

Solutions in Extended Field Theories

Duality Symmetries in String and M-Theories

CERN

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Key Points

- ▶ Solutions to EoMs of DFT and EFT
- ▶ Impose section & pick duality frame \rightarrow supergravity objects
 - ▶ String, membrane, fivebrane, wave, monopole, D-branes, ...
- ▶ Isometries give ambiguity in frame choice \rightarrow T, S, U duality

Outline

Motivation

Fundamental and Solitonic Solutions in DFT

Self-dual Solutions in EFT

Extensions

Motivation

Defining Features

- ▶ Extended geometry by including dual directions
- ▶ Coordinates for momentum and winding modes of string
- ▶ Dualities become manifest
- ▶ Geometrizes B and C fields

Extended Field Theories

- ▶ Double Field Theory $O(D, D)$ - T duality
- ▶ Exceptional Field Theory E_d - U duality

Motivation

Kaluza-Klein Theory

- ▶ Massless, uncharged state in full theory
- ▶ States in reduced theory have **mass** and **charge**
- ▶ Given by **momentum** in KK direction

Example

- ▶ Null wave solution in M-theory gives D0-brane
- ▶ D0-brane is momentum mode in 11th direction
- ▶ Mass and charge given by momentum - BPS state

Double Field Theory

Ingredients

- ▶ Generalized coordinates

$$X^M = (x^\mu, \tilde{x}_\mu)$$

- ▶ $O(D, D)$ structure

$$\eta_{MN} = \begin{pmatrix} 0 & \mathbb{I} \\ \mathbb{I} & 0 \end{pmatrix}$$

- ▶ Dilaton

$$e^{-2d} = \sqrt{g} e^{-2\phi}$$

- ▶ Generalized metric

$$\mathcal{H}_{MN} = \begin{pmatrix} g_{\mu\nu} - B_{\mu\rho} g^{\rho\sigma} B_{\sigma\nu} & B_{\mu\rho} g^{\rho\nu} \\ -g^{\mu\sigma} B_{\sigma\nu} & g^{\mu\nu} \end{pmatrix}$$

- ▶ Constraint

$$\mathcal{H}_{MK} \eta^{KL} \mathcal{H}_{LN} = \eta_{MN}$$

- ▶ Section condition

$$\eta^{MN} \partial_M \Phi \partial_N \Psi = 0$$

Double Field Theory

Dynamics

- ▶ Action $S = \int d^{2D} X e^{-2d} R$
- ▶ Equations of motion

$$P_{MN}{}^{KL} K_{KL} = 0 \quad R = 0$$

- ▶ **Projector** $P_{MN}{}^{KL} = \frac{1}{2} \left[\delta_M^{(K} \delta_N^{L)} - \mathcal{H}_{MP} \eta^{P(K} \eta_{NQ} \mathcal{H}^{L)Q} \right]$

- ▶ Ricci scalar

$$R = \frac{1}{8} \mathcal{H}^{MN} \partial_M \mathcal{H}^{KL} \partial_N \mathcal{H}_{KL} - \frac{1}{2} \mathcal{H}^{MN} \partial_M \mathcal{H}^{KL} \partial_K \mathcal{H}_{NL} \\ + \text{dilaton terms}$$

The DFT Wave Solution

$$X^M = (t, z, y^m, \tilde{t}, \tilde{z}, \tilde{y}_m)$$

Generalized metric

$$\begin{aligned} ds^2 &= \mathcal{H}_{MN} dX^M dX^N \\ &= (H - 2) [dt^2 - dz^2] - H [d\tilde{t}^2 - d\tilde{z}^2] \\ &\quad + 2(H - 1) [dt d\tilde{z} + d\tilde{t} dz] \\ &\quad + \delta_{mn} dy^m dy^n + \delta^{mn} d\tilde{y}_m d\tilde{y}_n \end{aligned}$$

Rescaled dilaton

$$e^{-2d} = \text{const.}$$

The DFT Wave Solution

Harmonic Function H

$$H(r) = 1 + \frac{h}{r^{D-4}}$$

$$r^2 = \delta_{mn} y^m y^n, \quad h = \text{const.}$$

The DFT Wave Solution

Properties

- ▶ No mass, null-like
- ▶ Carries momentum in \tilde{z} direction
- ▶ Interpret as **null wave** in DFT
- ▶ Smeared over dual directions \rightarrow obeys section condition

The Supergravity Picture

KK-Ansatz to remove dual directions

- ▶ Get fundamental string solution (F1-string)
- ▶ Extended along z
- ▶ Mass and charge given by momentum in \tilde{z}

If z and \tilde{z} are exchanged

- ▶ Get pp-wave in z direction
- ▶ Expected as wave and string are T-dual

Key Result

The fundamental string is a massless wave in doubled space with momentum in a dual direction.

