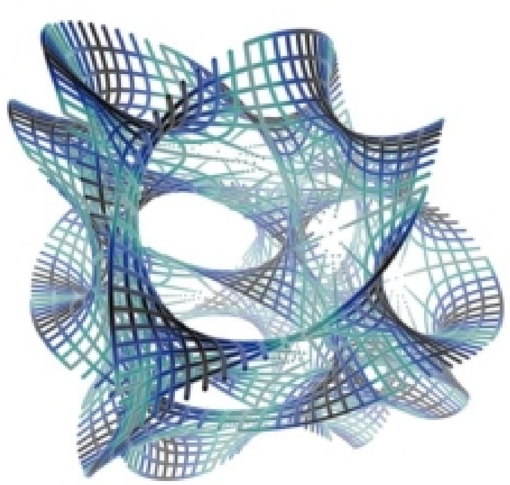
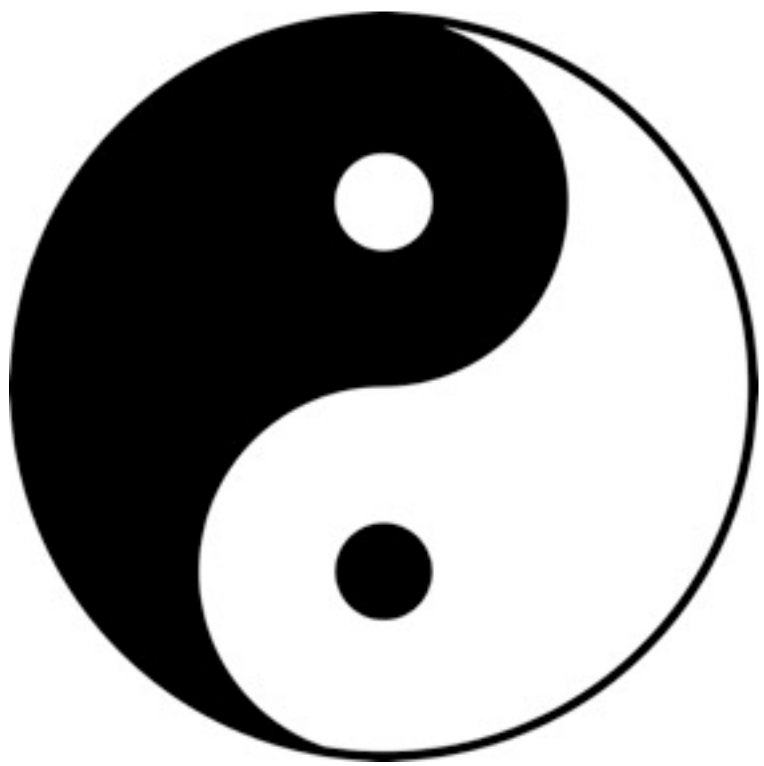
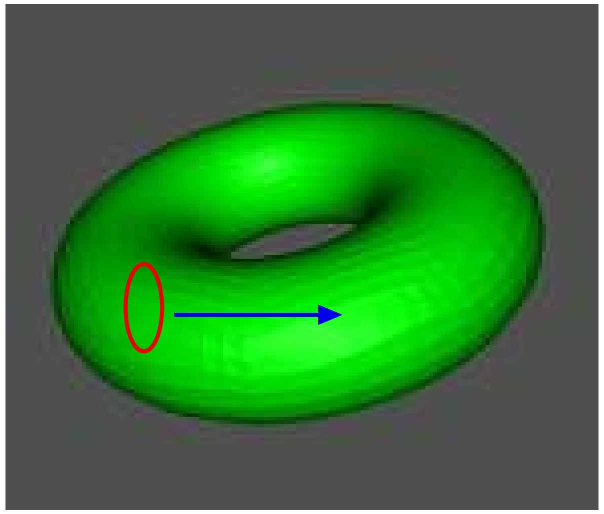
Double Field Theory and Stringy Geometry







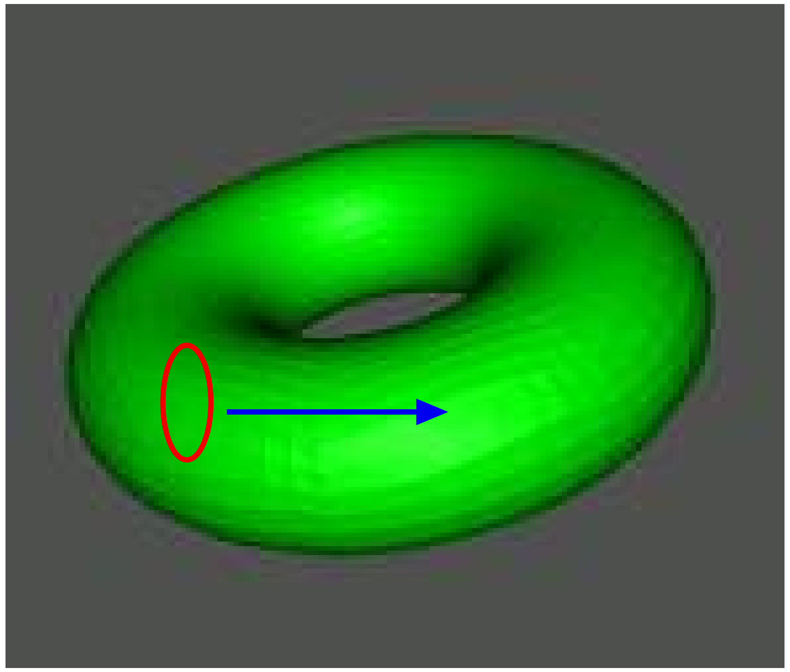
CERN-CKC TH Institute on Duality Symmetries in String and M-Theories

String/M Theory

- Rich theory, novel duality symmetries and exotic structures
- Fundamental formulation?
- Supergravity limit - misses stringy features
- Infinite set of fields: misses dualities
- Perturbative string - world-sheet theory
- String field theory: captures interactions, T-duality, algebraic structure

Strings in Geometric Background

Manifold, background tensor fields G_{ij}, H_{ijk}, Φ
Fluctuations: modes of string
Treat background and fluctuations the same?



