A Model of Flavor and Flavor-Changing

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OUTLINE:

1. I will briefly explain a model of flavor mixing proposed two years ago by SMB and Heng-Yu Chen, in which all flavor change is controlled by a single "master matrix."

2. I will review how the model gives testable predictions for neutrino masses/mixings as well as some precise post-dictions for quark properties that are at present imprecisely known. [SMB and H-Y Chen, JHEP 1211 (2012) 092]

   I will also review how it gives predictions for proton decay branching ratios [SMB and H-Y Chen, JHEP 1310 (2013) 039]

3. **NEW:** I will show that the model has predictions for the pattern of flavor-changing produced by the exchange of a SM singlet scalar field, specifically for the processes $\tau \to \mu\gamma$, $\tau \to e\gamma$, $\mu \to e\gamma$.

Basic Ideas of the model:

(1) ∃ an underlying SU(5) symmetry relates quark and lepton properties.

(2) All family mixing is due to the 3 chiral fermion families mixing with extra vector-like fermion multiplets.

Three chiral families: $\mathbf{10}_i + \overline{\mathbf{5}}_i$ $(i = 1{,}2\ 3)$

An abelian family symmetry makes the Yukawa couplings of SM Higgs to these families diagonal in family space ⟹ DIAGONAL MASS MATRICES

Extra vectorlike fermion multiplets: $\mathbf{5}'_A + \overline{\mathbf{5}}'_A$ $(A = 1{,}2, \ldots)$

These have no family symmetry.

The key is that the $\overline{\mathbf{5}}_i$ and $\overline{\mathbf{5}}'_A$ mix with each other. This does not break SU(5), but it involves the spontaneous breaking of the family symmetry. **This mixing is characterized by a 3x3 matrix we call A.**

This induces mixing among the three chiral families.

Because $\overline{\mathbf{5}}$ fermion multiplets contain **LH leptons,** and **RH quarks,**
LH mixing is greater for quarks than leptons: MNS mixing is larger than CKM mixing ("lopsided" mixing idea).

All flavor mixing is controlled by the "master matrix" A.

Recall that in SU(5) fermion multiplets look like this: $10 = \begin{pmatrix} \ell^+ & \begin{pmatrix} u \\ d \end{pmatrix} & u^c \end{pmatrix} \quad \bar{5} = \begin{pmatrix} \begin{pmatrix} \nu \\ \ell^- \end{pmatrix} & d^c \end{pmatrix}$

The mass matrix for the down-type quarks is

$$(d(10_i) \;\; D(5'_A)) \begin{pmatrix} (m_d)_{ij} & 0 \\ \Delta_{Aj} & M_{AB} \end{pmatrix} \begin{pmatrix} d^c(\bar{5}_j) \\ D^c(\bar{5}'_B) \end{pmatrix}$$

diagonal: $(m_d)_{ij}$

Δ_{Aj}: $y_{Aj}\, 5'_A \bar{5}_j \langle \Omega_j \rangle$

M_{AB}: $Y_{AB}\, 5'_A \bar{5}'_B \langle \Omega \rangle$

Ω, Ω_j are SU(5) and SM singlets with large VEVs.

$\langle \Omega_j \rangle$ spontaneously breaks the abelian family symmetries

This gets block diagonalized into the known three families and the heavy extra fermions by unitary matrices as follows:

$$\underbrace{\begin{pmatrix} I & G^\dagger \\ -G & I \end{pmatrix}}_{U_L{}^\dagger} \begin{pmatrix} m_d & 0 \\ \Delta & M \end{pmatrix} \underbrace{\begin{pmatrix} A & B \\ C & D \end{pmatrix}}_{U_R} = \begin{pmatrix} \bar{m}_d & 0 \\ 0 & \bar{M} \end{pmatrix},$$