The DFT Monopole Solution

$$X^M = (z, y^i, x^a, \tilde{z}, \tilde{y}_i, \tilde{x}_a), \quad i = 1, 2, 3$$

Generalized metric

$$\begin{aligned} ds^2 &= \mathcal{H}_{MN} dX^M dX^N \\ &= H(1 + H^{-2}A^2) dz^2 + H^{-1} d\tilde{z}^2 + 2H^{-1} A_i [dy^i d\tilde{z} - \delta^{ij} d\tilde{y}_j dz] \\ &\quad + H(\delta_{ij} + H^{-2}A_i A_j) dy^i dy^j + H^{-1} \delta^{ij} d\tilde{y}_i d\tilde{y}_j \\ &\quad + \eta_{ab} dx^a dx^b + \eta^{ab} d\tilde{x}_a d\tilde{x}_b \end{aligned}$$

Rescaled dilaton

$$e^{-2d} = H$$

The DFT Monopole Solution

Harmonic Function H

$$H(r) = 1 + \frac{h}{r}$$

Magnetic Potential

$$\partial_{[i} A_{j]} = \frac{1}{2} \epsilon_{ij}{}^k \partial_k H$$

$$r^2 = \delta_{ij} y^i y^j,$$

$$h = \text{const.}$$

The DFT Monopole Solution

Properties

- ▶ Hopf fibration $S^2 \times S^1$
- ▶ Monopole circle in \tilde{z} direction
- ▶ Interpret as **monopole** in DFT
- ▶ Smeared over dual directions \rightarrow obeys section condition

The Supergravity Picture

KK-Ansatz to remove dual directions

- ▶ Get solitonic fivebrane solution (NS5-brane)
- ▶ Delocalized in z (infinite periodic array)
- ▶ Couples magnetically to $B_{iz} = A_i$

If z and \tilde{z} are exchanged

- ▶ Get KK-monopole with S^1 in z direction
- ▶ Expected as monopole and fivebrane are T-dual

Key Result

The solitonic fivebrane is a monopole in doubled space with the monopole circle in a dual direction.

T Duality in DFT

Section Condition vs Frame Choice

- ▶ Section condition: depend only on **half** the coordinates
- ▶ Frame choice: which set of coordinates is physical spacetime

Ambiguity in choice

- ▶ Isometries: **fewer** coordinate dependencies than required by section condition
- ▶ Different frame choices possible
- ▶ Get the different T-duality frames in supergravity

DFT Solutions

- ▶ DFT wave: F1-string and pp-wave
- ▶ DFT monopole: NS5-brane and KK-monopole

Similar results in truncated EFT

- ▶ $SL(5)$: wave \leftrightarrow membrane
- ▶ E_7 : monopole \leftrightarrow fivebrane
- ▶ E_7 : wave \leftrightarrow fivebrane

Duality between electric and magnetic solutions

DFT \rightarrow EFT

Exceptional Field Theory

Features

- ▶ M-Theory analogue of DFT
- ▶ Exceptional group E_d - U duality manifest
- ▶ Split 11-dim. supergravity

$$M^{11} = M^{11-d} \times M^d \longrightarrow M^{11-d} \times M^{\dim E_d}$$

- ▶ Extend by including membrane and fivebrane wrappings

$$TM^d \oplus \Lambda^2 T^* M^d \oplus \Lambda^5 T^* M^d \oplus \Lambda^6 TM^d$$

Exceptional Field Theory

Work with E_7

- ▶ Fundamental representation **56**: Y^M
- ▶ Adjoint representation **133**: t_α
- ▶ Invariant symplectic form of $Sp(56) \supset E_7$

$$\Omega_{MN} = \begin{pmatrix} 0 & \mathbb{I} \\ -\mathbb{I} & 0 \end{pmatrix}$$

Field content

$$\{g_{\mu\nu}, \mathcal{M}_{MN}, \mathcal{A}_\mu{}^M, B_{\mu\nu}{}_\alpha, B_{\mu\nu}{}^M\}$$

$$\frac{g_{\mu\nu} \quad | \quad \mathcal{A}_\mu{}^N}{\cdot \quad | \quad \mathcal{M}_{MN}}$$

External sector

- ▶ 4-dim. spacetime: metric $g_{\mu\nu}$

Internal sector

- ▶ 56-dim. exceptional extended space: generalized metric \mathcal{M}_{MN}
- ▶ parametrizes coset $E_7/SU(8)$

