New gaugings and non-geometry

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10th August 2015

Based on work with Kanghoon Lee and Daniel Waldram

arxiv: 1401.3360, 1506.03457

Background / Introduction

- Flux compactifications $\rightarrow$ gauged supergravity
- E.g. 11d supergravity on S^7 [de Wit & Nicolai '86]

  $\rightarrow SO(8)$ gauged $N = 8$ supergravity in 4d
- Reductions ansatz complicated!

  $\rightarrow$ is there a simpler way to understand them?
- Recently: $SO(8)$ theory not unique! [Dall'Agata, Inverso & Trigiante 2012]

  $\rightarrow \exists$ 1-parameter family labelled by $\omega \in [0, \pi/8)$
- Do these "new gaugings" have a higher dimensional origin?

Outline of talk

- Generalised geometry
- Consistent truncations $\simeq$ Generalised Scherk-Schwarz
- No-go result for new gaugings
- (Minimally) Extended (non)-geometry construction
- Conclusions & outlook

Generalised Geometry

- Tangent Bundle TM [$GL(d,\mathbb{R})$ structure]

  $\rightarrow$ Generalised Tangent Bundle $E \simeq TM \oplus \dots$

- Hitchin and Gualtieri : $E \simeq T \oplus T^*$ [$O(d,d)$ structure]

  $\lambda_{(1)} \in T^*$ is gauge trans of $B_{(2)}$ $\rightarrow$ Flux $H_{(3)} = \mathrm{d}B_{(2)}$

  $\rightarrow$ NS-NS sector of Type II

- Spheres : $E \simeq T \oplus \Lambda^{d-2} T^*$ [$GL(d+1,\mathbb{R})$ structure]

  $\lambda_{(d-2)} \in \Lambda^{d-2} T^*$ gauge for $A_{(d-1)}$ $\rightarrow$ Flux $F_{(d)} = \mathrm{d}A_{(d-1)}$

- Full 11D or Type II : $E_{d(d)} \times \mathbb{R}^+$ generalised geometry

$$E \simeq TM \oplus \Lambda^2 T^*M \oplus \Lambda^5 T^*M \oplus (T^*M \otimes \Lambda^7 T^*M)$$

[Hull '07, Pacheco & Waldram '08]

Generalised Geometry

- $V \in E$ locally $V = v + \lambda_{(\alpha)}$ on patches $U_{(\alpha)} \subset M$
- E twisted also by gauge transformations $\Lambda_{(\alpha\beta)}$

$$\lambda_{(\alpha)} = \lambda_{(\beta)} - i_v \mathrm{d}\Lambda_{(\alpha\beta)}$$

- Lie derivative by $v \in TM \sim$ "diffeo"

$\rightarrow$ Dorfman derivative by $V \in E \sim$ "diffeo" $+$ "gauge"

- Generalised metric $G \leftrightarrow$ SUGRA fields $(g_{mn}, A_{m_1 \dots m_{d-1}}, \Delta)$
- Components of G parameterise coset, e.g.

$$\frac{O(d,d)}{O(d) \times O(d)} \qquad \frac{GL(d+1,\mathbb{R})}{SO(d+1)} \qquad \frac{E_{d(d)} \times \mathbb{R}^+}{H_d}$$

Generalised Scherk-Schwarz reduction

- Want (M, E) with Leibniz Generalised Parallelisation
- This means $\exists$ global frame $\{\hat{E}_A\}$ for E s.t.

$$L_{\hat{E}_A}\hat{E}_B = X_{AB}{}^C\hat{E}_C \qquad\qquad X_{AB}{}^C \text{ constant}$$

- Structure group of E trivial $\Rightarrow$ Preserves all SUSY
- Background generalised metric $G^{-1} = \delta^{AB}(\hat{E}_A \otimes \hat{E}_B)$
- Moduli $U_A{}^B(x)$ provide new frame $\hat{E}'_A = U_A{}^B(x)\hat{E}_A$

$$\rightarrow G^{-1}(x) = \delta^{AB}U_A{}^{A'}(x)U_B{}^{B'}(x)(\hat{E}_{A'} \otimes \hat{E}_{B'})$$

- Provides non-linear scalar ansatz for reduction

Generalised Scherk-Schwarz and the Embedding Tensor

- $\{\hat{E}_A\}$ define a Leibniz algebra

$$L_{\hat{E}_A}\hat{E}_B = X_{AB}{}^C\hat{E}_C$$

- But $[X_A, X_B] = -X_{AB}{}^C X_C$ is a Lie algebra
- $X_{AB}{}^C$ is the embedding tensor of reduced theory!
- Expanding vectors as $A_\mu = A_\mu{}^A\hat{E}_A$ we see

$$\delta A_\mu = \partial_\mu\Lambda - L_{A_\mu}\Lambda = \left[\partial_\mu\Lambda^A - (\Lambda^C X_{BC}{}^A)A_\mu^B\right]\hat{E}_A$$

