U-gravity : SL(N)

Yoonji Suh

Department of Physics, Sogang University

Workshop on duality symmetry in string and M-theories

CERN, Aug. 2015

Talk based on works with Jeong-Hyuck Park and Minwoo Suh*

- **U-geometry : SL**(5) arXiv:1302.1652 JHEP
- **U-gravity : SL**(N) arXiv:1402.5027 JHEP
- **Supersymmetric extension of U-gravity : SL**(11)* work in progress

Introduction

- Lorentz symmetry unifies space and time into spacetime
- Duality requires further extension of the spacetime
- T-duality in string theory becomes a manifest $\mathbf{O}(D, D)$ rotation in doubled spacetime and so do various $\mathcal{M}$-theory U-dualities in extended spacetime

Introduction

- Lorentz symmetry unifies space and time into spacetime
- Duality requires further extension of the spacetime
- T-duality in string theory becomes a manifest $\mathbf{O}(D,D)$ rotation in doubled spacetime and so do various $\mathcal{M}$-theory U-dualities in extended spacetime

Introduction

- Lorentz symmetry unifies space and time into spacetime
- Duality requires further extension of the spacetime
- T-duality in string theory becomes a manifest $\mathbf{O}(D,D)$ rotation in doubled spacetime and so do various $\mathcal{M}$-theory U-dualities in extended spacetime

Motivation for U-gravity

- Construct geometric structure of $\mathcal{M}$-theory

  $\Rightarrow$ Proper connection, curvature tensor and scalar curvature

- Extend semi-covariant approach to U-duality group

  $\Rightarrow$ Focus on $\mathbf{SL}(N)$ duality rotation

c.f. DFT: Jeon, Lee and Park

Motivation for U-gravity

- Construct geometric structure of $\mathcal{M}$-theory

  $\Rightarrow$ Proper connection, curvature tensor and scalar curvature

- Extend semi-covariant approach to U-duality group

  $\Rightarrow$ Focus on $\mathbf{SL}(N)$ duality rotation

c.f. DFT: Jeon, Lee and Park

Motivation for U-gravity

- Construct geometric structure of $\mathcal{M}$-theory

  $\Rightarrow$ Proper connection, curvature tensor and scalar curvature

- Extend semi-covariant approach to U-duality group

  $\Rightarrow$ Focus on $\mathbf{SL}(N)$ duality rotation

c.f. DFT: Jeon, Lee and Park

M-theory and duality

D	d	G	H
3	8	E_8	$SO(16)$
4	7	E_7	$SU(8)$
5	6	E_6	$USp(8)$
6	5	$SO(5,5)$	$SO(5)\times SO(5)$
7	4	$SL(5)$	$SO(5)$
8	3	$SL(3)\times SL(2)$	$SO(3)\times SO(2)$
9	2	$SL(2)$	$SO(2)$
10	1	$SO(1,1)$	1

M-theory and duality

D	d	G	H
3	8	E_8	$SO(16)$
4	7	E_7	$SU(8)$
5	6	E_6	$USp(8)$
6	5	$SO(5,5)$	$SO(5) \times SO(5)$
7	4	$SL(5)$	$SO(5)$
8	3	$SL(3) \times SL(2)$	$SO(3) \times SO(2)$
9	2	$SL(2)$	$SO(2)$
10	1	$SO(1,1)$	1

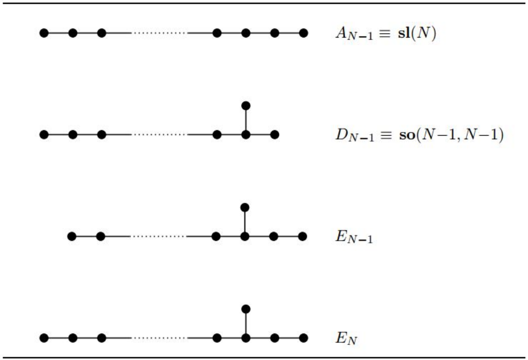
M-theory and duality

D	d	G	H
3	8	E_8	$SO(16)$
4	7	E_7	$SU(8)$
5	6	E_6	$USp(8)$
6	5	$SO(5,5)$	$SO(5)\times SO(5)$
7	4	$SL(5)$	$SO(5)$
8	3	$SL(3)\times SL(2)$	$SO(3)\times SO(2)$
9	2	$SL(2)$	$SO(2)$
10	1	$SO(1,1)$	1

$\Rightarrow$ **SL(N)** U-gravity

Dynkin diagrams and duality groups

- $A_{N-1}, D_{N-1}, E_{N-1}$ and E_N



D. Baraglia (2012), C. Strictland-Constable (2013)

E_{11} subgroups

- $A_{10} \equiv \mathbf{sl}(11)$

  $\Rightarrow$ **SL**(11) U-gravity

- $D_{10} \equiv \mathbf{so}(10, 10)$

  $\Rightarrow$ **O**(10, 10) T-duality DFT or U-duality U-gravity?

- E_{10}

  $\Rightarrow$ Exceptional Field Theory or Extended Geometry?

E_{11}: P. West (2001)

E_{11} subgroups

- $A_{10} \equiv \mathbf{sl}(11)$

  $\Rightarrow$ **SL**(11) U-gravity

- $D_{10} \equiv \mathbf{so}(10, 10)$

  $\Rightarrow$ **O**(10, 10) T-duality DFT or U-duality U-gravity?

- E_{10}

  $\Rightarrow$ Exceptional Field Theory or Extended Geometry?

E_{11}: P. West (2001)

E_{11} subgroups

- $A_{10} \equiv \mathbf{sl}(11)$

  $\Rightarrow$ $\mathbf{SL}(11)$ U-gravity

- $D_{10} \equiv \mathbf{so}(10, 10)$

  $\Rightarrow$ $\mathbf{O}(10, 10)$ T-duality DFT or U-duality U-gravity?

- E_{10}

  $\Rightarrow$ Exceptional Field Theory or Extended Geometry?

E_{11}: P. West (2001)

E_{11} subgroups

- $A_{10} \equiv \mathbf{sl}(11)$

  $\Rightarrow$ $\mathbf{SL}(11)$ U-gravity

- $D_{10} \equiv \mathbf{so}(10, 10)$

  $\Rightarrow$ $\mathbf{O}(10, 10)$ T-duality DFT or U-duality U-gravity?

- E_{10}

  $\Rightarrow$ Exceptional Field Theory or Extended Geometry?

E_{11}: P. West (2001)

Riemannian geometry

- Ordinary spacetime
- Symmetry : Diffeomorphism
- Field contents : Metric
- Covariant derivative

  $\Rightarrow$ Covariant Riemann curvatures

U-gravity

- Extended-yet-gauged spacetime
- Generalized diffeomorphism
  & **SL**(N) U-duality
- U-metric (Generalized metric)
- Semi-covariant derivative

  $\Rightarrow$ Semi-covariant **SL**(N) curvatures

Riemannian geometry

- Ordinary spacetime
- Symmetry : Diffeomorphism
- Field contents : Metric
- Covariant derivative

  $\Rightarrow$ Covariant Riemann curvatures

U-gravity

- Extended-yet-gauged spacetime
- Generalized diffeomorphism
  & **SL**(N) U-duality
- U-metric (Generalized metric)
- Semi-covariant derivative

  $\Rightarrow$ Semi-covariant **SL**(N) curvatures

Riemannian geometry

- Ordinary spacetime
- Symmetry : Diffeomorphism
- Field contents : Metric
- Covariant derivative

  $\Rightarrow$ Covariant Riemann curvatures

U-gravity

- Extended-yet-gauged spacetime
- Generalized diffeomorphism

  & **SL**(N) U-duality
- U-metric (Generalized metric)
- Semi-covariant derivative

  $\Rightarrow$ Semi-covariant **SL**(N) curvatures

Riemannian geometry

- Ordinary spacetime
- Symmetry : Diffeomorphism
- Field contents : Metric
- Covariant derivative

  ⇒ Covariant Riemann curvatures

U-gravity

- Extended-yet-gauged spacetime
- Generalized diffeomorphism
  & **SL**(N) U-duality
- U-metric (Generalized metric)
- Semi-covariant derivative

  ⇒ Semi-covariant **SL**(N) curvatures

Riemannian geometry

- Ordinary spacetime
- Symmetry : Diffeomorphism
- Field contents : Metric
- Covariant derivative

