# Symmetries in cosmology and LSS

Lam Hui
Columbia University

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# Outline:

1. Equivalence principle: a generic test of modified gravity
   - with Alberto Nicolis.

2. Parity in measurements of large scale structure (LSS)
   - with Camille Bonvin & Enrique Gaztanaga.

3. Spontaneously broken symmetry in the theory of LSS
   - with Kurt Hinterbichler & Justin Khoury;
     Walter Goldberger & Alberto Nicolis;
     Creminelli, Gleyzes, Simonovic &Vernizzi;
     Bart Horn & Xiao Xiao.

## Idea 1: a generic test of scalar-tensor gravity

- Modifying gravity necessarily introduces new d.o.f. such as a scalar, i.e. a long range scalar force in addition to usual gravitational force (Weinberg/Deser thm.).

- Assume black holes have no scalar hair. More generally, compact objects have Q/M (scalar-charge/mass ratio) $\rightarrow$ 0. Normal stars like the Sun have Q/M $\approx$ 1. Thus, in the same environment a black hole and a star fall differently (Nordvedt).

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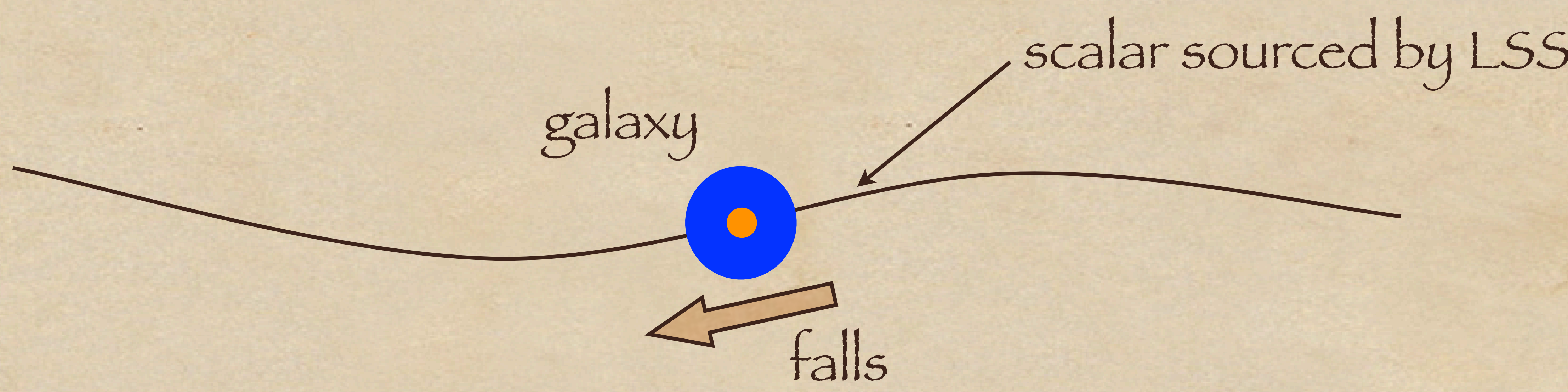

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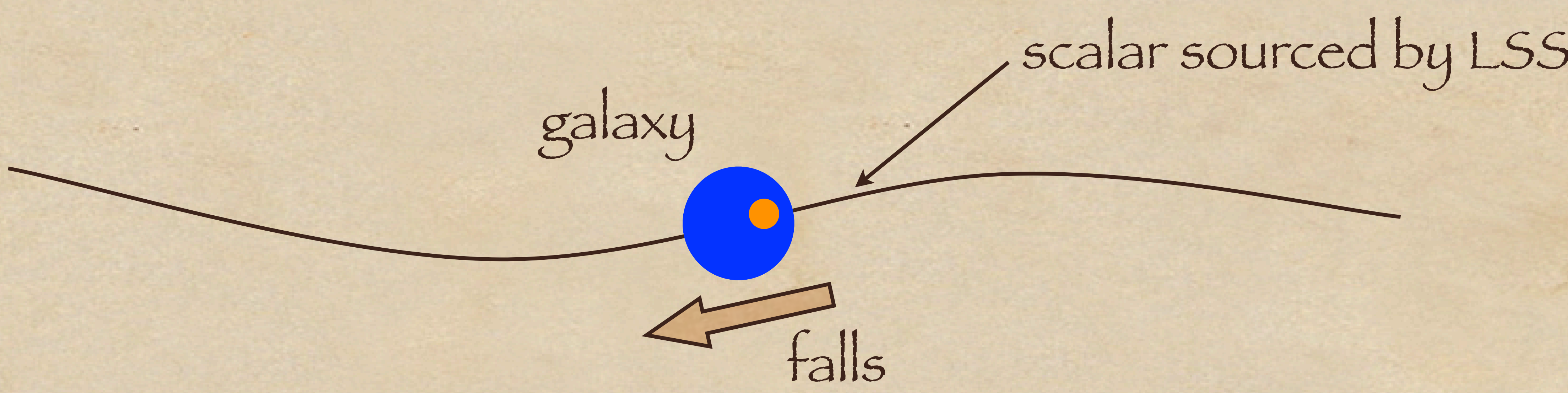