[c.f. Hohm & Samtleben]

- Analogue of unimodular condition $X_{CA}{}^C = 0$

Generalised Torsion Interpretation

- Can define generalised connections D and torsion T
- Find that torsion $T \in E^* \oplus K \subset E^* \otimes \mathrm{ad}(E_{d(d)} \times \mathbb{R}^+)$
- K the usual embedding tensor representation
- Find for Weitzenböck connection D s.t. $D\hat{E}_A = 0$

$$X_{AB}{}^{C} = -T_A{}^{C}{}_{B}$$

- $X_{AB}{}^{C}$ the intrinsic torsion of identity structure
- Constraint $X_{CA}{}^{C} = 0 \Rightarrow X \in K$

Spheres: General results

- $GL(d+1,\mathbb{R})$ generalised tangent space $E \simeq T \oplus \Lambda^{d-2} T^*$
  is Leibnitz generalised parallelisable on any sphere S^d
- $\{\hat{E}_{ij}\}$ = Globally defined basis for E
- $SO(d+1)$ algebra in Dorfman derivative

$$L_{\hat{E}_{ij}} \hat{E}_{kl} = R^{-1} \left(\delta_{ik} \hat{E}_{lj} - \delta_{il} \hat{E}_{kj} - \delta_{jk} \hat{E}_{li} + \delta_{jl} \hat{E}_{ki} \right)$$

Spheres: S^7 Example in 11D sugra

- Full S^7 reduction requires $E_{7(7)} \times \mathbb{R}^+$ geometry
- $\mathbf{56} \to \mathbf{28} + \mathbf{28}'$ under $SL(8,\mathbb{R})$ so two parts $\hat{E}_A \to (\hat{E}_{ij}, \hat{E}'^{ij})$
- Result: Frame $\hat{E}_A$ with Leibniz algebra

$$
\begin{aligned}
L_{\hat{E}_{ij}} \hat{E}_{kl} &= 2R^{-1}\big(\delta_{i[k}\hat{E}_{l]j} - \delta_{j[k}\hat{E}_{l]i}\big), \\
L_{\hat{E}_{ij}} \hat{E}'_{kl} &= 2R^{-1}\big(\delta_{i[k}\hat{E}'_{l]j} - \delta_{j[k}\hat{E}'_{l]i}\big), \\
L_{\hat{E}'_{ij}} \hat{E}_{kl} &= L_{\hat{E}'_{ij}} \hat{E}'_{kl} = 0
\end{aligned}
$$

- Can also write under $SU(8)$ decomposition $\hat{E}_A \to (\hat{E}_{\alpha\beta}, \bar{\hat{E}}^{\alpha\beta})$

Spheres: S^3 Example in NS-NS

- $SL(4,\mathbb{R}) \sim SO(3,3)$ so this is $E \sim T \oplus T^*$ geometry
- Here $\hat{E}_A \to (\hat{E}^R_a, \hat{E}^L_{\bar{a}})$ under $SO(3) \times SO(3)$
- Frame has a $SO(3) \times SO(3)$ algebra

$$L_{\hat{E}^R_a} \hat{E}^R_b = \frac{2}{R} \epsilon_{ab}{}^c \hat{E}^R_c \qquad L_{\hat{E}^L_{\bar{a}}} \hat{E}^L_{\bar{b}} = \frac{2}{R} \epsilon_{\bar{a}\bar{b}}{}^{\bar{c}} \hat{E}^L_{\bar{c}}$$

- Can express as left and right invariant vectors on $S^3 \sim SU(2)$

$$\hat{E}^R_a = r_a + \rho_a - i_{r_a} B$$
$$\hat{E}^L_{\bar{a}} = l_{\bar{a}} - \lambda_{\bar{a}} - i_{l_{\bar{a}}} B$$

Other Generalised Parallelisations

- Some spaces admit more than one generalised parallelisation
- Ordinary Scherk-Schwarz is also a Generalised Scherk-Schwarz
- E.g.) on $S^3 \cong SU(2)$ can build basis for $E \simeq T \oplus T^*$

$$\hat{E}_a = \hat{e}_a + i_{\hat{e}_a} B$$
$$\hat{E}^a = e^a$$

- This gives only an $SU(2)$ gauging, before had $SO(4)$
- Different generalised parallelisations give different gaugings