Stringy geometry? Singularity resolution?
Dualities: mix geometric and stringy modes

Non-Geometric Background?

String theory: solutions that are not "geometric"
Moduli stabilisation. Richer landscape?

Generalised Geometry

- Much recent work in string theory, supergravity... GG and its generalisations
- Geometric framework for metric + p-form gauge fields
- Unification of various geometric structures
- Organizing principle : Duality symmetries
- New mathematics for old geometries
- New 'non-geometries' in string theory

Generalised Geometry

Studies structures on a d-dimensional manifold M on which there is a natural action of O(d,d) Hitchin

Manifold with metric + B-field, natural action of O(d,d), tangent space "doubled" to $T \oplus T^*$

Extended/Exceptional Geometry: O(d,d) replaced by E_{d+1} acting on extended tangent bundle

Hull; Pacheco & Waldram

Organizing principle : Duality symmetries e.g. O(d,d), E_d

Duality Symmetries

- String theory: discrete quantum duality symmetries not present in field theories
- T-duality: perturbative symmetry on torus, mixes momentum modes and winding states
- U-duality: non-perturbative symmetry of type II on torus, mixes momentum modes and wrapped brane states
- Supergravities: continuous classical symmetry, broken to discrete quantum symmetry

Symmetry & Geometry

- Spacetime constructed from local patches
- All symmetries of physics used in patching
- Patching with diffeomorphisms, gives manifold
- Patching with gauge symmetries: bundles
- String theory has new symmetries, not present in field theory. New <u>non-geometric</u> string backgrounds

Hull

- Patching with T-duality: **T-FOLDS**
- Patching with U-duality: **U-FOLDS**

Extra Dimensions

- Torus compactified theory has charges arising in SUSY algebra, carried by BPS states
- P_M: Momentum in extra dimensions
- Z_A: wrapped brane & wound string charges
- But P_M ,Z_A related by dualities. Can Z_A be thought of as momenta for extra dimensions?
- Space with coordinates X^M, Y^A ?

Extended Spacetime

- Supergravity can be rewritten in extended space with coordinates X^M, Y^A. Duality symmetry manifest.
- But fields depend only on X^M, (or coords related to these by duality). Gives generalised geometry formulation locally.
- Gives extended geometry for T-folds and U-folds: a geometry for non-geometry
- In string theory, can do better... DOUBLE FIELD THEORY, fields depending on X^M, Y_M.

Double Field Theory

Hull & Zwiebach

- From sector of String Field Theory. Features some stringy physics, including T-duality, in simpler setting
- Strings see a doubled space-time
- Necessary consequence of string theory
- Needed for non-geometric backgrounds
- What is geometry and physics of doubled space?

Strings on a Torus

- States: momentum p, winding w
- String: Infinite set of fields $\psi(p, w)$
- Fourier transform to doubled space: $\psi(x, \tilde{x})$
- "Double Field Theory" from closed string field theory. Some non-locality in doubled space
- Subsector? e.g. $g_{ij}(x, \tilde{x}),\ b_{ij}(x, \tilde{x}),\ \phi(x, \tilde{x})$

Double Field Theory

- Double field theory on doubled torus
- General solution of string theory: involves doubled fields $\psi(x, \tilde{x})$
- *Real* dependence on *full* doubled geometry, dual dimensions not auxiliary or gauge artifact. Double geom. *physical* and *dynamical*
- *Strong constraint* restricts to subsector in which extra coordinates auxiliary: get conventional field theory locally. Recover Siegel's duality covariant formulation of (super)gravity

Generalised Geometry

Conventional manifold M, doubled tangent space

$GL(d,\mathbb{R})$ acting on TM ⟶ $O(d,d)$ acting on $T \oplus T^*$

Natural structure (M,g,B) instead of (M,g)

Doubled Geometry

$GL(d,\mathbb{Z})$ acting on T^d ⟶ $O(d,d,\mathbb{Z})$ acting on T^{2d}

Doubled manifold

M-Theory

- 11-d sugra can be written in extended space.
- Extension to full M-theory?
- If M-theory were a perturbative theory of membranes, would have extended fields depending on X^M and 2-brane coordinates Y_{MN}
- But it's not.
- Don't have e.g. formulation as infinite no. of fields. Only implicit construction as a limit.
- What is M-theory?

Strings on Circle

$M = S^1 \times X$

Discrete momentum p=n/R

If it winds m times round S^1, winding energy w=mRT

Energy = p^2+w^2+....

T-duality: Symmetry of string theory

p $\leftrightarrow$ w

m $\leftrightarrow$ n

R $\leftrightarrow$ 1/RT

- Fourier transf of discrete p,w gives periodic coordinates
  $X, \tilde{X}$ Circle + dual circle
- Stringy symmetry, not in field theory
- On d torus, T-duality group $O(d, d; \mathbb{Z})$

Strings on T^d

$$X = X_L(\sigma + \tau) + X_R(\sigma - \tau), \qquad \tilde{X} = X_L - X_R$$

X conjugate to momentum, $\tilde{X}$ to winding no.

$$dX = *d\tilde{X} \qquad\qquad \partial_a X = \epsilon_{ab}\partial^b \tilde{X}$$

Strings on T^d

$$X = X_L(\sigma+\tau) + X_R(\sigma-\tau), \qquad \tilde{X} = X_L - X_R$$

X conjugate to momentum, $\tilde{X}$ to winding no.

$$dX = *d\tilde{X} \qquad\qquad \partial_a X = \epsilon_{ab}\partial^b \tilde{X}$$

Need "auxiliary" $\tilde{X}$ for interacting theory

i) Vertex operators $e^{ik_L \cdot X_L}, \quad e^{ik_R \cdot X_R}$

ii) String field Kugo & Zwiebach $\Phi[x, \tilde{x}, a, \tilde{a}]$

Strings on T^d

$$X = X_L(\sigma+\tau) + X_R(\sigma-\tau), \qquad \tilde{X} = X_L - X_R$$

X conjugate to momentum, $\tilde{X}$ to winding no.

$$dX = *d\tilde{X} \qquad \partial_a X = \epsilon_{ab}\partial^b \tilde{X}$$

Strings on torus see **DOUBLED GEOMETRY**!

