Enhanced gauge symmetry and winding states in Double Field Theory

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13/08/2015

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Goals

1. The study of the compactified boson, $M_d = M_D \times S^1$, from the world sheet perspective and from the target space (DFT) one.
2. Make the comparison between the world sheet scattering amplitudes (string action) and the effective action of double field theory.
3. Is the (string action) $=$ (effective action of DFT)?
4. Give as good as possible a geometric interpretation of the internal (violating section condition) sector of the theory.

I will be dealing with states that satisfy the level matching but not the section condition.

Introduction and motivation

Momentum state on flat non-compact manifold

$$e^{ik\left(x^L(z)+x^R(\bar{z})\right)} \quad ; \quad k \in \mathbb{R}$$

CFT field

$$x(z,\bar{z}) = x^L(z) + x^R(\bar{z})$$

Non compact coordinate

$$x = x^L + x^R$$

Introduction and motivation

Momentum+winding state on TORUS

$$e^{ik_L x^L(z)+ik_R x^R(\bar{z})} \quad ; \quad k_{L,\,R} = \frac{p}{R} \pm \frac{\tilde{p}}{\tilde{R}} \quad ; \quad p,\ \tilde{p} \in \mathbb{Z} \quad ; \quad \boxed{\tilde{R} = \frac{\alpha'}{R}}$$

CFT field

$$x(z,\bar{z}) = x^L(z) + x^R(\bar{z}) \quad ; \quad \tilde{x}(z,\bar{z}) = x^L(z) - x^R(\bar{z})$$

Compact coordinate

$$x = x^L + x^R \quad ; \quad \tilde{x} = x^L - x^R$$

$$\boxed{M_D \times S^1 \to M_D \times S^1 \times \tilde{S}^1}$$

"Double Field Theory," C.Hull and B.Zwiebach.
Talk by C. M. Hull.

Introduction and motivation

- Holomorphic anti-holomorphic current operators

$$\begin{aligned}
\bar{J}^{+}(\bar{z}) &= \exp[+2i/\sqrt{\alpha'}x^R(z)]d\bar{z} \\
\bar{J}^{-}(\bar{z}) &= \exp[-2i/\sqrt{\alpha'}x^R(z)]d\bar{z} \\
\bar{J}^{3}(\bar{z}) &= i/\sqrt{\alpha'}\partial_z x^R(z)d\bar{z}
\end{aligned}$$

Current OPE and algebra

$$J^a(z)J^b(0) \sim \frac{f^{abc}}{z}J^c(0) \quad ; \quad J^a(z) = \sum J_m{}^a z^{-(m+1)}$$

$$[J_m{}^a, J_n{}^b] = \frac{1}{2}m\delta^{ab}\delta_{m+n,\,0} + f^{abc}J_{m+n}{}^c$$

Introduction and motivation

- Going beyond ordinary DFT or GG relaxing the section condition.
- Including not only winding modes but also winding states with $N \neq \bar{N}$.

$$m^2 = \frac{2}{\alpha'}(N + \bar{N} - 2) + (\frac{p}{R})^2 + (\frac{\tilde{p}}{\tilde{R}})^2$$

$$N - \bar{N} = -p\tilde{p}$$

$$R = \tilde{R} = \sqrt{\alpha'}$$

At special points in the torus moduli space there are extra states with $m^2 = 0$ giving enhanced gauge symmetry, while near these special points these states will have small m^2. These have $(N, \bar{N}) = (1, 0)$ or $(N, \bar{N}) = (0, 1)$ and so have $m^2 = -\frac{2}{\alpha'}$; we will not include these here.

Taken from: "Double Field Theory," C.Hull and B.Zwiebach.

Introduction and motivation

- Going beyond ordinary DFT or GG relaxing the section condition.
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$$m^2 = \frac{2}{\alpha'}(N + \bar{N} - 2) + (\frac{p}{\sqrt{\alpha'}})^2 + (\frac{\tilde{p}}{\sqrt{\alpha'}})^2$$

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Introduction and motivation

- Going beyond ordinary DFT or GG relaxing the section condition.
- Including not only winding modes but also winding states with $N \neq \bar{N}$.

$$m^2 = \frac{2}{\alpha'}(0 + 1 - 2) + \left(\frac{p}{\sqrt{\alpha'}}\right)^2 + \left(\frac{\tilde{p}}{\sqrt{\alpha'}}\right)^2 = 0$$

$$0 - 1 = -p\tilde{p}$$

$$R = \tilde{R} = \sqrt{\alpha'}$$

At special points in the torus moduli space there are extra states with $m^2 = 0$ giving enhanced gauge symmetry, while near these special points these states will have small m^2. These have $(N, \bar{N}) = (1, 0)$ or $(N, \bar{N}) = (0, 1)$ and so have $m^2 = -\frac{2}{\alpha'}$; we will not include these here.

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Introduction and motivation

- Going beyond ordinary DFT or GG relaxing the section condition.
- Including not only winding modes but also winding states with $N \neq \bar{N}$.

$$m^2 = \frac{2}{\alpha'}(0 + 1 - 2) + \frac{1}{\alpha'} + \frac{1}{\alpha'} = 0$$

$$0 - 1 = -1 \cdot 1$$

$$R = \tilde{R} = \sqrt{\alpha'}$$

At special points in the torus moduli space there are extra states with $m^2 = 0$ giving enhanced gauge symmetry, while near these special points these states will have small m^2. These have $(N, \bar{N}) = (1, 0)$ or $(N, \bar{N}) = (0, 1)$ and so have $m^2 = -\frac{2}{\alpha'}$; we will not include these here.

