T-duality and α'-corrections

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Duality Symmetries in String and M-Theories

Based on DM and Nuñez, 1507.00652

What?

- Explore the T-duality structure of the first order α'-corrections in the string effective actions.
- Find a T-duality covariant symmetry principle that organizes the corrections.

Why?

Scattering amplitudes
$\beta-$functions

Duality?

How?

- Start with T-duality covariant formalism (DFT) in which the degrees of freedom are T-duality multiplets and the gauge transformations are duality covariant.
- Propose a $\mathcal{O}(\alpha')$ consistent duality covariant deformation of the gauge transformations.
- Write the duality and gauge invariant action.
- Compare with first-order actions obtained through other methods.

T-duality and the generalized Lie derivative

DFT is based on a duality covariant gauge principle

$$\left.\begin{array}{l}\delta g = L_\xi g \\ \delta B = L_\xi B + d\tilde{\xi} \\ \delta \phi = L_\xi \phi\end{array}\right\} \quad \rightarrow \quad \left\{\begin{array}{l}\delta \mathcal{H} \;=\; \widehat{\mathcal{L}}_\xi \mathcal{H} \\ \delta d \;=\; \xi \cdot d - \frac{1}{2}\,\partial \cdot \xi\end{array}\right.$$

that fixes the two-derivative universal action

$$S = \int dx \sqrt{-g}\, e^{-2\phi} \left(R + \alpha\, (\partial\phi)^2 + \beta\, H^2\right)$$

T-duality and the generalized Lie derivative

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that fixes the two-derivative universal action

$$S = \int dx \sqrt{-g} e^{-2\phi} \left(R + 4\, (\partial\phi)^2 - \frac{1}{12}\, H^2 \right)$$

Outline

Can the gauge principle be deformed to require and fix the four-derivative corrections?

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- Review of first-order α'-corrections in string effective actions (Metsaev and Teytlin and Bergshoeff and de Roo).
- The Green-Schwarz mechanism as a first-order gauge principle.
- Duality in dimensionally reduced theories and field redefinitions.
- Duality covariant Green-Schwarz mechanism: a gauge principle that requires and fixes the four-derivative string effective actions.

I. Review of first-order string effective actions.

First-order string effective actions

Metsaev and Tseytlin, 1987

$$S_{MT} = \int dx \sqrt{-g} e^{-2\phi} \left[R + 4(\partial\phi)^2 - \frac{1}{12} H^2 + \frac{a-b}{4} H^{\mu\nu\rho}\Omega_{\mu\nu\rho} \right.$$
$$\left. - \frac{a+b}{8} \left(\text{Riem}^2 - \frac{1}{2} H\, H\, \text{Riem} + \frac{1}{24} H^4 - \frac{1}{8} H^2_{\mu\nu} H^{2\mu\nu} \right) \right]$$

$$H_{\mu\nu\rho} = 3\partial_{[\mu} B_{\nu\rho]} \ , \qquad \Omega_{\mu\nu\rho} = \omega_{[\mu a}{}^{b} \partial_\nu \omega_{\rho]b}{}^{a} + \frac{2}{3} \omega_{[\mu a}{}^{b} \omega_{\nu b}{}^{c} \omega_{\rho]c}{}^{a}$$

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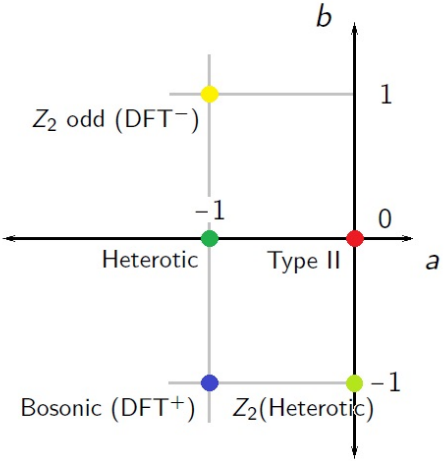
$$a + b = \begin{cases} -2\alpha' \\ -\alpha' \\ 0 \end{cases} , \qquad a - b = \begin{cases} 0 & \text{bosonic} \\ -\alpha' & \text{heterotic} \\ 0 & \text{type II} \end{cases}$$