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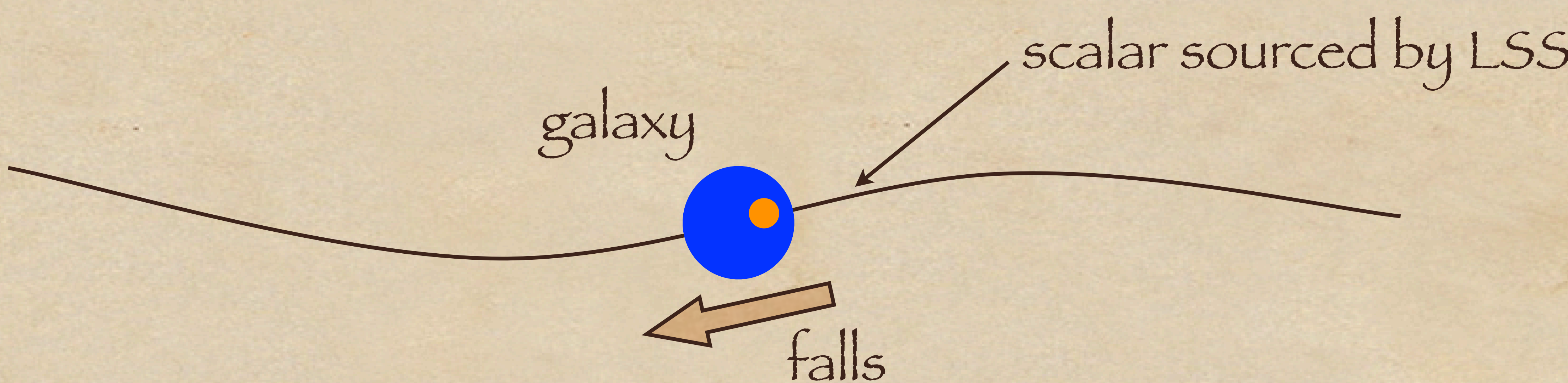


- Black hole offset up to 100 pc (use local, small Seyfert galaxies). Known offset: 7 pc for M87; Batcheldor et al. 2010 - beware astrophys. effects.

## Idea 2: ~~parity~~ in the measurement of LSS

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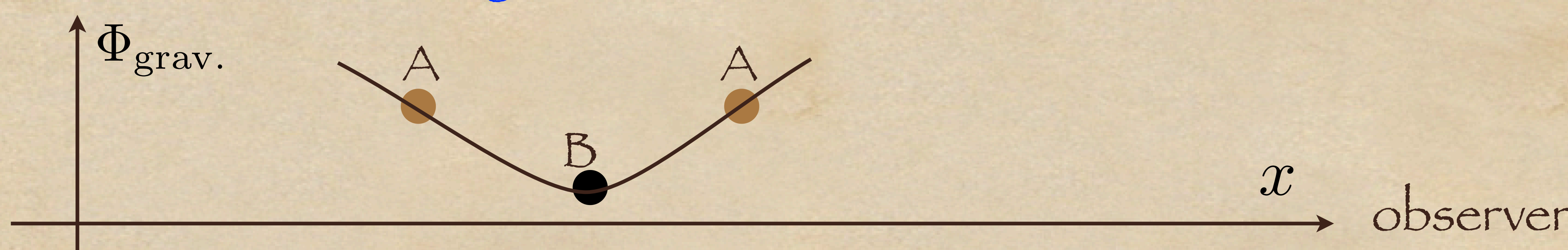

