Conserved currents of double field theory

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CERN-CKC TH Institute on Duality Symmetries in String and M-Theories
August 2015

Based on arXiv 1507.07541

(Those who viewed this paper also viewed:
arXiv 1507.07545 by J-H Park, S-J Rey, W Rim, Y Sakatani (poster in common room)
arXiv 1508.00844 by U Naseer)

Introduction and outline

- Look for conserved currents in DFT associated to invariance under generalised diffeomorphisms
- Define conserved charges - study for solutions
- This talk:
  - Derivation of conserved current
  - Application to F1/pp-wave in DFT
  - Electric dual to 5_2^2
  - RR fields
  - Conclusions and discussion

Reminder of Noether's theorem: global

- Consider Lagrangian $L(\phi_A, \partial_i \phi_A)$, global symmetry $\delta L = \partial_i f^i$. Then

$$\delta S = \int \left(\left(\frac{\partial L}{\partial \phi_A} - \partial_i \frac{\partial L}{\partial(\partial_i \phi_A)} \right) \delta\phi_A + \partial_i \left(\frac{\partial L}{\partial(\partial_i \phi_A)} \delta\phi_A \right) \right)$$
$$= \int \partial_i f^i \,.$$

- Hence on shell the current

$$J_1^i = \frac{\partial L}{\partial(\partial_i \phi_A)} \delta\phi_A - f^i$$

is conserved.

Reminder of Noether's theorem: local

- Now assume local symmetry, then $\delta\phi_A$ contains derivatives of the symmetry parameters ξ^I. Integration by parts implies

$$0 = \int \left(\xi^I D_I \frac{\delta S}{\delta \phi} + \partial_i J^i \right) ,$$

  where D_I represent some specific derivative operators acting on the equations of motion.
- Terms proportional to ξ^I give Bianchi identities.
- Rest gives off-shell conserved current, $\partial_i J^i = 0$. ($J^i = \partial_j J^{ij}$ with J^{ij} antisymmetric.)
- In GR,

$$J^i = \sqrt{|g|} \nabla_j J^{ij} \quad , \quad J^{ij} = 2\nabla^{[j}\xi^{i]} \, .$$

  Defines charges

$$Q = \int_\Sigma d\Sigma_i J^i = \int_{\partial\Sigma} d\Sigma_{ij} 2\nabla^j \xi^i \, .$$

DFT

- DFT generalised metric $\mathcal{H}_{MN}$, dilaton d, $O(D,D)$ structure η_{MN}, projectors

$$P_M^N = \frac{1}{2}\left(\delta_M^N - S_M^N\right) \quad , \quad \bar{P}_M^N = \frac{1}{2}\left(\delta_M^N + S_M^N\right) ,$$

  with $S_M^N \equiv \mathcal{H}_{MP}\eta^{PN} = \eta_{MP}\mathcal{H}^{PN}$.
- Connection ${\Gamma_{MN}}^P$ with covariant derivative ∇_M, annihilates $\mathcal{H}_{MN}$, η_{MN}, d, vanishing generalised torsion (Levi-Civita-like). Either semi-covariant or contains undetermined components - no ambiguities when appropriately projected. Jeon, Lee, Park; Hohm, Zwiebach
- Generalised Ricci tensor $\mathcal{R}_{MN}$ of this connection gives the equations of motion of $\mathcal{H}_{MN}$, equation of motion of d is the generalised Ricci scalar $\mathcal{R}$.

The DFT action

- These eom are derived from an action Hohm, Hull, Zwiebach

$$S_{DFT} = \int dx d\tilde{x} e^{-2d} \mathcal{R} \,,$$

with

$$\begin{aligned}\mathcal{R} = & \frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{PQ}\partial_N\mathcal{H}_{PQ} - \frac{1}{2}\mathcal{H}^{MN}\partial_M\mathcal{H}^{PQ}\partial_P\mathcal{H}_{QN} \\ & + 4\partial_M\mathcal{H}^{MN}\partial_N d - 4\mathcal{H}^{MN}\partial_M d\partial_N d \\ & - \partial_M\partial_N\mathcal{H}^{MN} + 4\mathcal{H}^{MN}\partial_M\partial_N d \,.\end{aligned}$$