Taken from: "Double Field Theory," C.Hull and B.Zwiebach.

Introduction and motivation

- String theory on $M_d = M_D \times S^1$

$$\{G, \;\; B, \;\; \Phi\} \cup \{A_L, \;\; A_R, \;\; M\}$$

$$\{A_L, \;\; A_R\} \in \{\mathfrak{su}(2)_L, \;\; \mathfrak{su}(2)_R\}$$

$$M \in \mathfrak{su}(2)_L \times \mathfrak{su}(2)_R$$

Introduction and motivation

- T-duality symmetry group (counting the massless fields)

Field content

$$\begin{aligned} G,\ \ B &\rightarrow D^2 \\ A_L,\ \ A_R &\rightarrow 6D \\ M &\rightarrow 3^2 \end{aligned}$$

Dimension of the coset

$$\dim\left[\frac{O(D+3,D+3)}{O(D+3)\times O(D+3)}\right] = (D+3)^2$$

Structure of the talk

Structure of the talk

Structure of the talk

1. Action from string theory
2. Effective action from DFT
3. The geometry of the internal space
4. Conclusions

Structure of the talk

String theory side

World sheet amplitudes at the self dual radius

Massless $\mathfrak{su}(2)_R$ gauge boson states

$$N = 1 \quad \bar{N} = 1 \quad p = 0 \quad \tilde{p} = 0 \quad \rightarrow \quad \bar{A}_\mu^3 dx^\mu$$

$$N = 1 \quad \bar{N} = 0 \quad p = \pm 1 \quad \tilde{p} = \mp 1 \quad \rightarrow \quad \bar{A}_\mu^\pm dx^\mu$$

Vertex operator

$$\bar{V}(z, \bar{z}) = i\sqrt{2}\frac{g_c'}{\alpha'^{\,1/2}} \sum_{a=\pm,\,3} \bar{\epsilon}_\mu{}^a : e^{ik\cdot x}\partial x^\mu \bar{J}^{\,a}(\bar{z}) :$$

Modes expansion

$$: \bar{A}^{\,a}\bar{J}^{\,a}(\bar{z}) :=: (\bar{A}^{\,+}e^{(+2i/\sqrt{\alpha'}x^R)} + \bar{A}^{\,-}e^{(-2i/\sqrt{\alpha'}x^R)})d\bar{z} + \bar{A}^3 dx^R(\bar{z}) :$$

World sheet amplitudes at the self dual radius

Massless $\mathfrak{su}(2)_L$ gauge boson states

$$N=1 \quad \bar{N}=1 \quad p=0 \quad \tilde{p}=0 \quad \rightarrow \quad A_\mu^3 dx^\mu$$
$$N=0 \quad \bar{N}=1 \quad p=\pm 1 \quad \tilde{p}=\pm 1 \quad \rightarrow \quad A_\mu^\pm dx^\mu$$

Vertex operator

$$V(z,\bar{z}) = i\sqrt{2}\frac{g_c'}{\alpha'^{\,1/2}} \sum_{a=\pm,\,3} \epsilon_\mu{}^a : e^{ik\cdot x}\bar{\partial}x^\mu J^{\,a}(z) :$$

Modes expansion

$$: A^{\,a} J^{\,a}(z) :=: (A^{\,+} e^{(+2i/\sqrt{\alpha'}x^L)} + A^{-} e^{(-2i/\sqrt{\alpha'}x^L)})dz + A^3 dx^L(z) :$$

World sheet amplitudes at the self dual radius

Massless $\mathfrak{su}(2)_L \times \mathfrak{su}(2)_R$ scalars states

$$
\begin{array}{lllll}
N=1 & \bar{N}=1 & p=0 & \tilde{p}=0 & \rightarrow M_{33} \\
N=1 & \bar{N}=0 & p=\pm 1 & \tilde{p}=\mp 1 & \rightarrow M_{3\pm} \\
N=0 & \bar{N}=1 & p=\pm 1 & \tilde{p}=\pm 1 & \rightarrow M_{\pm 3} \\
N=0 & \bar{N}=0 & p=\pm 2 & \tilde{p}=0 & \rightarrow M_{\pm\pm} \\
N=0 & \bar{N}=0 & p=0 & \tilde{p}=\pm 2 & \rightarrow M_{\pm\mp}
\end{array}
$$

Vertex operator

$$V_S(z,\bar{z}) = g_c' \sum_{a,\, b=\pm,\, 3} \epsilon^{ab} : e^{ik\cdot x} J^a(z) \bar{J}^b(\bar{z}) :$$

World sheet amplitudes at the self dual radius

Current OPE and algebra

$$J^a(z)J^b(0) \sim \frac{f^{abc}}{z} J^c(0) \quad ; \quad J^a(z) = \sum J_m{}^a z^{-(m+1)}$$

$$[J_m{}^a, J_n{}^b] = \frac{1}{2} m\delta^{ab}\delta_{m+n,\,0} + f^{abc} J_{m+n}{}^c$$