First-order string effective actions



First-order string effective actions

Bergshoeff and de Roo, 1989, for heterotic $(a\,,\,b) = (-\alpha'\,,\,0)$

$$S_{BdR} = \int dx\sqrt{-g}e^{-2\phi}\left(R + 4(\partial\phi)^2 - \frac{1}{12}\widetilde{H}^{\mu\nu\rho}\widetilde{H}_{\mu\nu\rho}\right.$$
$$\left. + \frac{a}{8}R^{(-)}_{\mu\nu a}{}^{b}R^{(-)\mu\nu}{}_{b}{}^{a} + \frac{b}{8}R^{(+)}_{\mu\nu a}{}^{b}R^{(+)\mu\nu}{}_{b}{}^{a}\right)$$

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$$\widetilde{H}_{\mu\nu\rho} = H_{\mu\nu\rho} - \frac{3}{2}\,a\,\Omega^{(-)}_{\mu\nu\rho} + \frac{3}{2}\,b\,\Omega^{(+)}_{\mu\nu\rho}$$

$$\Omega^{(\pm)}_{\mu\nu\rho} = \omega^{(\pm)}_{[\mu a}{}^{b}\partial_\nu\omega^{(\pm)}_{\rho]b}{}^{a} + \frac{2}{3}\omega^{(\pm)}_{[\mu a}{}^{b}\omega^{(\pm)}_{\nu b}{}^{c}\omega^{(\pm)}_{\rho]c}{}^{a}\,, \quad \omega^{(\pm)}_{\mu a}{}^{b} = \omega_{\mu a}{}^{b} \pm \frac{1}{2}H_{\mu a}{}^{b}$$

First-order string effective actions

Chemissany, de Roo and Panda, 2007, for heterotic
$(a\,,\ b) = (-\alpha'\,,\, 0)$

$$S_{MT} = S_{BdR}$$

modulo field redefinitions. Also true in general.

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Z_2-parity transformations

$$Z_2(B) = -B\ , \qquad a \leftrightarrow b$$

$$a - b = \begin{cases} 0 & \text{bosonic} \\ -\alpha' & \text{heterotic} \end{cases}$$

The Green-Schwarz mechanism as a gauge principle

The BdR action is invariant and

$$\widetilde{H}_{\mu\nu\rho} = 3\partial_{[\mu} B_{\nu\rho]} - \frac{3}{2}\, a\, \Omega^{(-)}_{\mu\nu\rho} + \frac{3}{2}\, b\, \Omega^{(+)}_{\mu\nu\rho}$$

is covariant due to the Green-Schwarz transformation of B

$$\begin{aligned}
\delta\phi &= L_\xi \phi \\
\delta e_\mu{}^a &= L_\xi e_\mu{}^a + e_\mu{}^b \Lambda_b{}^a \\
\delta\Omega^{(\pm)}_{\mu\nu\rho} &= L_\xi \Omega^{(\pm)}_{\mu\nu\rho} - \partial_{[\mu}\left(\partial_\nu \Lambda_a{}^b \omega^{(\pm)a}_{\rho]b}\right) \\
\delta B_{\mu\nu} &= L_\xi B_{\mu\nu} + 2\partial_{[\mu}\tilde{\xi}_{\nu]} - \frac{a}{2}\partial_{[\mu}\Lambda_a{}^b \omega^{(-)a}_{\nu]b} + \frac{b}{2}\partial_{[\mu}\Lambda_a{}^b \omega^{(+)a}_{\nu]b}
\end{aligned}$$

The Green-Schwarz mechanism as a gauge principle

Green-Schwarz transformation of the two-form

$$\delta_\Lambda B_{\mu\nu} = \frac{a-b}{2}\omega_{[\mu a}{}^b\partial_{\nu]}\Lambda_b{}^a - \frac{a+b}{4}H_{[\mu a}{}^b\partial_{\nu]}\Lambda_b{}^a$$