  $\Rightarrow$ Covariant Riemann curvatures

U-gravity

- Extended-yet-gauged spacetime
- Generalized diffeomorphism

  & **SL**(N) U-duality
- U-metric (Generalized metric)
- Semi-covariant derivative

  $\Rightarrow$ Semi-covariant **SL**(N) curvatures

Riemannian geometry

- Ordinary spacetime
- Symmetry : Diffeomorphism
- Field contents : Metric
- Covariant derivative

  $\Rightarrow$ Covariant Riemann curvatures

U-gravity

- Extended-yet-gauged spacetime
- Generalized diffeomorphism & **SL**(N) U-duality
- U-metric (Generalized metric)
- Semi-covariant derivative

  $\Rightarrow$ Semi-covariant **SL**(N) curvatures

Riemannian geometry

- Ordinary spacetime
- Symmetry : Diffeomorphism
- Field contents : Metric
- Covariant derivative

  $\Rightarrow$ Covariant Riemann curvatures

U-gravity

- Extended-yet-gauged spacetime
- Generalized diffeomorphism

  & **SL**(N) U-duality
- U-metric (Generalized metric)
- Semi-covariant derivative

  $\Rightarrow$ Semi-covariant **SL**(N) curvatures

Riemannian geometry

- Ordinary spacetime
- Symmetry : Diffeomorphism
- Field contents : Metric
- Covariant derivative

  ⇒ Covariant Riemann curvatures

U-gravity

- Extended-yet-gauged spacetime
- Generalized diffeomorphism

  & **SL**(N) U-duality
- U-metric (Generalized metric)
- Semi-covariant derivative

  ⇒ Semi-covariant **SL**(N) curvatures

Riemannian geometry

- Ordinary spacetime
- Symmetry : Diffeomorphism
- Field contents : Metric
- Covariant derivative

  ⇒ Covariant Riemann curvatures

U-gravity

- Extended-yet-gauged spacetime
- Generalized diffeomorphism
  & **SL**(N) U-duality
- U-metric (Generalized metric)
- Semi-covariant derivative

  ⇒ Semi-covariant **SL**(N) curvatures

Riemannian geometry

- Ordinary spacetime
- Symmetry : Diffeomorphism
- Field contents : Metric
- Covariant derivative

  ⇒ Covariant Riemann curvatures

U-gravity

- Extended-yet-gauged spacetime
- Generalized diffeomorphism
  & **SL**(N) U-duality
- U-metric (Generalized metric)
- Semi-covariant derivative

  ⇒ Semi-covariant **SL**(N) curvatures

Extended-yet-gauged spacetime

- Anti-symmetric **SL**(N) vector indices

$$x^{ab} = -x^{ba} = x^{[ab]}$$

where $a, b, c, \cdots = 1, 2, \cdots, N$.

- Section condition

$$\partial_{[ab}\partial_{cd]} = 0$$

- Weak constraint and strong constraint

$$\partial_{[ab}\partial_{cd]}\Phi = 0\,, \qquad \partial_{[ab}\Phi\partial_{cd]}\Phi' = 0$$

c.f. **SL**(5): Berman, Godazgar, Godazgar, Perry (2011)

Extended-yet-gauged spacetime

- Anti-symmetric **SL**(N) vector indices

$$x^{ab} = -x^{ba} = x^{[ab]}$$

where $a, b, c, \cdots = 1, 2, \cdots, N$.

- Section condition

$$\partial_{[ab}\partial_{cd]} = 0$$

- Weak constraint and strong constraint

$$\partial_{[ab}\partial_{cd]}\Phi = 0\,, \qquad \partial_{[ab}\Phi\partial_{cd]}\Phi' = 0$$

c.f. **SL**(5): Berman, Godazgar, Godazgar, Perry (2011)

Extended-yet-gauged spacetime

- Anti-symmetric **SL**(N) vector indices

$$x^{ab} = -x^{ba} = x^{[ab]}$$

where $a, b, c, \cdots = 1, 2, \cdots, N$.

- Section condition

$$\partial_{[ab}\partial_{cd]} = 0$$

- Weak constraint and strong constraint

$$\partial_{[ab}\partial_{cd]}\Phi = 0\,, \qquad \partial_{[ab}\Phi\partial_{cd]}\Phi' = 0$$

c.f. **SL**(5): Berman, Godazgar, Godazgar, Perry (2011)

Extended-yet-gauged spacetime

- Gauged spacetime

$$x^{ab} \sim x^{ab} + \Delta^{ab}$$

- Realization of coordinate gauge symmetry

$$\Phi(x+\Delta) = \Phi(x)\,, \qquad \Delta^{ab} = \tfrac{1}{(N-4)!}\epsilon^{abc_1\cdots c_{N-4}de}\phi_{c_1\cdots c_{N-4}}\partial_{de}\varphi$$

- Dimension of the extended spacetime is $\frac{1}{2}N(N-1)$.

- Yet, the physical dimension is reduced to either $N-1$ or 3.

c.f. DFT coordinate gauge symmetry: Park (2013)

Extended-yet-gauged spacetime

- Gauged spacetime

$$x^{ab} \sim x^{ab} + \Delta^{ab}$$

- Realization of coordinate gauge symmetry

$$\Phi(x+\Delta) = \Phi(x)\,, \qquad \Delta^{ab} = \tfrac{1}{(N-4)!}\epsilon^{abc_1\cdots c_{N-4}de}\phi_{c_1\cdots c_{N-4}}\partial_{de}\varphi$$

- Dimension of the extended spacetime is $\frac{1}{2}N(N-1)$.
- Yet, the physical dimension is reduced to either $N-1$ or 3.

c.f. DFT coordinate gauge symmetry: Park (2013)

Extended-yet-gauged spacetime

- Gauged spacetime

$$x^{ab} \sim x^{ab} + \Delta^{ab}$$

- Realization of coordinate gauge symmetry

$$\Phi(x+\Delta) = \Phi(x)\,, \qquad \Delta^{ab} = \tfrac{1}{(N-4)!}\epsilon^{abc_1\cdots c_{N-4}de}\phi_{c_1\cdots c_{N-4}}\partial_{de}\varphi$$

- Dimension of the extended spacetime is $\frac{1}{2}N(N-1)$.

- Yet, the physical dimension is reduced to either $N-1$ or 3.

c.f. DFT coordinate gauge symmetry: Park (2013)

Extended-yet-gauged spacetime

- Gauged spacetime

$$x^{ab} \sim x^{ab} + \Delta^{ab}$$

- Realization of coordinate gauge symmetry

$$\Phi(x+\Delta) = \Phi(x)\,, \qquad \Delta^{ab} = \tfrac{1}{(N-4)!}\epsilon^{abc_1\cdots c_{N-4}de}\phi_{c_1\cdots c_{N-4}}\partial_{de}\varphi$$

- Dimension of the extended spacetime is $\frac{1}{2}N(N-1)$.

- Yet, the physical dimension is reduced to either $N-1$ or 3.

c.f. DFT coordinate gauge symmetry: Park (2013)

Generalized Diffeomorphism

- Generalized Lie derivative

$$\hat{\mathcal{L}}_X T^{a_1\cdots a_p}{}_{b_1\cdots b_q} := \tfrac{1}{2} X^{cd}\partial_{cd} T^{a_1\cdots a_p}{}_{b_1\cdots b_q} + \tfrac{1}{2}(\tfrac{1}{2}p - \tfrac{1}{2}q + \omega)\partial_{cd} X^{cd} T^{a_1\cdots a_p}{}_{b_1\cdots b_q} - \textstyle\sum_{i=1}^{p} T^{a_1\cdots c\cdots a_p}{}_{b_1\cdots b_q}\partial_{cd}X^{a_i d} + \textstyle\sum_{j=1}^{q}\partial_{b_j d}X^{cd}T^{a_1\cdots a_p}{}_{b_1\cdots c\cdots b_q}$$

where ω is a weight.