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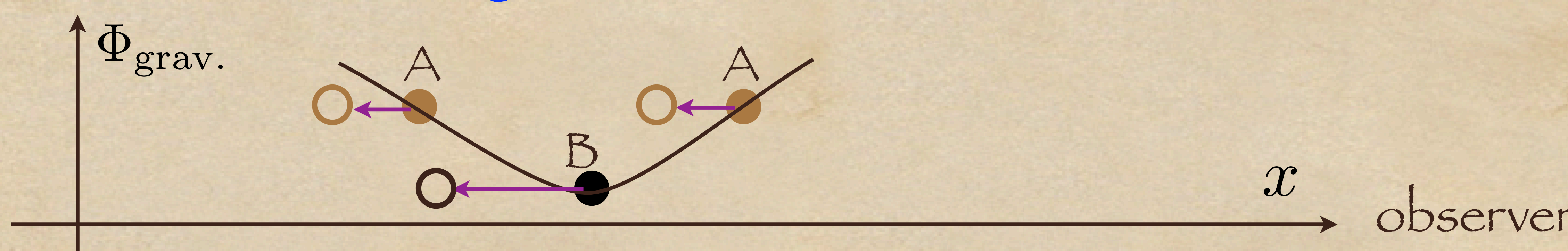

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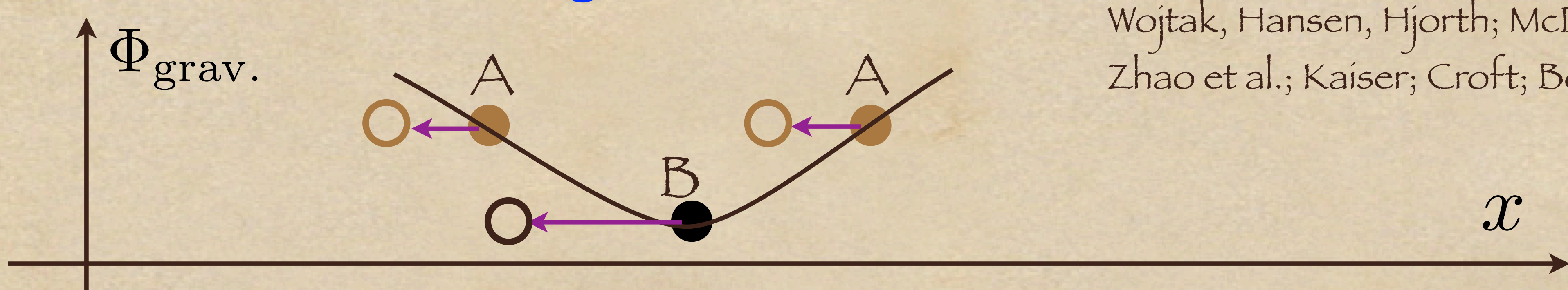
Wojtak, Hansen, Hjorth; McDonald; Yoo et al., Zhao et al.; Kaiser; Croft; Bonvin, LH, Gaztanaga