- $a-b$ term non-removable unless $a=b$.
- $a+b$ term removable through

$$B_{\mu\nu} \to B_{\mu\nu} + \frac{a+b}{4}H_{[\mu a}{}^b\omega_{\nu]b}{}^a$$

II. Duality and field redefinitions.

Duality in lower dimensions

Meissner, 1996, compactified the MT action ($a = b = -\alpha'$) to one-dimension

$$S = \int dt e^{-2d}\left[-4\dot{d}^2 - \frac{1}{8}Tr(\dot{\mathcal{H}}\eta)^2 + \frac{\alpha'}{4}\left(\frac{1}{16}Tr(\dot{\mathcal{H}}\eta)^4 - \frac{1}{64}(Tr(\dot{\mathcal{H}}\eta)^2)^2 - Tr(\dot{\mathcal{H}}\eta)^2\dot{d}^2 - \frac{16}{3}\dot{d}^4\right)\right]$$

$$\mathcal{H} = \begin{pmatrix} \bar{g}^{-1} & -\bar{g}^{-1}\bar{B} \\ \bar{B}\bar{g}^{-1} & \bar{g} - \bar{B}\bar{g}^{-1}\bar{B} \end{pmatrix} \rightarrow \begin{cases} \bar{g} = g + \frac{\alpha'}{4}\dot{g}g^{-1}\dot{g} - \frac{\alpha'}{4}\dot{B}g^{-1}\dot{B} \\ \bar{B} = B + \frac{\alpha'}{4}\dot{g}g^{-1}\dot{B} + \frac{\alpha'}{4}\dot{B}g^{-1}\dot{g} \end{cases}$$

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Similar results by Hohm and Zwiebach, 2011 and Godazgar and Godazgar, 2013.

Duality versus diff/Lorentz covariance

Hull and Townsed, 1986 and Bergshoeff, Janssen and Ortin, 1996, heterotic ($a = -\alpha'$, $b = 0$)

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + \frac{\alpha'}{4}\omega_{\mu a}^{(-)\,b}\omega_{\nu b}^{(-)\,a}$$

T-duality (Buscher) acts as usual $\bar{g}'_{00} = \frac{1}{\bar{g}_{00}}$, . . .

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T-duality (Buscher) acts as usual $\bar{g}'_{00} = \frac{1}{\bar{g}_{00}}$, . . .

- $\bar{g}$ is duality covariant but receives a non-standard Lorentz transformation.
- g is Lorentz invariant but receives a non-standard T-duality transformation.

III. Recapitulation.

Recap

- Two-parameter family of theories (a, b)
  - Z_2-parity exchanges $a \leftrightarrow b$
  - Z_2-even is closed bosonic string ($a = b$)
  - Z_2-odd is related to HSZ theory ($a = -b$)
  - Heterotic breaks Z_2-parity $a = -\alpha'$ and $b = 0$
- Green-Schwarz transformation of $\bar{B}$

$$\delta_\Lambda \bar{B} = -\frac{a}{2}\partial_{[\mu}\Lambda_a{}^b\omega^{(-)}_{\nu]b}{}^a + \frac{b}{2}\partial_{[\mu}\Lambda_a{}^b\omega^{(+)}_{\nu]b}{}^a$$

- Indications of Green-Schwarz-like transformation of $\bar{g}$

$$\delta_\Lambda \bar{g} = -\frac{a}{2}\partial_{(\mu}\Lambda_a{}^b\omega^{(-)}_{\nu)b}{}^a - \frac{b}{2}\partial_{(\mu}\Lambda_a{}^b\omega^{(+)}_{\nu)b}{}^a$$

IV. Double Field Theory and the Generalized Green-Schwarz transformation.

Generalized frame formulation of DFT

Siegel, 1993 and Hohm and Kwak, 2010. Symmetries:

- Global $G = O(d,d)$

$$h_M{}^P \eta_{PQ} h_N{}^Q = \eta_{MN} \qquad h \in G$$

- Double-Lorentz local $H = O(1, d-1) \times O(d-1, 1)$ generated infinitesimally by $\Lambda^B{}_A$

$$\begin{aligned} \delta_\Lambda \eta_{AB} &= \eta_{CB} \Lambda^C{}_A + \eta_{AC} \Lambda^C{}_B = 0 \\ \delta_\Lambda \mathcal{H}_{AB} &= \mathcal{H}_{CB} \Lambda^C{}_A + \mathcal{H}_{AC} \Lambda^C{}_B = 0 \end{aligned}$$

- Generalized diffeomorphisms $\widehat{\mathcal{L}}$ generated infinitesimally by ξ^M

Generalized frame formulation of DFT

Two type of indices:

- $M, N, \ldots$ global G
- $A, B, \ldots$ local H

raised and lowered with η_{MN} and η_{AB} respectively.

