Natural Inflation and Quantum Gravity

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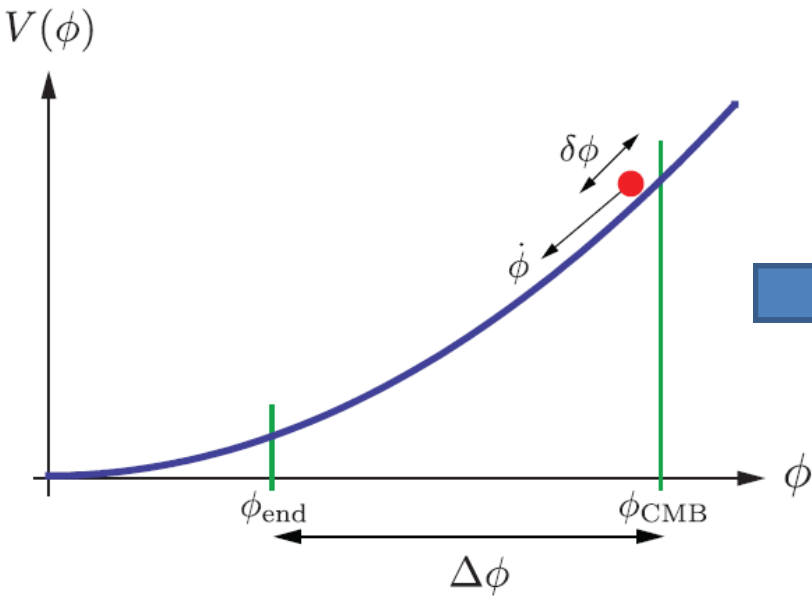
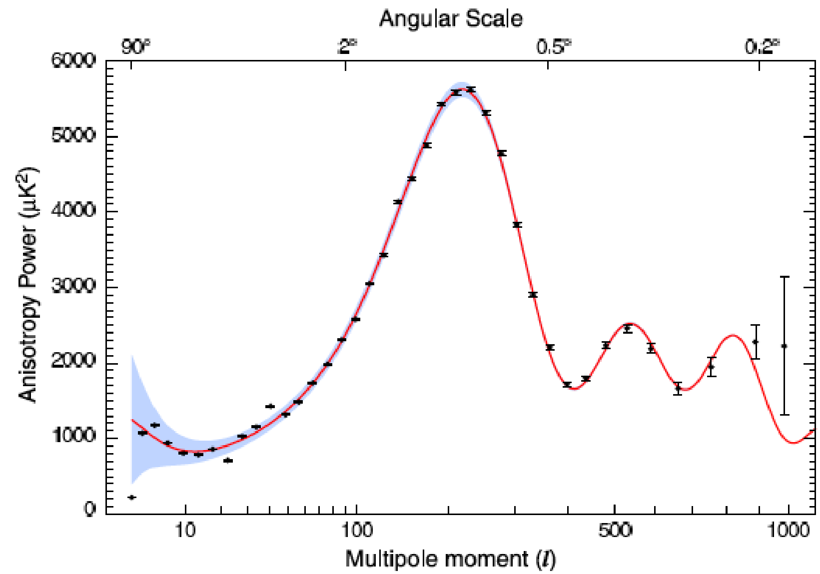
Based on arXiv:1412.3457 with Anton de la Fuente and Raman Sundrum

Outline

- Intro: the “Transplanckian Problem”
- Simple EFT of inflation: foiled by the Weak Gravity Conjecture
- A controlled model: “winding” in field space (despite recent claims in the literature!)
- Potentially striking CMB observables

Slow-roll Inflation





Quantum fluctuations of the inflation generate density perturbations, CMB anisotropies

→ We can observe the scalar field dynamics with data!

Lyth Bound

$$\mathcal{N}_{\text{e-folds}} = \int \frac{d\phi}{M_{\text{pl}}} \frac{1}{\sqrt{2\epsilon}} \approx 60 \int \frac{d\phi}{M_{\text{pl}}} \sqrt{\frac{2 \times 10^{-3}}{r}}$$

Flatness of Universe requires $\mathcal{N}_{\text{e-folds}} \gtrsim 60$

$\Rightarrow$ Observable tensor-to-scalar ratio implies

$$\boxed{\Delta\phi > M_{\text{pl}}}$$

Do we need to work in a full UV theory, e.g. string theory, to understand large-field inflation?

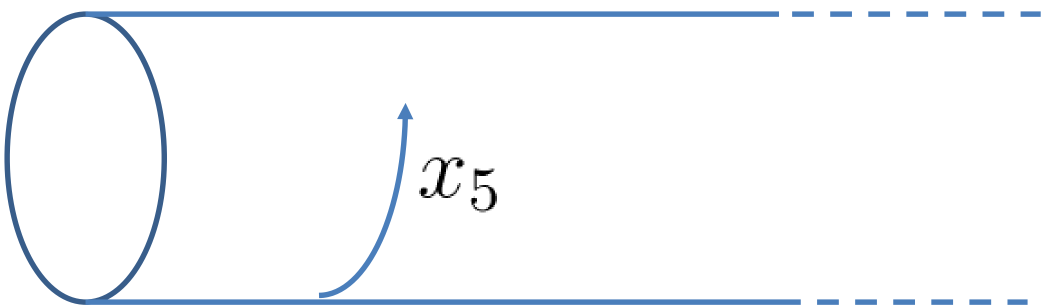
Or can we still find an effective field theory model?

The EFT way: Assume symmetries of the UV theory, but not detailed dynamics. Can this work for inflation?

Extranatural Inflation: Protecting the inflaton with gauge symmetry

Arkani-Hamed, Cheng, Creminelli, Randall hep-th/0301218

U(1) gauge field in the bulk of an extra dimension S^1



A_5 component gives a light scalar field in 4D; gauge-invariant observable is the Wilson loop: $e^{ig \oint_{S^1} dx^5 A_5}$