New gaugings: Desired generalised frame

- To realise new $SO(8)$ gauging, need a frame satisfying

$$
\begin{aligned}
L_{\hat{E}_{ij}} \hat{E}_{kl} &= 2R^{-1} \cos\omega \left(\delta_{i[k} \hat{E}_{l]j} - \delta_{j[k} \hat{E}_{l]i}\right), \\
L_{\hat{E}_{ij}} \hat{E}'_{kl} &= 2R^{-1} \cos\omega \left(\delta_{i[k} \hat{E}'_{l]j} - \delta_{j[k} \hat{E}'_{l]i}\right), \\
L_{\hat{E}'_{ij}} \hat{E}_{kl} &= -2R^{-1} \sin\omega \left(\delta_{i[k} \hat{E}_{l]j} - \delta_{j[k} \hat{E}_{l]i}\right), \\
L_{\hat{E}'_{ij}} \hat{E}'_{kl} &= -2R^{-1} \sin\omega \left(\delta_{i[k} \hat{E}'_{l]j} - \delta_{j[k} \hat{E}'_{l]i}\right),
\end{aligned}
$$

- Also $G(\hat{E}_A, \hat{E}_B) = \delta_{AB}$

No-Go Result: i) Background locally S^7

- All gaugings have $\mathcal{N} = 8$ AdS$_4$ vacuum
- (Locally) geometric uplift must be $\mathcal{N} = 8$ (maximal SUSY)

  $\Rightarrow$ local geometry is AdS$_4 \times S^7$ [Figueroa-O'Farrill & Papadopoulos '02]
- This fixes generalised metric $G = G^{(S^7)}$

No-Go Result: ii) Properties of vector parts

Write:

$$\hat{E}_A = u_A + \dots \qquad \hat{F}_A^{(S^7)} = v_A + \dots$$

Frame algebra $\Rightarrow$

- $u_A \not\equiv 0$
- $L_{\hat{E}_A} G = 0 \Rightarrow u_A$ are Killing vectors
- Further $u_{\alpha\beta} = \mathrm{e}^{\mathrm{i}\omega} v_{\alpha\beta}$ $\qquad \alpha, \beta$ are $SU(8)$ indices

No-Go Result iii) Contradiction

- $G = G^{(S^7)} \Rightarrow \exists U_\alpha{}^\beta \in SU(8)$ s.t.

$$\hat{E}_{\alpha\alpha'} = U_\alpha{}^\beta U_{\alpha'}{}^{\beta'} \hat{F}^{(S^7)}_{\beta\beta'}$$

- But this $\Rightarrow u_{\alpha\alpha'} = \mathrm{e}^{\mathrm{i}\omega} v_{\alpha\alpha'} = U_\alpha{}^\beta U_{\alpha'}{}^{\beta'} v_{\beta\beta'}$
- This has no solutions for $\omega \in (0, \pi/8)$
- $\Rightarrow$ there is no $\hat{E}_A$ which locally realises new gaugings

Similar family of $SO(4)$ gaugings

- $\exists$ $SO(4)$ gaugings of 7d $N = 1$ sugra for $\omega \in [0, \pi/4]$
- $\omega = \pi/4$ case $\leftrightarrow$ Type I on S^3 with H_3 flux
- Working in $O(3,3)$ geometry $E \sim T \oplus T^*$ need

$$L_{\hat{E}^R_a} \hat{E}^R_b = \frac{2}{R} \epsilon_{ab}{}^c \hat{E}^R_c \qquad L_{\hat{E}^L_{\bar{a}}} \hat{E}^L_{\bar{b}} = \frac{2}{R} (\cot \omega) \epsilon_{\bar{a}\bar{b}}{}^{\bar{c}} \hat{E}^L_{\bar{c}}$$

- S^3 parallelisation explicitly known...
- View S^3 as Hopf fibration $S^1 \to S^3 \to S^2$