T-duality group $O(d,d;\mathbb{Z})$

Doubled Torus 2d coordinates

Transform linearly under $O(d,d;\mathbb{Z})$

$$X \equiv \begin{pmatrix} \tilde{x}_i \\ x^i \end{pmatrix}$$

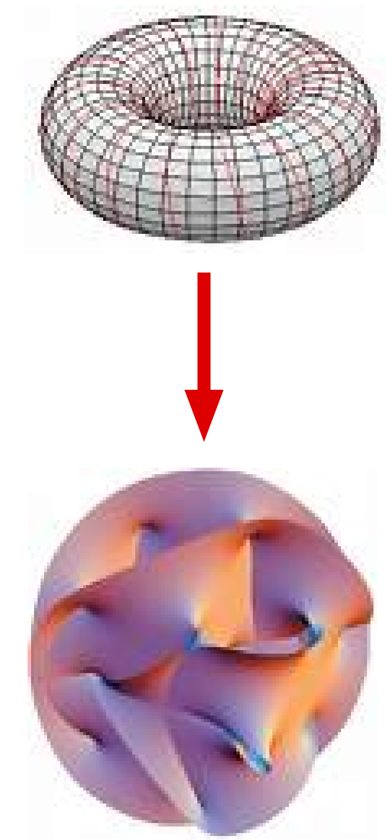
Sigma model on doubled torus Tseytlin; Hull

T-Duality

$G(Y), B(Y)$

X^i

Y^m



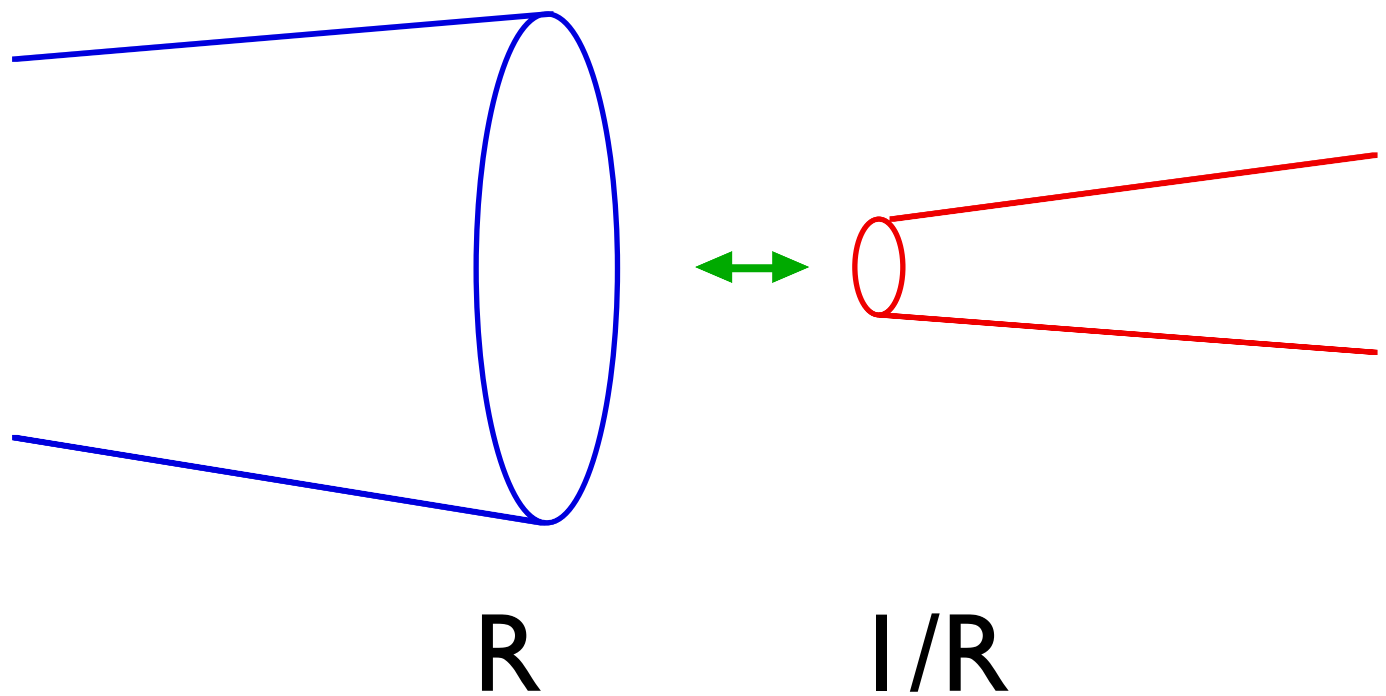
- Space has d-torus fibration
- G,B on fibres
- T-Duality O(d,d;Z), mixes G,B
- Mixes Momentum and Winding
- Changes geometry and topology

$$E \to (aE+b)(cE+d)^{-1}$$

$$h = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in O(d,d;Z) \qquad E_{ij} = G_{ij} + B_{ij}$$