- With the section condition, the commutator of the generalized Lie derivative is closed by a generalized bracket.

$$\left[\hat{\mathcal{L}}_X, \hat{\mathcal{L}}_Y\right] = \hat{\mathcal{L}}_{[X,Y]_G}$$

where

$$[X,Y]_G^{ab} = \tfrac{1}{2} X^{cd}\partial_{cd}Y^{ab} - \tfrac{3}{2}X^{[ab}\partial_{cd}Y^{cd]} - (X \leftrightarrow Y)$$

c.f. **SL**(5): Berman, Godazgar, Godazgar, Perry (2011)

Generalized Diffeomorphism

- Generalized Lie derivative

$$\hat{\mathcal{L}}_X T^{a_1\cdots a_p}{}_{b_1\cdots b_q} := \frac{1}{2}X^{cd}\partial_{cd}T^{a_1\cdots a_p}{}_{b_1\cdots b_q} + \frac{1}{2}(\frac{1}{2}p - \frac{1}{2}q + \omega)\partial_{cd}X^{cd}T^{a_1\cdots a_p}{}_{b_1\cdots b_q} - \sum_{i=1}^{p} T^{a_1\cdots c\cdots a_p}{}_{b_1\cdots b_q}\partial_{cd}X^{a_i d} + \sum_{j=1}^{q}\partial_{b_j d}X^{cd}T^{a_1\cdots a_p}{}_{b_1\cdots c\cdots b_q}$$

where ω is a weight.

- With the section condition, the commutator of the generalized Lie derivative is closed by a generalized bracket.

$$\left[\hat{\mathcal{L}}_X, \hat{\mathcal{L}}_Y\right] = \hat{\mathcal{L}}_{[X,Y]_G}$$

where

$$[X,Y]^{ab}_G = \frac{1}{2}X^{cd}\partial_{cd}Y^{ab} - \frac{3}{2}X^{[ab}\partial_{cd}Y^{cd]} - (X \leftrightarrow Y)$$

c.f. **SL**(5): Berman, Godazgar, Godazgar, Perry (2011)

U-metric

- **SL**(N) U-metric

$$M_{ab} = M_{(ab)}\,,$$

$$M = \det(M_{ab})\,, \qquad \omega_M = 4 - N$$

- Duality invariant integral measure

$$|M|^{\frac{1}{4-N}}$$

Semi-covariant derivative

- Semi-covariant derivative

$$\nabla_{cd}T^{a_1\cdots a_p}{}_{b_1\cdots b_q} := \partial_{cd}T^{a_1\cdots a_p}{}_{b_1\cdots b_q} + \tfrac{1}{2}(\tfrac{1}{2}p - \tfrac{1}{2}q + \omega)\Gamma_{cde}{}^{e}T^{a_1\cdots a_p}{}_{b_1\cdots b_q} - \textstyle\sum_{i=1}^{p} T^{a_1\cdots e\cdots a_p}{}_{b_1\cdots b_q}\Gamma_{cde}{}^{a_i} + \textstyle\sum_{j=1}^{q}\Gamma_{cdb_j}{}^{e}T^{a_1\cdots a_p}{}_{b_1\cdots e\cdots b_q}$$

- **SL**(N) connection

$$\Gamma_{abcd} = A_{abcd} + \tfrac{1}{2}(A_{acbd} - A_{adbc} + A_{bdac} - A_{bcad}) + \tfrac{1}{N-2}(M_{ac}A^{e}{}_{(bd)e} - M_{ad}A^{e}{}_{(bc)e} + M_{bd}A^{e}{}_{(ac)e} - M_{bc}A^{e}{}_{(ad)e})$$

where

$$A_{abcd} := -\frac{1}{2}\partial_{ab}M_{cd} + \frac{1}{2(N-4)}M_{cd}\partial_{ab}\ln|M|$$

Semi-covariant derivative

- Semi-covariant derivative

$$\begin{aligned}\nabla_{cd}T^{a_1\cdots a_p}{}_{b_1\cdots b_q} :=& \partial_{cd}T^{a_1\cdots a_p}{}_{b_1\cdots b_q} + \tfrac{1}{2}(\tfrac{1}{2}p - \tfrac{1}{2}q + \omega)\Gamma_{cde}{}^{e}T^{a_1\cdots a_p}{}_{b_1\cdots b_q} \\ & - \textstyle\sum_{i=1}^{p} T^{a_1\cdots e\cdots a_p}{}_{b_1\cdots b_q}\Gamma_{cde}{}^{a_i} + \textstyle\sum_{j=1}^{q}\Gamma_{cdb_j}{}^{e}T^{a_1\cdots a_p}{}_{b_1\cdots e\cdots b_q}\end{aligned}$$

- **SL**(N) connection

$$\begin{aligned}\Gamma_{abcd} =& A_{abcd} + \tfrac{1}{2}(A_{acbd} - A_{adbc} + A_{bdac} - A_{bcad}) \\ & + \tfrac{1}{N-2}(M_{ac}A^{e}{}_{(bd)e} - M_{ad}A^{e}{}_{(bc)e} + M_{bd}A^{e}{}_{(ac)e} - M_{bc}A^{e}{}_{(ad)e})\end{aligned}$$

where

$$A_{abcd} := -\frac{1}{2}\partial_{ab}M_{cd} + \frac{1}{2(N-4)}M_{cd}\partial_{ab}\ln|M|$$

Semi-covariant derivative

- The connection is the unique solution to the following constraints

$$\nabla_{ab} M_{cd} = 0$$

$$\nabla_{(ab)} = 0$$

$$\hat{\mathcal{L}}_X(\nabla) = \hat{\mathcal{L}}_X(\partial)$$

$$\mathcal{P}_{abcd}{}^{efgh}\Gamma_{efgh} = 0$$

Semi-covariant derivative

- The connection is the unique solution to the following constraints

$$\nabla_{ab}M_{cd} = 0 \qquad \Rightarrow \quad \Gamma_{abcd} + \Gamma_{abdc} = 2A_{abcd}$$

$$\nabla_{(ab)} = 0$$

$$\hat{\mathcal{L}}_X(\nabla) = \hat{\mathcal{L}}_X(\partial)$$

$$\mathcal{P}_{abcd}{}^{efgh}\Gamma_{efgh} = 0$$

Semi-covariant derivative

- The connection is the unique solution to the following constraints

$$\nabla_{ab} M_{cd} = 0 \qquad \Rightarrow \qquad \Gamma_{abcd} + \Gamma_{abdc} = 2A_{abcd}$$

$$\nabla_{(ab)} = 0 \qquad \Rightarrow \qquad \Gamma_{abc}{}^{d} + \Gamma_{bac}{}^{d} = 0$$

$$\hat{\mathcal{L}}_X(\nabla) = \hat{\mathcal{L}}_X(\partial)$$

$$\mathcal{P}_{abcd}{}^{efgh}\Gamma_{efgh} = 0$$

Semi-covariant derivative

- The connection is the unique solution to the following constraints

$$\nabla_{ab}M_{cd} = 0 \qquad \Rightarrow \qquad \Gamma_{abcd} + \Gamma_{abdc} = 2A_{abcd}$$

$$\nabla_{(ab)} = 0 \qquad \Rightarrow \qquad \Gamma_{abc}{}^{d} + \Gamma_{bac}{}^{d} = 0$$

$$\hat{\mathcal{L}}_X(\nabla) = \hat{\mathcal{L}}_X(\partial) \qquad \Rightarrow \qquad \begin{cases} \Gamma_{abc}{}^{d} + \Gamma_{bca}{}^{d} + \Gamma_{cab}{}^{d} = 0 \\ \Gamma_{cab}{}^{c} + \Gamma_{cba}{}^{c} = 0 \end{cases}$$

$$\mathcal{P}_{abcd}{}^{efgh}\Gamma_{efgh} = 0$$

Semi-covariant derivative

- The connection is the unique solution to the following constraints

$$\nabla_{ab} M_{cd} = 0 \quad \Rightarrow \quad \Gamma_{abcd} + \Gamma_{abdc} = 2A_{abcd}$$

$$\nabla_{(ab)} = 0 \quad \Rightarrow \quad \Gamma_{abc}{}^{d} + \Gamma_{bac}{}^{d} = 0$$

$$\hat{\mathcal{L}}_X(\nabla) = \hat{\mathcal{L}}_X(\partial) \quad \Rightarrow \quad \begin{cases} \Gamma_{abc}{}^{d} + \Gamma_{bca}{}^{d} + \Gamma_{cab}{}^{d} = 0 \\ \Gamma_{cab}{}^{c} + \Gamma_{cba}{}^{c} = 0 \end{cases}$$

$$\mathcal{P}_{abcd}{}^{efgh} \Gamma_{efgh} = 0 \quad \Rightarrow \quad \text{Kernal condition}$$