Three points functions

$< VVV >$ $< V\bar{V}V_S >$ $< \bar{V}\bar{V}\bar{V} >$

$< VVV_S >$ $< V_S V_S V_S >$ $< \bar{V}\bar{V}V_S >$

$< VV_S V_S >$ $< \bar{V}V_S V_S >$

World sheet amplitudes at the self dual radius

String action

$$
\begin{aligned}
S^{\text{string}} \quad = \quad & \int d^D x \sqrt{g} e^{-2\Phi} \Big[R + 4(\partial\Phi)^2 - \frac{1}{12} H^2 \\
& -\frac{1}{4}\delta^{ij}\, F_{\mu\nu}{}^{i} F^{\mu\nu\, j} - \frac{1}{4}\delta^{ij}\, \bar{F}_{\mu\nu}{}^{i} \bar{F}^{\mu\nu\, j} - \frac{1}{2} M_{ij}\, F_{\mu\nu}{}^{i} \bar{F}^{\mu\nu\, j} \\
& + \frac{1}{8} g^{\mu\nu} (D_\mu M)_{ij} (D_\nu M)_{ij} - \det(M) \Big]
\end{aligned}
$$

Covariant derivative

$$
(D_\mu M)_{ij} = \partial_\mu M_{ij} - f_{ik}{}^{l} A_\mu{}^{k} M_{lj} - \bar{f}_{jk}{}^{l} \bar{A}_\mu{}^{k} M_{il}
$$

$$
H = dB_2 + C.S_L + C.S_R
$$

Double Field Theory side

Circle reduction

Generalised frame

$$
\begin{aligned}
E_{\hat{a}} &= \hat{e}_a - \iota_{\hat{e}_{\hat{a}}} \hat{B} \\
E^{\hat{a}} &= \hat{e}^{\hat{a}}
\end{aligned}
$$

KK decomposition 1-form frame

$$\hat{e}^{\hat{a}} = \begin{pmatrix} e^a \\ \phi(dx^d + V_1) \end{pmatrix}$$

KK decomposition vector frame

$$\hat{e}_{\hat{a}} = \begin{pmatrix} e_a - \iota_{e^a} V_1 \partial_{x^d} \\ \phi^{-1} \partial_{x^d} \end{pmatrix}$$

KK decomposition B-field

$$\hat{B}_2 = B_2 + B_1 \wedge (dx^d + V_1) \quad ; \quad \iota_{x^d} B_2 = 0$$

"Generalised geometry for string corrections," Coimbra, Minasian, Triendl and Waldram.

Circle reduction

KK decomposition generalised frame

$$
\begin{aligned}
E_a &= e_a - (\iota_{e_a} V_1)\,\partial_{x^d} - (\iota_{e_a} B_1) dx^d - \iota'_{e_a} C^+ \\
E_d &= \phi^{-1}(\partial_{x^d} + B_1) \\
E^d &= \phi(dx^d + V_1) \\
E^a &= e^a
\end{aligned}
$$

$$
C^+ = (B_2 + V_1 \wedge B_1) + V_1 B_1 = B'_2 + V_1 B_1
$$

Circle reduction

Internal U(1) × U(1) generalised frame

$$\begin{pmatrix} E_d \\ E^d \end{pmatrix} = \begin{pmatrix} \phi^{-1} & 0 \\ 0 & \phi \end{pmatrix} \begin{pmatrix} \partial_{x^d} + B_1 \\ dx^d + V_1 \end{pmatrix}$$

Scherk-Schwarz generalised form

$$E_A = U_A{}^B(x) E'_B$$

Generalised tangent bundle

$$TS^1 \oplus T^*S^1 \to \mathcal{C}_+ \oplus \mathcal{C}_-$$

Matrix rotation

$$R = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

Circle reduction

Scalar matrix

$$U_A{}^B(x) = \begin{pmatrix} \mathbb{I}_D & 0 & 0 & 0 \\ 0 & U^+ & -U^- & 0 \\ 0 & -U^- & U^+ & 0 \\ 0 & 0 & 0 & \mathbb{I}_D \end{pmatrix} \in \frac{O(D+1, D+1)}{O(D+1) \times O(D+1)}$$

$$\begin{pmatrix} E'{}_a \\ E'_- \\ E'_+ \\ E'{}^a \end{pmatrix} = \begin{pmatrix} e_a - (\iota_{e_a}\bar{A})\bar{J} - (\iota_{e_a}A)J - \iota'_{e_a}C^+ \\ \bar{J} - \bar{A} \\ J + A \\ e^a \end{pmatrix}$$

invi

Circle reduction

Scalar matrix

$$U_A{}^B(x) = \begin{pmatrix} \mathbb{I}_D & 0 & 0 & 0 \\ 0 & U^+ & -U^- & 0 \\ 0 & -U^- & U^+ & 0 \\ 0 & 0 & 0 & \mathbb{I}_D \end{pmatrix} \in \frac{O(D+1, D+1)}{O(D+1) \times O(D+1)}$$

$$\begin{pmatrix} E'{}_a \\ E'_- \\ E'_+ \\ E'{}^a \end{pmatrix} = \begin{pmatrix} e_a - (\iota_{e_a}\bar{A})\bar{J} - (\iota_{e_a}A)J - \iota'_{e_a}C^+ \\ \bar{J} - \bar{A} \\ J + A \\ e^a \end{pmatrix}$$

invi

Circle reduction

Left-Right gauge boson and basis

$$A = \frac{1}{\sqrt{2}}(V_1 + B_1) \qquad J = \frac{1}{\sqrt{2}}(\partial_{x^d} + dx^d)$$

$$\bar{A} = \frac{1}{\sqrt{2}}(V_1 - B_1) \qquad \bar{J} = \frac{1}{\sqrt{2}}(\partial_{x^d} - dx^d)$$