On circle, radius R: $O(1,1;\mathbb{Z}) = \mathbb{Z}_2 : R \mapsto \frac{1}{R}$

T-fold patching

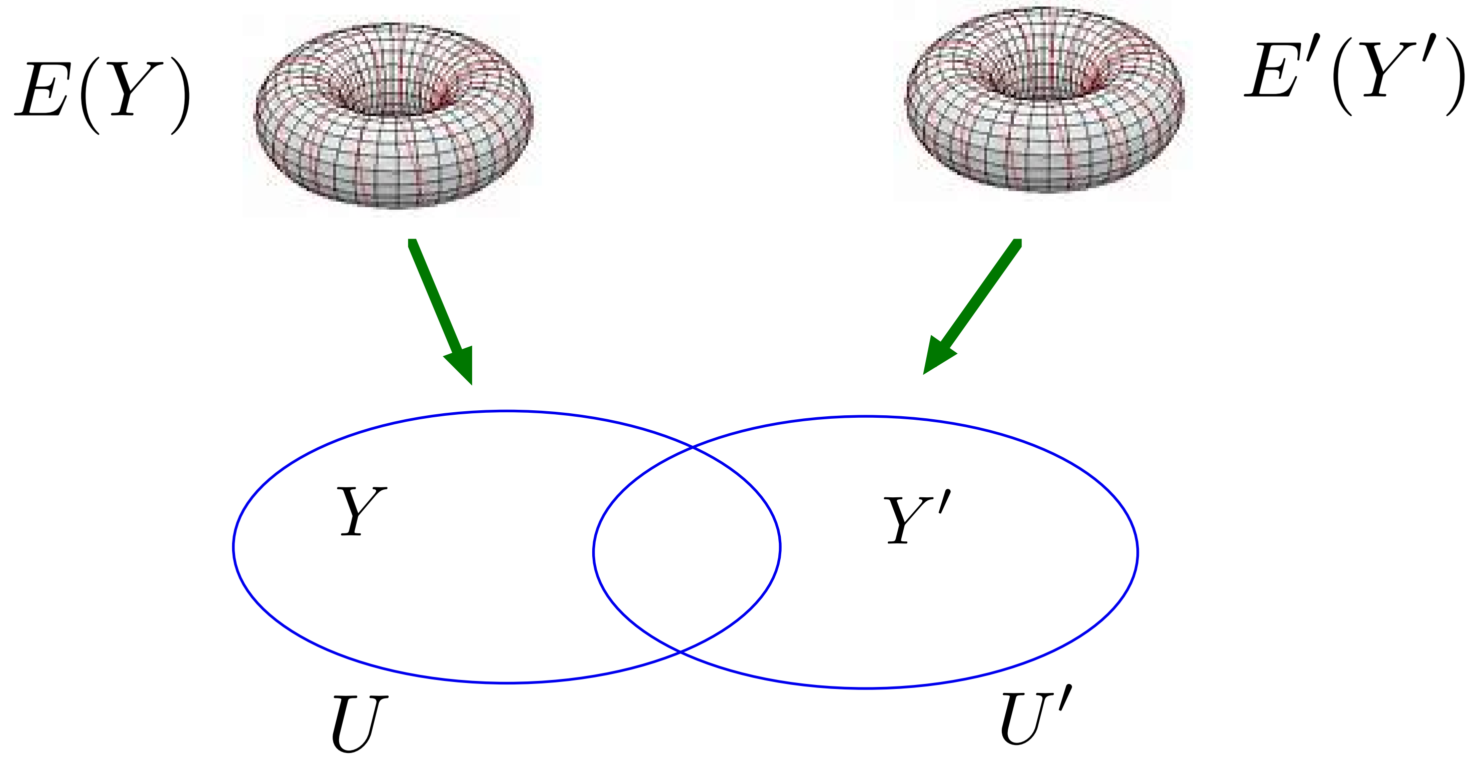


Glue big circle (R) to small (1/R)

Glue momentum modes to winding modes

(or linear combination of momentum and winding)

Not conventional smooth geometry



Torus fibration

Geometric background: G, H=dB tensorial

T-fold: Transition functions involve T-dualities (as well as diffeomorphisms and 2-form gauge transformations)

E=G+B Non-tensorial

$O(d,d;\mathbb{Z})$ $\qquad E' = (aE+b)(cE+d)^{-1} \text{ in } U \cap U'$

Glue using T-dualities also → **T-fold**

Physics smooth, as T-duality a symmetry

Not conventional smooth geometry

Doubled Geometry for T-fold

Hull

T^d torus fibres have doubled coords $\mathbb{X}^I = \begin{pmatrix} X^i \\ \widetilde{X}_i \end{pmatrix}$ $I = 1, ..., 2d$

Transforms linearly under $O(d, d; \mathbb{Z})$

T-fold transition: mixes $X, \tilde{X}$

No global way of separating "real" space coordinate X from "auxiliary" $\widetilde{X}$

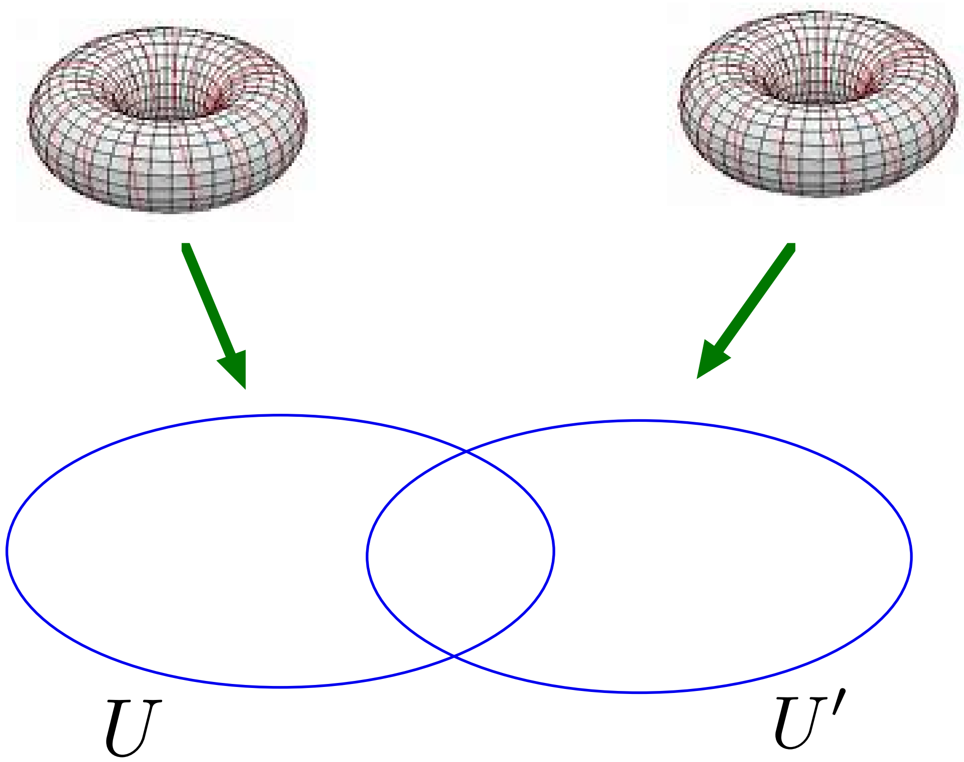
Duality covariant formulation in terms of $\mathbb{X}$

Transition functions $O(d, d; \mathbb{Z}) \subset GL(2d; \mathbb{Z})$ can be used to construct bundle with fibres T^{2d}

Doubled space is smooth manifold!