Projection

- Definition of projection

$$\mathcal{P}_{abcd}{}^{klmn} = \frac{1}{2}\delta_{[a}^{[k}\delta_{b]}{}^{l]}\delta_{[c}^{[m}\delta_{d]}{}^{n]} + \frac{1}{2}\delta_{[c}^{[k}\delta_{d]}{}^{l]}\delta_{[a}^{[m}\delta_{b]}{}^{n]} + \frac{1}{2}M_{c[a}\delta_{b]}^{m}M^{n[k}\delta_{d}{}^{l]} - \frac{1}{2}M_{c[a}\delta_{b]}^{[k}M^{l]n}\delta_{d}{}^{m}$$
$$+ \frac{1}{N-2}\left(\delta_{[a}^{n}M_{b][c}M^{m[k}\delta_{d]}{}^{l]} + \delta_{[c}^{n}M_{d][a}M^{m[k}\delta_{b]}{}^{l]} - M_{c[a}M_{b]d}M^{m[k}M^{l]n}\right)$$

- Properties

$$\mathcal{P}_{abcd}{}^{pqrs}\mathcal{P}_{pqrs}{}^{klmn} = \mathcal{P}_{abcd}{}^{klmn}\,,$$
$$\mathcal{P}_{abs}{}^{sklmn} = 0\,,$$
$$\mathcal{P}_{[abc]d}{}^{klmn} = \mathcal{P}_{[abcd]}{}^{[klmn]} = \delta_{[a}{}^{[k}\delta_{b}{}^{l}\delta_{c}{}^{m}\delta_{d]}{}^{n]}$$

Projection

- Definition of projection

$$\mathcal{P}_{abcd}{}^{klmn} = \frac{1}{2}\delta_{[a}^{[k}\delta_{b]}^{l]}\delta_{[c}^{[m}\delta_{d]}^{n]} + \frac{1}{2}\delta_{[c}^{[k}\delta_{d]}{}^{l]}\delta_{[a}^{[m}\delta_{b]}^{n]} + \frac{1}{2}M_{c[a}\delta_{b]}^{m}M^{n[k}\delta_{d}{}^{l]} - \frac{1}{2}M_{c[a}\delta_{b]}^{[k}M^{l]n}\delta_{d}{}^{m}$$
$$+ \frac{1}{N-2}\left(\delta_{[a}^{n}M_{b][c}M^{m[k}\delta_{d]}{}^{l]} + \delta_{[c}^{n}M_{d][a}M^{m[k}\delta_{b]}{}^{l]} - M_{c[a}M_{b]d}M^{m[k}M^{l]n}\right)$$

- Properties

$$\mathcal{P}_{abcd}{}^{pqrs}\mathcal{P}_{pqrs}{}^{klmn} = \mathcal{P}_{abcd}{}^{klmn},$$
$$\mathcal{P}_{abs}{}^{sklmn} = 0,$$
$$\mathcal{P}_{[abc]d}{}^{klmn} = \mathcal{P}_{[abcd]}{}^{[klmn]} = \delta_{[a}{}^{[k}\delta_{b}{}^{l}\delta_{c}{}^{m}\delta_{d]}{}^{n]}$$

Covariantization

- The projection dictates the anomalous terms in the diffeomorphic transformations

$$(\delta_X - \hat{\mathcal{L}}_X)\nabla_{ab}T^{c_1\cdots c_p}{}_{d_1\cdots d_q} = -\sum_{i=1}^{p} T^{c_1\cdots e\cdots c_p}{}_{d_1\cdots d_q}\Omega_{abe}{}^{c_i} + \sum_{j=1}^{q}\Omega_{abd_j}{}^{e}T^{c_1\cdots c_p}{}_{d_1\cdots e\cdots d_q}$$

where

$$\Omega_{abcd} = \mathcal{P}_{abcd}{}^{klm}{}_{n}\partial_{kl}\partial_{me}X^{ne}$$

- Properties of anomalous term

$$\Omega_{abcd} = \Omega_{[ab][cd]} = \Omega_{cdab}\,, \quad \Omega_{[abc]d} = 0\,,$$

$$\Omega_{abcd} = \mathcal{P}_{abcd}{}^{efgh}\Omega_{efgh}\,, \quad \Omega_{acb}{}^{c} = 0$$

Covariantization

- The projection dictates the anomalous terms in the diffeomorphic transformations

  $$(\delta_X - \hat{\mathcal{L}}_X)\nabla_{ab}T^{c_1\cdots c_p}{}_{d_1\cdots d_q} = -\sum_{i=1}^{p} T^{c_1\cdots e\cdots c_p}{}_{d_1\cdots d_q}\Omega_{abe}{}^{c_i} + \sum_{j=1}^{q}\Omega_{abd_j}{}^{e}T^{c_1\cdots c_p}{}_{d_1\cdots e\cdots d_q}$$

  where

  $$\Omega_{abcd} = \mathcal{P}_{abcd}{}^{klm}{}_{n}\partial_{kl}\partial_{me}X^{ne}$$

- Properties of anomalous term

  $$\Omega_{abcd} = \Omega_{[ab][cd]} = \Omega_{cdab}\,, \quad \Omega_{[abc]d} = 0\,,$$

  $$\Omega_{abcd} = \mathcal{P}_{abcd}{}^{efgh}\Omega_{efgh}\,, \quad \Omega_{acb}{}^{c} = 0$$

Covariantization

- Complete covariantizations

$$
\begin{array}{ll}
\nabla_{[ab}T_{c_1\cdots c_q]}\,, & \nabla_{ab}T^{a}\,, \\
\nabla^{a}{}_{b}T_{[ca]}+\nabla^{a}{}_{c}T_{[ba]}\,, & \nabla^{a}{}_{b}T_{(ca)}-\nabla^{a}{}_{c}T_{(ba)}\,, \\
\nabla_{ab}T^{[abc_1\cdots c_q]} & \text{Divergence}\,, \\
\nabla_{ab}\nabla^{[ab}T^{c_1\cdots c_q]} & \text{Laplacian}
\end{array}
$$

where $q=0,1,\cdots N-2$.

SL(N) curvatures

- Semi-covariant *Riemann* curvature

$$S_{abcd} := 3\nabla_{[ab}\Gamma_{e][cd]}{}^{e} + 3\nabla_{[cd}\Gamma_{e][ab]}{}^{e} - \tfrac{1}{4}\Gamma_{abe}{}^{e}\Gamma_{cdf}{}^{f} - \tfrac{1}{2}\Gamma_{abe}{}^{f}\Gamma_{cdf}{}^{e}$$
$$- \Gamma_{ab[c}{}^{e}\Gamma_{d]ef}{}^{f} - \Gamma_{cd[a}{}^{e}\Gamma_{b]ef}{}^{f} - \Gamma_{ea[c}{}^{f}\Gamma_{d]fb}{}^{e} + \Gamma_{eb[c}{}^{f}\Gamma_{d]fa}{}^{e}$$

- Under an arbitrary transformation of the connection,

$$\delta S_{abcd} = 3\nabla_{[ab}\delta\Gamma_{e][cd]}{}^{e} + 3\nabla_{[cd}\delta\Gamma_{e][ab]}{}^{e}$$

SL(N) curvatures

- Semi-covariant *Riemann* curvature

$$S_{abcd} := 3\nabla_{[ab}\Gamma_{e][cd]}{}^{e} + 3\nabla_{[cd}\Gamma_{e][ab]}{}^{e} - \tfrac{1}{4}\Gamma_{abe}{}^{e}\Gamma_{cdf}{}^{f} - \tfrac{1}{2}\Gamma_{abe}{}^{f}\Gamma_{cdf}{}^{e}$$
$$- \Gamma_{ab[c}{}^{e}\Gamma_{d]ef}{}^{f} - \Gamma_{cd[a}{}^{e}\Gamma_{b]ef}{}^{f} - \Gamma_{ea[c}{}^{f}\Gamma_{d]fb}{}^{e} + \Gamma_{eb[c}{}^{f}\Gamma_{d]fa}{}^{e}$$

- Under an arbitrary transformation of the connection,

$$\delta S_{abcd} = 3\nabla_{[ab}\delta\Gamma_{e][cd]}{}^{e} + 3\nabla_{[cd}\delta\Gamma_{e][ab]}{}^{e}$$

SL(N) curvatures

- The semi-covariant Riemann curvature satisfies the same symmetric properties as the ordinary Riemann curvature, including the Bianchi identity,

$$S_{abcd} = S_{[ab][cd]} = S_{cdab}\,, \qquad S_{[abc]d} = 0$$

- Diffeomorphic transformations

$$(\delta_X - \hat{\mathcal{L}}_X)S_{abcd} = 2\nabla_{e[a}\Omega_{b][cd]}{}^{e} + 2\nabla_{e[c}\Omega_{d][ab]}{}^{e}$$

where

$$\Omega_{abcd} = \mathcal{P}_{abcd}{}^{klm}{}_{n}\partial_{kl}\partial_{me}X^{ne}$$