- Local symmetry: generalised diffeomorphisms on vector weight w

$$\delta_\xi V^M = \xi^N\partial_N V^M - V^N\partial_N\xi^M + \eta^{MN}\eta_{PQ}\partial_N\xi^P V^Q + w\partial_N\xi^N V^M \,.$$

Variation of the action

- Expect

$$\delta_\xi S_{DFT} \equiv \int \partial_M \left(\xi^M e^{-2d}\mathcal{R}\right) = \int \delta_\xi \left(e^{-2d}\mathcal{R}\right) \, .$$

- Varying the Lagrangian density explicitly and integrating by parts we obtain an identity

$$0 = \int \xi^M e^{-2d}\mathcal{Z}_M + \int \partial_M J^M \, .$$

- The Bianchi identities come out as

$$\mathcal{Z}_P \equiv -P_P^Q\nabla_Q\mathcal{R} + 4\bar{P}^{MN}\nabla_M\mathcal{R}_{NP} - \bar{P}_P^Q\nabla_Q\mathcal{R} - 4P^{MN}\nabla_M\mathcal{R}_{NP} = 0\,,$$

agrees with Siegel; Kwak; Hohm, Zwiebach.

The current

- The current is

$$J^M = \nabla_N J^{MN} - 2\nabla_N \left(\eta^{MN} e^{-2d} S_P^{\,Q} \nabla_Q \xi^P \right) ,$$

where J^{MN} is antisymmetric

$$J^{MN} \equiv 4e^{-2d} \nabla_Q \xi^P \left(P_P^{\,[M} \bar{P}^{N]Q} - \bar{P}_P^{\,[M} P^{N]Q} \right) .$$

- If we expand the covariant derivatives,

$$J^M = \partial_N J^{MN} + \frac{1}{4} \eta^{MN} \partial_N \mathcal{H}_{PK} S_L^{\,P} J^{KL} - 2\eta^{MN} e^{-2d} \partial_N \left(\partial_Q - 2\partial_Q d \right) \left(S_P^{\,Q} \xi^P \right) .$$

Manifestly conserved off-shell (using section), $\partial_M J^M = 0$.

The current

- Note the general form
$$J^M = \partial_N J^{MN} + \phi\eta^{MN}\partial_N\phi' \,.$$
- Solving section condition
$$\partial_M J^M = \partial_i J^i \quad , \quad J^i = \partial_j J^{ij} \,,$$
"section condition terms" modify J_i - no physical relevance of these components?
- Define charge by integrating over spatial hypersurface of physical section
$$Q \sim \int_{\Sigma_{D-1}} J^t \,.$$

The current and generalised Killing vectors

- For a generalised Killing vector $\delta_\xi \mathcal{H}_{MN} = 0, \delta_\xi d = 0$, the current simplifies to
$$J^M = -4\xi^P \mathcal{H}^{MN} \mathcal{R}_{NP} \,.$$
- Zero on-shell (in absence of sources)
- This is expected, need to add boundary terms Berman, Musaev, Perry, see further discussions in Park, Rey, Rim, Sakatani; Naseer

Evaluating the current

- Solve the strong constraint so that $\partial_M J^M = \partial_i J^i = 0$. Usual parametrisation of metric and dilaton gives

$$J^i = \partial_j \left(\sqrt{|g|} e^{-2\phi} \left[\nabla^j \xi^i - \nabla^i \xi^j - (\lambda_k + \xi^p B_{pk}) H^{ijk} \right] \right) .$$

  - $\partial_j \left(\sqrt{|g|} e^{-2\phi} \left[\nabla^j \xi^i - \nabla^i \xi^j \right] \right)$: usual current associated to diffeos in GR.
  - $-\partial_j \left(\sqrt{|g|} e^{-2\phi} \lambda_k H^{ijk} \right)$: electric current associated to gauge invariance.