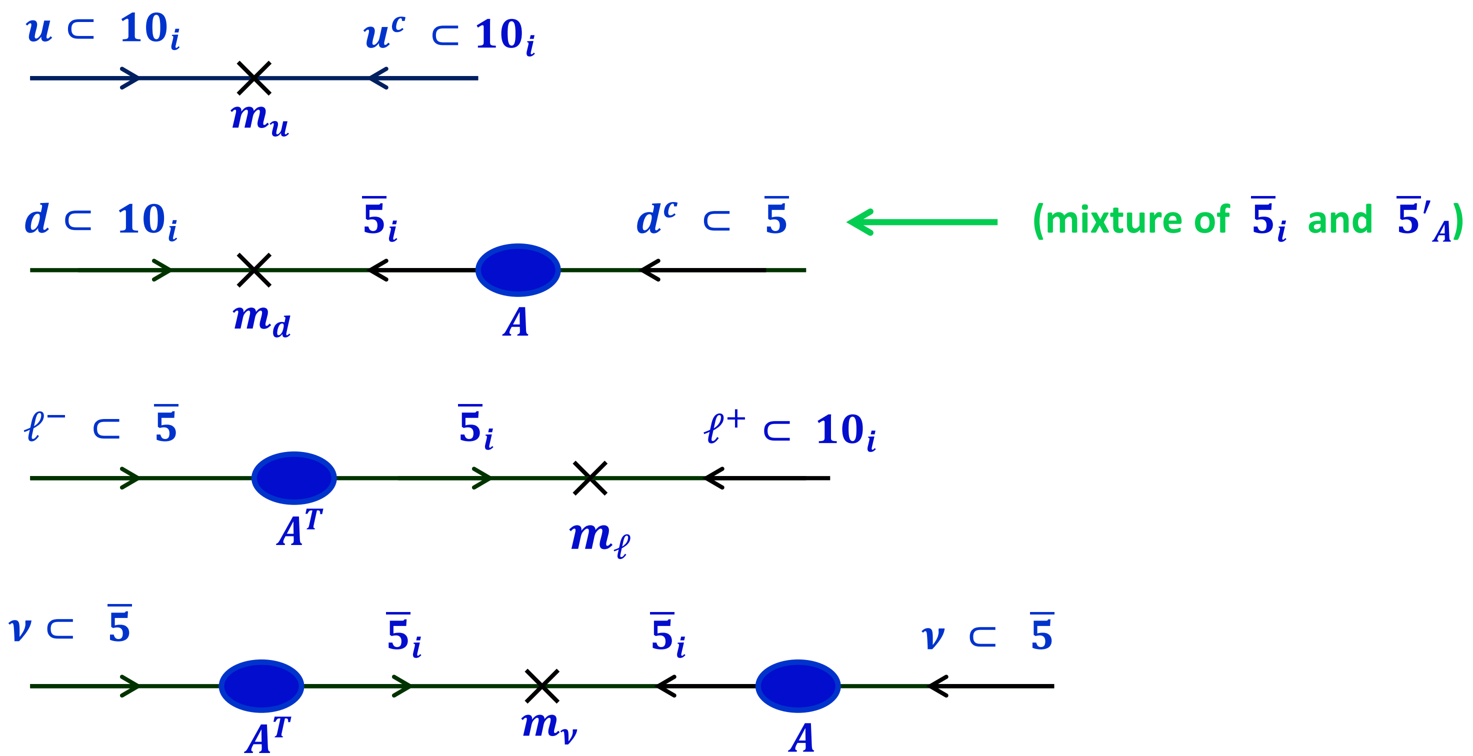
where

$$A = \left[I + \Delta^\dagger M^{-1\dagger} M^{-1} \Delta\right]^{-1/2}$$
$$B = \left[I + \Delta^\dagger M^{-1\dagger} M^{-1} \Delta\right]^{-1/2} \Delta^\dagger M^{-1\dagger}$$
$$C = -M^{-1}\Delta\left[I + \Delta^\dagger M^{-1\dagger} M^{-1} \Delta\right]^{-1/2}$$
$$D = \left[I + M^{-1}\Delta\Delta^\dagger M^{-1\dagger}\right]^{-1/2}$$
$$G^\dagger = -m_d \Delta^\dagger M^{-1\dagger} D^2 M^{-1}$$

So that the mass matrix for ***d, s,*** and ***b*** is

$$\bar{m}_d = m_d A$$

m_d: diagonal; A: "master matrix"



$$\bar{m}_u = m_u$$
$$\bar{m}_d = m_d A$$
$$\bar{m}_\ell = A^T m_\ell$$
$$\bar{m}_\nu = A^T m_\nu A$$

$$\bar{m}_u = m_u$$

are diagonal

$$\bar{m}_d = m_d A$$

so CKM mixing comes from A

A is a hermitian 3x3 matrix $\Longrightarrow$ 9 parameters. Too many?