Generalized frame ${E_M}^A \in G/H$ and dilaton d

$$\eta_{MN} = {E_M}^A \eta_{AB} {E_N}^B \ , \qquad \mathcal{H}_{MN} = {E_M}^A \mathcal{H}_{AB} {E_N}^B$$

Constraints

$$\mathcal{H}\, \eta^{-1} \mathcal{H} = \eta$$

$$\partial_M \partial^M \cdots = 0 \ , \qquad \partial_M \ldots \ \partial^M \cdots = 0$$

Generalized frame formulation of DFT

Generalized fluxes

$$\mathcal{F}_{ABC} = 3E_{M[A}\partial^M E^N{}_B E^P{}_{C]}\eta_{NP}$$

Projectors

$$P = \frac{1}{2}(\eta - \mathcal{H}) \ , \qquad \bar{P} = \frac{1}{2}(\eta + \mathcal{H})$$

Projected generalized fluxes

$$\begin{aligned} \mathcal{F}^{(-)}_{MAB} &= \mathcal{F}_{\overline{M}\underline{AB}} = \bar{P}_M{}^N E_N{}^C \mathcal{F}_{CDE} P_A{}^D P_B{}^E \\ \mathcal{F}^{(+)}_{MAB} &= \mathcal{F}_{\underline{M}\overline{AB}} = P_M{}^N E_N{}^C \mathcal{F}_{CDE} \bar{P}_A{}^D \bar{P}_B{}^E \end{aligned}$$

Generalized Green-Schwarz transformation

Duality covariant gauge transformations

$$\begin{aligned} \delta d &= \xi^P \partial_P d - \frac{1}{2}\partial_P \xi^P \\ \delta E_M{}^A &= \widehat{\mathcal{L}}_\xi E_M{}^A + \delta_\Lambda E_M{}^A + \widetilde{\delta}_\Lambda E_M{}^A \end{aligned}$$

include a two-parameter generalized Green-Schwarz transformation, DM and Nunez, 2015

$$\begin{aligned} \delta_\Lambda E_M{}^A &= E_M{}^B \Lambda_B{}^A \\ \widetilde{\delta}_\Lambda E_M{}^A &= \left(a\, \partial_{[\underline{M}} \Lambda_C{}^B \, \mathcal{F}^{(-)\ C}_{\overline{N}]B} - b\, \partial_{[\overline{M}} \Lambda_C{}^B \, \mathcal{F}^{(+)\ C}_{\underline{N}]B} \right) E^{NA} \end{aligned}$$

where a and b of $\mathcal{O}(\alpha')$ are the same as before.

Generalized Green-Schwarz transformation

For the generalized metric this implies

$$\delta \mathcal{H}_{MN} = \widehat{\mathcal{L}}_\xi \mathcal{H}_{MN} + \widetilde{\delta}_\Lambda \mathcal{H}_{MN}$$

$$\widetilde{\delta}_\Lambda \mathcal{H}_{MN} = 2a\, \partial_{(\underline{M}} \Lambda_A{}^B\, \mathcal{F}^{(-)}_{\overline{N})B}{}^A + 2b\, \partial_{(\overline{M}} \Lambda_A{}^B\, \mathcal{F}^{(+)}_{\underline{N})B}{}^A$$

and the projected fluxes transform as generalized connections to lowest order

$$\begin{aligned} \delta \mathcal{F}^{(-)}_{MA}{}^B &= \widehat{\mathcal{L}}_\xi \mathcal{F}^{(-)}_{MA}{}^B - \partial_{\overline{M}} \Lambda_{\underline{A}}{}^{\underline{B}} + \mathcal{F}^{(-)}_{MA}{}^C \Lambda_C{}^B - \Lambda_A{}^C \mathcal{F}^{(-)}_{MC}{}^B \\ \delta \mathcal{F}^{(+)}_{MA}{}^B &= \widehat{\mathcal{L}}_\xi \mathcal{F}^{(+)}_{MA}{}^B - \partial_{\underline{M}} \Lambda_{\overline{A}}{}^{\overline{B}} + \mathcal{F}^{(+)}_{MA}{}^C \Lambda_C{}^B - \Lambda_A{}^C \mathcal{F}^{(+)}_{MC}{}^B \end{aligned}$$

