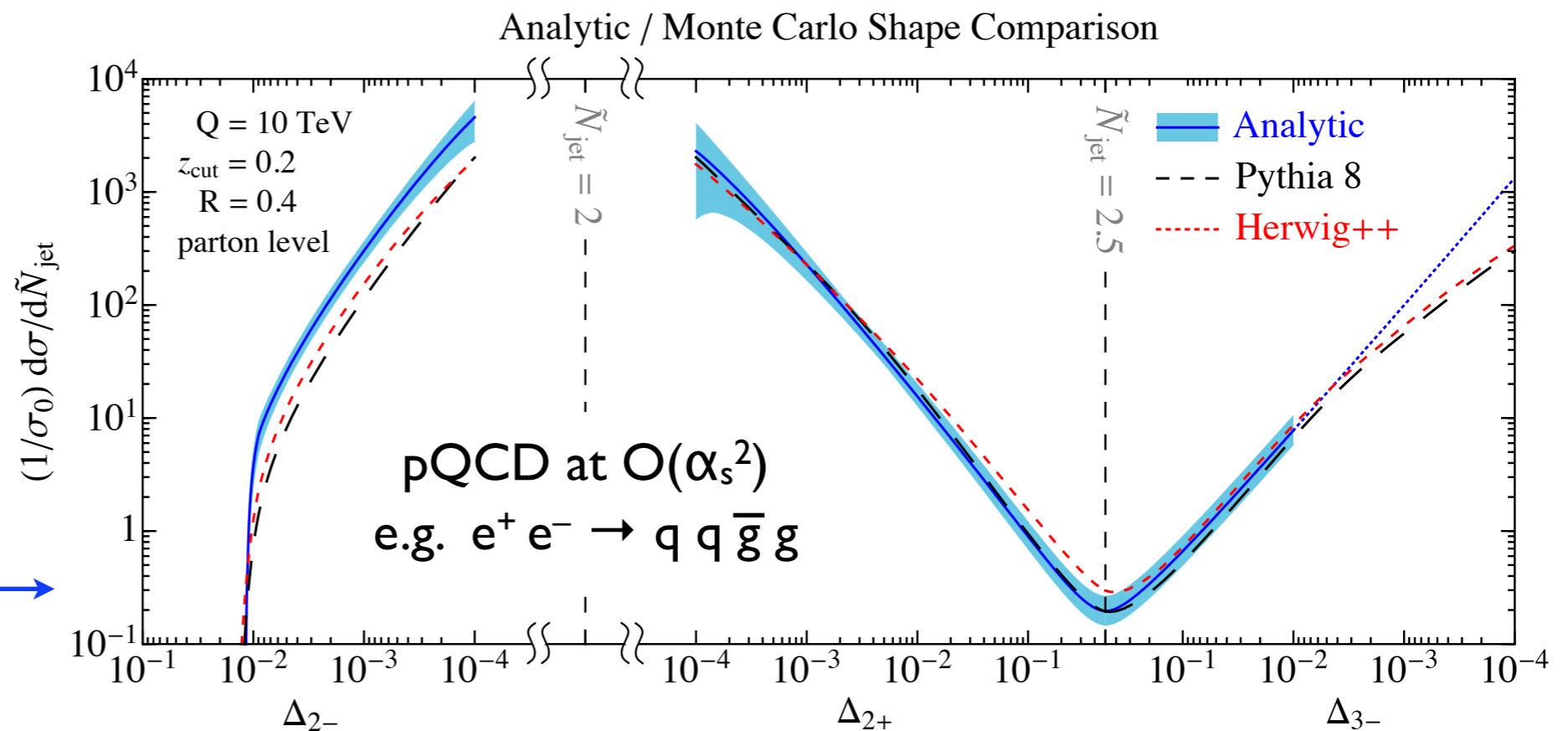
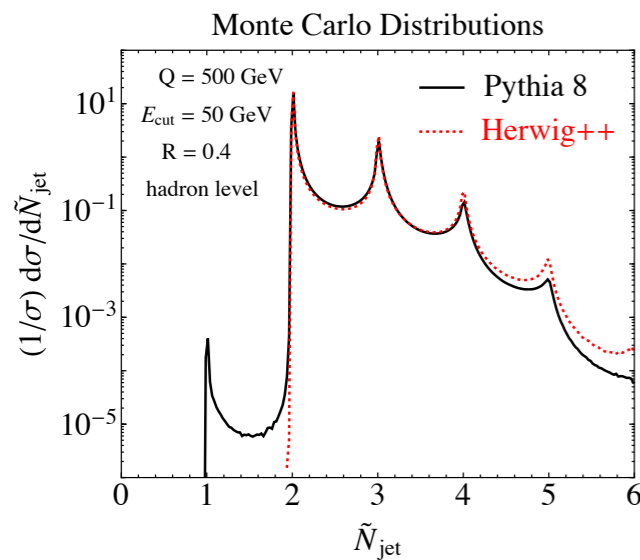
## First N<sup>3</sup>LO Calculation at a Hadron Collider