S^3 frame

$$\hat{F}^R_+ = e^{i\phi}\Big[\left(2R^{-1}\partial_\theta + \tfrac{1}{2}R\mathrm{d}\theta\right) + i\cot\theta\left(2R^{-1}\partial_\phi - \tfrac{1}{2}R\mathrm{d}\phi\right) - i\csc\theta\left(2R^{-1}\partial_\psi + \tfrac{1}{2}R\mathrm{d}\psi\right)\Big],$$

$$\hat{F}^R_3 = 2R^{-1}\partial_\phi + \tfrac{1}{2}R\mathrm{d}\phi,$$

and

$$\hat{F}^L_+ = e^{-i\psi}\Big[\left(2R^{-1}\partial_\theta - \tfrac{1}{2}R\mathrm{d}\theta\right) + i\csc\theta\left(2R^{-1}\partial_\phi - \tfrac{1}{2}R\mathrm{d}\phi\right) - i\cot\theta\left(2R^{-1}\partial_\psi + \tfrac{1}{2}R\mathrm{d}\psi\right)\Big],$$

$$\hat{F}^L_3 = 2R^{-1}\partial_\psi - \tfrac{1}{2}R\mathrm{d}\psi,$$

- (θ, ϕ) coords on S^2 base. $\psi \sim \psi + 4\pi$ Hopf fibre coord

T duality and dual coordinate

- T dualise along Hopf fibre
- S^3 with q units H_3 flux $\leftrightarrow$ $S^3/\mathbb{Z}_q$ with one unit H_3 flux
- Hopf fibre of $S^3/\mathbb{Z}_q$ has dual coordinate $\tilde{\psi} \sim \tilde{\psi} + \frac{4\pi}{q}$
- Vectors $\hat{F}^L$ not globally defined on $S^3/\mathbb{Z}_q$

  $\rightarrow$ appears no global frame and less SUSY
- But still have S^3 frame depending on dual coordinate ψ
- Suggests should look at geometry on a correspondence space

$$S^1_\psi \times S^1_{\tilde{\psi}} \rightarrow X_4 \xrightarrow{\pi} S^2$$

T fold double geometry: Informal version

- Work in coordinates $(\theta, \phi, \psi, \tilde{\psi})$ on X_4
- Identify $\frac{R}{2}\mathrm{d}\psi \to \frac{2}{R}\partial_{\tilde{\psi}}$ in generalised vectors
- So $V^M = (v^\theta, v^\phi, v^\psi, v^{\tilde{\psi}}, \lambda_\theta, \lambda_\phi)$
- Set $\partial_M = (\partial_\theta, \partial_\phi, \partial_\psi, \partial_{\tilde{\psi}}, 0, 0)$ and allow $\tilde{\psi}$ dependence
- Define generalised Lie derivative via usual DFT expression

$$L_V W^M = V^N \partial_N W^M + \left(\partial^M V^N - \partial^N V^M\right) W_N$$

T fold double geometry: Formal version

- Define a generalised tangent space on X_4

$$\pi^*(T^*S^2) \to E \to TX_4$$

- Possible to define $O(3,3)$ metric, generalised Lie derivative etc.

- Manifest covariance only under

  $\to$ diffeo $+$ gauge on S^2

  $\to$ $U(1) \times U(1)$ gauge transformations of ψ and $\tilde{\psi}$

Recovering physical background

- Take quotient by ψ or $\tilde{\psi} \to S^3$ or $S^3/\mathbb{Z}_q$
- Connection 1-forms $\tilde{A}$ or A for the Hopf fibres

  $\to$ B-field component $B = \tilde{A} \wedge \mathrm{d}\psi$ or $B = A \wedge \mathrm{d}\tilde{\psi}$
- Bundle E on X_4 $\to$ Generalised tangent space

Natural generalisation of S^3 frame

- Consider frames depending on both ψ and $\tilde{\psi}$
- E.g. could modify "left frame" to

$$\hat{E}^L_+ = e^{-i(a\psi + b\tilde{\psi})}\Big[\left(2R^{-1}\partial_\theta - \tfrac{1}{2}R\mathrm{d}\theta\right) + i\csc\theta\left(2R^{-1}\partial_\phi - \tfrac{1}{2}R\mathrm{d}\phi\right) - i\cot\theta\, 2R^{-1}\left(\partial_\psi + \partial_{\tilde{\psi}}\right)\Big]$$

$$\hat{E}^L_3 = 2R^{-1}\left(\partial_\psi - \partial_{\tilde{\psi}}\right).$$

- This works:

If $a + b = 1 \rightarrow$ Gauge algebra with $\cot\omega = a - b$

Comments

- Frame is globally defined if $a \in \mathbb{Z}$ and $\frac{b}{q} \in \mathbb{Z}$

  $\to$ Discrete set $\cot\omega = 1 - 2nq$ for $n \in \mathbb{Z}$

- Have $\partial_\psi \partial_{\tilde{\psi}} (\mathrm{e}^{-a\psi - b\tilde{\psi}}) \neq 0$

  $\to$ Violates both strong and weak constraints of DFT

- Generalised metric for $\hat{E}_A$ same as for S^3 (geometric!)