C^+ tensor redefinition

$$C^+ = B_2' + \frac{1}{2}(AA - \bar{A}\bar{A})$$

Extension

Extended scalar matrix

$$U_A{}^B(x) = \begin{pmatrix} \mathbb{I}_D & 0 & 0 & 0 \\ 0 & U^{++\,ij} & -U^{+-\,ij} & 0 \\ 0 & -U^{-+\,ij} & U^{--\,ij} & 0 \\ 0 & 0 & 0 & \mathbb{I}_D \end{pmatrix} \in \frac{O(D+3, D+3)}{O(D+3) \times O(D+3)}$$

$$\begin{pmatrix} E'{}_a \\ E'{}_-{}^j \\ E'{}_+{}^j \\ E'{}^a \end{pmatrix} = \begin{pmatrix} e_a - (\iota_{e_a}\bar{A}^j)\bar{J}^j - (\iota_{e_a}A^j)J^j - \iota'_{e_a}C^+ \\ \bar{J}^j - \bar{A}^j \\ J^j + A^j \\ e^a \end{pmatrix}$$

"Generalised geometry for string corrections," Coimbra, Minasian, Triendl and Waldram.
"Spheres, generalised parallelisability and consistent truncations,"Lee, Strickland-Constable and Waldram.

Extension

Useful and sufficient information

$$\bar{A} = \bar{A}^i \bar{J}^i \in \mathfrak{su}(2)_R \qquad A = A^i J^i \in \mathfrak{su}(2)_L$$

$$\left[\bar{J}^i, \bar{J}^j\right] = \epsilon^{ijk} \bar{J}^k \qquad \left[J^i, J^j\right] = \epsilon^{ijk} J^k$$

$$\left[\bar{J}^i, J^j\right] = 0$$

C^+ tensor redefinition

$$C^+ = B_2' + \frac{1}{2}\left(\kappa_{ij} A^i A^j - \bar{\kappa}_{ij} \bar{A}^i \bar{A}^j\right)$$

Generalised metric

Generalised metric

$$H = \mathcal{S}^{AB} E_A \otimes E_B \quad , \quad \mathcal{S}^{AB} = \mathrm{diag}(s^{ab}, s_{ab})$$

$$H = \mathcal{S}^{AB} \bar{U}_A{}^{A'} \bar{U}_B{}^{B'} \bar{E}_{A'} \otimes \bar{E}_{B'} = \bar{\mathcal{H}}^{A'B'} \bar{E}_{A'} \otimes \bar{E}_{B'}$$

$O(3,3)/O(3) \times O(3)$ scalar metric parametrization

$$\frac{O(3,3)}{O(3) \times O(3)} \ni \bar{\mathcal{H}}^{in} = \begin{pmatrix} h^{-1} & -h^{-1}b \\ bh^{-1} & h - bh^{-1}b \end{pmatrix}$$

$$\bar{\mathcal{H}}^{in\,T} \bar{\eta} \bar{\mathcal{H}}^{in} = \bar{\eta} \quad , \quad \bar{\eta} = \frac{1}{2} \begin{pmatrix} 0 & \mathbb{I}_3 \\ \mathbb{I}_3 & 0 \end{pmatrix}$$

Generalised metric

$O(3,3)/O(3)\times O(3)$ scalar metric parametrization $\mathcal{C}_+ \oplus \mathcal{C}_-$

$$\frac{O(3,3)}{O(3)\times O(3)} \ni \mathcal{H}^{in} = R\bar{\mathcal{H}}^{in}R^T = \begin{pmatrix} \mathcal{H}^{++} & \mathcal{H}^{+-} \\ \mathcal{H}^{-+} & \mathcal{H}^{--} \end{pmatrix}$$

$$\mathcal{H}^{in\,T}\eta\mathcal{H}^{in} = \eta \quad , \quad \eta = \frac{1}{2}\begin{pmatrix} -\mathbb{I}_3 & 0 \\ 0 & \mathbb{I}_3 \end{pmatrix}$$

Generalised metric

Scalar metric components

$$\mathcal{H}^{++} = (\mathcal{H}^{--})^T = \frac{1}{2}\left[(h + h^{-1}) + (h^{-1}b - bh^{-1}) - bh^{-1}b\right]$$

$$\mathcal{H}^{+-} = (\mathcal{H}^{-+})^T = -\frac{1}{2}\left[(h - h^{-1}) + (h^{-1}b + bh^{-1}) + bh^{-1}b\right]$$