[Anastasiou, Duhr, Dulat, Herzog, Mistlberger, 1503.06056; + Furlan, Gehrmann, 1403.4616;  
see also Li, Manteuffel, Schabinger, Zhu, 1412.2771]

# Differential Cross Sections

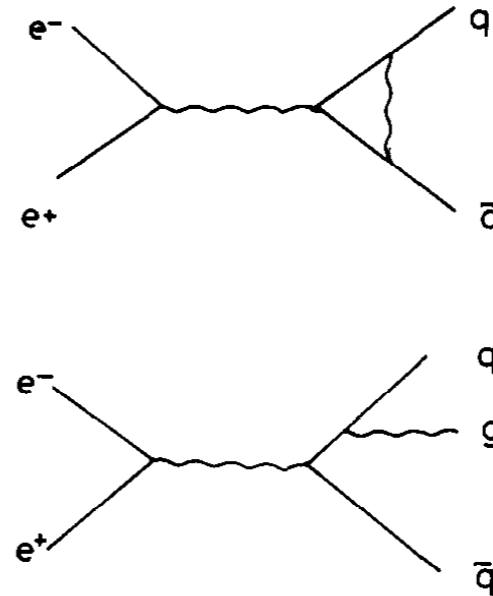
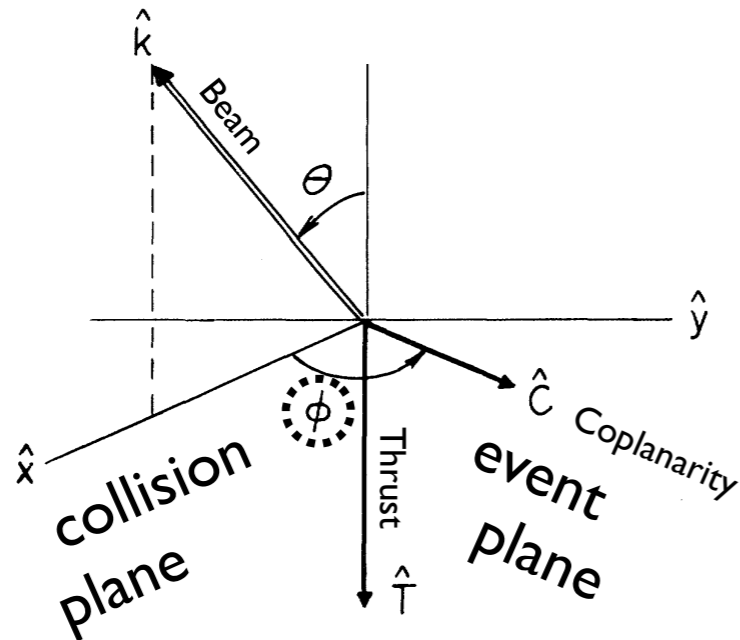
e.g. Fractional Jet Counting with “Jets Without Jets”

$$\tilde{N}_{\text{jet}}(p_{T\text{cut}}, R) = \sum_{i \in \text{event}} \frac{p_{Ti}}{p_{Ti,R}} \Theta(p_{Ti,R} - p_{T\text{cut}}) \quad \text{IRC Safe}$$



[Bertolini, Chan, JDT, 1310.7584; Bertolini, JDT, Walsh, 1501.01965]

# 0. Learn from Our Elders



$\varphi$  ambiguous

$\varphi$  well-defined

Me: “ $\varphi$  is IRC unsafe”

My Elder: “We explicitly calculated  $d\sigma/d\varphi$  in 1978”

$$\frac{2\pi}{\sigma_0} \frac{d\sigma}{d\varphi} = 1 + O(\alpha_s(Q^2)) + \frac{\alpha_s(Q^2)}{\pi} \left( \frac{16}{3} \ln \frac{3}{2} - 2 \right) \cos 2\varphi$$

↑ Born cross section despite ambiguity (!)

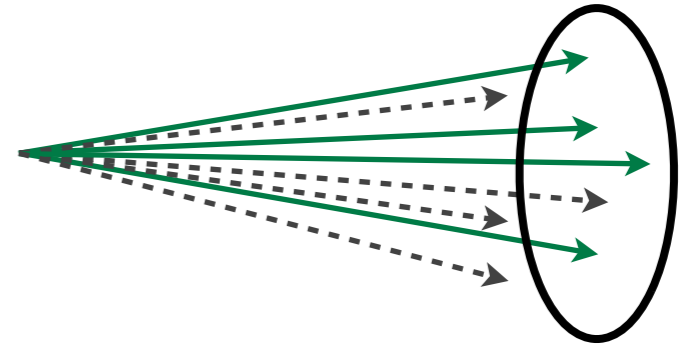
Lesson: Use IRC limit to resolve ambiguities