→ Periodic potential for A_5 field

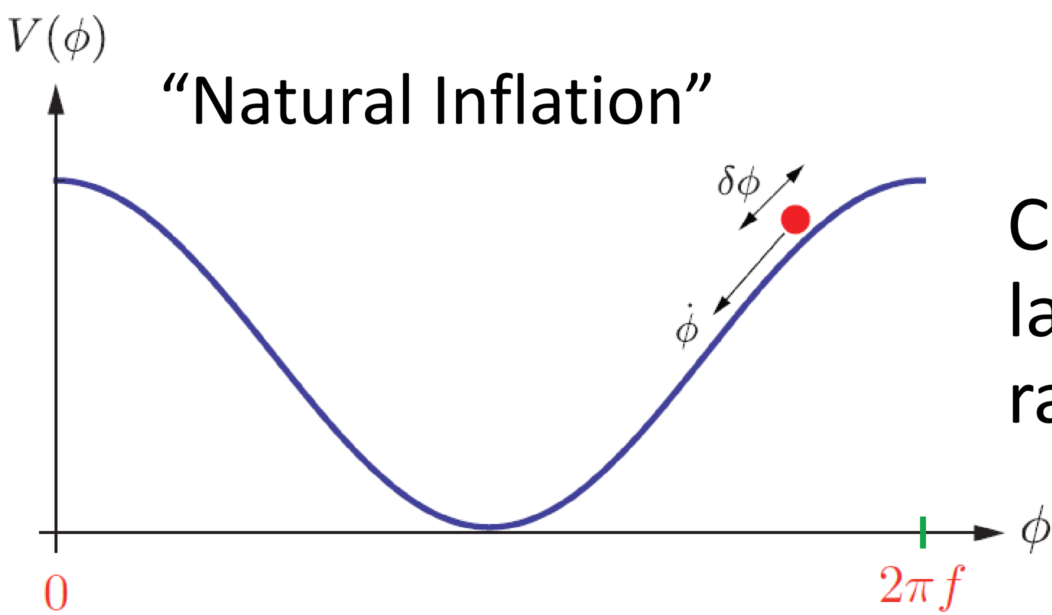
$$V \sim \frac{1}{R^4} \cos\left(2\pi R g A_5\right)$$

1-loop potential from a charged field is:

$$V(\phi) = \frac{3(-1)^S}{4\pi^2}\frac{1}{(2\pi R)^4}\sum_n c_n e^{-2\pi n R m_5} e^{in\phi/f} \quad \text{+ h.c.}$$

$$c_n(2\pi R m_5) = \frac{(2\pi R m_5)^2}{3n^3} + \frac{2\pi R m_5}{n^4} + \frac{1}{n^5}$$

$$\boxed{f = \frac{1}{2\pi R g}}$$



"Natural Inflation"

Can get arbitrarily large inflaton field range by taking $g \ll 1$

Extranatural Inflation: Success?

5D gauge symmetry and locality guarantee that physics above the compactification scale gives small corrections to the A_5 potential

…but doesn't the limit $g \to 0$ bring us to a global symmetry, which was problematic?

Weak Gravity Conjecture

Arkani-Hamed, Motl, Nicolis, Vafa hep-th/0601001

Claim: In any theory with gravity and a gauge field with coupling strength *g*, effective field theory must break down at a scale Λ, where

$$\Lambda < g M_{\mathrm{pl}}$$

So small *g* limits the validity of EFT!

Familiar in string theory: string states are below Planck scale at weak coupling

Downfall of Extranatural Inflation

Recall that extranatural inflation required

$$f \sim \frac{1}{gR} > M_{\rm pl}$$

But the WGC tells us EFT is only valid up to $\Lambda < gM_{\rm pl}$

This implies

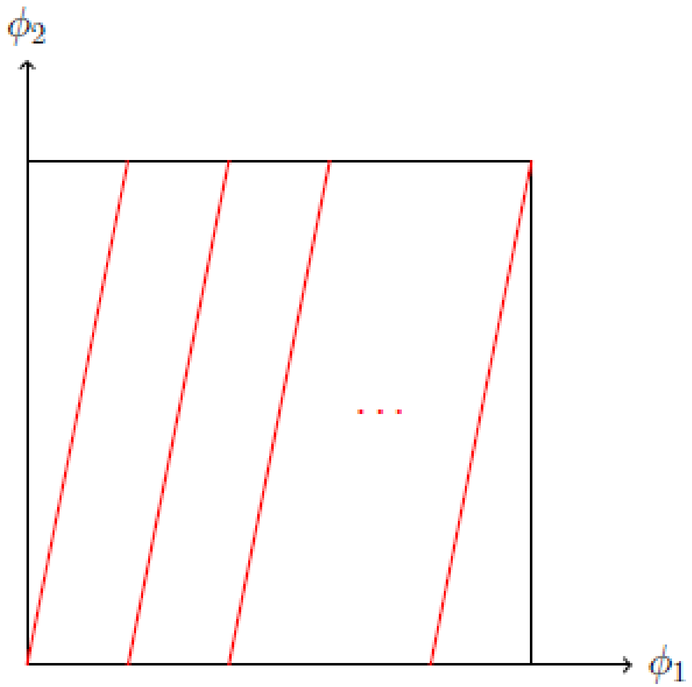
$$\Lambda < 1/R$$

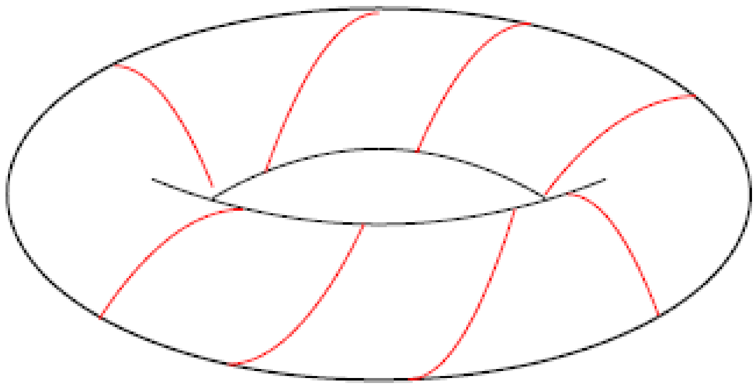
→ 5D theory is not within EFT control!

Biaxion Models

Kim, Nilles, Peloso hep-ph/0409138

Even if the radius of scalar field space subplanckian, there are paths with long distance which one can traverse