Closure

The gauge transformations preserve the constraints and close

$$\left[\delta_{(\xi_1\,,\Lambda_1)},\delta_{(\xi_2\,,\Lambda_2)}\right]=\delta_{(\xi_{21}\,,\Lambda_{21})}$$

w.r.t the brackets

$$\begin{aligned}\xi_{12}^M &= [\xi_1\,,\,\xi_2]^M_{(C)}+\frac{1}{2}\left(\gamma^{(+)}\mathcal{H}^{AB}-\gamma^{(-)}\eta^{AB}\right)\eta^{CD}\Lambda_{[1AC}\partial^M\Lambda_{2]BD}\\ \Lambda_{12\,A}{}^B &= 2\xi^P_{[1}\partial_P\Lambda_{2]\,A}{}^B-2\Lambda_{[1\,A}{}^C\Lambda_{2]\,C}{}^B\end{aligned}$$

where

$$\gamma^{(\pm)}=-\frac{a\pm b}{2}$$

Gauge invariant action

The zeroth order DFT action is

$$S = \int dX e^{-2d} \mathcal{R}$$

where (coefficients fixed by $\widehat{\mathcal{L}}$)

$$\begin{aligned} \mathcal{R} \;=\; & 4\mathcal{H}^{MN}\partial_{MN}d - \partial_{MN}\mathcal{H}^{MN} - 4\mathcal{H}^{MN}\partial_M d\,\partial_N d + 4\partial_M\mathcal{H}^{MN}\partial_N d \\ & + \frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_K\mathcal{H}_{NL} \end{aligned}$$

Gauge invariance under generalized GS transformation requires corrections

$$\delta_\xi \mathcal{R} = \widehat{\mathcal{L}}_\xi \mathcal{R} \quad \text{but} \quad \widetilde{\delta}_\Lambda \mathcal{R} \neq 0$$

Gauge invariant action

The first-order gauge invariant action is fixed

$$S = \int dX e^{-2d} \left(\mathcal{R} + a\,\mathcal{R}^{(-)} + b\,\mathcal{R}^{(+)} \right)$$

where

$$\begin{aligned} \mathcal{R}^{(-)} &= -4\,\mathcal{F}^{(-)}_{MAB}\,\mathcal{F}^{(-)BA}_{N}\,\partial^{MN} d \\ &\quad + \frac{1}{2}\,\mathcal{H}^{MN}\,\partial_M \mathcal{F}^{(-)}_{RAB}\,\partial_N \mathcal{F}^{(-)RBA} + \dots \end{aligned}$$

and similar for $\mathcal{R}^{(+)}[\mathcal{F}^{(+)}]$, see 1507.00652 for more details. Now

$$\delta \left(\mathcal{R} + a\mathcal{R}^{(-)} + b\mathcal{R}^{(+)} \right) = \widehat{\mathcal{L}}_\xi \left(\mathcal{R} + a\mathcal{R}^{(-)} + b\mathcal{R}^{(+)} \right)$$

Z_2-parity

Z_2-parity acts as

$$Z_2\left(\eta_{\bullet\bullet}\right) = -\eta_{\bullet\bullet}\ , \quad Z_2\left(P_\bullet{}^\bullet\right) = \bar{P}_\bullet{}^\bullet\ , \quad Z_2\left(\bar{P}_\bullet{}^\bullet\right) = P_\bullet{}^\bullet$$

and leaves the following quantities invariant

$$\partial_\bullet\ , \quad \mathcal{E}_\bullet{}^\bullet\ , \quad \mathcal{H}_{\bullet\bullet}\ , \quad \mathcal{F}_{\bullet\bullet}{}^\bullet\ , \quad \xi^\bullet\ , \quad \Lambda_\bullet{}^\bullet$$