Expansion

$$\begin{aligned} h &\approx \mathbb{I}_3 + h' \\ h^{-1} &\approx \mathbb{I}_3 - h' \\ b &\approx b \end{aligned}$$

Scalar metric fluctuations

$$\mathcal{H}^{in} = \begin{pmatrix} \mathbb{I}_3 & -M_{ij} \\ -M_{ji} & \mathbb{I}_3 \end{pmatrix}$$

Identification

$$M = h' + b$$

Generalised metric

Scalar metric components

$$
\begin{aligned}
\mathcal{H}^{++} &= (\mathcal{H}^{--})^T = \frac{1}{2}\left[(h + h^{-1}) + (h^{-1}b - bh^{-1}) - bh^{-1}b\right] \\
\mathcal{H}^{+-} &= (\mathcal{H}^{-+})^T = -\frac{1}{2}\left[(h - h^{-1}) + (h^{-1}b + bh^{-1}) + bh^{-1}b\right]
\end{aligned}
$$

Expansion

$$
\begin{aligned}
h &\approx \mathbb{I}_3 + h' \\
h^{-1} &\approx \mathbb{I}_3 - h' \\
b &\approx b
\end{aligned}
$$

Scalar metric fluctuations

$$
\mathcal{H}^{in} = \begin{pmatrix} \mathbb{I}_3 & -M_{ij} \\ -M_{ji} & \mathbb{I}_3 \end{pmatrix}
$$

Identification

$$
M = h' + b
$$

A small digression

$$\boxed{G, \quad B, \quad \Phi, \quad A_L, \quad A_R, \quad M}$$

Non-linear sigma model

$$
\begin{aligned}
S \;=\; & \int \Big(G_{\mu\nu}(x)\partial_z x^\mu \partial_{\bar{z}} x^\nu + B_{\mu\nu}(x)\partial_z x^\mu \partial_{\bar{z}} x^\nu \\
& + \Phi(x) R^{(2)} \Big) dz d\bar{z} \\
& + \int \Big(A_{L\,\mu}{}^{a}(x)\partial_z x^\mu J_R{}^a dz + A_{R\,\mu}{}^{a}(x)\partial_{\bar{z}} x^\mu J_L{}^a d\bar{z} \Big) \\
& + \int M_{ab}(x) J_L{}^a(z) J_R{}^b(\bar{z}) dz d\bar{z}
\end{aligned}
$$

D. Karabali, H. J. Schnitzer and K. Tsokos, "Low-energy Effective Action for Closed Bosonic Strings on Group Manifolds," Nucl. Phys. B **294***, 412 (1987).*

A small digression

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Non-linear sigma model

$$\begin{aligned} S \quad = \quad & \int \Big(G_{\mu\nu}(x)\partial_z x^\mu \partial_{\bar{z}} x^\nu + B_{\mu\nu}(x)\partial_z x^\mu \partial_{\bar{z}} x^\nu \\ & +\Phi(x) R^{(2)} \Big) dz d\bar{z} \\ & + \int \Big(A_{L\ \mu}{}^a(x)\partial_z x^\mu J_R{}^a dz + A_{R\ \mu}{}^a(x)\partial_{\bar{z}} x^\mu J_L{}^a d\bar{z} \Big) \\ & + \int \Big(M_{(ab)}(x) J_L{}^a(z) J_R{}^b(\bar{z}) + M_{[ab]}(x) J_L{}^a(z) J_R{}^b(\bar{z}) \Big) dz d\bar{z} \end{aligned}$$

D. Karabali, H. J. Schnitzer and K. Tsokos, "Low-energy Effective Action for Closed Bosonic Strings on Group Manifolds," Nucl. Phys. B **294***, 412 (1987).*

A small digression

$$\boxed{G, \quad B, \quad \Phi, \quad A_L, \quad A_R, \quad M}$$

Non-linear sigma model

$$\begin{aligned} S \;=\; & \int \Big(G_{\mu\nu}(x)\partial_z x^\mu \partial_{\bar z} x^\nu + B_{\mu\nu}(x)\partial_z x^\mu \partial_{\bar z} x^\nu \\ & \quad + \Phi(x) R^{(2)} \Big) dz d\bar z \\ & + \int \Big(A_{L\ \mu}{}^a(x)\partial_z x^\mu J_R{}^a dz + A_{R\ \mu}{}^a(x)\partial_{\bar z} x^\mu J_L{}^a d\bar z \Big) \\ & + \int \Big(h'_{ab}(x) J_L{}^a(z) J_R{}^b(\bar z) + b_{ab}(x) J_L{}^a(z) J_R{}^b(\bar z) \Big) dz d\bar z \end{aligned}$$

D. Karabali, H. J. Schnitzer and K. Tsokos, "Low-energy Effective Action for Closed Bosonic Strings on Group Manifolds," Nucl. Phys. B **294***, 412 (1987).*

Generalised Lie derivative and fluxes

Splitting generalised vectors

$$V = \hat{v} + \hat{\xi} \in TM_d \oplus T^*M_d$$

$$V = v + \left(v^d + \xi^d\right) + \xi \in TM_D \oplus \left(TS^1 \oplus T^*S^1\right) \oplus T^*M_D$$