- Of course, could calculate J^i directly in SUGRA. In DFT have access to: different parametrisations e.g. β-supergravity (talk by Andriot), non-geometric backgrounds, generalised Scherk-Schwarz, new interpretation of charges associated to gauge parameters.

The fundamental string in DFT

- Fundamental string appears in DFT as a generalised pp-wave. Berkeley, Berman, Rudolph
- Doubled coordinates $X^M = (t, z, y^\mu, \tilde{t}, \tilde{z}, \tilde{y}_\mu)$, with $\mu = 1, \ldots, D-2$. Generalised line element:

$$
\begin{aligned}
ds^2 = (H-2)(dt^2 - dz^2) + 2(H-1)(dt d\tilde{z} + d\tilde{t} dz) \\
- H(d\tilde{t}^{\,2} - d\tilde{z}^2) + \delta_{\mu\nu} dy^\mu dy^\nu + \delta^{\mu\nu} d\tilde{y}_\mu d\tilde{y}_\nu \,,
\end{aligned}
$$

with $H = 1 + \frac{h}{r^{D-4}}$, $r = \sqrt{\delta_{\mu\nu} y^\mu y^\nu}$. Take $e^{-2d} = 1$.

The fundamental string: section choices

- Section choice: (t, z, y^μ) gives F1

$$
\begin{aligned}
ds^2 &= -H^{-1}dt^2 + H^{-1}dz^2 + \delta_{\mu\nu}dy^\mu dy^\nu \\
B &= (1 - H^{-1})dt \wedge dz \\
e^{-2\phi} &= H\,.
\end{aligned}
$$

- Section choice: $(t, \tilde{z}, y^\mu)$ gives pp-wave:

$$
\begin{aligned}
ds^2 &= -(2 - H)dt^2 + 2(H - 1)dt d\tilde{z} + H d\tilde{z}^2 + \delta_{\mu\nu}dy^\mu dy^\nu\,, \\
B &= 0\,, \\
e^{-2\phi} &= 1\,.
\end{aligned}
$$

Related by Buscher $z \leftrightarrow \tilde{z}$.

The fundamental string: current

- Assume isometry $\partial_t = \partial_z = 0$ (to compare with pp).
- In the F1 frame, we evaluate

$$J^t = \partial_\mu \left(\partial^\mu \xi^t - \partial^\mu H (\xi^t + \lambda_z) \right) .$$

  If we specialise to the case where the gauge parameters are constant then

$$J^t = -(\xi^t + \lambda_z) \partial_\mu \partial^\mu H .$$

- $\partial_\mu \partial^\mu H \sim \delta(r)$ so when integrated this gives a non-zero charge.

The fundamental string: charge

- Take $D = 10$ here. Define

$$Q = \frac{e^{2\phi_0}}{16\pi G_N^{(10)}} \int_{\Sigma_9} J^t \,.$$

(Here $G_N^{(10)} = 8\pi^6 l_s^8 e^{2\phi_0}$, in DFT should have $e^{d_0} = e^{\phi_0}$, assumptions on asymptotics.)

- For the F1, we obtain

$$Q = (\xi^t + \lambda_z)\frac{R_z}{l_s^2} \,.$$

- Associated to (generalised) Killing vectors:
  - $\xi = \partial_t$, $\xi^t = 1, \lambda_z = 0$, **mass**
  - $\xi = \partial^z$, $\xi^t = 0, \lambda_z = 1$, **momentum in dual direction / winding charge**

The pp wave: charge

- Using $R_{\tilde{z}} = l_s^2/R_z$, or calculating carefully in the pp-wave frame, find there
$$Q = (\xi^t + \xi^{\tilde{z}})\frac{1}{R_{\tilde{z}}}\,,$$
as expected for a KK state.
- T-duality gives $\xi^{\tilde{z}} = \lambda_z$.