Biaxion Models

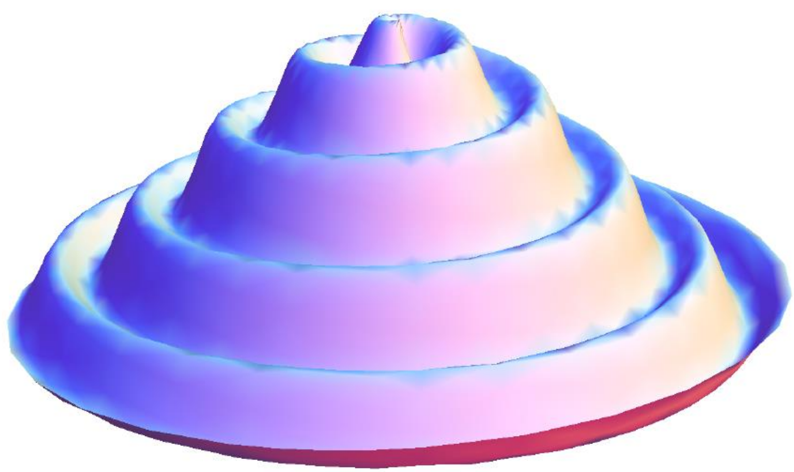
Consider two *U(1)* gauge fields *A* and *B* and two light particles with charges (*N*, 1) and (1, 0) under (*A*, *B*)

$$V = V_0 \left[\cos\left(\frac{NA}{f_A} + \frac{B}{f_B} \right) + \cos\frac{A}{f_A} \right]$$

$$V_0 = \frac{3}{4\pi^2}\frac{1}{(2\pi R)^4}$$

"Groove" potential

"Hill" potential



Radial direction: *A*
Angular direction: *B*

Constraints for EFT Control

5D gauge theory is non-renormalizable, with strong coupling scale

$$\Lambda_g = \frac{8\pi}{N^2 g^2}\frac{1}{R}$$

WGC implies an EFT cutoff

$$\Lambda_{\rm WGC} = g M_{\rm pl}$$

Requiring both of these to be above the compactification scale 1/R implies the bound

$$\frac{f_{\rm eff}}{M_{\rm pl}} \equiv N \frac{1}{2\pi R M_{\rm pl}} \lesssim M_{\rm pl} R$$

But 1/*R* also controls the Hubble scale:

$$\boxed{H \sim \frac{\sqrt{V}}{M_{\rm pl}} \sim M_{\rm pl}\frac{1}{(M_{\rm pl}R)^2} \lesssim \frac{M_{\rm pl}}{\mathcal{N}^2_{\text{e-folds}}}}$$

To fit the real world data we need

$$\frac{H}{M_{\rm pl}} \sim 10^{-4} \qquad \mathcal{N}_{\text{e-folds}} \gtrsim 60$$

On the edge of the controlled parameter space…

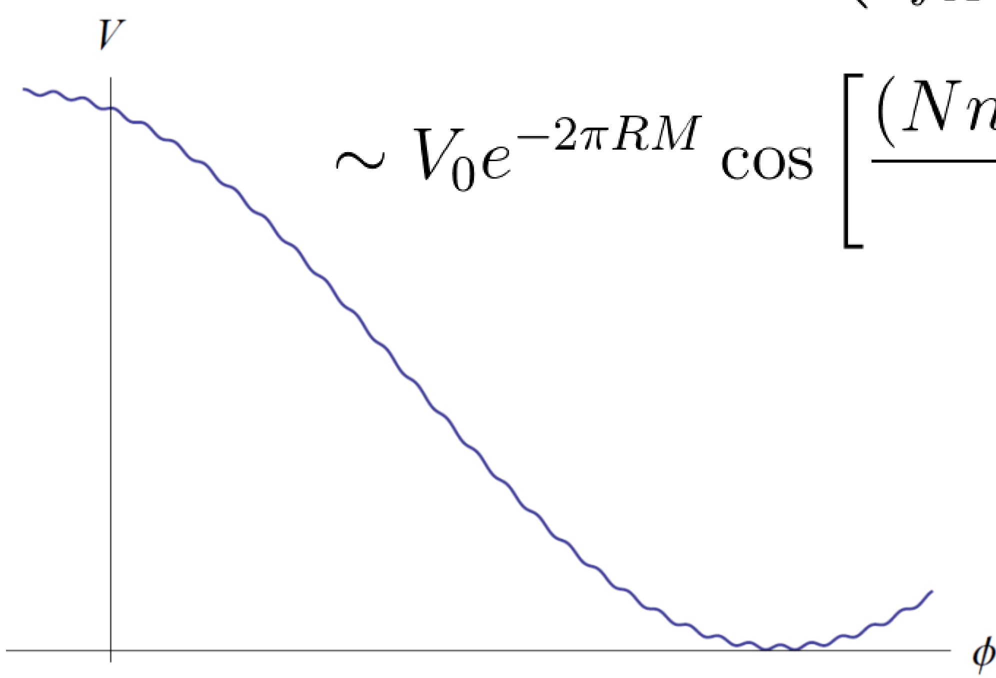
Effects of UV Physics

At the cutoff of EFT, new states with unknown quantum numbers may exist (possibly mandated by the quantum gravity theory), affecting the potential. For a particle of mass *M* with charges (n_A, n_B):

$$\delta V \sim V_0 e^{-2\pi RM} \cos\left(\frac{n_A A}{f_A} + \frac{n_B B}{f_B}\right)$$

Expect $M \sim \Lambda$ typically

$$\sim V_0 e^{-2\pi RM} \cos\left[\frac{(N n_B - n_A)\phi}{f_{\rm eff}}\right]$$



Potential generically receives small-amplitude but high-frequency perturbations

Fitting the data

Cosmological data can be fit by inflation with

$$V = V_0 \cos \frac{\phi}{f_{\rm eff}} \qquad \begin{array}{c} V_0 \sim 10^{-2} M_{\rm pl} \\ f_{\rm eff} \sim 10 M_{\rm pl} \end{array}$$

This can achieved in this model by choosing e.g.

$$M_{\rm pl} R = 8 \qquad N = 40 \qquad g = .08$$

Then additional charges with mass at EFT cutoff Λ give modulation of the slow-roll parameter:

$$\frac{\delta\epsilon}{\epsilon} \sim 2\%$$

(Current searches: $\frac{\delta\epsilon}{\epsilon} \lesssim 1 - 5\%$)

So if there are extra charges near the cutoff, we may observe a “smoking gun” signal with further data…

Claim in the literature: Axion inflation is inconsistent with Weak Gravity Conjecture?