Then one finds

$$Z_2\left(\mathcal{F}^{(\pm)}_{\bullet\bullet}{}^\bullet\right) = \mathcal{F}^{(\mp)}_{\bullet\bullet}{}^\bullet\ , \qquad Z_2\left(\mathcal{R}^{(\pm)}\right) = \mathcal{R}^{(\mp)}$$

so effectively

$$a \leftrightarrow b$$

Parameterization and the strong constraint

G-invariant metric

$$\eta_{MN} = \begin{pmatrix} 0 & \delta^{\mu}_{\nu} \\ \delta_{\mu}^{\nu} & 0 \end{pmatrix}$$

H-invariant metrics

$$\mathcal{H}_{AB} = \begin{pmatrix} g^{ab} & 0 \\ 0 & g_{ab} \end{pmatrix} \ , \qquad \eta_{AB} = \begin{pmatrix} g^{ab} & 0 \\ 0 & -g_{ab} \end{pmatrix}$$

Good bye dual coordinates

$$\partial_M = \left(\tilde{\partial}^{\mu} \, , \, \partial_{\mu}\right) = (0 \, , \, \partial_{\mu})$$

Parameterization and the strong constraint

Double Lorentz parameter

$$\Lambda_A{}^B = \begin{pmatrix} \Lambda^{(+)a}{}_b & 0 \\ 0 & \Lambda^{(-)}{}_a{}^b \end{pmatrix}$$

Parameterization of generalized frame

$$E_M{}^A = \frac{1}{\sqrt{2}} \begin{pmatrix} \bar{e}_a^{(+)\mu} & -g^{ab}\bar{e}_b^{(-)\mu} \\ \bar{e}_\mu^{(+)b} g_{ba} - \bar{e}_a^{(+)\rho} \bar{B}_{\rho\mu} & \bar{e}_\mu^{(-)a} + g^{ab}\bar{e}_b^{(-)\rho} \bar{B}_{\rho\mu} \end{pmatrix}$$

Gauge fixing

$$\Lambda^{(+)} = \Lambda^{(-)} = \Lambda \ , \quad \bar{e}^{(+)} = \bar{e}^{(-)} = \bar{e} \ , \quad \bar{g} = \bar{e}^a g_{ab} \bar{e}^b$$

Parameterization and the strong constraint

Other parameterizations

$$
\begin{aligned}
e^{-2d} &= \sqrt{-\bar{g}}\,e^{-2\bar{\phi}} = \sqrt{-g}\,e^{-2\phi} \\
\mathcal{H}_{MN} &= \begin{pmatrix} \bar{g}^{\mu\nu} & -\bar{g}^{\mu\rho}\bar{B}_{\rho\nu} \\ \bar{B}_{\mu\rho}\bar{g}^{\rho\nu} & \bar{g}_{\mu\nu} - \bar{B}_{\mu\rho}\bar{g}^{\rho\sigma}\bar{B}_{\sigma\nu} \end{pmatrix} \\
\mathcal{F}^{(-)}_{Mab} &= \frac{1}{2}\begin{pmatrix} g^{\mu\nu}\omega^{(-)}_{\nu ab} \\ B_{\mu\nu}g^{\nu\rho}\omega^{(-)}_{\rho ab} + \omega^{(-)}_{\mu ab} \end{pmatrix} \\
\mathcal{F}^{(+)ab}_{M} &= \frac{1}{2}\begin{pmatrix} g^{\mu\nu}\omega^{(+)ab}_{\nu} \\ B_{\mu\nu}g^{\nu\rho}\omega^{(+)ab}_{\rho} - \omega^{(+)ab}_{\mu} \end{pmatrix}
\end{aligned}
$$