But A can be written in the form

$$\boxed{A = \mathcal{D} A_\Delta U}$$

where $\mathcal{D}$ is diagonal, U is unitary, and

$$A_\Delta = \begin{pmatrix} 1 & b & ce^{i\theta} \\ 0 & 1 & a \\ 0 & 0 & 1 \end{pmatrix}$$

So THIS matrix controls all flavor mixing

$$\bar{m}_d = (m_d \mathcal{D}) \begin{pmatrix} 1 & b & ce^{i\theta} \\ 0 & 1 & a \\ 0 & 0 & 1 \end{pmatrix} U \longrightarrow \cong \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} \begin{pmatrix} 1 & b & ce^{i\theta} \\ 0 & 1 & a \\ 0 & 0 & 1 \end{pmatrix}$$

U can be absorbed into a redefinition of the right-handed quarks

$$\Longrightarrow \quad a \cong V_{cb}(m_b/m_s) \cong (0.04)(50) \cong 2$$

$$b \cong V_{us}(m_s/m_d) \cong (0.2)(20) \cong 4$$

$$c \cong V_{ub}(m_b/m_d) \cong \left(0.003\, e^{i\delta}\right)\left(10^3\right) \cong 3e^{i\delta}$$

One can straightforwardly show that in the basis in which the charged-lepton mass matrix is diagonal the mass matrix of the three light neutrinos is

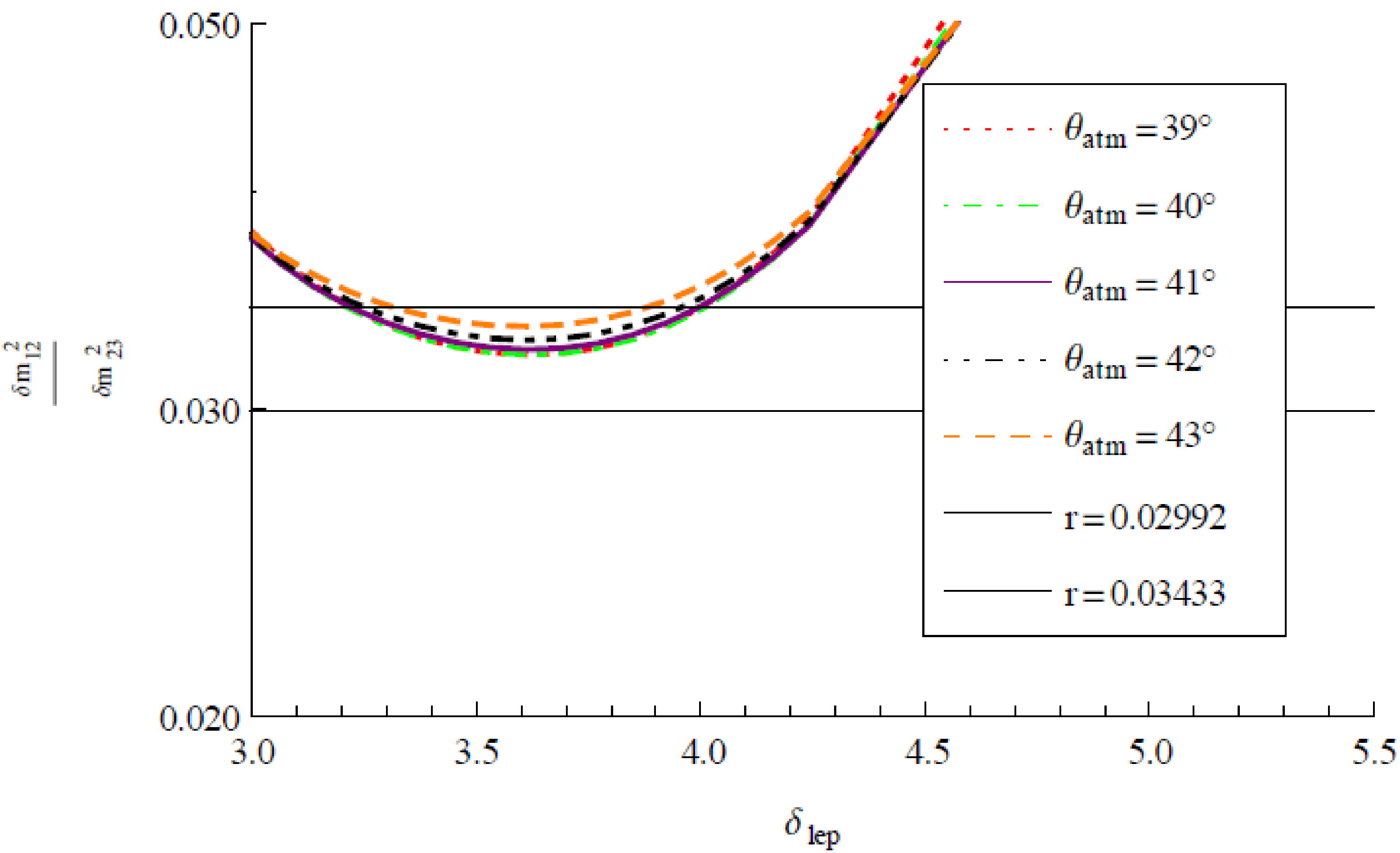
$$\bar{m}_\nu = A_\Delta{}^T\,(diag)\,A_\Delta = \mu_\nu \begin{pmatrix} 1 & 0 & 0 \\ b & 1 & 0 \\ ce^{i\theta} & a & 1 \end{pmatrix} \begin{pmatrix} qe^{i\beta} & 0 & 0 \\ 0 & pe^{i\alpha} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & b & ce^{i\theta} \\ 0 & 1 & a \\ 0 & 0 & 1 \end{pmatrix}$$