Argument made in 1503.00795 (T. Rudelius), 1503.04783, 1504.00659 (J. Brown et. al.):

Generic form of axion potential from instanton with action *S*:

$$V(\phi) \sim \sum_n e^{-nS} \cos \frac{n\phi}{f}$$

“0-form” WGC: $S < M_{\rm pl}/f$

Claim: Need $S < 1$ to suppress higher harmonics. Therefore

$$f < M_{\rm pl}$$

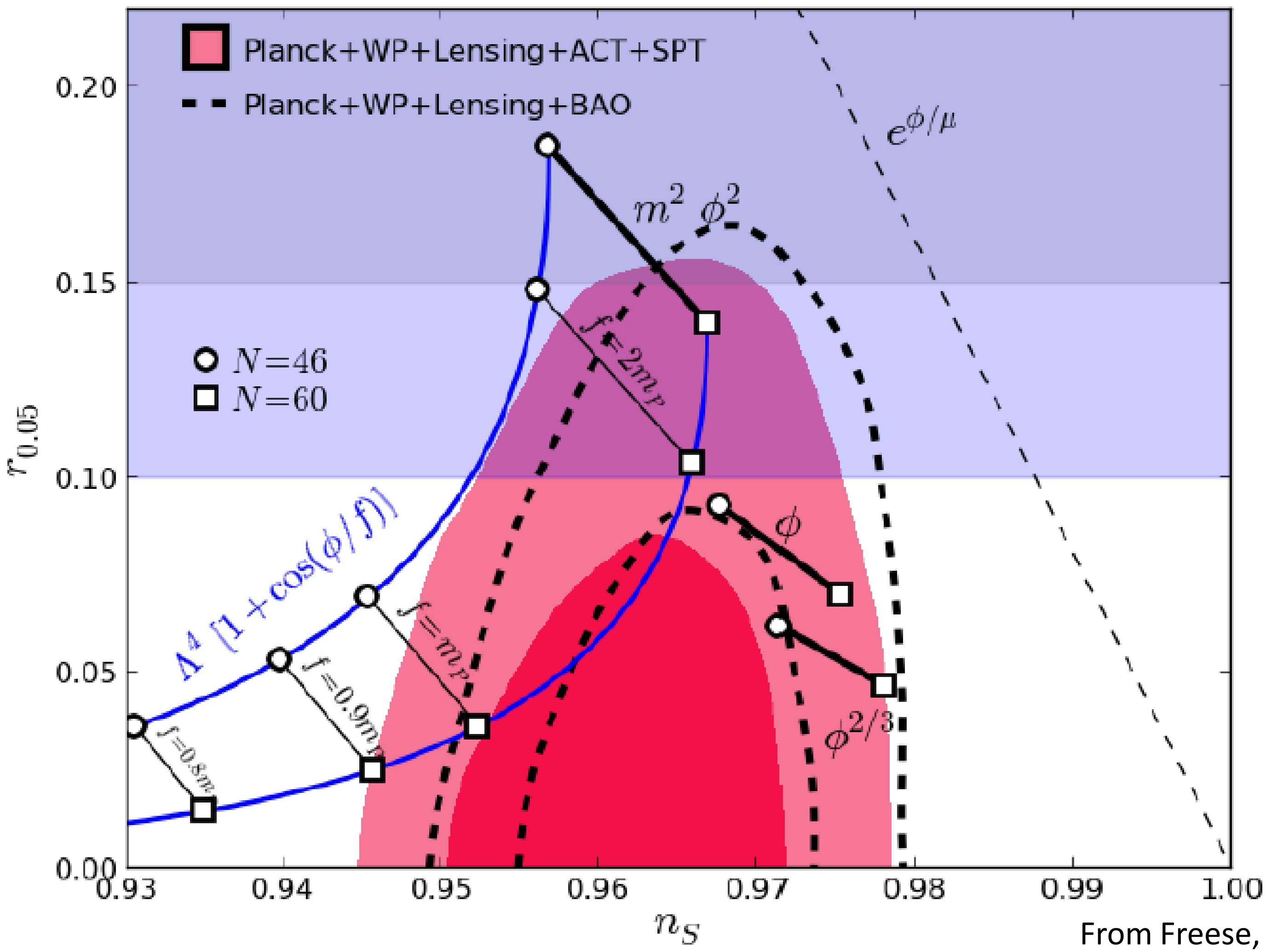
Completely evaded in our model! We have

$$V(\phi) = \sum_n \frac{1}{n^5} e^{-nS} \cos\frac{n\phi}{f} \qquad S = 2\pi R m_5$$

But even for *S* = 0 we have complete control over higher harmonics, inflation potential is still sufficiently flat.

The “0-form” WGC places no bound!

If true: Gravitational Waves in the next few years



From Freese, Kinney 1403.5277

Conclusions

- In a theory with gravity, there are limits to how effectively global or even gauge symmetries can protect a scalar potential
- “Winding” models with axions from gauge fields can be theoretically controlled and suggest high-frequency oscillations of the power spectrum
- Contrary to recent claims, such models can be fully consistent with the Weak Gravity Conjecture

Backup Slides

Slow-Roll Condition

Accelerated expansion requires

$$\epsilon = -\frac{\dot{H}}{H^2} \approx \frac{M_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2 < 1$$

Consider a “generic” scalar field potential

$$V = m^2\phi^2 + g\phi^3 + \ldots \Rightarrow V'/V \approx 1/\phi$$

Inflation then occurs only when $\boxed{\phi \gtrsim M_{\rm pl}}$

M_{Pl} is the scale at which both GR and field theory break down! Generically expect higher-dimension operators:

$$V \supset \frac{\phi^5}{\Lambda} + \frac{\phi^6}{\Lambda^2} + \ldots \qquad \Lambda \lesssim M_{\mathrm{pl}}$$

For $\phi \gtrsim M_{\mathrm{pl}}$, potential is completely out of control!

Usual trick: Assume some symmetry of the theory to forbid unwanted operators.

But quantum gravity does not seem to allow continuous global symmetries: black holes violate them

Inflaton as a PNGB

Consider a Nambu-Goldstone boson ϕ :

$$\psi = \rho e^{i\phi/f} \quad U(1) : \phi \to \phi + c$$

$$\langle\psi\rangle = f \Longrightarrow V(\phi) = 0$$

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$$\psi = \rho e^{i\phi/f} \quad U(1) : \phi \to \phi + c$$