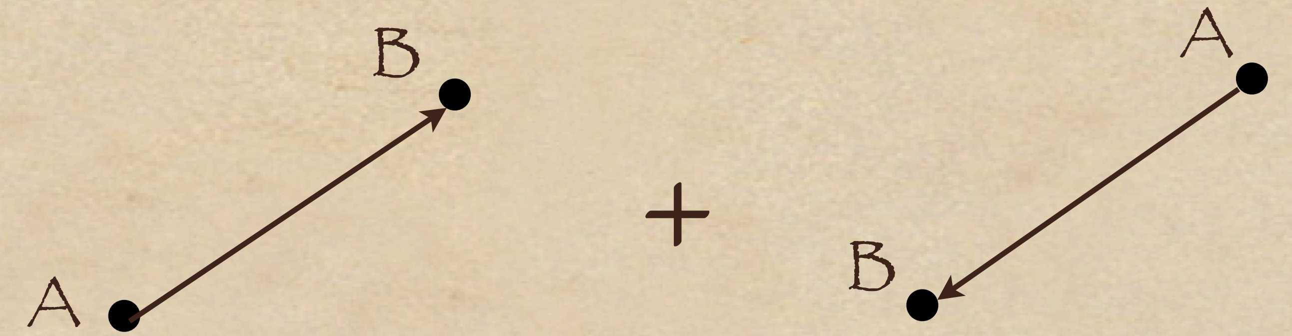



observer

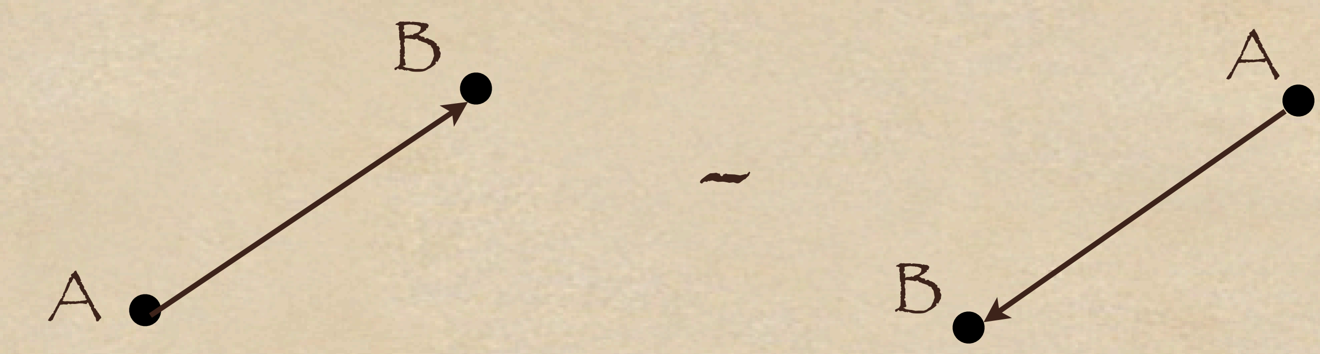
- Several additional (apparent) parity-violating effects. Possible to separate.

## Lessons for LSS measurement:

- Don't just add:




Subtract too:

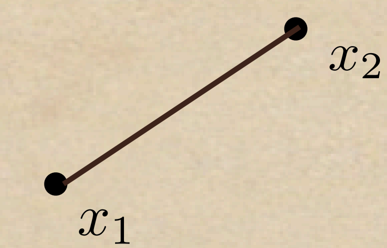



Or, more generally: combine different orientations appropriately.

# Idea 3: non-perturbative consistency relations in LSS

- 1. Consider a familiar example of symmetry: spatial translation.

$$x \to x + \Delta x \quad , \quad \text{where } \Delta x = \text{const.}$$




Its consequence for correlation function is well known:

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\rangle = \langle \phi(x_1+\Delta x)\phi(x_2+\Delta x)\phi(x_3+\Delta x)\rangle$$

For small $\Delta x$, we have:

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Thus, alternatively, we say:

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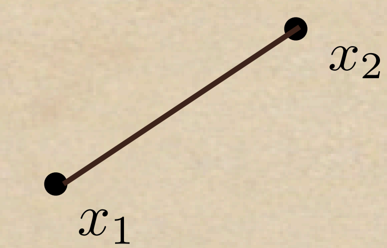


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- 2. Consider a different symmetry: shift in gravitational potential.

$$\phi \to \phi + c \quad , \quad \text{where } c = \text{const.}$$

For small $c$, we have:

$$\langle (\phi_1 + c)(\phi_2 + c)(\phi_3 + c)\rangle \sim \langle \phi_1\phi_2\phi_3\rangle + c\langle \phi_1\phi_2\rangle + c\langle \phi_2\phi_3\rangle + c\langle \phi_1\phi_3\rangle$$

Thus, saying $\langle \phi_1\phi_2\phi_3\rangle = \langle (\phi_1 + c)(\phi_2 + c)(\phi_3 + c)\rangle$ is equiv. to saying :

$$c(\langle \phi_1\phi_2\rangle + \langle \phi_2\phi_3\rangle + \langle \phi_1\phi_3\rangle) = 0 \longleftarrow \text{clearly false!}$$

Conclude : $\langle \phi_1\phi_2\phi_3\rangle$ is not invariant under $\phi \to \phi + c$

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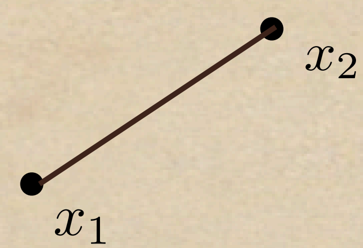