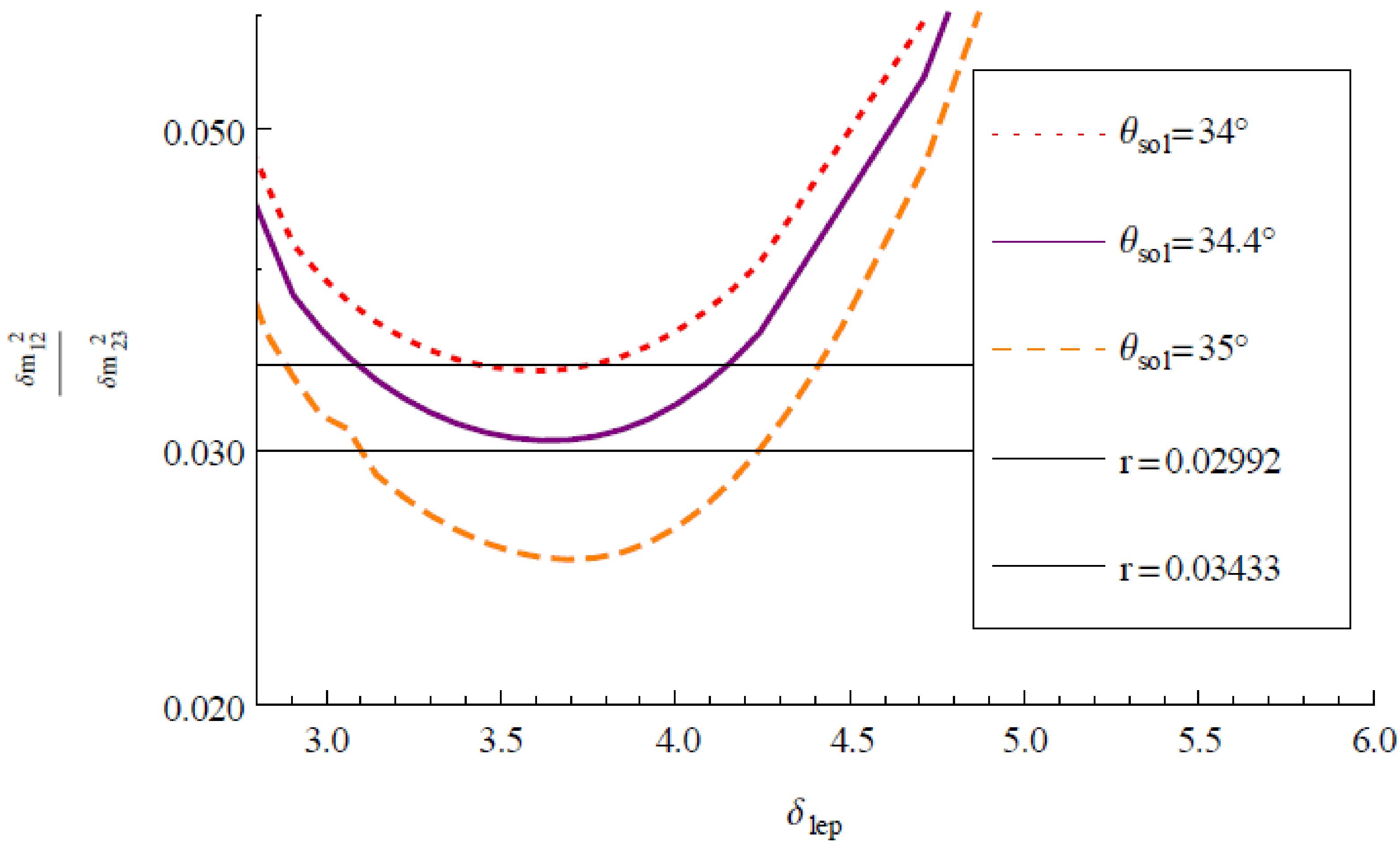
Quantity	Values in fit	Experiment
μ_ν	0.1428 eV	—
$pe^{i\alpha}$	$0.1525e^{-2.734i}$	—
$qe^{i\beta}$	$0.01405e^{-0.352i}$	—
m_b/m_s	52.9	52.9 ± 2.6
m_s/m_d	19	17 to 22
$\lvert V_{us}\rvert$	0.2252	0.2252 ± 0.0009
$\lvert V_{cb}\rvert$	0.0409	0.0409 ± 0.0011
$\lvert V_{ub}\rvert$	0.00415	0.00415 ± 0.00049
δ	1.30 rad	$1.187^{+0.175}_{-0.192}$ rad
θ_{sol}	34.1°	$33.89^\circ\ ^{+0.976^\circ}_{-0.971^\circ}$
θ_{atm}	40°	$45^\circ \pm 6.5^\circ$
θ_{13}	9.12°	$9.122^\circ\ ^{+0.609^\circ}_{-0.647^\circ}$
δm^2_{23}	2.32×10^{-3} eV^2	$2.32^{+0.12}_{-0.08} \times 10^{-3}$ eV^2
δm^2_{12}	7.603×10^{-5} eV^2	$(7.5 \pm 0.2) \times 10^{-5}$ eV^2
δ_{lep}	1.15π rad	$1.1\pi^{+0.3\pi}_{-0.4\pi}$ rad
$(M_\nu)_{ee}$	0.0020 eV	