$$\langle \psi \rangle = f \Rightarrow V(\phi) = 0$$

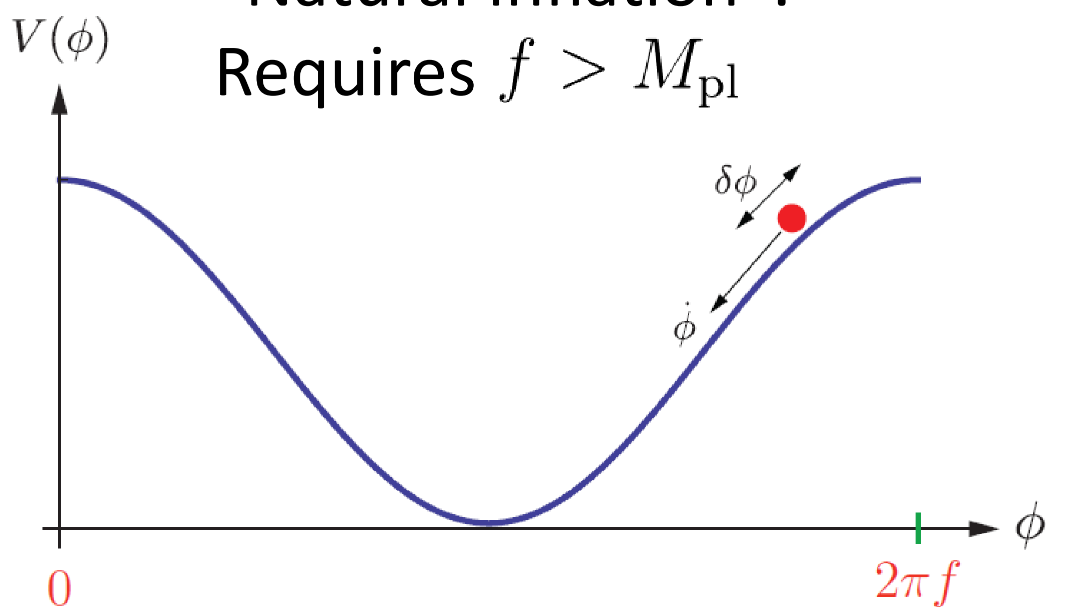
If *U*(1) symmetry is broken by small term

$$\delta \mathcal{L} = \epsilon \mu^3 \psi$$

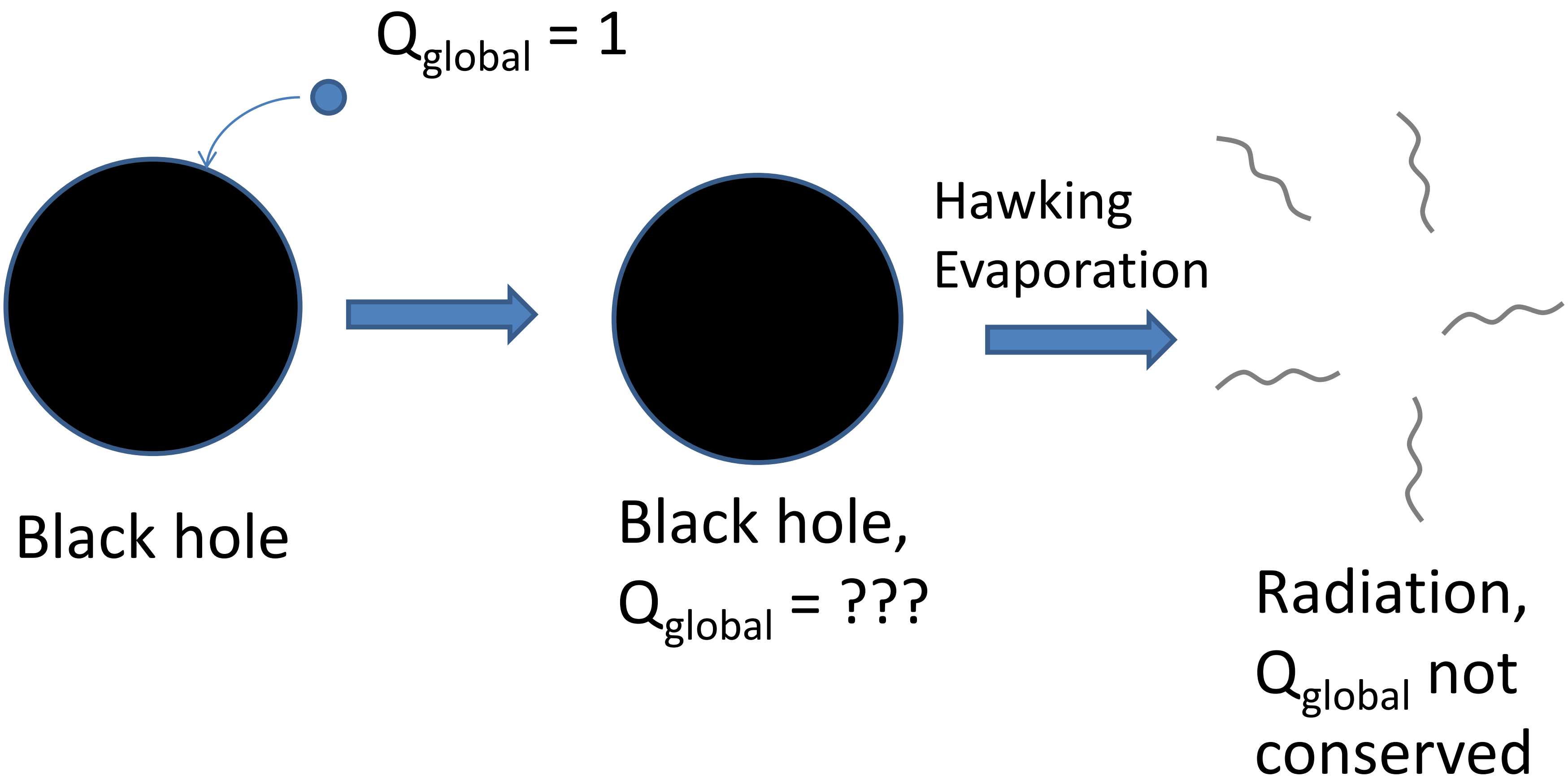
Then ϕ gets a potential

$$V(\phi) = \epsilon \mu^3 f \cos\left(\frac{\phi}{f}\right)$$

"Natural Inflation":
Requires $f > M_{\rm pl}$



However, black holes seem to violate all continuous global symmetries!