SL(N) curvatures

- The semi-covariant Riemann curvature satisfies the same symmetric properties as the ordinary Riemann curvature, including the Bianchi identity,

$$S_{abcd} = S_{[ab][cd]} = S_{cdab}\,, \qquad S_{[abc]d} = 0$$

- Diffeomorphic transformations

$$(\delta_X - \hat{\mathcal{L}}_X)S_{abcd} = 2\nabla_{e[a}\Omega_{b][cd]}{}^{e} + 2\nabla_{e[c}\Omega_{d][ab]}{}^{e}$$

where

$$\Omega_{abcd} = \mathcal{P}_{abcd}{}^{klm}{}_{n}\partial_{kl}\partial_{me}X^{ne}$$

SL(N) curvatures

- Covariant Ricci curvature and scalar curvature

$$S_{ab} := S_{acb}{}^{c} = S_{(ab)}\,, \quad S := M^{ab} S_{ab} = S_{ab}{}^{ab}$$

- Action

$$\int_{\Sigma} M^{\frac{1}{4-N}} S$$

where the integral is taken over a section, Σ.

SL(N) curvatures

- *Einstein* equation of motion

$$S_{ab} + \frac{1}{2(N-4)} M_{ab} S = 0$$

- G-diffeomorphism implies a conservation relation

$$\nabla^{c}{}_{[a} S_{b]c} + \tfrac{3}{8} \nabla_{ab} S = 0$$

Duality inequivalent sections

- $(N-1)$-dimensional (physical) section : Σ_{N-1}

$$\partial_{\alpha\beta} = 0\,, \quad \partial_{\alpha} := \partial_{\alpha N} \neq 0$$

where $\alpha, \beta = 1, 2, \cdots, N-1$.

- Three-dimensional (physical) section : Σ_3

$$\partial_{\mu i} = 0\,, \quad \partial_{ij} = 0\,, \quad \partial_{\mu\nu} \neq 0$$

where $\mu, \nu = 1, 2, 3$ and $i, j = 4, 5, \cdots, N$.

* Note that the section condition is $\partial_{[ab}\partial_{cd]} = 0$.

Duality inequivalent sections

- **SL**(N) covariant expression

$$\partial_{[ab}\Phi\partial_{c][d}\Phi'\partial_{ef]}\Phi''$$

Duality inequivalent sections

- **SL**(N) covariant expression

$$\partial_{[ab}\Phi\partial_{c][d}\Phi'\partial_{ef]}\Phi'' \quad \Rightarrow \quad \left\{ \begin{array}{ll} \partial_{[N\alpha}\Phi\partial_{\beta][N}\Phi'\partial_{\gamma\lambda]}\Phi'' = 0 & \text{On } \Sigma_{N-1} \\ & \end{array} \right.$$

Duality inequivalent sections

- **SL**(N) covariant expression

$$\partial_{[ab}\Phi\partial_{c][d}\Phi'\partial_{ef]}\Phi'' \quad \Rightarrow \quad \begin{cases} \partial_{[N\alpha}\Phi\partial_{\beta][N}\Phi'\partial_{\gamma\lambda]}\Phi'' = 0 & \text{On } \Sigma_{N-1} \\ \partial_{[\mu\nu}\Phi\partial_{\rho][\mu'}\Phi'\partial_{\nu'\rho']}\Phi'' \neq 0 & \text{On } \Sigma_3 \end{cases}$$

c.f. **SL**(5): Blair, Malek and Park (2013), EFT: Hohm and Samtleben (2013)

Riemannian reduction to $N-1$ dimensions

- Σ_{N-1} parametrization

$$M_{ab} = \begin{pmatrix} \frac{g_{\alpha\beta}}{\sqrt{|g|}} & v_\alpha \\ v_\beta & \sqrt{|g|}(-e^\phi + v^2) \end{pmatrix}, \quad |M|^{\frac{1}{4-N}} = e^{\frac{1}{4-N}\phi}\sqrt{|g|}, \quad v = *C_{N-2}$$

- Parameter of generalized Lie derivative

$$X^{ab} = (X^{\alpha\beta}, X^{\alpha N}) = \left(\tfrac{1}{(N-3)!}\epsilon^{\alpha\beta\rho_1\cdots\rho_{N-3}}\Lambda_{\rho_1\cdots\rho_{N-3}},\ \xi^\alpha\right)$$

- Generalized Lie derivative

$$\hat{\mathcal{L}}_X M_{ab} \Rightarrow \begin{cases} \delta\phi = \mathcal{L}_\xi \phi\,, \\ \delta g_{\alpha\beta} = \mathcal{L}_\xi g_{\alpha\beta} \\ \delta C_{N-2} = \mathcal{L}_\xi C_{N-2} + \mathrm{d}\Lambda_{N-3} \end{cases}$$

Riemannian reduction to $N-1$ dimensions

- Σ_{N-1} parametrization

$$M_{ab} = \begin{pmatrix} \frac{g_{\alpha\beta}}{\sqrt{|g|}} & v_\alpha \\ v_\beta & \sqrt{|g|}(-e^\phi + v^2) \end{pmatrix}, \quad |M|^{\frac{1}{4-N}} = e^{\frac{1}{4-N}\phi}\sqrt{|g|}, \quad v = *C_{N-2}$$

- Parameter of generalized Lie derivative

$$X^{ab} = (X^{\alpha\beta}, X^{\alpha N}) = \left(\tfrac{1}{(N-3)!}\epsilon^{\alpha\beta\rho_1\cdots\rho_{N-3}}\Lambda_{\rho_1\cdots\rho_{N-3}}, \, \xi^\alpha\right)$$

- Generalized Lie derivative

$$\hat{\mathcal{L}}_X M_{ab} \Rightarrow \begin{cases} \delta\phi = \mathcal{L}_\xi \phi \,, \\ \delta g_{\alpha\beta} = \mathcal{L}_\xi g_{\alpha\beta} \\ \delta C_{N-2} = \mathcal{L}_\xi C_{N-2} + \mathrm{d}\Lambda_{N-3} \end{cases}$$

Riemannian reduction to $N-1$ dimensions

- Σ_{N-1} parametrization

$$M_{ab} = \begin{pmatrix} \frac{g_{\alpha\beta}}{\sqrt{|g|}} & v_\alpha \\ v_\beta & \sqrt{|g|}(-e^\phi + v^2) \end{pmatrix}, \quad |M|^{\frac{1}{4-N}} = e^{\frac{1}{4-N}\phi}\sqrt{|g|}, \quad v = *C_{N-2}$$

- Parameter of generalized Lie derivative

$$X^{ab} = (X^{\alpha\beta}, X^{\alpha N}) = \left(\frac{1}{(N-3)!}\epsilon^{\alpha\beta\rho_1\cdots\rho_{N-3}}\Lambda_{\rho_1\cdots\rho_{N-3}},\ \xi^\alpha\right)$$

- Generalized Lie derivative

$$\hat{\mathcal{L}}_X M_{ab} \Rightarrow \begin{cases} \delta\phi = \mathcal{L}_\xi \phi\,, \\ \delta g_{\alpha\beta} = \mathcal{L}_\xi g_{\alpha\beta} \\ \delta C_{N-2} = \mathcal{L}_\xi C_{N-2} + \mathrm{d}\Lambda_{N-3} \end{cases}$$

Riemannian reduction to $N-1$ dimensions

- Σ_{N-1} parametrization

$$M_{ab} = \begin{pmatrix} \frac{g_{\alpha\beta}}{\sqrt{|g|}} & v_\alpha \\ v_\beta & \sqrt{|g|}(-e^\phi + v^2) \end{pmatrix}, \quad |M|^{\frac{1}{4-N}} = e^{\frac{1}{4-N}\phi}\sqrt{|g|}, \quad v = *C_{N-2}$$

- U-gravity scalar curvature

$$S\Big|_{\Sigma_{N-1}} = 2e^{-\phi}\left[R_g - \tfrac{(N-3)(3N-8)}{4(N-4)^2}\partial_\alpha\phi\partial^\alpha\phi + \tfrac{N-2}{N-4}\triangle\phi + \tfrac{1}{(N-1)!}(\mathrm{d}C_{N-2})^2\right]$$

Riemannian reduction to three dimensions

- Σ_3 parametrization

$$M_{ab}=\begin{pmatrix}\frac{\tilde{g}^{\mu\nu}}{\sqrt{|\tilde{g}|}} & -\tilde{v}^{j\mu}\\ -\tilde{v}^{i\nu} & \sqrt{|\tilde{g}|}(-e^{\tilde{\phi}}\tilde{\mathcal{M}}^{ij}+\tilde{v}^{i\lambda}\tilde{v}^{j}{}_{\lambda})\end{pmatrix}\,,\quad |M|^{\frac{1}{4-N}}=e^{\frac{N-3}{4-N}\tilde{\phi}}\sqrt{|\tilde{g}|}\,,\quad \tilde{v}^{i}=*\tilde{C}_{2}^{i}$$

- Dual coordinates

$$\tilde{x}_{\mu}\equiv\varepsilon_{\mu\nu\rho}x^{\nu\rho}\,,\quad \tilde{\partial}^{\mu}\tilde{x}_{\nu}=\delta^{\mu}{}_{\nu}$$

where a three-dimensional Levi-Civita symbol, $\varepsilon_{123}\equiv 1$.