There are 9 neutrino observables: 3 masses,
3 mixing angles, Dirac phase,
2 Majorana phases.

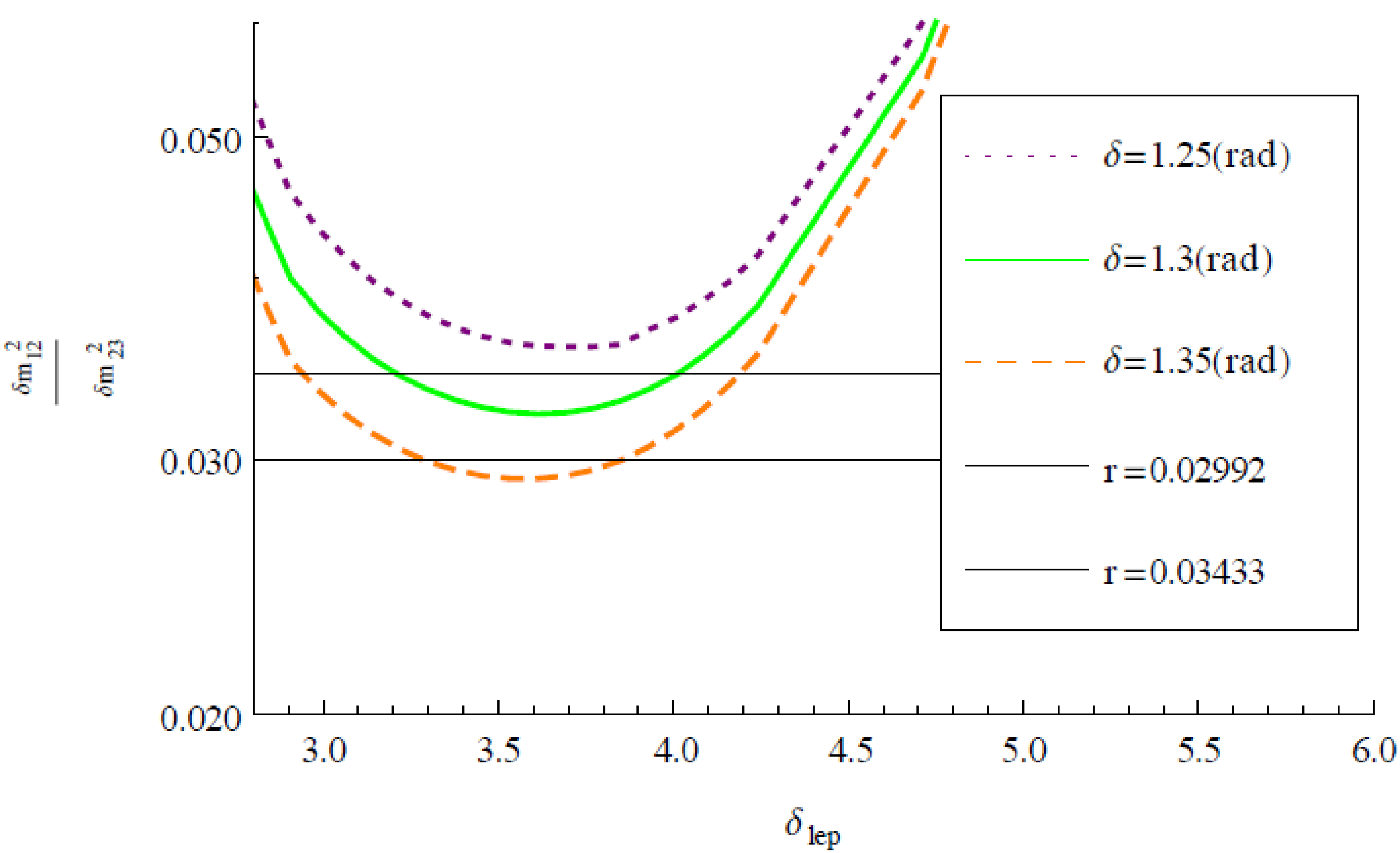
There are 5 model parameters.