Deformed gauge transformations of components

The duality covariant Green-Schwarz transformation

$$\delta \mathcal{H}_{MN} = \widehat{\mathcal{L}}_\xi \mathcal{H}_{MN} + \widetilde{\delta}_\Lambda \mathcal{H}_{MN}$$

$$\widetilde{\delta}_\Lambda \mathcal{H}_{MN} = 2a\, \partial_{(\underline{M}} \Lambda_A{}^B\, \mathcal{F}^{(-)}_{\overline{N})B}{}^A + 2b\, \partial_{(\overline{M}} \Lambda_A{}^B\, \mathcal{F}^{(+)}_{\underline{N})B}{}^A$$

induces a Lorentz transformation of the components

$$\begin{aligned} \delta \bar{g}_{\mu\nu} &= L_\xi \bar{g}_{\mu\nu} - \frac{a}{2} \omega^{(-)}_{(\mu a}{}^b \partial_{\nu)} \Lambda_b{}^a - \frac{b}{2} \omega^{(+)}_{(\mu a}{}^b \partial_{\nu)} \Lambda_b{}^a \\ \delta \bar{B}_{\mu\nu} &= L_\xi \bar{B}_{\mu\nu} + 2\partial_{[\mu} \tilde{\xi}_{\nu]} + \frac{a}{2} \omega^{(-)}_{[\mu a}{}^b \partial_{\nu]} \Lambda_b{}^a - \frac{b}{2} \omega^{(+)}_{[\mu a}{}^b \partial_{\nu]} \Lambda_b{}^a \end{aligned}$$

Induced non-covariant field redefinitions

The non-standard Lorentz transformation of $\bar{g}$ can be removed through a first-order Lorentz non-covariant field redefinition

$$\bar{g}_{\mu\nu} = g_{\mu\nu} - \frac{a}{4}\omega_{\mu a}^{(-)\,b}\omega_{\nu b}^{(-)\,a} - \frac{b}{4}\omega_{\mu a}^{(+)\,b}\omega_{\nu b}^{(+)\,a}$$

two-parameter generalization of Hull and Townsed, 1986 and Bergshoeff, Janssen and Ortin, 1996, heterotic ($a = -\alpha'$, $b = 0$).

Induced non-covariant field redefinitions

Generically for $\bar{B}$ this is not possible. Not surprising: Green-Schwarz mechanism in heterotic string.

But! for the bosonic string $a = b$ it is possible

$$\begin{aligned} \bar{g}_{\mu\nu} &= g_{\mu\nu} - \frac{a}{2}\omega_{\mu a}{}^{b}\omega_{\nu b}{}^{a} - \frac{a}{8}H_{\mu a}{}^{b}H_{\nu b}{}^{a} \\ \bar{B}_{\mu\nu} &= B_{\mu\nu} - \frac{a}{2}H_{[\mu a}{}^{b}\omega_{\nu]b}{}^{a} \end{aligned}$$

Similar to Meissner, 1996, remember...

$$\begin{aligned} \bar{g} &= g + \frac{\alpha'}{4}\dot{g}g^{-1}\dot{g} - \frac{\alpha'}{4}\dot{B}g^{-1}\dot{B} \\ \bar{B} &= B + \frac{\alpha'}{4}\dot{g}g^{-1}\dot{B} + \frac{\alpha'}{4}\dot{B}g^{-1}\dot{g} \end{aligned}$$

Back to the first-order string effective actions

Re-writing the first-order T-duality invariant action in terms of the redefined fields, one obtains precisely the two-parameter BdR action

$$\mathcal{R} + a\mathcal{R}^{(-)} + b\mathcal{R}^{(+)} = R + 4(\partial\phi)^2 - \frac{1}{12}\widetilde{H}^2 + \frac{a}{8} R^{(-)}_{\mu\nu a}{}^{b} R^{(-)\mu\nu}{}_{b}{}^{a} + \frac{b}{8} R^{(+)}_{\mu\nu a}{}^{b} R^{(+)\mu\nu}{}_{b}{}^{a}$$

where *all* the coefficients are fixed by the generalized Green-Schwarz transformation.

V. Other duality covariant deformations of the generalized Lie derivative.

Double α'-Geometry

Hohm, Siegel and Zwiebach, 2013, first duality covariant deformation of $\widehat{\mathcal{L}}$

$$\delta \mathcal{M}_{MN} = \widehat{\mathcal{L}}_\xi \mathcal{M}_{MN} - \alpha' \partial_M \mathcal{M}^{PQ} \partial_{P[Q}\xi_{N]} + \ldots$$

Double metric $\mathcal{M}_{MN}$ unconstrained, but duality covariant. Corrected bracket:

$$[\xi_1 \, , \, \xi_2]^M_{\alpha'} = [\xi_1 \, , \, \xi_2]^M_{(C)} + \alpha' \, \partial_P \xi^Q_{[1} \partial^M \partial_Q \xi^P_{2]}$$

Exact to all orders and odd under Z_2-parity $Z_2(B) = -B$.