Riemannian reduction to three dimensions

- Σ_3 parametrization

$$M_{ab} = \begin{pmatrix} \frac{\tilde{g}^{\mu\nu}}{\sqrt{|\tilde{g}|}} & -\tilde{v}^{j\mu} \\ -\tilde{v}^{i\nu} & \sqrt{|\tilde{g}|}(-e^{\tilde{\phi}}\tilde{\mathcal{M}}^{ij} + \tilde{v}^{i\lambda}\tilde{v}^{j}{}_{\lambda}) \end{pmatrix}, \quad |M|^{\frac{1}{4-N}} = e^{\frac{N-3}{4-N}\tilde{\phi}}\sqrt{|\tilde{g}|}, \quad \tilde{v}^i = *\tilde{C}^i_2$$

- Dual coordinates

$$\tilde{x}_\mu \equiv \varepsilon_{\mu\nu\rho}x^{\nu\rho}, \quad \tilde{\partial}^\mu \tilde{x}_\nu = \delta^\mu{}_\nu$$

where a three-dimensional Levi-Civita symbol, $\varepsilon_{123} \equiv 1$.

Riemannian reduction to three dimensions

- Σ_3 parametrization

$$M_{ab} = \begin{pmatrix} \frac{\tilde{g}^{\mu\nu}}{\sqrt{|\tilde{g}|}} & -\tilde{v}^{j\mu} \\ -\tilde{v}^{i\nu} & \sqrt{|\tilde{g}|}(-e^{\tilde{\phi}}\tilde{\mathcal{M}}^{ij} + \tilde{v}^{i\lambda}\tilde{v}^{j}{}_{\lambda}) \end{pmatrix}, \quad |M|^{\frac{1}{4-N}} = e^{\frac{N-3}{4-N}\tilde{\phi}}\sqrt{|\tilde{g}|}, \quad \tilde{v}^i = *\tilde{C}_2^i$$

- U-gravity scalar curvature

$$S\Big|_{\Sigma_3} = -2R_{\tilde{g}} + \frac{(N-3)(3N-8)}{2(N-4)^2}\tilde{\partial}_\mu\tilde{\phi}\tilde{\partial}^\mu\tilde{\phi} - \frac{4(N-3)}{N-4}\tilde{\triangle}\tilde{\phi} - \frac{1}{2}\tilde{\partial}^\mu\tilde{\mathcal{M}}^{ij}\tilde{\partial}_\mu\tilde{\mathcal{M}}_{ij} + e^{\tilde{\phi}}\tilde{\mathcal{M}}_{ij}\tilde{\nabla}^\mu\tilde{v}^i{}_\mu\tilde{\nabla}^\nu\tilde{v}^j{}_\nu$$

which manifests **SL**$(N-3)$ S-duality.

c.f. **SL**(5): Blair, Malek and Park (2013)

Supersymmetric extension of SL(11) U-gravity

Work in progress with Jeong-Hyuck Park, Minwoo Suh and YS

- Ten-dimensional reformulation of Romans massive type IIA supergravity.
- Also it gives a connection to three-dimensional new maximal supergravity.

Supersymmetric extension of SL(11) U-gravity

Work in progress with Jeong-Hyuck Park, Minwoo Suh and YS

- Ten-dimensional reformulation of Romans massive type IIA supergravity.
- Also it gives a connection to three-dimensional new maximal supergravity.

Review on Romans massive type IIA supergravity

- Field contents

$$\phi, \quad g_{\mu\nu}, \quad B_{\mu\nu}, \quad A_{\mu\nu\lambda}, \quad \lambda, \quad \psi_\mu$$

- The field strengths for the tensor fields

$$G_{\mu\nu\lambda} \equiv 3\partial_{[\mu} B_{\nu\lambda]}\,, \quad F_{\mu\nu\lambda\rho} \equiv F^0_{\mu\nu\lambda\rho} + 6mB_{[\mu\nu}B_{\lambda\rho]}\,, \quad F^0_{\mu\nu\lambda\rho} := 4\partial_{[\mu}A_{\nu\lambda\rho]}$$

where $\mu = 0, \cdots 9$.

Romans (1986)

Review on Romans massive type IIA supergravity

- Romans Lagrangian for $m \neq 0$

$$
\begin{aligned}
e^{-1}\mathcal{L}_{\mathrm{R}} = & -\tfrac{1}{4}R_g + \tfrac{1}{2}D^\mu\phi D_\mu\phi + \tfrac{1}{8}m^2 e^{-5\phi} \\
& + \tfrac{1}{2}\bar{\psi}_\mu\gamma^{\mu\nu\lambda}D_\nu\psi_\lambda + \tfrac{1}{2}\bar{\lambda}\gamma^\mu D_\mu\lambda - \tfrac{1}{\sqrt{2}}\partial_\nu\phi\bar{\lambda}\gamma^\mu\gamma^\nu\psi_\mu \\
& + \tfrac{1}{8}me^{-\frac{5}{2}\phi}\left(\bar{\psi}_\mu\gamma^{\mu\nu}\psi_\nu + \tfrac{5}{\sqrt{2}}\bar{\lambda}\gamma^\mu\psi_\mu - \tfrac{21}{4}\bar{\lambda}\lambda\right) \\
& + \tfrac{1}{48}e^{-\phi'}F^{\mu\nu\lambda\kappa}F_{\mu\nu\lambda\kappa} + \tfrac{1}{12}e^{2\phi'}G^{\mu\nu\lambda}G_{\mu\nu\lambda} + \tfrac{1}{4}m^2e^{-3\phi'}B^{\mu\nu}B_{\mu\nu} \\
& - \tfrac{1}{24\cdot 48}(\epsilon FFB) + \cdots
\end{aligned}
$$

where

$$
\begin{aligned}
(\epsilon FFB) = & e^{-1}\epsilon^{\mu_1\cdots\mu_{10}}\left(F^0_{\mu_1\mu_2\mu_3\mu_4}F^0_{\mu_5\mu_6\mu_7\mu_8}B_{\mu_9\mu_10} + 4mF^0_{\alpha\beta\gamma\lambda}B_{\kappa\sigma}B_{\rho\mu}B_{\nu\eta}\right. \\
& \left. + \tfrac{36}{5}m^2B_{\alpha\beta}B_{\gamma\lambda}B_{\kappa\sigma}B_{\rho\mu}B_{\nu\eta}\right).
\end{aligned}
$$

Supersymmetric extension of SL(11) U-gravity

Symmetries

- **SL**(11) U-duality
- Gauge symmetries
  - Generalized diffeomorphism
  - Local Lorentz symmetry
  - Local $\mathcal{N} = 1$ maximal supersymmetry
- Metric for the symmetries
  - M_{ab} for SL(11) & G-diffeomorphism
  - $\bar{\eta}_{\bar{a}\bar{b}}$ for vector local Lorentz
  - $C_{\alpha\beta}$ for spinor local Lorentz

Supersymmetric extension of SL(11) U-gravity

Symmetries

- **SL**(11) U-duality
- Gauge symmetries
  - Generalized diffeomorphism
  - Local Lorentz symmetry
  - Local $\mathcal{N} = 1$ maximal supersymmetry

- Metric for the symmetries
  - M_{ab} for SL(11) & G-diffeomorphism
  - $\bar{\eta}_{\bar{a}\bar{b}}$ for vector local Lorentz
  - $C_{\alpha\beta}$ for spinor local Lorentz