Extending generalised vectors

$$V = v + \left(\bar{\Lambda}^i \bar{J}^i + \Lambda^i J^i\right) + \xi \in TM_D \oplus \left(\mathfrak{su}(2)_R \oplus \mathfrak{su}(2)_L\right) \oplus T^*M_D$$

Generalised Lie derivative

$$\mathcal{L}_{V_1} V_2 = L_{v_1} v_2 + \left(L_{v_1}\Lambda_2 - L_{v_2}\Lambda_1 + [\Lambda_1, \Lambda_2]\right) + \left(L_{v_1}\xi_2 - \iota_{v_2} d\xi_1\right)$$

Generalised Lie derivative and fluxes

Generalised Lie derivative local expression

$$
\begin{aligned}
(\mathcal{L}_{V_1} V_2)^{\mathcal{M}} &= V_1^{\mathcal{P}} \partial_{\mathcal{P}} V_2^{\mathcal{M}} - V_2^{\mathcal{P}} \partial_{\mathcal{P}} V_1^{\mathcal{M}} + \eta^{\mathcal{M}\mathcal{N}} \eta_{\mathcal{P}\mathcal{Q}} \partial_{\mathcal{P}} V_1^{\mathcal{P}} V_2^{\mathcal{Q}} \\
&\quad - f_{\mathcal{P}\mathcal{Q}}{}^{\mathcal{M}} V_1^{\mathcal{P}} V_2^{\mathcal{Q}}
\end{aligned}
$$

Generalised fluxes

$$
\mathcal{L}_{E_{\mathcal{A}}} E_{\mathcal{B}} = F_{\mathcal{A}\mathcal{B}}{}^{\mathcal{C}} E_{\mathcal{C}}
$$

"Double Field Theory Formulation of Heterotic Strings," Hohm and Kwak.
"Exploring Double Field Theory," Geissbuhler, Marques, Nunez and Penas.

Can S^{DFT} reproduce S^{string} ?

Effective action

$$
\begin{aligned}
S^{DFT} \;=\; & \int d^Dx\sqrt{g}e^{-2\Phi}\Big[R + 4(\partial\Phi)^2 - \frac{1}{12}H^2 \\
& -\frac{1}{4}g^{\mu\mu'}g^{\nu\nu'}\mathcal{H}_{IJ}F_{\mu\nu}{}^{I}F_{\mu'\nu'}{}^{J} + \frac{1}{8}g^{\mu\nu}(D_\mu\mathcal{H})_{IJ}(D_\nu\mathcal{H})^{IJ} \\
& -\frac{1}{12}F_{AB}{}^{C}F_{A'B'}{}^{C'}\mathcal{H}^{AA'}\mathcal{H}^{BB'}\mathcal{H}_{CC'} \\
& +\frac{1}{4}F_{AB}{}^{C}F_{A'B'}{}^{C'}\mathcal{H}^{AA'}\eta^{BB'}\eta_{CC'} \\
& -\frac{1}{6}F_{AB}{}^{C}F_{A'B'}{}^{C'}\eta^{AA'}\eta^{BB'}\eta_{CC'}\Big]
\end{aligned}
$$

"The effective action of Double Field Theory," Aldazabal, Baron, Marques and Nunez.
"Double Field Theory and N=4 Gauged Supergravity," D. Geissbuhler.

Can S^{DFT} reproduce S^{string} ?

Effective action

$$
\begin{aligned}
S^{DFT} \;=\; & \int d^D x \sqrt{g} e^{-2\Phi} \Big[R + 4(\partial\Phi)^2 - \frac{1}{12} H^2 \\
& - \frac{1}{4} g^{\mu\mu'} g^{\nu\nu'} \mathcal{H}_{IJ} F_{\mu\nu}{}^{I} F_{\mu'\nu'}{}^{J} + \frac{1}{8} g^{\mu\nu} (D_\mu \mathcal{H})_{IJ} (D_\nu \mathcal{H})^{IJ} \\
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\end{aligned}
$$

"The effective action of Double Field Theory," Aldazabal, Baron, Marques and Nunez.
"Double Field Theory and N=4 Gauged Supergravity," D. Geissbuhler.

Discussion-gauge term

Scalar metric fluctuations

$$\mathcal{H}^{IJ} = \begin{pmatrix} \mathbb{I}_3 & -M_{ij} \\ -M_{ji} & \mathbb{I}_3 \end{pmatrix}$$

Scalar metric fluctuations

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Can S^{DFT} reproduce S^{string} ?

Effective action

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\begin{aligned}
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Discussion-scalar term

Scalar metric fluctuations

$$\mathcal{H}_{IJ} = \begin{pmatrix} \mathbb{I}_3 & M_{ij} \\ M_{ji} & \mathbb{I}_3 \end{pmatrix}$$

$$\frac{1}{8} g^{\mu\nu} \left(D_\mu \mathcal{H}\right)_{IJ} \left(D_\nu \mathcal{H}\right)^{IJ}$$

Covariant derivative

$$\left(D_\mu \mathcal{H}\right)_{IJ} = \partial_\mu \mathcal{H}_{IJ} - F_{IK}{}^{L} A_\mu{}^{K} \mathcal{H}_{LJ} - F_{JK}{}^{L} A_\mu{}^{K} \mathcal{H}_{IL}$$

$$(D_\mu \mathcal{H})_{ij} = (D_\mu \mathcal{H})_{\bar{i}\,\bar{j}} = 0$$

$$(D_\mu \mathcal{H})_{ij} = \partial_\mu M_{ij} - f_{ik}{}^{l} A_\mu{}^{k} M_{lj} - \bar{f}_{jk}{}^{l} \bar{A}_\mu{}^{k} M_{il}$$