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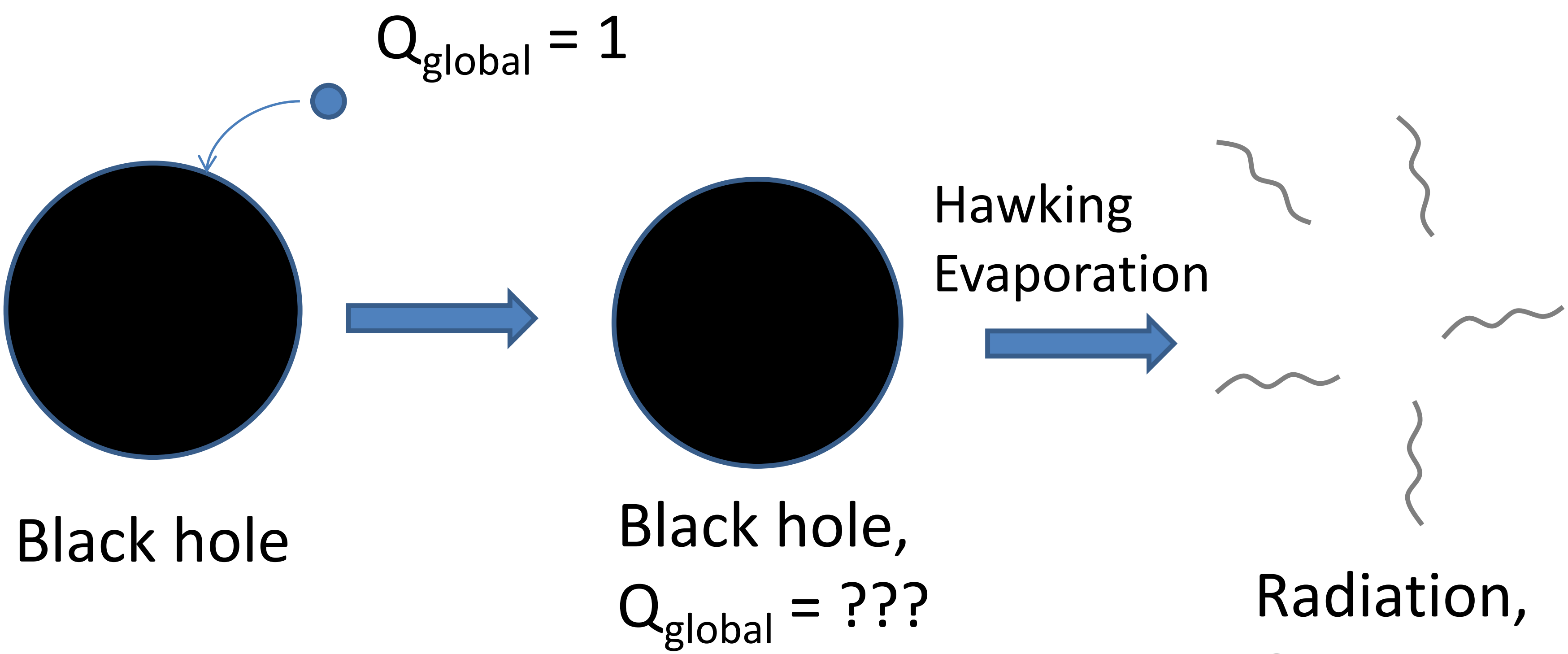


If a black hole remembers its charge, infinitely many microstates for each black hole → thermodynamic problems, violation of entropy bounds

No global symmetries in UV

If QG ultimately respects no global symmetries, no reason not to write down terms like $\frac{\psi|\psi|^4}{M_{\rm pl}}$

Inflaton potential gets corrections

$$V(\phi) \supset \frac{f^5}{M_{\rm pl}} \cos\frac{\phi}{f} + \ldots$$

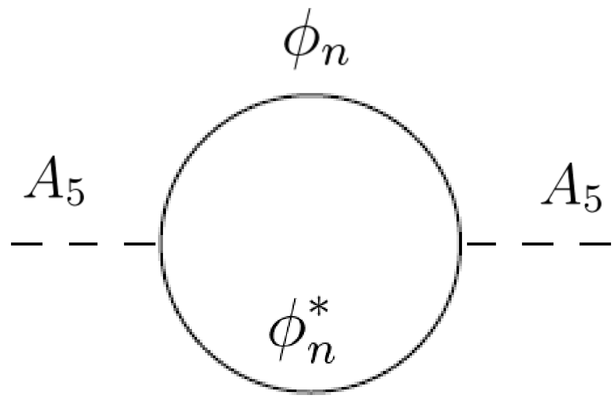
Which are uncontrolled for $f > M_{\rm pl}$!

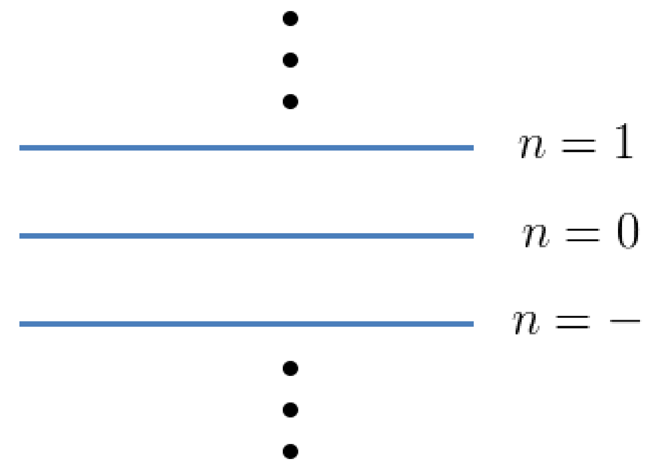
Related: in string theory, axions with $f > M_{\rm pl}$ tend to have unsuppressed higher harmonics

Banks, Dine, Fox, Gorbatov hep-th/0303252

Potential from charged KK tower

$$V = \sum_n$$





Coleman-Weinberg potential from a KK mode is a function of the field-dependent mass:

$$m^2_{\mathrm{KK,n}} = m_5^2 + \left(\frac{n}{R} - gA_5\right)^2$$

$A_5 \to A_5 + \dfrac{1}{gR}$ simply shifts the whole KK tower

“Lemma”: Gravity implies charge quantization (compact gauge groups)

Suppose there exist incommensurate electric charges, e.g. $q_{\mathrm{A}} = 1$ and $q_{\mathrm{B}} = \pi$

Then in addition to electric charge there exists an exactly conserved global symmetry, $A - B$ number

Once again, issues with entropy bounds etc.

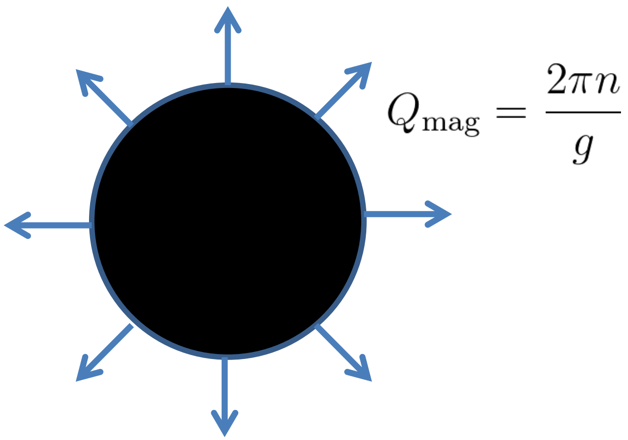
Entropy of magnetic black holes

The gauge + gravity EFT includes magnetically charged black hole solutions

Minimal (extremal) magnetic BH has finite entropy:

$$M \sim Q_{\text{mag}} M_{\text{pl}} = \frac{2\pi}{g} M_{\text{pl}}$$

$$S \sim 1/g^2$$



Conjecture: There must be a fundamental monopole that is not a black hole to explain this entropy in terms of microstates