Cross-terms

- ▶ EFT vector potential $\mathcal{A}_\mu{}^M$ with **self-dual** field strength $\mathcal{F}_{\mu\nu}{}^M$

$$e = \sqrt{\det g_{\mu\nu}}$$

Action

$$S = \int d^4x d^{56}Y e \left[\hat{R} + \frac{1}{48} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} + e^{-1} \mathcal{L}_{\text{top}} \right. \\ \left. - \frac{1}{8} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}_{\mu\nu}{}^N - V(\mathcal{M}_{MN}, g_{\mu\nu}) \right]$$

Twisted self-duality

$$\mathcal{F}_{\mu\nu}{}^M = \frac{1}{2} e \epsilon_{\mu\nu\rho\sigma} \Omega^{MN} \mathcal{M}_{NK} \mathcal{F}^{\rho\sigma K}$$

Embedding Supergravity in EFT

Section condition

- ▶ $(t_\alpha)^{MN} \partial_M \partial_N \Phi = 0$ and $(t_\alpha)^{MN} \partial_M \Phi \partial_N \Psi = 0$
- ▶ $\Omega^{MN} \partial_M \Phi \partial_N \Psi = 0$

Solution to section condition

- ▶ Decomposition $56 \rightarrow 7 + 21 + 7 + 21$
- ▶ Coordinates $Y^M = (y^m, y_{mn}, y_m, y^{mn})$
- ▶ Dependence $\partial^{mn} \rightarrow 0, \quad \partial^m \rightarrow 0, \quad \partial_{mn} \rightarrow 0$

Generalized metric

$$\mathcal{M}_{MN}(g_{mn}) = g^{1/2} \text{diag}[g_{mn}, g^{mn,kl}, g^{-1} g^{mn}, g^{-1} g_{mn,kl}]$$

The Self-dual EFT Solution

$$H(r) = 1 + \frac{h}{r}$$

External spacetime

- ▶ Coordinates: $x^\mu = (t, w^i)$ and $r^2 = \delta_{ij} w^i w^j$
- ▶ Metric: $g_{\mu\nu} = \text{diag}[-H^{-1/2}, H^{1/2} \delta_{ij}]$

Internal exceptional extended space

- ▶ Coordinates: $Y^M = (y^m, y_{mn}, y_m, y^{mn})$
- ▶ Generalized metric:

$$\mathcal{M}_{MN} = \text{diag}[H^{3/2}, H^{1/2} \delta_6, H^{-1/2} \delta_6, H^{1/2} \delta_{15}, \\ H^{-3/2}, H^{-1/2} \delta_6, H^{1/2} \delta_6, H^{-1/2} \delta_{15}]$$

The Self-dual EFT Solution

Vector Potential \mathcal{A}_μ^M

$$\mathcal{A}_t^M = \frac{H-1}{H} a^M \qquad \mathcal{A}_i^M = A_i \tilde{a}^M$$

Dual vectors a^M and \tilde{a}^M - give direction

$$\hat{a}^M = \Omega^{MN} \mathcal{M}_{NK} \hat{a}^K$$

Other fields trivial

Properties

Charge in KK-theory

- ▶ Electric charge: $q_e = n/R_e$ $n, k \in \mathbb{Z}$
- ▶ Magnetic charge: $q_m = kR_m$
- ▶ Self-duality: $q_e = q_m \implies n/k = R_e R_m$

Radii given in terms of a^M and \tilde{a}^M

- ▶ $R_e = |a^M| = H^{3/4}$
- ▶ $R_m = |\tilde{a}^M| = H^{-3/4}$
- ▶ $R_e R_m = 1 \implies n = k$

The Supergravity Picture

Get supergravity solutions in $4 + 7$ split

- ▶ \mathcal{M}_{MN} gives internal metric g_{mn}
- ▶ \mathcal{A}_t^M and \mathcal{A}_i^M give C_{mnk} and $C_{m_1\dots m_6}$ or KK-vector
- ▶ $g_{\mu\nu}$ carries over to external metric

Pick a direction for a^M

- | | | | |
|--------------|----------|------------|-------------|
| ▶ y^m : | pp-wave | y_m : | KK-monopole |
| ▶ y_{mn} : | M2-brane | y^{mn} : | M5-brane |

Example

The M2-brane

- ▶ Let a^M be in the y_{12} direction
- ▶ \mathcal{A}_t^M becomes $\mathcal{A}_t{}_{12} = C_{ty^1y^2} = -(H^{-1} - 1)$
- ▶ \mathcal{A}_i^M becomes $\mathcal{A}_i{}^{12} = \frac{1}{5!}\epsilon^{1\dots 7}C_{iy^3\dots y^7} = A_i$
- ▶ \mathcal{M}_{MN} gives $g_{mn} = H^{1/3}\text{diag}[H^{-1}\delta_2, \delta_5]$
- ▶ Combine with $g_{\mu\nu}$ to get

$$ds^2 = H^{-2/3}[-dt^2 + (dy^1)^2 + (dy^2)^2] + H^{1/3}[d\vec{w}_{(3)}^2 + d\vec{y}_{(5)}^2]$$