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Scalar potential

Scalar metric fluctuations

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Mismatch

$$V = -\det(M) + \frac{2}{\alpha'}$$

Scalar potential

Scalar metric fluctuations

$$\mathcal{H}_{IJ} = \begin{pmatrix} \mathbb{I}_3 & M_{ij} \\ M_{ji} & \mathbb{I}_3 \end{pmatrix}$$

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Mismatch

$$V = -\det(M) + \frac{2}{\alpha'}$$

Scalar potential

DFT- action

$$S^{DFT} = \int dx^D e^{-2d}\left(\mathcal{R} + \Lambda\right)$$

Compactified boson

$$\Lambda = -\frac{2}{\alpha'}$$

The geometry of the internal space

Internal generalised Lie derivative

$$\mathcal{L}_{E_A} E_B = \left[E_A{}^P \partial_P E_B{}^M - E_B{}^P \partial_P E_A{}^M + \eta^{MN} \eta_{PQ} \partial_N E_A{}^P E_A{}^Q \right] D_M$$

$$\begin{aligned} D_M &= \left(t_1,\ t_2,\ \frac{1}{\sqrt{\alpha'}} dx^R,\ t_3,\ t_4,\ \frac{1}{\sqrt{\alpha'}} dx^L \right)^T \\ \partial_P &= \left(0,\ 0,\ \sqrt{\alpha'}\partial_{x^R},\ 0,\ 0,\ \sqrt{\alpha'}\partial_{x^L} \right) \end{aligned}$$

Local (non-geometric) expression of η

$$\eta_{MN} = \frac{1}{2} \begin{pmatrix} -\mathbb{I}_3 & 0 \\ 0 & \mathbb{I}_3 \end{pmatrix} \quad ; \quad \boxed{\partial_{x^d} \to d\tilde{x}^d}$$

"Duality orbits of non-geometric fluxes," Dibitetto, Fernandez-Melgarejo, Marques and Roest.
"New gaugings and non-geometry," Lee, Strickland-Constable and Waldram.

The geometry of the internal space

Contraction operation

$$V = v + \xi$$

$$\eta(V_1, V_2) = \frac{1}{2}(\iota_{v_1}\xi_2 + \iota_{v_2}\xi_1) = {V_1}^M \eta'_{MN} {V_2}^N$$

Local (geometric) expression of η

$$\eta'_{MN} = \frac{1}{2}\begin{pmatrix} 0 & \mathbb{I}_3 \\ \mathbb{I}_3 & 0 \end{pmatrix} \quad ; \quad \eta(E_A, E_B) = \eta_{AB}$$

Non-geometric change of coordinates

$$x^L = \frac{1}{2}(x^d + \tilde{x}^d) \quad ; \quad x^R = \frac{1}{2}(x^d - \tilde{x}^d)$$

The geometry of the internal space

Internal frame

$$E_A = \Psi_A{}^{A'}(x^L, x^R)\delta_{A'}{}^M D_M$$

$$E_{\bar{a}} = \begin{pmatrix} \cos(2\,x^R/\sqrt{\alpha'}) & \sin(2\,x^R/\sqrt{\alpha'}) & 0 \\ -\sin(2\,x^R/\sqrt{\alpha'}) & \cos(2\,x^R/\sqrt{\alpha'}) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \\ dx^R \end{pmatrix}$$

$\mathfrak{su}(2) \times \mathfrak{su}(2)$ algebra

$$\mathcal{L}_{E_a} E_b = \epsilon_{abc} E_c$$

$$\mathcal{L}_{E_{\bar{a}}} E_{\bar{b}} = \epsilon_{\bar{a}\bar{b}\bar{c}} E_{\bar{c}}$$

$$\mathcal{L}_{E_a} E_{\bar{b}} = \mathcal{L}_{E_{\bar{a}}} E_b = 0$$

$$R' = \begin{pmatrix} 1/\sqrt{2} & -i/\sqrt{2} & 0 \\ 1/\sqrt{2} & i/\sqrt{2} & 0 \\ 0 & 0 & i \end{pmatrix}$$

$$\begin{pmatrix} J_{\bar{a}} \\ J_a \end{pmatrix} = \begin{pmatrix} R'_{\bar{a}}{}^{\bar{b}} & 0 \\ 0 & R'_a{}^b \end{pmatrix} \begin{pmatrix} E_{\bar{b}} \\ E_b \end{pmatrix}$$