[Pi, Jaffe, Low, 1978;  
Kramer, Schierholz, Willrodt, 1978]

# Track-Based Observables

Track Jets:

$$x = \frac{E_{\text{tracks}}}{E_{\text{jet}}}$$

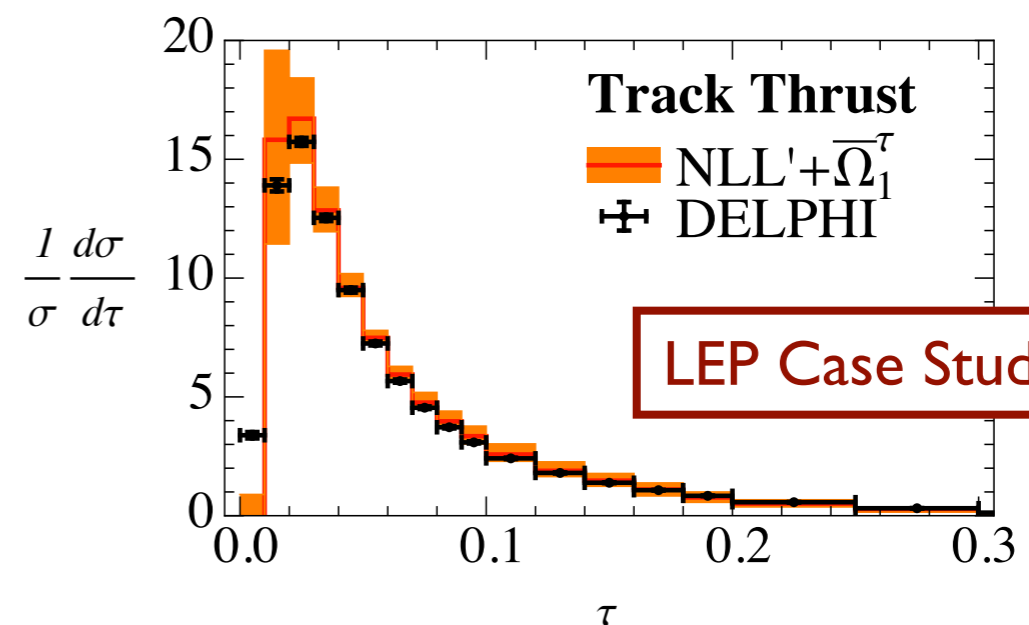
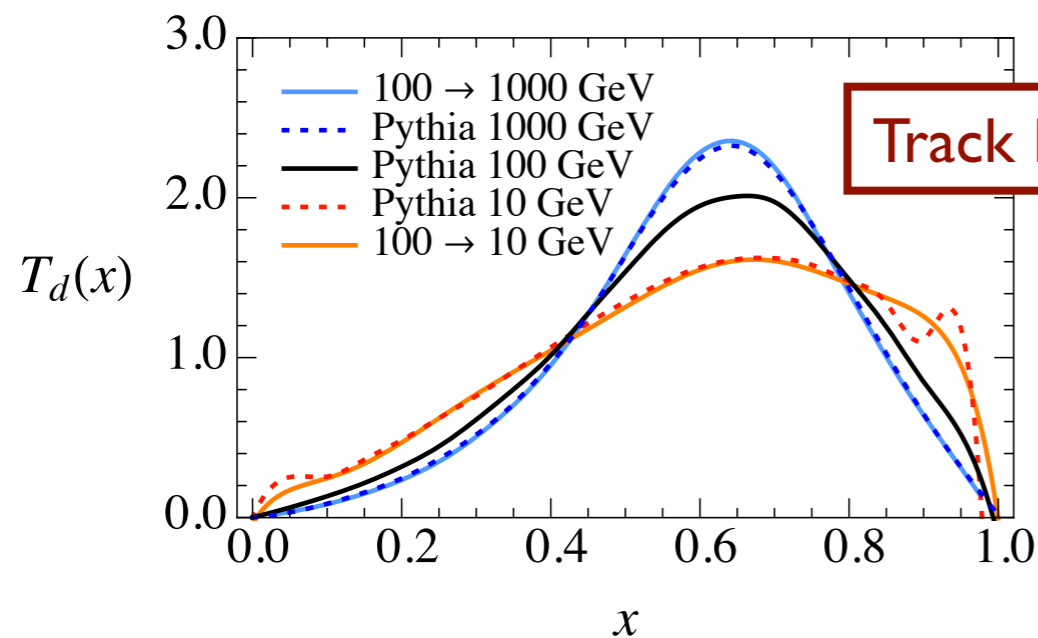


Track Functions:

$$T_i(x; \mu)$$

Non-perturbative object with perturbative evolution (just like PDFs!)

↑ track fraction    ↑ Non-linear DGLAP



[Chang, Procura, JDT, Waalewijn, 1303.6637, 1306.6630]