Sigma Model on doubled space. T-duality manifest.

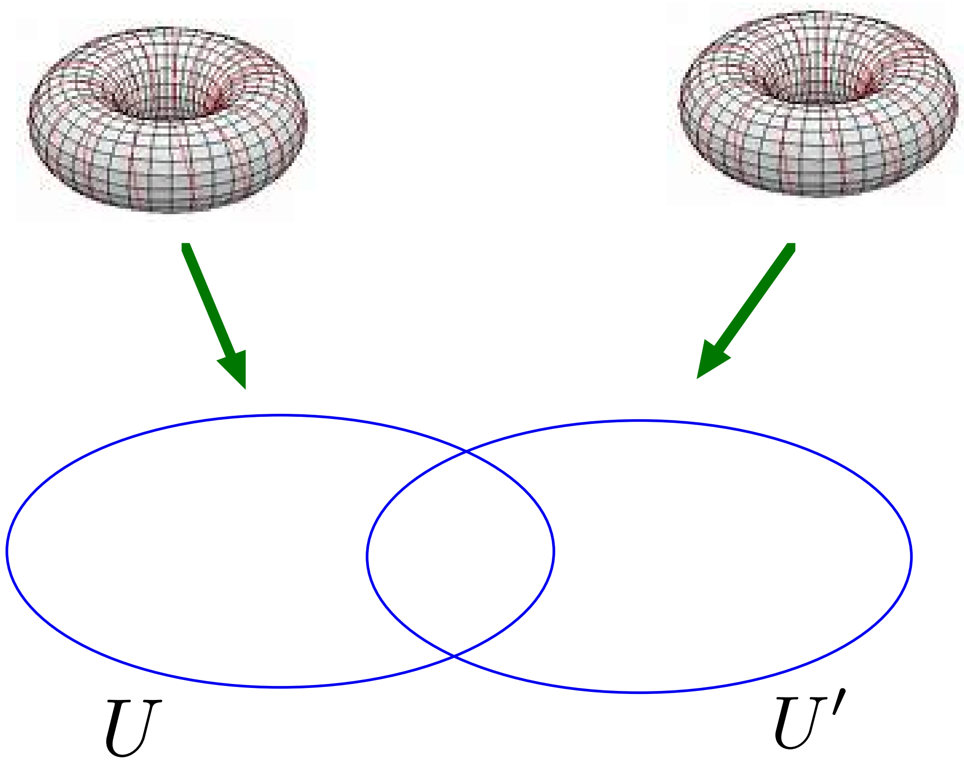
Transition Functions and Non-Geometry



U, U' patches in $\mathbb{R}^{2n}$
Fibres T^{2d}

Transition functions:
DFT gauge transformations and
$O(d,d;\mathbb{Z})$

Transition Functions and Non-Geometry

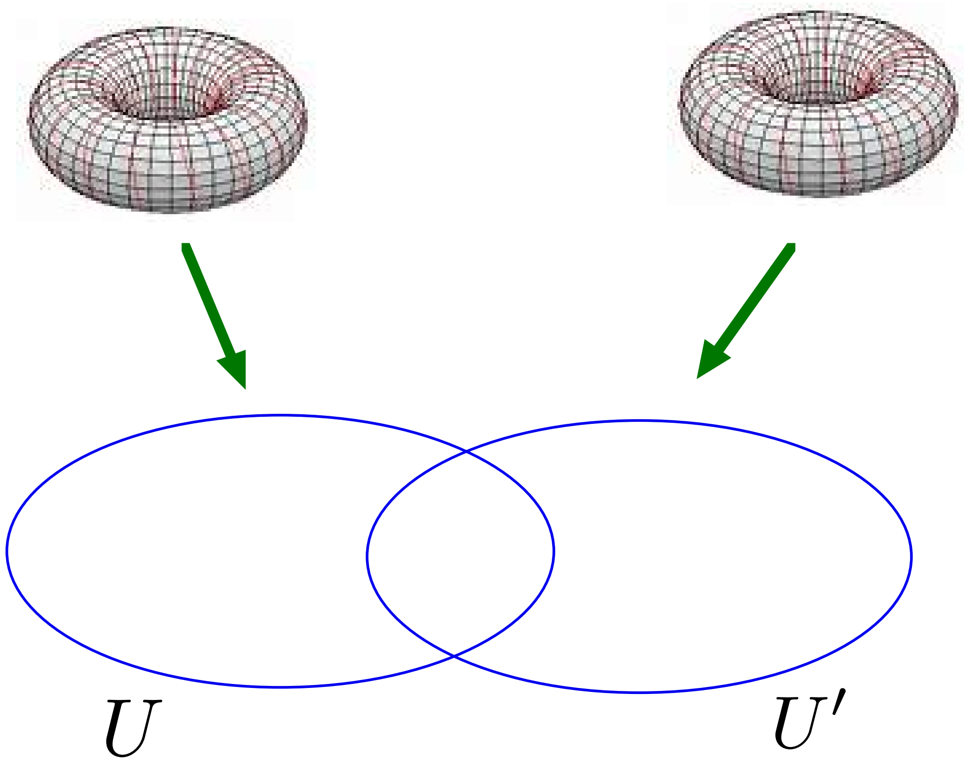


U, U' patches in $\mathbb{R}^{2n}$
Fibres T^{2d}

Transition functions:
DFT gauge transformations and $O(d,d;\mathbb{Z})$

If transition functions include T-duality, then can construct T-folds. As $O(d,d;\mathbb{Z}) \subset GL(2d;\mathbb{Z})$ coordinate transition functions are a diffeomorphism on doubled space, so doubled space is a manifold