Electric vs magnetic

- F1 charged electrically under B_2, pp-wave under $A_1^{(1)}$ (KK vector).
- NS5 charged magnetically under B_2, KKM under $A_1^{(1)}$, exotic 5_2^2 under $\beta^{(2)}$ (bivector).
- Is there a solution charged electrically under the bivector? Bergshoeff, Ortin, Riccioni
- An aside: note that we measure magnetic charge using generalised flux Ellwood; many others

$$Q_m = \int_{\Sigma_3} f_{MNP} dX^M \wedge dX^N \wedge dX^P \,,$$

with $f_{MNP} = 3E_{\alpha[M}\partial_N E^\alpha{}_{P]}$ (generalised fluxes/generalised torsion of Weitzenböck connection).
- Gives

$$\int H_{ijk} dX^i dX^j dX^k \to \int T_{ij}{}^k dX^i dX^j d\tilde{X}_k$$
$$\to \int Q_i^{jk} dX^i d\tilde{X}_j d\tilde{X}_k \to \int R^{ijk} d\tilde{X}_i d\tilde{X}_j d\tilde{X}_k \,.$$

Electric dual to 5^2_2

- Pick $(\tilde{t}, \tilde{z}, y^\mu)$ section

$$
\begin{aligned}
ds^2 &= -\frac{1}{2-H} d\tilde{t}^{\,2} + \frac{1}{2-H} d\tilde{z}^2 + \delta_{\mu\nu} dy^\mu dy^\nu \,, \\
B &= \frac{H-1}{2-H} d\tilde{t} \wedge d\tilde{z} \,, \\
e^{-2\phi} &= |2-H| \,.
\end{aligned}
$$

The metric, B-field and its field strength are now singular at $r = h^{1/(D-4)}$.

- Alternatively express using metric and bivector (Sakatani)

$$
\begin{aligned}
ds^2 &= -H d\tilde{t}^{\,2} + H d\tilde{z}^2 + \delta_{\mu\nu} dy^\mu dy^\nu \,, \\
\beta &= (1 - H^{-1}) \partial_{\tilde{t}} \wedge \partial_{\tilde{z}} \,, \\
e^{-2\phi} &= H^{-1} \,.
\end{aligned}
$$

(Note $H \to \infty$ for $r \to 0$, c.f. Berman, Rudolph)

Electric dual to 5^2_2: charge

- Charge comes from $J^{\tilde{t}}$ (originally the J_t component) get assuming $\partial^{\tilde{t}} = 0$
$$J^{\tilde{t}} = \partial_\mu \left(\partial^\mu \xi^{\tilde{t}} + \partial^\mu H(-\lambda_{\tilde{z}} + \xi^{\tilde{t}}) \right) .$$
- Thus
$$Q \sim \int \partial_\mu \partial^\mu H(-\lambda_{\tilde{z}} + \xi^{\tilde{t}}) \, .$$

  These charges correspond to momentum in the dual direction z, and the mass of this solution. Note $\xi^{\tilde{t}}$ appears with the opposite sign to ξ^t in the original $J^t \Rightarrow$ negative mass.

RR fields

- Can also include RR fields as $O(D,D)$ spinor Hohm, Kwak, Zwiebach; Geissbuhler; Geissbuhler, Marques, Nunez, Penas (see also Jeon, Lee, Park), and find conserved charges associated to generalised diffeomorphisms and also RR gauge transformations
- Exact expressions in paper, similar general form $J^M = \partial_N J^{MN} + \eta^{MN}\partial_N(\dots)$.
- Reduction to spacetime should give more complicated charges relevant to situations with multiple sources (Page charge?)
- Relevant for studying configurations involving exotic branes (subtleties with charge conservations). de Boer, Shigemori; Okada, Sakatani
- These are naturally suited to a DFT description - these charges may be interesting to use here.

Conclusions and applications: DFT

- Found conserved charge associated to generalised diffeomorphism in DFT.
- Can be used to study the properties of solutions - interesting interpretation of winding charge as momentum in dual direction, as conjectured previously.
- Could extend to SUSY DFT, find also supercharges.
- Algebra of asymptotic symmetries?

Conclusions and applications: EFT

- Obviously, could and should carry out same analysis of EFT.
- Particularly interesting as there electric and magnetic solutions (e.g. M2 and M5) are related by duality, so should appear on the same footing in the charge.

Thanks for listening.