Supersymmetric extension of SL(11) U-gravity

Symmetries

- **SL**(11) U-duality
- Gauge symmetries
  - Generalized diffeomorphism
  - Local Lorentz symmetry
  - Local $\mathcal{N} = 1$ maximal supersymmetry

- Metric for the symmetries
  - M_{ab} for **SL**(11) & G-diffeomorphism
  - $\bar{\eta}_{\bar{a}\bar{b}}$ for vector local Lorentz
  - $C_{\alpha\beta}$ for spinor local Lorentz

Supersymmetric extension of SL(11) U-gravity

- U-vielbein

$$M_{ab} = E_a{}^{\bar{a}} E_b{}^{\bar{b}} \bar{\eta}_{\bar{a}\bar{b}}$$

- Master semi-covariant derivative

$$\mathcal{D}_{ab} = \partial_{ab} + \Gamma_{ab} + \Phi_{ab}$$

Note that

$$\nabla_{ab} = \partial_{ab} + \Gamma_{ab}\,, \qquad D_{ab} = \partial_{ab} + \Phi_{ab}$$

- Metric compatibility conditions

$$\mathcal{D}_{ab} M_{cd} = 0\,, \qquad \mathcal{D}_{ab} E_c{}^{\bar{d}} = 0\,, \qquad \mathcal{D}_{ab} \bar{\eta}_{\bar{c}\bar{d}} = 0\,, \qquad \mathcal{D}_{ab} \mathcal{C}_{\alpha\beta} = 0\,, \qquad \mathcal{D}_{ab} (\Gamma^{\bar{c}})_\alpha{}^\beta = 0$$

Supersymmetric extension of SL(11) U-gravity

- U-vielbein

$$M_{ab} = E_a{}^{\bar{a}} E_b{}^{\bar{b}} \bar{\eta}_{\bar{a}\bar{b}}$$

- Master semi-covariant derivative

$$\mathcal{D}_{ab} = \partial_{ab} + \Gamma_{ab} + \Phi_{ab}$$

Note that

$$\nabla_{ab} = \partial_{ab} + \Gamma_{ab} \,, \qquad D_{ab} = \partial_{ab} + \Phi_{ab}$$

- Metric compatibility conditions

$$\mathcal{D}_{ab} M_{cd} = 0 \,, \qquad \mathcal{D}_{ab} E_c{}^{\bar{d}} = 0 \,, \qquad \mathcal{D}_{ab} \bar{\eta}_{\bar{c}\bar{d}} = 0 \,, \qquad \mathcal{D}_{ab} \mathcal{C}_{\alpha\beta} = 0 \,, \qquad \mathcal{D}_{ab} (\Gamma^{\bar{c}})_\alpha{}^\beta = 0$$

Supersymmetric extension of SL(11) U-gravity

- U-vielbein

$$M_{ab} = E_a{}^{\bar{a}} E_b{}^{\bar{b}} \bar{\eta}_{\bar{a}\bar{b}}$$

- Master semi-covariant derivative

$$\mathcal{D}_{ab} = \partial_{ab} + \Gamma_{ab} + \Phi_{ab}$$

Note that

$$\nabla_{ab} = \partial_{ab} + \Gamma_{ab} \,, \qquad D_{ab} = \partial_{ab} + \Phi_{ab}$$

- Metric compatibility conditions

$$\mathcal{D}_{ab} M_{cd} = 0 \,, \qquad \mathcal{D}_{ab} E_c{}^{\bar{d}} = 0 \,, \qquad \mathcal{D}_{ab} \bar{\eta}_{\bar{c}\bar{d}} = 0 \,, \qquad \mathcal{D}_{ab} \mathcal{C}_{\alpha\beta} = 0 \,, \qquad \mathcal{D}_{ab} (\Gamma^{\bar{c}})_\alpha{}^\beta = 0$$

Supersymmetric extension of SL(11) U-gravity

- Spin connection

$$\Phi_{ab\bar{c}\bar{d}} = (E^{-1})_{\bar{c}}{}^{e}\nabla_{ab}E_{e\bar{d}}\,, \qquad \Phi_{ab\alpha}{}^{\beta} = \frac{1}{4}\Phi_{ab\bar{c}\bar{d}}(\Gamma^{\bar{c}\bar{d}})_{\alpha}{}^{\beta}$$

- **SL**(11) connection

$$\Gamma_{abcd} = A_{abcd} + \tfrac{1}{2}(A_{acbd} - A_{adbc} + A_{bdac} - A_{bcad}) + \tfrac{1}{9}(M_{ac}A^{e}{}_{(bd)e} - M_{ad}A^{e}{}_{(bc)e} + M_{bd}A^{e}{}_{(ac)e} - M_{bc}A^{e}{}_{(ad)e})$$

where

$$A_{abcd} := -\frac{1}{2}\partial_{ab}M_{cd} + \frac{1}{14}M_{cd}\partial_{ab}\ln|M|$$

Supersymmetric extension of SL(11) U-gravity

- Spin connection

$$\Phi_{ab\bar{c}\bar{d}} = (E^{-1})_{\bar{c}}{}^{e}\nabla_{ab}E_{e\bar{d}}\,, \qquad \Phi_{ab\alpha}{}^{\beta} = \frac{1}{4}\Phi_{ab\bar{c}\bar{d}}(\Gamma^{\bar{c}\bar{d}})_{\alpha}{}^{\beta}$$

- **SL**(11) connection

$$\begin{aligned}\Gamma_{abcd} =& A_{abcd} + \tfrac{1}{2}(A_{acbd} - A_{adbc} + A_{bdac} - A_{bcad}) \\ & + \tfrac{1}{9}(M_{ac}A^{e}{}_{(bd)e} - M_{ad}A^{e}{}_{(bc)e} + M_{bd}A^{e}{}_{(ac)e} - M_{bc}A^{e}{}_{(ad)e})\end{aligned}$$

where

$$A_{abcd} := -\frac{1}{2}\partial_{ab}M_{cd} + \frac{1}{14}M_{cd}\partial_{ab}\ln|M|$$

Supersymmetric extension of SL(11) U-gravity

- Field contents

$$\{g_{\mu\nu}, v^{\mu}, \phi\} \Rightarrow M_{ab}\,, \qquad \{\psi_{\mu}, \lambda\} \Rightarrow \Psi_a\,, \qquad \{B_{\mu\nu}, A_{\mu\nu\lambda}\} \Rightarrow \text{U-Flux ?}$$

- Supersymmetric **SL**(11) U-gravity Lagrangian

$$\mathcal{L}_{\rm U} = |M|^{-\frac{1}{7}} S_{ab} M^{ab} + \mathcal{L}_{\rm Fermion} + (\mathcal{L}_{\rm Flux}?)$$

Supersymmetric extension of SL(11) U-gravity

- Field contents

$$\{g_{\mu\nu}, v^{\mu}, \phi\} \Rightarrow M_{ab}\,, \qquad \{\psi_{\mu}, \lambda\} \Rightarrow \Psi_{a}\,, \qquad \{B_{\mu\nu}, A_{\mu\nu\lambda}\} \Rightarrow \text{U-Flux ?}$$

- Supersymmetric **SL**(11) U-gravity Lagrangian

$$\mathcal{L}_{\text{U}} = |M|^{-\frac{1}{7}} S_{ab} M^{ab} + \mathcal{L}_{\text{Fermion}} + (\mathcal{L}_{\text{Flux}}?)$$

Supersymmetric extension of SL(11) U-gravity

- Supersymmetric **SL**(11) U-gravity Lagrangian

$$\mathcal{L}_{\mathrm{U}} = |M|^{-\frac{1}{7}} S_{ab} M^{ab} + \mathcal{L}_{\mathrm{Fermion}} + (\mathcal{L}_{\mathrm{Flux}}?)$$