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  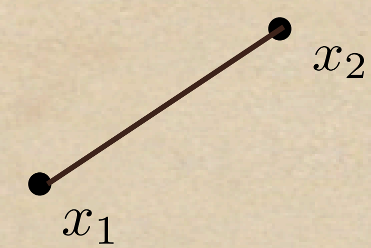
  

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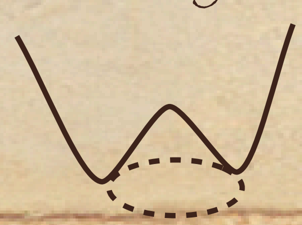
- What makes the second case so different? We generally choose some expectation value for $\phi$ e.g. $\langle\phi\rangle = 0$ . The choice breaks the shift symmetry i.e. spontaneous symm. breaking.

  1. Unbroken symmetries $\longrightarrow$ invariant correlation functions.
  2. Spontaneously broken symmetries $\longrightarrow$ consistency relations.

References:

Maldacena; Creminelli & Zaldarriaga; Creminelli, Norena, Simonovic; Assassi, Baumann & Green; Flauger, Green & Porto; Pajer, Schmidt, Zaldarriaga; Kehagias & Riotto; Peloso & Pietronni; Berezhiani & Khoury; Pimentel; Creminelli, Norena, Simonovic, Vernizzi; Goldberger, LH, Nicolis; Hinterbichler, LH, Khoury; Horn, LH, Xiao.

# Symmetries and consistency relations

comoving gauge $\delta\phi = 0$ $\quad ds^2_{\text{spatial}} = a^2 e^{2\zeta} [e^\gamma]_{ij} dx^i dx^j$

dilation symm. $\quad x \to e^{-2\lambda} x \quad , \quad \zeta \to \zeta + \lambda$

$$\lim_{q\to 0} \frac{1}{P_\zeta(q)} \langle \zeta(q) \zeta_{k_1} ... \zeta_{k_m} \rangle' \sim k \cdot \partial_k \langle \zeta_{k_1} ... \zeta_{k_m} \rangle'$$

Maldacena

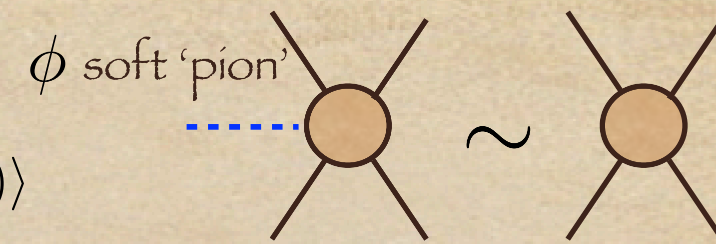
generalization $\quad x \to x + M \cdot x^{N+1} \quad , \quad \zeta \to \zeta + M \cdot x^N \quad , \quad \gamma \to \gamma + M \cdot x^N$

$$\lim_{q\to 0} \partial_q^N \left( \frac{1}{P_\zeta(q)} \langle \zeta(q) \zeta_{k_1} ... \zeta_{k_m} \rangle' + \frac{1}{P_\gamma(q)} \langle \gamma(q) \zeta_{k_1} ... \zeta_{k_m} \rangle' \right) \sim k \cdot \partial_k^{N+1} \langle \zeta_{k_1} ... \zeta_{k_m} \rangle'$$

Note:

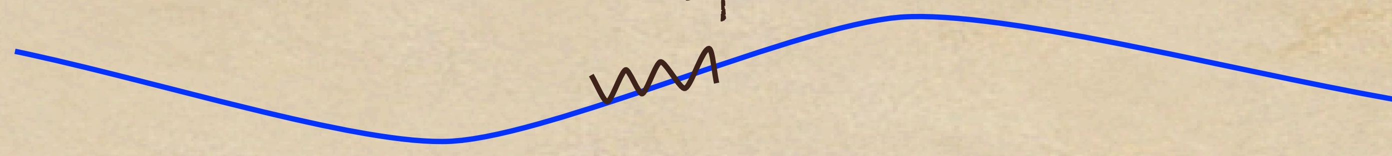
1. The symmetries originate as diff. But consistency relations are not empty statements i.e. they can be violated (e.g. curvaton); they are a test of initial conditions (e.g. single clock, etc).
2. They are non-perturbative, derived from Ward identities.
3. Testing these requires seeing general relativistic effects, but there exists a Newtonian consistency relation (Peloso & Pietroni; Kehagias & Riotto).