Magnetic monopole cannot be pointlike; its size defines a cutoff length scale 1/Λ

Mass of monopole (magnetic self-energy) is

$$M_{\mathrm{monopole}} \sim \int d^3x B^2 \sim \frac{\Lambda}{g^2}$$

Require Schwarzschild radius to be less than 1/Λ:

$$\frac{\Lambda}{g^2 M_{\mathrm{pl}}^2} \lesssim \frac{1}{\Lambda} \rightarrow \boxed{\Lambda \lesssim g M_{\mathrm{pl}}}$$

Contrast to usual argument in the literature, that there must exist a magnetic monopole light enough that extremal black holes can decay into it, otherwise there are infinitely many stable extremal black holes

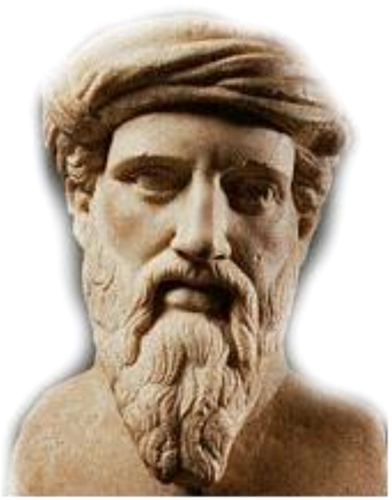
But:

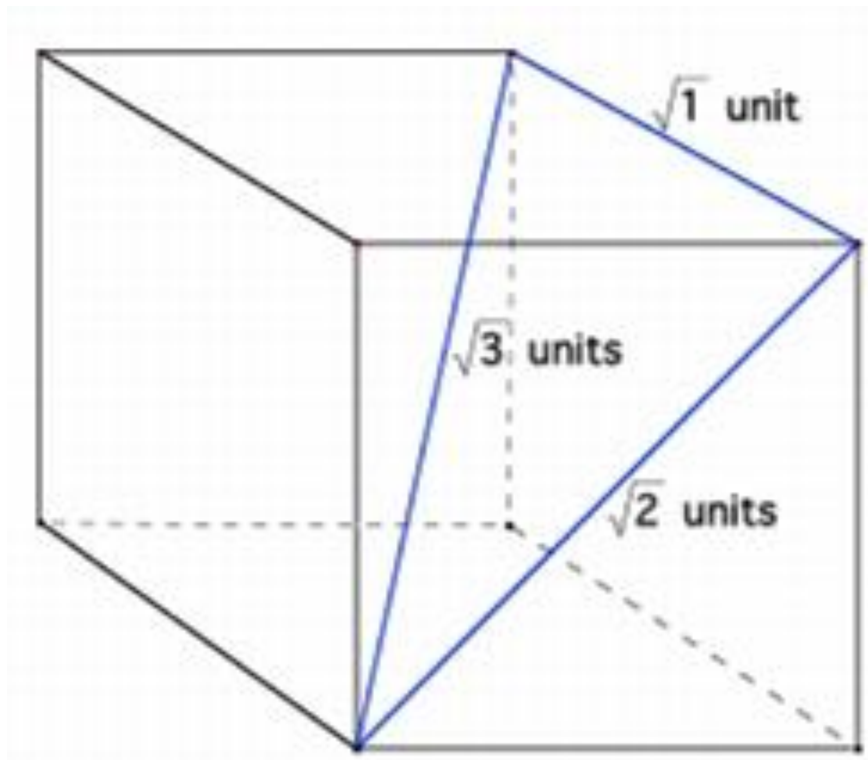
1) An infinite tower of stable states does not immediately seem problematic; there are finitely many states below any mass threshold

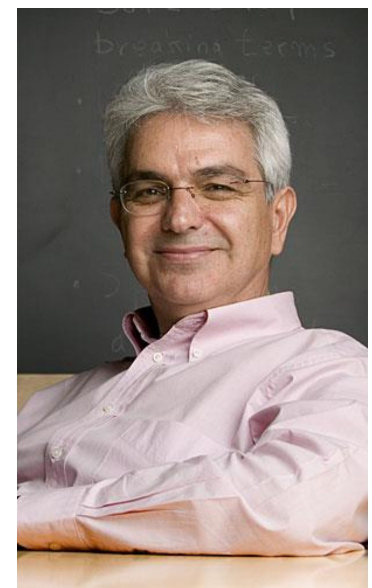
2) Corrections to extremal relation $M = Q$ from UV physics can allow the tower to decay

Multiple Fields?

N-flation: With *N* axion fields, radius of field space is increased by factor $\sim \sqrt{N}$; can achieve transplanckian range with $f \sim M_{pl}/\sqrt{N}$







Attempt #3: Extranatural *N*-flation

But for *N* U(1) gauge fields, there is a stronger WGC!
Imagine breaking U(1)'s to the diagonal:

$$U(1) \times U(1) \times \ldots \to U(1)_D$$

Coupling of $U(1)_D$ is $g/\sqrt{N}$. But then WGC requires

$$\Lambda \lesssim \frac{g}{\sqrt{N}} M_{\rm pl}$$

See e.g. Cheung, Remmen 1402.2287

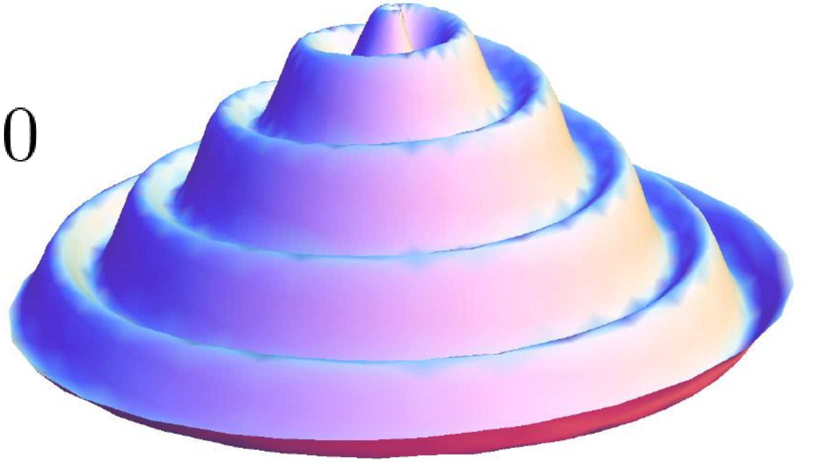
Then for each axion has $f < M_{pl}/\sqrt{N}$, so even with *N*-flation we must have $f_{\rm eff} < M_{pl}$!