- Romans Lagrangian for $m \neq 0$

$$\begin{aligned} e^{-1}\mathcal{L}_{\mathrm{R}} = & -\tfrac{1}{4}R_g + \tfrac{1}{2}D^{\mu}\phi D_{\mu}\phi + \tfrac{1}{8}m^2 e^{-5\phi} \\ & + \tfrac{1}{2}\bar{\psi}_{\mu}\gamma^{\mu\nu\lambda}D_{\nu}\psi_{\lambda} + \tfrac{1}{2}\bar{\lambda}\gamma^{\mu}D_{\mu}\lambda - \tfrac{1}{\sqrt{2}}\partial_{\nu}\phi\bar{\lambda}\gamma^{\mu}\gamma^{\nu}\psi_{\mu} \\ & + \tfrac{1}{8}me^{-\frac{5}{2}\phi}\left(\bar{\psi}_{\mu}\gamma^{\mu\nu}\psi_{\nu} + \tfrac{5}{\sqrt{2}}\bar{\lambda}\gamma^{\mu}\psi_{\mu} - \tfrac{21}{4}\bar{\lambda}\lambda\right) \\ & + \tfrac{1}{48}e^{-\phi'}F^{\mu\nu\lambda\kappa}F_{\mu\nu\lambda\kappa} + \tfrac{1}{12}e^{2\phi'}G^{\mu\nu\lambda}G_{\mu\nu\lambda} + \tfrac{1}{4}m^2e^{-3\phi'}B^{\mu\nu}B_{\mu\nu} \\ & - \tfrac{1}{24\cdot 48}(\epsilon FFB) + \cdots \end{aligned}$$

Supersymmetric extension of SL(11) U-gravity

- Supersymmetric **SL**(11) U-gravity Lagrangian

$$\mathcal{L}_{\mathrm{U}} = |M|^{-\frac{1}{7}} S_{ab} M^{ab} + \mathcal{L}_{\mathrm{Fremion}} + (\mathcal{L}_{\mathrm{Flux}}?)$$

- Romans Lagrangian for $m \neq 0$

$$\begin{aligned} e^{-1}\mathcal{L}_{\mathrm{R}} = & -\tfrac{1}{4}R_g + \tfrac{1}{2}D^\mu\phi D_\mu\phi + \tfrac{1}{8}m^2 e^{-5\phi} \\ & + \tfrac{1}{2}\bar{\psi}_\mu\gamma^{\mu\nu\lambda}D_\nu\psi_\lambda + \tfrac{1}{2}\bar{\lambda}\gamma^\mu D_\mu\lambda - \tfrac{1}{\sqrt{2}}\partial_\nu\phi\bar{\lambda}\gamma^\mu\gamma^\nu\psi_\mu \\ & + \tfrac{1}{8}me^{-\frac{5}{2}\phi}\left(\bar{\psi}_\mu\gamma^{\mu\nu}\psi_\nu + \tfrac{5}{\sqrt{2}}\bar{\lambda}\gamma^\mu\psi_\mu - \tfrac{21}{4}\bar{\lambda}\lambda\right) \\ & + \tfrac{1}{48}e^{-\phi'}F^{\mu\nu\lambda\kappa}F_{\mu\nu\lambda\kappa} + \tfrac{1}{12}e^{2\phi'}G^{\mu\nu\lambda}G_{\mu\nu\lambda} + \tfrac{1}{4}m^2e^{-3\phi'}B^{\mu\nu}B_{\mu\nu} \\ & - \tfrac{1}{24\cdot 48}(\epsilon FFB) + \cdots \end{aligned}$$

Summary

- On the extended-yet-gauged spacetime, we have constructed a duality manifest $\frac{1}{2}N(N-1)$-dimensional gravitational theory for the special linear group, $\mathbf{SL}(N)$ with $N \neq 4$,

$$x^{ab} \sim x^{ab} + \Delta^{ab}$$

- $\mathbf{SL}(N)$ U-gravity unifies both $(N-1)$-dimensional and three-dimensional gravitational theories as different aspects of the section condition.
- Romans type IIA massive supergravity will be obtained from supersymmetric extension of $\mathbf{SL}(11)$ U-gravity.

Summary

- On the extended-yet-gauged spacetime, we have constructed a duality manifest $\frac{1}{2}N(N-1)$-dimensional gravitational theory for the special linear group, $\mathbf{SL}(N)$ with $N \neq 4$,

$$x^{ab} \sim x^{ab} + \Delta^{ab}$$

- $\mathbf{SL}(N)$ U-gravity unifies both $(N-1)$-dimensional and three-dimensional gravitational theories as different aspects of the section condition.

- Romans type IIA massive supergravity will be obtained from supersymmetric extension of $\mathbf{SL}(11)$ U-gravity.

Summary

- On the extended-yet-gauged spacetime, we have constructed a duality manifest $\frac{1}{2}N(N-1)$-dimensional gravitational theory for the special linear group, $\mathbf{SL}(N)$ with $N \neq 4$,

$$x^{ab} \sim x^{ab} + \Delta^{ab}$$

- $\mathbf{SL}(N)$ U-gravity unifies both $(N-1)$-dimensional and three-dimensional gravitational theories as different aspects of the section condition.

- Romans type IIA massive supergravity will be obtained from supersymmetric extension of $\mathbf{SL}(11)$ U-gravity.

Comments and future works

- Using our construction, internal part of EFT can be manifestly geometrized for various U-dualities.
- The other section of supersymmetric **SL**(11) U-gravity shall provide three-dimensional maximal supergravity.
- We expect that generalized Scherk-Schwarz compactification can be constructed with covariant definition for U-duality via semi-covariant approach.
- In U-gravity, invariant measure contains an exponential of scalar term naturally, cosmological constant contributes differently from ordinary cosmology.

$$\int_{\Sigma} |M|^{\frac{1}{4-N}} \Lambda = \begin{cases} \int_{\Sigma_{N-1}} e^{\frac{1}{4-N}\phi} \sqrt{|g|}\Lambda \\ \\ \int_{\Sigma_3} e^{\frac{N-3}{4-N}\tilde{\phi}} \sqrt{|g|}\Lambda \end{cases}$$

Comments and future works

- Using our construction, internal part of EFT can be manifestly geometrized for various U-dualities.
- The other section of supersymmetric **SL**(11) U-gravity shall provide three-dimensional maximal supergravity.
- We expect that generalized Scherk-Schwarz compactification can be constructed with covariant definition for U-duality via semi-covariant approach.
- In U-gravity, invariant measure contains an exponential of scalar term naturally, cosmological constant contributes differently from ordinary cosmology.

$$\int_{\Sigma} |M|^{\frac{1}{4-N}} \Lambda = \begin{cases} \int_{\Sigma_{N-1}} e^{\frac{1}{4-N}\phi} \sqrt{|g|}\Lambda \\ \int_{\Sigma_3} e^{\frac{N-3}{4-N}\tilde{\phi}} \sqrt{|g|}\Lambda \end{cases}$$

Comments and future works

- Using our construction, internal part of EFT can be manifestly geometrized for various U-dualities.
- The other section of supersymmetric $\mathbf{SL}(11)$ U-gravity shall provide three-dimensional maximal supergravity.
- We expect that generalized Scherk-Schwarz compactification can be constructed with covariant definition for U-duality via semi-covariant approach.
- In U-gravity, invariant measure contains an exponential of scalar term naturally, cosmological constant contributes differently from ordinary cosmology.

$$\int_{\Sigma} |M|^{\frac{1}{4-N}}\Lambda = \begin{cases} \int_{\Sigma_{N-1}} e^{\frac{1}{4-N}\phi}\sqrt{|g|}\Lambda \\ \int_{\Sigma_3} e^{\frac{N-3}{4-N}\tilde{\phi}}\sqrt{|g|}\Lambda \end{cases}$$

Comments and future works

- Using our construction, internal part of EFT can be manifestly geometrized for various U-dualities.
- The other section of supersymmetric **SL**(11) U-gravity shall provide three-dimensional maximal supergravity.
- We expect that generalized Scherk-Schwarz compactification can be constructed with covariant definition for U-duality via semi-covariant approach.
- In U-gravity, invariant measure contains an exponential of scalar term naturally, cosmological constant contributes differently from ordinary cosmology.

$$\int_{\Sigma} |M|^{\frac{1}{4-N}} \Lambda = \begin{cases} \int_{\Sigma_{N-1}} e^{\frac{1}{4-N}\phi} \sqrt{|g|}\Lambda \\ \int_{\Sigma_3} e^{\frac{N-3}{4-N}\tilde{\phi}} \sqrt{|g|}\Lambda \end{cases}$$

- It has been said that string theory is a piece of 21st century physics that happened to fall into the 20th century – *Anonymous Italian*
- Our U-gravity may provide a clue toward a new framework beyond Riemannian geometry.

Thank you!

- It has been said that string theory is a piece of 21st century physics that happened to fall into the 20th century – *Anonymous Italian*

- Our U-gravity may provide a clue toward a new framework beyond Riemannian geometry.

Thank you!