Transition Functions and Non-Geometry



U, U' patches in $\mathbb{R}^{2n}$

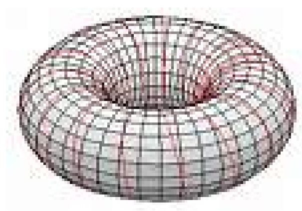
Fibres T^{2d}

Transition functions:
DFT gauge transformations and
$O(d, d; \mathbb{Z})$

Can construct explicit doubled geometries of
Dabholkar & Hull; Hull & Reid-Edwards
in this way, including those with 'R-flux'

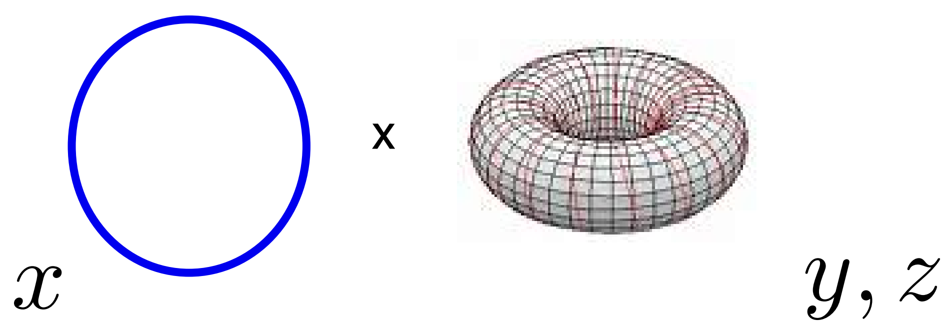
Example: T^3 with H-flux

$$H = N \times (Vol)$$



$$H_{xyz} = N$$

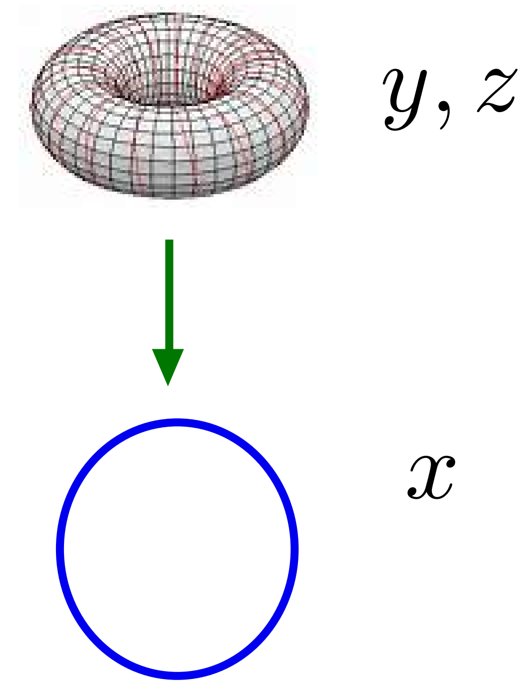
Regard as product $S^1 \times T^2$



$$B_{yz} = B_0 + \frac{1}{2\pi} N x$$

T-dual on z-circle:

Torus bundle over circle, H=0



$$\tau(x) = \tau_0 + \frac{1}{2\pi} N x$$

Nilfold: Heisenberg group manifold identified under discrete subgroup

Next, T-dual on y-circle

No global Killing vector. Do fibrewise duality, use Buscher rules locally, using local gauging

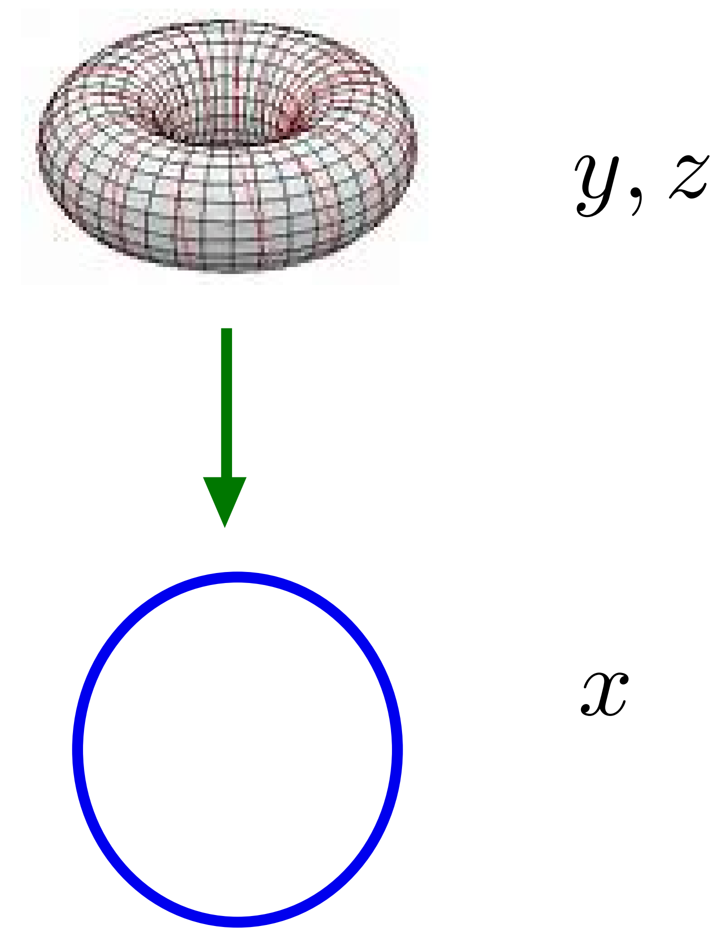
CMH

T-dual of T^3 with flux:

Torus bundle over circle?

$$ds^2 = dx^2 + \frac{1}{1+N^2x^2}\left(dy^2 + dz^2\right)$$

$$B_{yz} = \frac{Nx}{1+N^2x^2}$$



But x periodic

$$E(x+2\pi) = (aE+b)(cE+d)^{-1}$$

Monodromy $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in O(2,2;Z)$ T-duality

T-fold. Realise as doubled torus T^4 bundle over S^1.

CMH

- Doubled geometry: non-compact group identified under discrete subgroup

CMH & Reid-Edwards

- Gives T-fold and its T-duals

- Transition functions are diffeomorphisms of base space + T-duality, so allowed in DFT

- Not solution. Solution obtained by adding dependence on a 4th dimension.