# Consistency relations from SSB




- Schematic form: $\lim_{q\to 0}\frac{1}{P_\phi(q)}\langle\phi(q)\mathcal{O}(k_1)...\mathcal{O}(k_N)\rangle \sim \langle\mathcal{O}(k_1)...\mathcal{O}(k_N)\rangle$

They are (momentum space) statements about how correlations of observables $\mathcal{O}$ behave in the presence of a long wave-mode Goldstone boson/pion.



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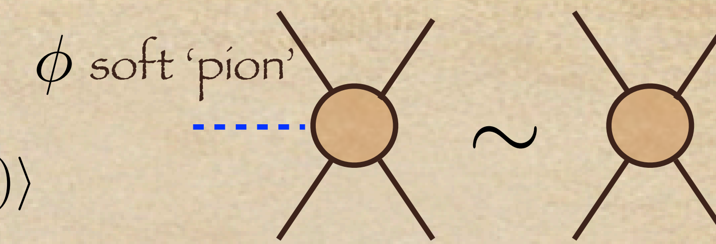


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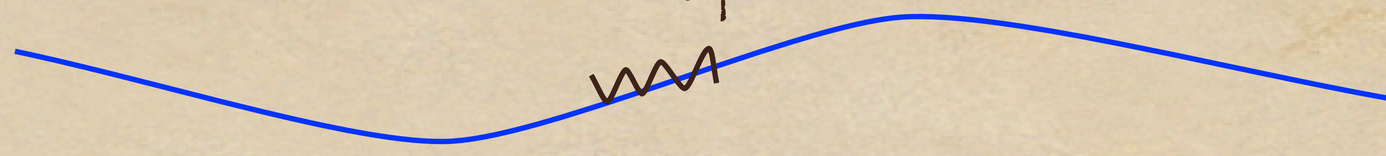

- Why are they interesting?

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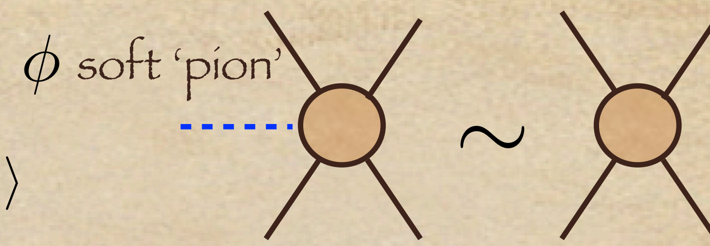


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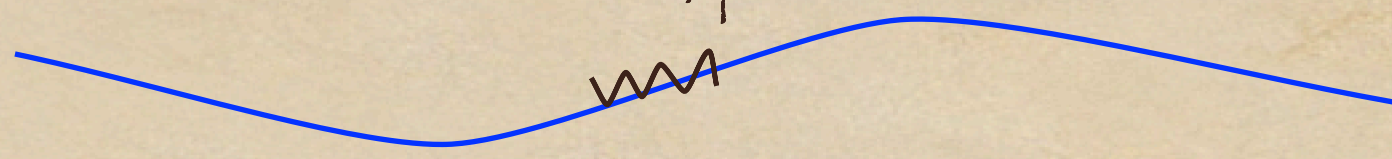

- Why are they interesting?

1. These are symmetry statements, and are therefore exact, non-perturbative i.e. they hold even if the observables $\mathcal{O}$ are highly nonlinear, and even if they involve astrophysically complex objects, such as galaxies. The main input necessary is how they transform under the symmetry of interest (robust against galaxy mergers, birth, etc.)

2. In the fully relativistic context, there is an infinite number of consistency relations. Two of them have interesting Newtonian limits (shift and time-dependent translation).

3. Two assumptions go into these consistency relations, which can be experimentally tested (using highly nonlinear observables!): Gaussian initial condition (or more precisely, single-clock initial condition such as provided by inflation), and the equivalence principle (that all objects fall at the same rate under gravity). $10^{-4}$ constraint possible.