Biaxion Models

Heavy mode (orthogonal to groove):

$$\phi_H \sim \frac{NA}{f_A} + \frac{B}{f_B}$$

Integrate out: frozen at $\phi_H \approx 0$



Light mode (inflaton):

$$\phi_L \sim B - \frac{f_A A}{N f_B}$$

$$V(\phi_L) \sim \cos\frac{\phi_L}{N f_B} \Rightarrow f_{\text{eff}} = N f_B$$

Potentially transplanckian

Constraints on Inflationary Phenomenology

$$\mathcal{N}_{\text{e-folds}} \sim \frac{f_{\text{eff}}}{M_{\text{pl}}} \lesssim M_{\text{pl}} R$$

But $1/R$ also controls the Hubble scale:

$$\boxed{H \sim \frac{\sqrt{V}}{M_{\text{pl}}} \sim M_{\text{pl}} \frac{1}{(M_{\text{pl}} R)^2} \lesssim \frac{M_{\text{pl}}}{\mathcal{N}_{\text{e-folds}}^2}}$$

To fit the real world data we need

$$\frac{H}{M_{\text{pl}}} \sim 10^{-4} \qquad \mathcal{N}_{\text{e-folds}} \gtrsim 60$$

On the edge of the controlled parameter space…

Corrections to CMB Power Spectrum

$$\epsilon = \frac{M_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2$$

Scalar power spectrum goes as 1/ε

$$\frac{\delta\epsilon}{\epsilon} \sim 2(Nn_B - n_A)e^{-2\pi RM}\sin\left[(Nn_B - n_A)\frac{\phi}{f_{\rm eff}}\right]$$

Searches for oscillations in the CMB power spectrum at the relevant frequencies require

$$\frac{\delta\epsilon}{\epsilon} \lesssim 1 - 5\%$$

$\Rightarrow$

Additional charged particles must have mass > few times compactification scale

Does the Theory Need to Have a Large N?

We required that the theory give us a light field with a parametrically large charge N in this model– looks strange. Perhaps the UV theory can’t actually realize this low-energy EFT?

With a slightly different model, we can avoid assuming that the dynamical theory has parametrically large integers “built-in.”

Chern-Simons model

Consider coupling the 5D gauge field to a non-Abelian sector:

$$S \supset \int d^5x \frac{N}{64\pi^2} \epsilon^{LMNPQ} G^a_{LM} G^a_{NP} A_Q.$$

In 4D:

$$S \supset \int d^4x \frac{N}{64\pi^2} \frac{A}{f} \epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma}. \qquad f = \frac{1}{2\pi R g}$$

If the non-Abelian group confines in the IR, one obtains an axion-like potential:

$$V(A) \sim V_0 \cos\left(\frac{NA}{f}\right)$$

Can recover the biaxion model without charged particles; the large *N* is in a coupling

Large Integer *N* from Flux

We can UV complete the 5D Chern-Simons model without introducing *N* in the action by considering a 7D model (in $\mathbb{R}^4 \times S^1 \times S^2$):

$$S \supset \int d^7x \frac{1}{32\pi^2} \mathrm{d}A \wedge A \wedge G \wedge G$$

A flux of $F = \mathrm{d}A$ can wrap the two-sphere:

$$\oint_{S^2} \mathrm{d}A = \frac{N}{2\pi}.$$

Integrating out the S^2 then gives the previous 5D coupling with a large *N*.

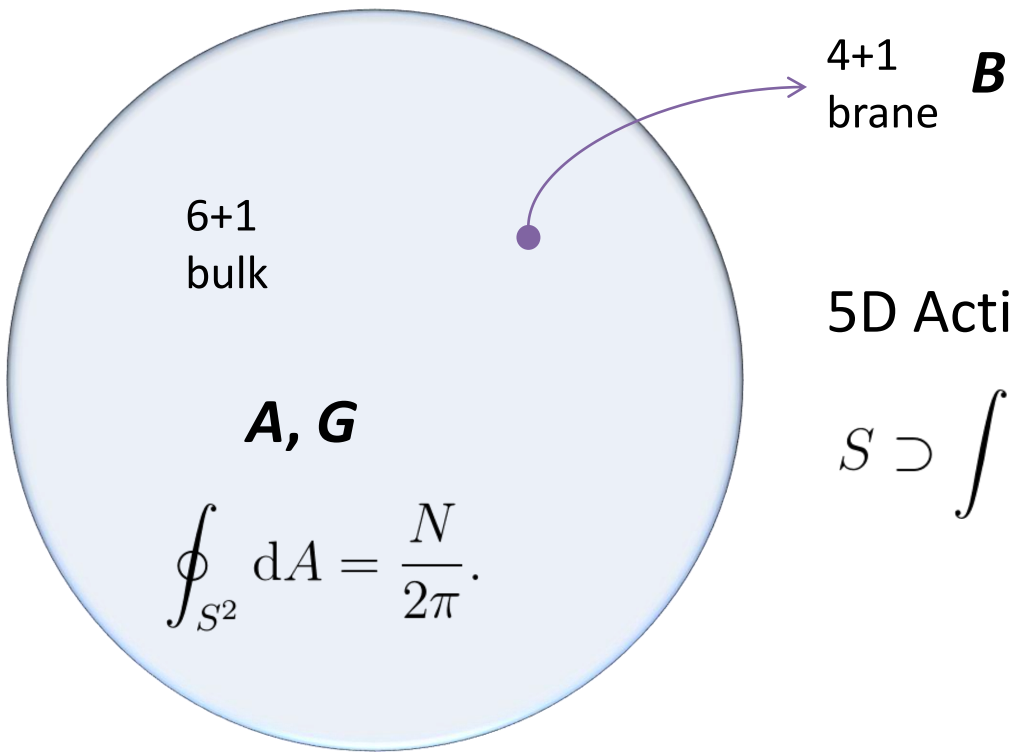
N is no longer in the action of the theory

Instead, there is a *landscape* of solutions with different values of *N*.

NOTE: “Anthropic selection” not necessary, since *N* does not need to be tuned!

Price for large *N*: large flux can destabilize the S^2

To obtain a large *N* for only one axion, one could imagine having one live in the 6+1 bulk while the other is localized to a 4+1 brane:



5D Action:

$$S \supset \int d^5x \frac{1}{32\pi^2} (NA + B) \wedge G \wedge G$$

→ "Charges" of the form $(n_A, n_B) = (N, 1)$
do not require tuning