- Gives explicit DFT solution with patching by DFT symmetries.

- More general non-geometric backgrounds. Gives uplift of GENERIC gauged Sugras

Dabholkar & Hull 2005 Shelton, Taylor & Wecht 2005

- Explicit doubled geometries constructed for T-folds and "spaces with R-flux"

Hull & Reid-Edwards 2008-9

- Sigma models on doubled spaces; constraints from gauging. Quantisation.

Hull 2004-6

- Other approaches to quantisation

Tseytlin; Berman, Thompson, Copland; Hackett-Jones & Motsopoulos
Lust et al; Bakas & Lust,....

Moduli stabilisation. Richer landscape

String Field Theory on Minkowski Space

Closed SFT: Zwiebach

String field $\Phi[X(\sigma), c(\sigma)]$

$$X^i(\sigma) \to x^i, \text{ oscillators}$$

Expand to get infinite set of fields

$$g_{ij}(x), b_{ij}(x), \phi(x), \ldots, C_{ijk\ldots l}(x), \ldots$$

Integrating out massive fields gives field theory for

$$g_{ij}(x), b_{ij}(x), \phi(x)$$

Double Field Theory

- String field theory gives complete formulation of perturbative closed string in CFT background Zwiebach
- Iterative construction of infinite number of interactions
- Non polynomial. Homotopy Lie algebra (violation of Jacobi's associativity etc)
- String field theory for torus gives infinite set of fields depending on doubled coordinates

String Field Theory on a torus

String field $\qquad \Phi[X(\sigma), c(\sigma)]$

$$X^i(\sigma) \to x^i, \tilde{x}_i, \text{ oscillators}$$

Expand to get infinite set of double fields

$$g_{ij}(x,\tilde{x}), b_{ij}(x,\tilde{x}), \phi(x,\tilde{x}), \ldots, C_{ijk\ldots l}(x,\tilde{x}), \ldots$$

Seek **double field theory** for

$$g_{ij}(x,\tilde{x}), b_{ij}(x,\tilde{x}), \phi(x,\tilde{x})$$

Free Field Equations (B=0)

$$L_0 + \bar{L}_0 = 2$$

$$p^2 + w^2 = N + \bar{N} - 2$$

$$L_0 - \bar{L}_0 = 0$$

$$p_i w^i = N - \bar{N}$$

Free Field Equations (B=0)

$$L_0 + \bar{L}_0 = 2$$

$$p^2 + w^2 = N + \bar{N} - 2$$

Treat as field equation, kinetic operator in doubled space

$$G^{ij}\frac{\partial^2}{\partial x^i \partial x^j} + G_{ij}\frac{\partial^2}{\partial \tilde{x}_i \partial \tilde{x}_j}$$

$$L_0 - \bar{L}_0 = 0$$

$$p_i w^i = N - \bar{N}$$

Treat as constraint on double fields

$$\Delta \equiv \frac{\partial^2}{\partial x^i \partial \tilde{x}_i} \qquad (\Delta - \mu)\psi = 0$$

Free Field Equations (B=0)

$$L_0 + \bar{L}_0 = 2$$

$$p^2 + w^2 = N + \bar{N} - 2$$

Treat as field equation, kinetic operator in doubled space

$$G^{ij} \frac{\partial^2}{\partial x^i \partial x^j} + G_{ij} \frac{\partial^2}{\partial \tilde{x}_i \partial \tilde{x}_j}$$

Laplacian for metric

$$ds^2 = G_{ij} dx^i dx^j + G^{ij} d\tilde{x}_i d\tilde{x}_j$$

$$L_0 - \bar{L}_0 = 0$$

$$p_i w^i = N - \bar{N}$$

Treat as constraint on double fields

$$\Delta \equiv \frac{\partial^2}{\partial x^i \partial \tilde{x}_i} \qquad (\Delta - \mu)\psi = 0$$

Laplacian for metric

$$ds^2 = dx^i d\tilde{x}_i$$

$$g_{ij}(x,\tilde{x}), b_{ij}(x,\tilde{x}), \phi(x,\tilde{x})$$

$$N = \bar{N} = 1$$

$$p^2 + w^2 = 0$$

$$p \cdot w = 0$$

"Double Massless"

DFT gives O(D,D) covariant formulation

O(D,D) Covariant Notation

$$X^M \equiv \begin{pmatrix} \tilde{x}_i \\ x^i \end{pmatrix} \qquad \partial_M \equiv \begin{pmatrix} \tilde{\partial}^i \\ \partial_i \end{pmatrix}$$

$$\eta_{MN} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \qquad M = 1, ..., 2D$$

$$\Delta \equiv \frac{\partial^2}{\partial x^i \partial \tilde{x}_i} = \frac{1}{2} \partial^M \partial_M$$

Constraint $\partial^M \partial_M A = 0$

on all fields and parameters

Weak Constraint or weak section condition

Arises from string theory constraint

$$(L_0 - \bar{L}_0)\Psi = 0$$

- Weakly constrained DFT non-local. Constructed to cubic order Hull & Zwiebach
- ALL doubled geometry dynamical, evolution in all doubled dimensions
- Restrict to simpler theory: STRONG CONSTRAINT
- Fields then depend on only half the doubled coordinates
- Locally, just conventional SUGRA written in duality symmetric form

Strong Constraint for DFT

Hohm, H &Z

$$\partial^M \partial_M (AB) = 0 \qquad\qquad (\partial^M A)\,(\partial_M B) = 0$$

on all fields and parameters

If impose this, then it implies weak form, but product of constrained fields satisfies constraint.

This gives **Restricted DFT,** a subtheory of DFT

Locally, it implies fields only depend on at most half of the coordinates, fields are restricted to null subspace N.
Looks like conventional field theory on subspace N

- If fields supported only on submanifold N of doubled space M, recover Siegel's duality covariant form of (super)gravity on N

- In general get this only locally. In each 2D-dim patch of doubled space, fields supported on D-dim sub-patch, but sub-patches don't fit together to form a manifold with smooth fields.