9 - 5 = 4 predictions.


0.050
0.030
0.020
3.0
3.5
4.0
4.5
5.0
5.5
$\delta m_{12}^2 / \delta m_{23}^2$
δ_{lep}
$\theta_{atm} = 39°$
$\theta_{atm} = 40°$
$\theta_{atm} = 41°$
$\theta_{atm} = 42°$
$\theta_{atm} = 43°$
r = 0.02992
r = 0.03433



$\frac{\delta m_{12}^2}{\delta m_{23}^2}$
0.050
0.030
0.020
3.0
3.5
4.0
4.5
5.0
5.5
6.0
δ_{lep}
$\theta_{sol}=34°$
$\theta_{sol}=34.4°$
$\theta_{sol}=35°$
$r=0.02992$
$r=0.03433$



0.050
0.030
0.020
3.0
3.5
4.0
4.5
5.0
5.5
6.0
δm²₁₂ / δm²₂₃
δ_lep
δ=1.25(rad)
δ=1.3(rad)
δ=1.35(rad)
r=0.02992
r=0.03433

Proton decay branching ratios

There are other mixing angles that describe how quarks and leptons are put together in SU(5) multiplets. These are also controlled by the matrix A_Δ . One can show that the fermion multiplets in this model look like this:

$$\mathbf{10} \cong \left[e^{-i\Theta} V_{\ell^+} \tilde{\ell}^c \quad \begin{pmatrix} \tilde{u} \\ V_{CKM} \tilde{d} \end{pmatrix} \quad e^{-i\Phi} \tilde{u}^c \right] \qquad \overline{\mathbf{5}} \cong \left[\begin{pmatrix} U_{MNS} \tilde{\nu} \\ \tilde{\ell}^- \end{pmatrix} \quad \tilde{d}^c \right]$$

where

$$V_{\ell^+} \cong \begin{pmatrix} 1 & -\frac{m_e}{m_\mu} b & -\frac{m_e}{m_\tau} c e^{i\theta} \\ \frac{m_e}{m_\mu} b & 1 & -\frac{m_\mu}{m_\tau} a \\ \frac{m_e}{m_\tau} c e^{-i\theta} & \frac{m_\mu}{m_\tau} a & 1 \end{pmatrix}$$