The 1/2 BPS Spectrum of Supergravity

theory	solution	orientation	EFT vector	\mathcal{A}_t^M	\mathcal{A}_i^M
$D = 11$	WM	y^m	\mathcal{A}_μ^m	KK-vector	dual graviton
	M2	y_{mn}	\mathcal{A}_μ^{mn}	C_3	C_6
	M2/M5	*	*	$C_3 \oplus C_6$	$C_6 \oplus C_3$
	M5	y^{mn}	\mathcal{A}_μ^{mn}	C_6	C_3
	KK7	y_m	\mathcal{A}_μ^m	dual graviton	KK-vector
$D = 10$ Type IIA	WA	$y^{\bar{m}}$	$\mathcal{A}_\mu^{\bar{m}}$	KK-vector	dual graviton
	D0	y^θ	\mathcal{A}_μ^θ	C_1	C_7
	D2	$y_{\bar{m}\bar{n}}$	$\mathcal{A}_\mu^{\bar{m}\bar{n}}$	C_3	C_5
	F1	$y_{\bar{m}\theta}$	$\mathcal{A}_\mu^{\bar{m}\theta}$	B_2	B_6
	KK6A	$y_{\bar{m}}$	$\mathcal{A}_\mu^{\bar{m}}$	dual graviton	KK-vector
	D6	y_θ	\mathcal{A}_μ^θ	C_7	C_1
	D4	$y^{\bar{m}\bar{n}}$	$\mathcal{A}_\mu^{\bar{m}\bar{n}}$	C_5	C_3
	NS5	$y^{\bar{m}\bar{\theta}}$	$\mathcal{A}_\mu^{\bar{m}\bar{\theta}}$	B_6	B_2
$D = 10$ Type IIB	WB	$y^{\bar{m}}$	$\mathcal{A}_\mu^{\bar{m}}$	KK-vector	dual graviton
	F1 / D1	$y_{\bar{m} a}$	$\mathcal{A}_\mu^{\bar{m} a}$	B_2 / C_2	B_6 / C_6
	D3	$y_{\bar{m}\bar{n}\bar{k}}$	$\mathcal{A}_\mu^{\bar{m}\bar{n}\bar{k}}$	C_4	C_4
	NS5 / D5	$y^{\bar{m} a}$	$\mathcal{A}_\mu^{\bar{m} a}$	B_6 / C_6	B_2 / C_2
	KK6B	$y_{\bar{m}}$	$\mathcal{A}_\mu^{\bar{m}}$	dual graviton	KK-vector

Summary

Solutions in DFT

- ▶ Wave in winding direction gives string
- ▶ Monopole in winding direction gives fivebrane
- ▶ Isometries \rightarrow ambiguity \rightarrow T duality

Solutions in EFT

- ▶ One self-dual solution gives full 1/2 BPS spectrum
- ▶ Orientation in extended space determines supergravity object
- ▶ Isometries \rightarrow ambiguity \rightarrow U duality

Bound States

M2/M5 System

- ▶ EFT solution in mixed direction - parameter ξ

$$a_{(M2/M5)}^M = \sin \xi a_{(M2)}^M + \cos \xi a_{(M5)}^M$$

- ▶ Recover metric and C-fields for bound state

Wave vs Monopole

Combine à la Kaluza-Klein

$$\mathcal{H}_{\hat{M}\hat{N}} = \begin{pmatrix} g_{\mu\nu} + \mathcal{A}_\mu^M \mathcal{A}_\nu^N \mathcal{M}_{MN} & \mathcal{A}_\mu^M \mathcal{M}_{MN} \\ \mathcal{M}_{MN} \mathcal{A}_\nu^N & \mathcal{M}_{MN} \end{pmatrix}$$

Find wave and monopole sector

$$\mathcal{H}_{\hat{M}\hat{N}} = \begin{pmatrix} H^{1/2} \mathcal{H}_{AB}^{\text{wave}} & 0 \\ 0 & H^{-1/2} \mathcal{H}_{\bar{A}\bar{B}}^{\text{mono}} \end{pmatrix}$$

Localisations

Monopole localised in winding space

- ▶ On \mathbb{R}^3 : $H(r) = 1 + \frac{h}{r}$
- ▶ On \mathbb{R}^4 : $H(r, z) = 1 + \frac{h}{r^2 + z^2}$
- ▶ On $\mathbb{R}^3 \times S^1$:

$$H(r, z) = 1 + \frac{h}{2Rr} \frac{\sinh r/R}{\cosh r/R - \cos z/R}.$$