Double α'-Geometry

Hohm and Zwiebach, 2014

Studying field perturbations around flat space, the α'-corrected transformation induces

$$\delta B_{\mu\nu} = L_\xi B_{\mu\nu} + 2\partial_{[\mu}\tilde{\xi}_{\nu]} + \frac{\alpha'}{2}\partial_{[\mu}(\partial_\rho \xi^\sigma)\,\Gamma^\rho_{\ \nu]\sigma}$$

so the correction induces a Green-Schwarz-like transformation of B w.r.t. diffeomorphisms.

The deformation of $\widehat{\mathcal{L}}$ is a generalized Green-Schwarz-like transformation!

Double Field Theory$^{\gamma}$

Hohm and Zwiebach, 2014

Field perturbations around flat space, two-parameter $\gamma^{(\pm)}$ deformation

$$\mathcal{M}_{MN} \to \mathcal{H}_{MN} = \bar{\mathcal{H}}_{MN} + m_{MN}$$

$$\delta m_{\underline{M}\overline{N}} = \ldots + \frac{\gamma^{(+)} + \gamma^{(-)}}{2} \partial_{\underline{M}} K^{\underline{PQ}} \Gamma_{\overline{N}\underline{QP}} - \frac{\gamma^{(+)} - \gamma^{(-)}}{2} \partial_{\overline{N}} K^{\overline{PQ}} \Gamma_{\underline{M}\overline{QP}}$$

$$K_{MN} = \partial_{[M}\xi_{N]}$$

$$\Gamma_{MNP} = \text{generalized connections}$$

Double Field Theory$^\gamma$

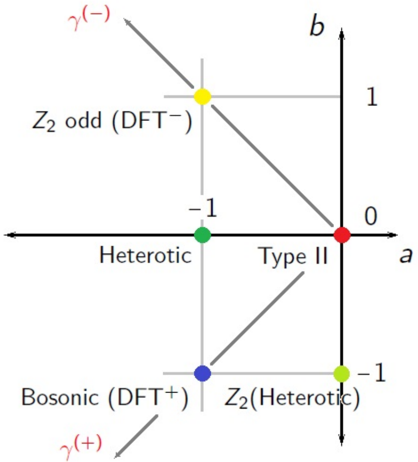
Hohm and Zwiebach, 2014
Brackets (field-dependent)

$$[\xi_1\,,\,\xi_2]^M_{\alpha'} = [\xi_1\,,\,\xi_2]^M_{(C)} + \frac{1}{2}\left(\gamma^{(+)}\bar{\mathcal{H}}^{KL} - \gamma^{(-)}\eta^{KL}\right)\eta^{PQ}K_{[1KP}\partial^M K_{2]LQ}$$

Two-paramter family of theories: action worked out to cubic order in field perturbations. Interesting cases:

- DFT^+ : $(\gamma^{(+)}\,,\,\gamma^{(-)}) = (1,0)$ CSFT (Bosonic)
- DFT^- : $(\gamma^{(+)}\,,\,\gamma^{(-)}) = (0,1)$ Double α'-Geom. (Z_2-odd)
- Heterotic : interpolates between both

Double Field Theory$^{\gamma}$



$$\gamma^{(\pm)} \leftrightarrow a \pm b$$

VI. Conclusions.

Conclusions

- We introduced a first-order duality covariant gauge principle that requires and fixes the four-derivative terms in the universal sector of the string effective actions.
- It consists of a two-parameter generalized Green-Schwarz transformation that leads to the Metsaev-Tseytlin and Bergshoeff-de Roo actions.
- It calls for first-order Lorentz non-covariant field redefinitions, that relate duality covariant with Lorentz covariant fields.
- Closely related to DFT$^{\gamma}$ by Hohm and Zwiebach, 2014.
- Many exciting results to come...!