From this it can be shown that

$$\frac{\Gamma(p \longrightarrow \pi^0 \mu^+)}{\Gamma(p \longrightarrow \pi^0 e^+)} \cong \frac{(b\kappa)^2 + \sin^2\theta_C}{(1+\kappa)^2 + 4\cos^2\theta_C}$$

$$R \equiv 2\left(1 - \frac{{m_K}^2}{{m_p}^2}\right)^2 \left(\frac{1 + \frac{m_p}{m_B}(D-F)}{1 + D + F}\right)^2 = 0.105 \pm 0.005$$

$$\frac{\Gamma(p \longrightarrow \kappa^0 e^+)}{\Gamma(p \longrightarrow \pi^0 e^+)} \cong R \frac{(b\kappa)^2 + 4\sin^2\theta_C}{(1+\kappa)^2 + 4\cos^2\theta_C}$$

$$\frac{\Gamma(p \longrightarrow \kappa^0 \mu^+)}{\Gamma(p \longrightarrow \pi^0 e^+)} \cong R \frac{(1 + b^2\kappa)^2 + \cos^2\theta_C}{(1+\kappa)^2 + 4\cos^2\theta_C}$$

New: Flavor-changing mediated by exchange of $\widetilde{\Omega}$

$$(d \quad D)\begin{pmatrix} m_d & 0 \\ \Delta & M \end{pmatrix}\begin{pmatrix} d^c \\ D^c \end{pmatrix} \Longrightarrow (d \quad D)\begin{pmatrix} 0 & 0 \\ 0 & \frac{M}{\langle\Omega\rangle}\widetilde{\Omega} \end{pmatrix}\begin{pmatrix} d^c \\ D^c \end{pmatrix}$$

$$D\left(Y\left[\langle\Omega\rangle + \widetilde{\Omega}\right]\right)D^c = D\left(M + \frac{M}{\langle\Omega\rangle}\widetilde{\Omega}\right)D^c$$

To see how $\widetilde{\Omega}$ couples to the three known families, one must go through the block-diagonalization we did before

$$\underbrace{\begin{pmatrix} I & G^\dagger \\ -G & I \end{pmatrix}}_{U_L{}^\dagger}\begin{pmatrix} 0 & 0 \\ 0 & \frac{M}{\langle\Omega\rangle}\widetilde{\Omega} \end{pmatrix}\underbrace{\begin{pmatrix} A & B \\ C & D \end{pmatrix}}_{U_R} = \begin{pmatrix} G^\dagger MC & G^\dagger MD \\ MC & MD \end{pmatrix}\left(\frac{\widetilde{\Omega}}{\langle\Omega\rangle}\right)$$

So, $\widetilde{\Omega}$ couples to the 3 known families by the effective Yukawa coupling matrix

$$\frac{1}{\langle\Omega\rangle}G^\dagger MC \quad = \quad \frac{1}{\langle\Omega\rangle}\left(-m_d\Lambda^\dagger M^{-1\dagger}D^2M^{-1}\right)^\dagger(M)(-M^{-1}\Delta A)$$

$$= \frac{1}{\langle\Omega\rangle}m_d\left[\Delta^\dagger M^{-1\dagger}\right]D^2\left[M^{-1}\Delta\right]A$$

$$\tilde{Y} = \frac{1}{\langle\Omega\rangle} m_d \left[\Delta^\dagger M^{-1\dagger}\right] D^2 \left[M^{-1}\Delta\right] A$$

$$= \frac{1}{\langle\Omega\rangle} m_d \left[\Delta^\dagger M^{-1\dagger}\right]\left[I + M^{-1}\Delta\Delta^\dagger M^{-1\dagger}\right]^{-1}\left[M^{-1}\Delta\right] A$$

$$= \frac{1}{\langle\Omega\rangle} m_d \left[\Delta^\dagger M^{-1\dagger} M^{-1}\Delta\right]\left[I + \Delta^\dagger M^{-1\dagger} M^{-1}\Delta\right]^{-1} A$$

$$= \frac{1}{\langle\Omega\rangle} m_d \left[A^{-2} - I\right] A^2 A = \frac{1}{\langle\Omega\rangle} m_d \left[A - AA^\dagger A\right]$$

$$A = \mathcal{D}\, A_\Delta U$$

$$= \frac{1}{\langle\Omega\rangle} m_d \mathcal{D}\left[A_\Delta - A_\Delta\, {A_\Delta}^\dagger\, |\mathcal{D}|^2\, A_\Delta\right] U$$

is absorbed by redef. of RH fields

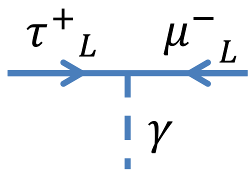
$$\tilde{Y} \cong \frac{1}{\langle\Omega\rangle}(m_d\, \mathcal{D} A_\Delta)\left[I - {A_\Delta}^\dagger\, |\mathcal{D}|^2\, A_\Delta\right] = \frac{1}{\langle\Omega\rangle} \bar{m}_d \left[I - {A_\Delta}^\dagger\, |\mathcal{D}|^2\, A_\Delta\right]$$

$$\tilde{Y}_{FC} \cong -\frac{1}{\langle\Omega\rangle} \bar{m}_d {A_\Delta}^\dagger\, |\mathcal{D}|^2\, A_\Delta$$