  $\to$ non-geometry appears at other points in moduli space

"Systematic" approach

- Embedding tensor $\cong$ torsion of generalised Weitzenböck
- Given $\hat{E}_A = U_A{}^B \hat{F}_B$ the two connections are related by

$$D'_A = D_A + K_A \qquad\qquad K_A = -U^{-1}\partial_A U$$

- If write new frame as rotation of old, then new torsion is

$$T(D') = T(D) + T(K)$$

"Systematic" approach

- Fibration breaks isometry group of S^3

$$SU(2)_R \times SU(2)_L \to SU(2)_R \times U(1)_H$$

- $U(1)_H$ acts on frame $\hat{F}_A$ simply by

$$\hat{F}^L_+ \to \hat{E}^L_+ = \mathrm{e}^{\mathrm{i}f} \hat{F}^L_+$$

- Say we choose $f = \frac{1}{2}c(\psi - \tilde{\psi})$ then

$$\hat{F}_A(f) = \begin{cases} 2c/R & \text{for } \hat{F}^L_3 \ (\leftarrow \text{singlet of } SO(3)_R \times U(1)_H) \\ 0 & \text{otherwise} \end{cases}$$

- Frame $\hat{E}_A$ gives $\cot\omega = 1 - c = 1 - 2qn$ (Same as before)

$SO(8)$ gaugings

- View S^7 as Hopf fibration $S^1 \to S^7 \to \mathbb{CP}^3$

  $\to$ Fibres preserving isometries $SU(4) \times U(1)_H \subset SO(8)$

- $SU(4) \times U(1)_H$ singlet parts of S^7 frame involve only

$$R^{-1}\partial_\psi + R^5 \mathrm{d}\psi \wedge \tfrac{1}{2}\omega^2 \qquad R^2\omega + R\mathrm{d}\psi \otimes \mathrm{vol}_g$$

- Introduce extended coordinates

$$R^{-1}\frac{\partial}{\partial\psi} = R^{-1}\frac{\partial}{\partial\psi_0}, \qquad R^2\omega = R^{-1}\frac{\partial}{\partial\psi_2},$$
$$\tfrac{1}{2}R^5\mathrm{d}\psi \wedge \omega^2 = R^{-1}\frac{\partial}{\partial\psi_4}, \qquad R\mathrm{d}\psi \otimes \mathrm{vol}_{S^7} = R^{-1}\frac{\partial}{\partial\psi_6}.$$

- Extended geometry $T^4 \to X_{10} \to \mathbb{CP}^3$

$SO(8)$ gaugings

- Say we choose $f = a(\psi_0 + 3\psi_4) + b(\psi_2 + \frac{1}{3}\psi_6)$ then

$$\hat{F}_{\alpha\beta}(f) = \begin{cases} 2c/R & \text{for } SU(4) \times U(1)_H \text{ singlet} \\ 0 & \text{otherwise} \end{cases}$$

- Define new frame $\hat{E}_A$ by action of $\mathrm{e}^{\mathrm{i}f} \in U(1)_H \subset SU(8)$
- Find desired algebra with $\mathrm{e}^{-4\mathrm{i}\omega} = (1 - 2a) - \frac{2}{3}\mathrm{i}b$
- This frame realises the algebra of the new gaugings!

Comments

- Effectively working in type IIA on $\mathbb{CP}^3$ here
- Coords ψ_n dual to $D(2n)$-branes wrapping $\mathbb{CP}^n$ cycles in $\mathbb{CP}^3$
- Brane masses are quantised $\rightarrow$ coords ψ_n have set periods
- Insisting frame respects periodicity excludes $\omega \neq 0$ gaugings
- Frame violates strong constraint!
- Background fields are geometric S^7 solution

Summary and Conclusions

Consistent reductions

- Maximally supersymmetric consistent truncations

  $\simeq$ generalised Leibniz parallelisations

- Embedding tensor $\rightarrow$ torsion / frame algebra
- Different parallelisations $\rightarrow$ different gaugings
- Generalised geometry is a useful tool for SUGRA!

Summary and Conclusions

New gaugings

- No (locally) geometric uplift $\rightarrow$ must violate strong constraint
- Algebra can be realised in extended geometry
- Naive quantisation condition still excludes $SO(8)$ gaugings
- Physical picture of this very unclear!!!

  $\rightarrow$ can one really introduce the extra coordinates?

  $\rightarrow$ violating strong and weak constraints?

The End

- Thanks for your attention!