4. Non-trivial constraints on analytic models.

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# Chan & Scoccimarro 2009



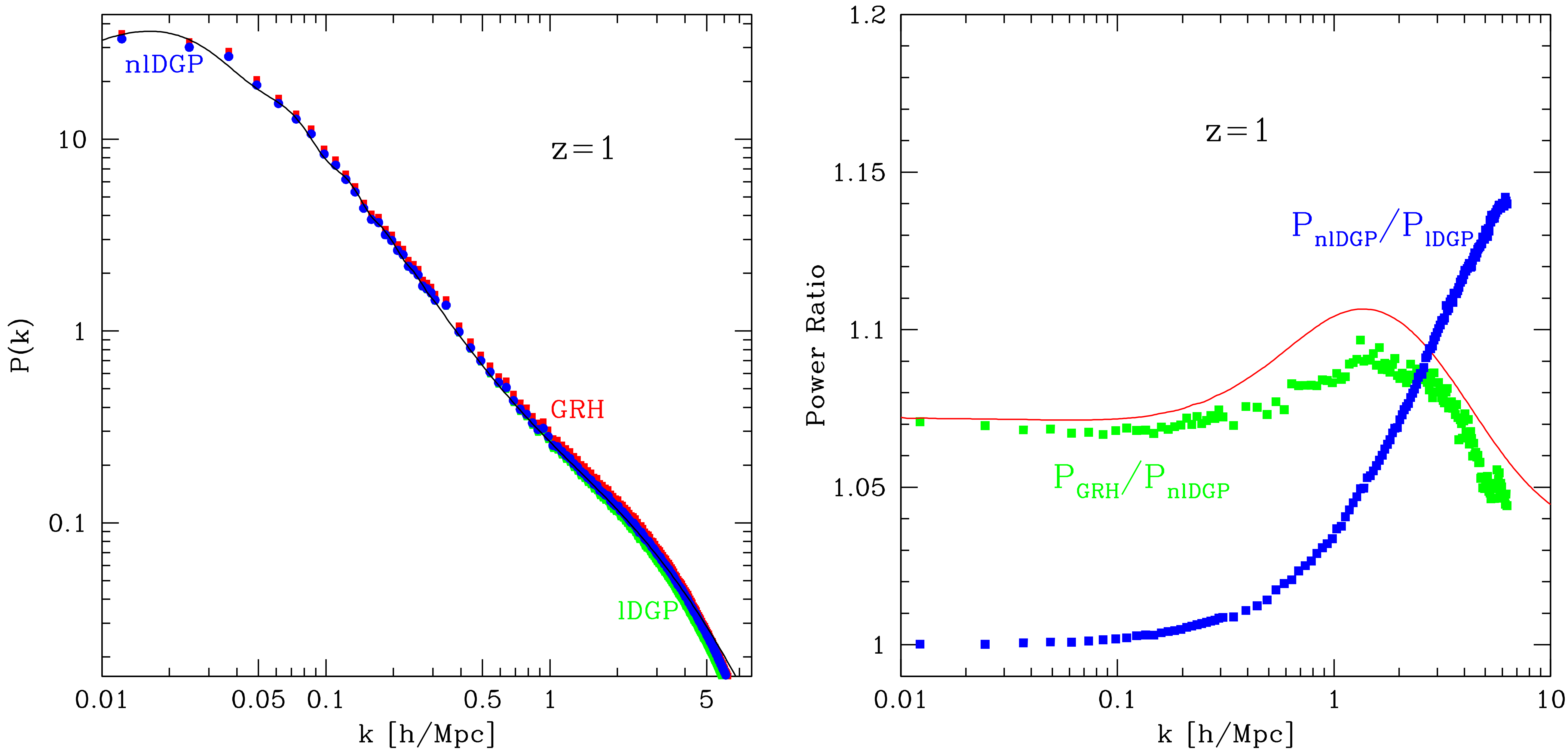



FIG. 5: Dark matter power spectra from the nonlinear DGP model (nlDGP) , linear DGP (lDGP), and GR perturbations with the same expansion history (GRH) at $z = 1$. The left panels show the power spectra, and the right panels shows ratios to better see the differences. Two sets of computational boxes are shown for each case, covering a different range in $k$ (see text). The solid line denotes the predictions from paper I for $P_{\rm nlDGP}$ (left panel) and $P_{\rm GRH}/P_{\rm nlDGP}$ (right panel).