- DFT 'background independent' HHZ. Can write on doubling of any space. What is double if not derived from string theory?

- Extension to WZW models   Blumenhagen, Hassler & Lust

Generalised T-duality transformations: HHZ

$$X'^M \equiv \begin{pmatrix} \tilde{x}'_i \\ x'^i \end{pmatrix} = hX^M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \tilde{x}_i \\ x^i \end{pmatrix}$$

h in O(d,d;Z) acts on toroidal coordinates only

$$\mathcal{E}_{ij} = g_{ij} + b_{ij}$$

$$\mathcal{E}'(X') = (a\mathcal{E}(X) + b)(c\mathcal{E}(X) + d)^{-1}$$

$$d'(X') = d(X)$$

Buscher if fields independent of toroidal coordinates
Generalisation to case without isometries

$$X^M = \begin{pmatrix} \tilde{x}_m \\ x^m \end{pmatrix} \qquad \xi^M = \begin{pmatrix} \tilde{\epsilon}_m \\ \epsilon^m \end{pmatrix}$$

Linearised Gauge Transformations

$$\delta h_{ij} = \quad \partial_i \epsilon_j + \partial_j \epsilon_i \; + \; \tilde{\partial}_i \tilde{\epsilon}_j + \tilde{\partial}_j \tilde{\epsilon}_i \, ,$$

$$\delta b_{ij} = -(\tilde{\partial}_i \epsilon_j - \tilde{\partial}_j \epsilon_i) - (\partial_i \tilde{\epsilon}_j - \partial_j \tilde{\epsilon}_i) \, ,$$

$$\delta d \; = -\, \partial \cdot \epsilon + \, \tilde{\partial} \cdot \tilde{\epsilon} \, .$$

Invariance needs constraint

Diffeos and B-field transformations mixed.

If fields indep of $\tilde{x}_m$, conventional theory $g_{ij}(x), b_{ij}(x), d(x)$

ϵ^m parameter for diffeomorphisms

$\tilde{\epsilon}_m$ parameter for B-field gauge transformations

Generalised Metric Formulation

Hohm, H &Z

$$\mathcal{H}_{MN} = \begin{pmatrix} g^{ij} & -g^{ik}b_{kj} \\ b_{ik}g^{kj} & g_{ij} - b_{ik}g^{kl}b_{lj} \end{pmatrix}.$$

2 Metrics on double space $\mathcal{H}_{MN},\ \eta_{MN}$

$$\mathcal{H}^{MN} \equiv \eta^{MP}\mathcal{H}_{PQ}\eta^{QN}$$

Constrained metric $\mathcal{H}^{MP}\mathcal{H}_{PN} = \delta^M{}_N$

Generalised Metric Formulation

Hohm, H &Z

$$\mathcal{H}_{MN} = \begin{pmatrix} g^{ij} & -g^{ik}b_{kj} \\ b_{ik}g^{kj} & g_{ij} - b_{ik}g^{kl}b_{lj} \end{pmatrix} .$$

2 Metrics on double space $\mathcal{H}_{MN}, \ \eta_{MN}$

$$\mathcal{H}^{MN} \equiv \eta^{MP}\mathcal{H}_{PQ}\eta^{QN}$$

Constrained metric $\mathcal{H}^{MP}\mathcal{H}_{PN} = \delta^M{}_N$

Covariant O(D,D) Transformation

$$h^P{}_M h^Q{}_N \mathcal{H}'_{PQ}(X') = \mathcal{H}_{MN}(X)$$

$$X' = hX \qquad h \in O(D,D)$$

O(D,D) covariant action

$$S = \int dx d\tilde{x}\, e^{-2d}\ L$$

$$L = \frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\,\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_N\mathcal{H}^{KL}\,\partial_L\mathcal{H}_{MK} - 2\,\partial_M d\,\partial_N\mathcal{H}^{MN} + 4\mathcal{H}^{MN}\,\partial_M d\,\partial_N d$$

Gauge Transformation

$$\delta_\xi\mathcal{H}^{MN} = \xi^P\partial_P\mathcal{H}^{MN} + (\partial^M\xi_P - \partial_P\xi^M)\,\mathcal{H}^{PN} + (\partial^N\xi_P - \partial_P\xi^N)\,\mathcal{H}^{MP}$$

Write as "Generalised Lie Derivative"

$$\delta_\xi\mathcal{H}^{MN} = \widehat{\mathcal{L}}_\xi\mathcal{H}^{MN}$$

Generalised Lie Derivative

$$A_{N_1 \dots}^{M_1 \dots}$$

$$\widehat{\mathcal{L}}_\xi A_M{}^N \equiv \xi^P \partial_P A_M{}^N$$
$$+(\partial_M \xi^P - \partial^P \xi_M) A_P{}^N + (\partial^N \xi_P - \partial_P \xi^N) A_M{}^P$$

Usual Lie derivative, plus terms involving η_{MN}

$$\widehat{\mathcal{L}}_\xi A_M{}^N = \mathcal{L}_\xi A_M{}^N$$
$$- \eta^{PQ} \eta_{MR}\, \partial_Q \xi^R\, A_P{}^N$$
$$+ \eta_{PQ} \eta^{NR}\, \partial_R \xi^Q\, A_M{}^P$$

DFT geometry

arXiv:1406.7794

- Simple explicit form of finite gauge transformations. Associative and commuative.
- Doubled space is a manifold, not flat, despite constant 'metric' η in DFT.
- Gives geometric understanding of 'generalised tensors' & relation to generalised geometry
- Transition functions give global picture
- T-folds: non-geometric backgrounds included

Conclusions

- DFT: conventional sugra in duality symmetric formulation, using generalised geometry on N
- Covariant formulation of generalised geometry, indep. of choice of duality frame
- More generally, this applies locally in patches. Use DFT gauge and O(D,D) symmetries in transition functions. Get T-folds etc.

- DFT extends field theory to non-geometric spaces: T-folds, with T-duality transition functions.
- What is full theory with weak constraint?
- Winding modes: double torus or torus fibres
- How much of this is special to tori?
- Other topologies may not have windings, or have different numbers of momenta and windings. No T-duality? No doubling?