$\mathfrak{su}(2) \times \mathfrak{su}(2)$ generalised frame

$$
\begin{aligned}
\bar{J}^{+} &= \exp[+2i/\sqrt{\alpha'}x^R]T_1^+ \quad ; \quad T_1^+ = \frac{1}{\sqrt{2}}(t_1 - it_2) \\
\bar{J}^{-} &= \exp[-2i/\sqrt{\alpha'}x^R]T_2^+ \quad ; \quad T_2^+ = \frac{1}{\sqrt{2}}(t_1 + it_2) \\
\bar{J}^{3} &= i/\sqrt{\alpha'}dx^R
\end{aligned}
$$

$$
\begin{aligned}
J^{+} &= \exp[+2i/\sqrt{\alpha'}x^L]T_3^+ \quad ; \quad T_3^+ = \frac{1}{\sqrt{2}}(t_3 - it_4) \\
J^{-} &= \exp[-2i/\sqrt{\alpha'}x^L]T_4^+ \quad ; \quad T_4^+ = \frac{1}{\sqrt{2}}(t_3 + it_4) \\
J^{3} &= i/\sqrt{\alpha'}dx^L
\end{aligned}
$$

$\mathfrak{su}(2) \times \mathfrak{su}(2)$ generalised frame

World sheet

$$x^R : \Omega X \to M_d \times S^1$$
$$x^L : \Omega X \to M_d \times S^1$$

Pullback

$$T_1^+ |_{S^1} = T_2^+ |_{S^1} = d\bar{z}$$
$$dx^R |_{S^1} = \partial_{\bar{z}} x^L(\bar{z}) d\bar{z}$$

Pullback

$$T_4^+ |_{S'^1} = T_5^+ |_{S'^1} = dz$$
$$dx^L |_{S'^1} = \partial_z x^L(z) dz$$

Geometric generalised metric

Internal frame

$$E_A = \Psi_A{}^{A'}(x^L, x^R)\delta_{A'}{}^M D_M$$

$$E_{\bar{a}} = \begin{pmatrix} \cos(2\,x^R/\sqrt{\alpha'}) & \sin(2\,x^R/\sqrt{\alpha'}) & 0 \\ -\sin(2\,x^R/\sqrt{\alpha'}) & \cos(2\,x^R/\sqrt{\alpha'}) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \\ dx^R \end{pmatrix}$$

Generalised metric

$$\begin{aligned} H^{in} &= \delta^{AB} E_A \otimes E_B = D^T(\Psi^T\Psi)D \quad ; \quad \boxed{d\tilde{x}^d \to \partial_{x^d}} \\ &= D^T \mathbb{I}_6 D \end{aligned}$$

Non-geometric generalised metric

Internal frame

$$E_A = U_A{}^{A''}(x)\Psi_{A''}{}^{A'}(x^L, x^R)\delta_{A'}{}^M D_M$$

non-geometric fluctuations

$$\begin{aligned} H^{in} &= \delta^{AB} E_A \otimes E_B = D^T(\Psi^T U^T U \Psi)D \\ &= D^T(\Psi^T \mathcal{H} \Psi)D \end{aligned}$$

Scalar metric fluctuations

$$\mathcal{H} = \begin{pmatrix} \mathbb{I}_3 & M_{ij} \\ M_{ji} & \mathbb{I}_3 \end{pmatrix}$$

"New gaugings and non-geometry," Lee, Strickland-Constable and Waldram.

Conclusions

1. We have built a sensitive DFT that captures windings effects.
2. Our construction has $N \neq \bar{N}$ satisfying the level matching condition and an explicitly dependence in the compact coordinate and the dual one, violating the section condition, otherwise it could be impossible to get the $\mathfrak{su}(2)_L \times \mathfrak{su}(2)_R$ enhanced symmetry.
3. There is no clear geometric interpretation but it seems to work at least at the self dual point.

Conclusions

1. It could be worth exploring how to go away from the self dual point , if it possible.
2. We think that extending the analysis from S^1 to T^6 it will be able to understand the generalised Scherk-Schwarz reduction introduced by Diego and Mariana and to understand (all) the $(\omega,\ P,\ Q,\ R)$ fluxes associating them with real winding configurations.

THANK YOU FOR YOUR ATTENTION

Circle reduction

Parametrization

$$\phi = \exp(\frac{1}{2}M^{33}) \quad , \quad M^{33} = v + M'^{\,33}$$

$$U^{+} = \frac{1}{2}(\phi + \phi^{-1}) = \cosh(\frac{1}{2}M^{33})$$

$$U^{-} = \frac{1}{2}(\phi - \phi^{-1}) = \sinh(\frac{1}{2}M^{33})$$

Fluctuation

$$U^{+} \approx 1 \qquad ; \qquad U^{-} \approx \frac{1}{2}M^{33}$$

Circle reduction

Scalar matrix fluctuations at $R = \tilde{R}$

$$U_A{}^B(x) = \begin{pmatrix} \mathbb{I}_D & 0 & 0 & 0 \\ 0 & 1 & -\frac{1}{2}M^{33} & 0 \\ 0 & -\frac{1}{2}M^{33} & 1 & 0 \\ 0 & 0 & 0 & \mathbb{I}_D \end{pmatrix} \in \frac{O(D+1, D+1)}{O(D+1) \times O(D+1)}$$

$$\begin{pmatrix} E'{}_a \\ E'_- \\ E'_+ \\ E'{}^a \end{pmatrix} = \begin{pmatrix} e_a - (\iota_{e_a}\bar{A})\bar{J} - (\iota_{e_a}A)J - \iota'_{e_a}C^+ \\ \bar{J} - \bar{A} \\ J + A \\ e^a \end{pmatrix}$$

Back to dious.