# Vainshtein screening   e.g. DGP

$$S_{\text{scalar}} \sim \int d^4x \left[ -\frac{1}{2}(\partial\varphi)^2 - \underline{\frac{1}{m^2}(\partial\varphi)^2 \Box\varphi} + \alpha\varphi T_m{}^{\mu}{}_{\mu} \right]$$ (Einstein frame)

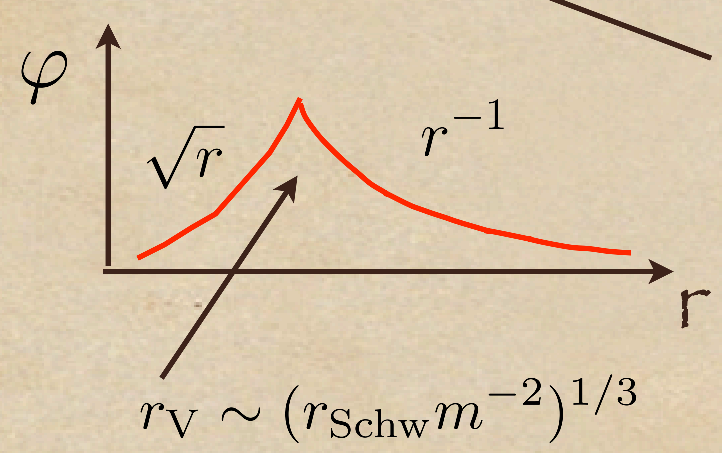
e.o.m.: $\Box\varphi + \frac{1}{m^2}\left[(\Box\varphi)^2 - \partial^\mu\partial^\nu\varphi\partial_\mu\partial_\nu\varphi\right] \sim \alpha\rho_m$

$\varphi \propto \frac{1}{r}$   large r

$\varphi \propto \sqrt{r}$   small r

point mass solution

graviton mass




$\alpha =$ scalar-matter coupling ≈ O(1) generically

Galileon symmetry (Nicolis, Rattazzi, Trincherini): $\varphi \to \varphi + c + b_\mu x^\mu$

- The idea is to look for the offset of massive black holes from the centers of galaxies (bottom of the gravitational potential well). Look at Seyfert galaxies where we can see both the stars and the black hole (active nucleus).

  The offset is estimated to be up to 0.1 kpc, for small galaxies.

- Sources of confusion:
  - asymmetric jets (case of M87: 7 pc offset, Batcheldor et al. 2010).
  - binary merger recoil.
  - Brownian motion.
  - disturbed galaxies.

- Distiniguishing feature: the spatial offset should be correlated with the direction of the streaming motion. Also: small velocity offset.

## What are other parity violating effects?

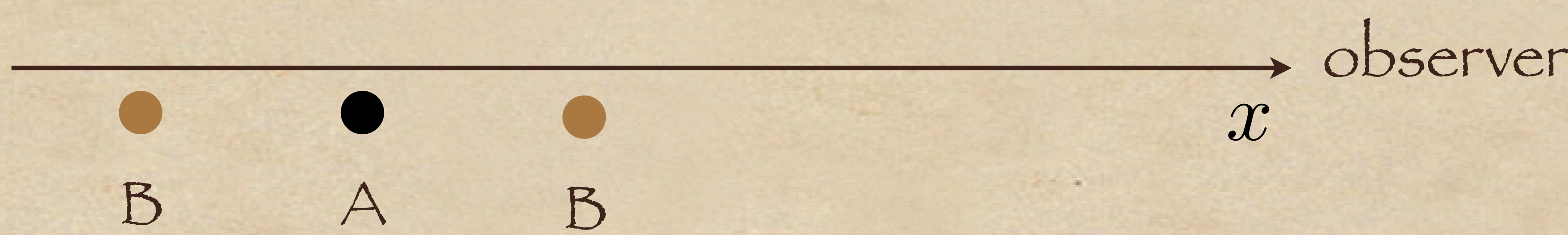
- Often grouped under the heading of general relativistic effects:

$$\delta_{\rm obs.} \sim \delta \left[ 1 + \frac{\mathcal{H}}{k} + \frac{\mathcal{H}^2}{k^2} \right]$$

parity violating

Yoo, Fitzpatrick, Zaldarriaga; Challinor, Lewis; Bonvin, Durrer; Raccanelli, Bertacca, Dore, Maartens.

- More mundane, but present: evolution.




- Can disentangle between the two.

Footnote 1: parity violation only in the z direction.

Footnote 2: $O(\mathcal{H}/k)$ terms can be derived in a 'Newtonian' manner.

Gravitational redshift term canceled, assuming geodesic motion.

Footnote 3: selection effects.