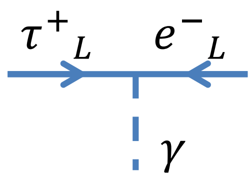
$$\mathcal{D} = \begin{pmatrix} |\delta|^2 & 0 & 0 \\ 0 & |\varepsilon|^2 & 0 \\ 0 & 0 & |\zeta|^2 \end{pmatrix}$$

$$A_\Delta = \begin{pmatrix} 1 & b & ce^{i\theta} \\ 0 & 1 & a \\ 0 & 0 & 1 \end{pmatrix} \qquad \bar{m}_d = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

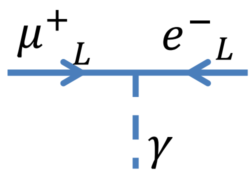
The expression for the Yukawa couplings of $\widetilde{\Omega}$ to the charged leptons looks the same, but with Left-handed and Right-handed fermions interchanged. It ends up looking as follows:



$$\frac{\boldsymbol{m_\tau}}{\langle\boldsymbol{\Omega}\rangle}\left[|\boldsymbol{\delta}|^2 bce^{-i\theta} + |\boldsymbol{\varepsilon}|^2 a\right] \equiv \frac{m_\tau}{\langle\Omega\rangle}\Delta_{\mu\tau}$$



$$\frac{\boldsymbol{m_\tau}}{\langle\boldsymbol{\Omega}\rangle}\left[|\boldsymbol{\delta}|^2 ce^{-i\theta}\right] \equiv \frac{m_\tau}{\langle\Omega\rangle}\Delta_{e\tau}$$



$$\frac{\boldsymbol{m_\mu}}{\langle\boldsymbol{\Omega}\rangle}\left[|\boldsymbol{\delta}|^2 b\right] \equiv \frac{m_\mu}{\langle\Omega\rangle}\Delta_{e\mu}$$

$$\frac{\Gamma(\tau \longrightarrow e\gamma)}{\Gamma(\mu \longrightarrow e\gamma)} = \left|\frac{\Delta_{e\tau}}{\Delta_{e\mu}}\right|^2 = \left|\frac{c}{b}\right|^2 \cong \frac{9}{16}$$

And if the δ^2 term dominates for $\tau \longrightarrow \mu\gamma$

$$\frac{\Gamma(\tau \longrightarrow \mu\gamma)}{\Gamma(\mu \longrightarrow e\gamma)} = \left|\frac{\Delta_{\mu\tau}}{\Delta_{e\mu}}\right|^2 = |c|^2 \cong 9$$

One can derive bounds on the unknown parameters $|\delta|^2, |\varepsilon|^2$, and $|\zeta|^2$.

$$AA^\dagger = \left[I + (M^{-1}\Delta)^\dagger (M^{-1}\Delta)\right]^{-1} = \begin{pmatrix} \delta & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \zeta \end{pmatrix} A_\Delta A_\Delta{}^\dagger \begin{pmatrix} \delta^* & 0 & 0 \\ 0 & \varepsilon^* & 0 \\ 0 & 0 & \zeta^* \end{pmatrix}$$

the elements of this matrix bounded from above

$$BR(\tau \longrightarrow \mu\gamma) = 24\left(\frac{\alpha}{\pi}\right)^3 \left(\frac{v}{\langle\Omega\rangle}\right)^4 f^2\left(\frac{M_F}{M_{\tilde{\Omega}}}\right) \underbrace{\left| |\boldsymbol{\delta}|^2 bce^{-i\theta} + |\boldsymbol{\varepsilon}|^2 a \right|}_{<2}{}^2$$

$$BR(\tau \longrightarrow \mu\gamma) < 10^{-8}\,(TeV/\langle\Omega\rangle)^4$$